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A Branch and Cut Algorithm for Two-Echelon Inventory Routing Problem with End-of-Tour Replenishment Policy

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Abstract

This study presents a two-echelon inventory routing problem (2E-IRP) with an end-of-tour replenishment (ETR) policy whose distribution network consists of a supplier, several distribution centers (DCs) and several retailers on a multi-period planning horizon. A formulation of the problem based on vehicle indices is proposed in the form of a mixed integer linear program (MILP). The mathematical model of the problem is solved using a branch and cut (B&C) algorithm. The results of the tests are compared to the results of a branch and price (B&P) algorithm from the literature on 2E-IRP with a classical distribution policy. The results of the tests show that the B&C algorithm solves 197 out of 200 instances (98.5%). The comparison of the B&C and B&P results shows that 185 best solutions are obtained with the B&C algorithm on 197 instances (93.9%). Overall, the B&C algorithm achieves cost reductions ranging from 0.26% to 41.44% compared to the classic 2E-IRP results solved with the B&P algorithm, with an overall average reduction of 18.08%.

Keywords

Multi-Depots, 2E-IRP, Branch and Cut Algorithm, End-of-Tour Replenishment Policy, Vendor Managed Inventory

1. Introduction

Since the first studies have shown that it is possible to make savings of 6% to 10% on the cost of operations in the field of gas distribution [1], many researchers are interested in supply chain planning with a view to further reducing the costs associated with the storage and distribution of products in various areas of economic activity. The inventory routing problem (IRP) is a very important class of problem in supply chain planning. It is essentially made up of the vehicle routing problem and the inventory management problem. The vehicle routing problem (VRP) is a special case of the traveling salesman problem (TSP). It focuses on satisfying customer demands while minimizing the cost of product distribution. As far as inventory management is concerned, it is a question of determining the quantities to be delivered to each customer considering its storage capacity. Thus, in an IRP, it is possible to give a customer a quantity of products greater than his request without exceeding his storage capacity. Unlike the problem of vehicle routing, this makes it possible to reduce the number of routes to a customer over a multi-period planning horizon. The problem of inventory and vehicle routing is therefore a generalization of the VRP in which we seek to determine the dates of delivery or collection of products on the one hand and the quantities to be delivered or collected on the other hand. In its basic formulation, the IRP is a multi-period problem [1] [2]. In addition, the work carried out over the last two decades has made it possible to consider new factors giving rise to variants of the IRP more adapted to the real problems of supply chain planning. Among the variants frequently studied in the literature, we have the IRP with the consideration of multi-depots [2]-[6], perishable products [7]-[10], multi-product [11]-[13], backhaul [9] [14]-[16], split delivery [17] [18], multiple tours [19], pick-up and delivery [20]-[22], with time windows [19] [21] [23] [24], heterogeneous fleet of vehicles [25], etc. These problems can be broadly classified into two main families. We have problems at one echelon and multi-echelons problems. A single-echelon IRP model is characterized by a single level of product distribution. Thus, in a one-echelon IRP, products are distributed directly to customers from one or more depots. In contrast, the multi-echelon models are built on a multi-echelon distribution network. In a multi-echelon IRP, products are not delivered directly to customers from the supplier. The products are first collected by vehicles that may have a large capacity for the supply of intermediate depots (central depots or satellite) and then collected in these depots by vehicles that may have a smaller capacity to supply customers. Thus, the transport of products in a multi-level IRP is generally done between one or more suppliers and one or more depots at the beginning of the echelon for the consolidation or storage of products in the first stage and the delivery of products to several customers from one or more depots at the end of the echelon in the second stage. For a better optimization of the costs related to the storage and distribution of products, the implementation of a good procurement or replenishment policy and an inventory management practice adapted to the type of supply chain is necessary. There are several types of procurement policies [26]. Maximum level (ML) is a replenishment policy in which any non-zero quantity can be delivered to a customer provided that this quantity does not exceed its storage capacity [27]. Another relevant policy is order-up-to-level (OU). In this policy, the quantity of products delivered to a customer is the complement of its current stock to reach its maximum storage capacity. Thus, in an OU replenishment policy, a customer's storage capacity is at its maximum level after a product delivery [28]. These procurement policies are implemented in inventory management practices, such as vendor-managed inventory (VMI) or customer/retailer-managed inventory (CMI or RMI). VMI is an inventory management practice in which the vendor decides when and how much to deliver to a customer over the planning horizon. The ML and the OU are therefore replenishment policies adapted to the context of VMI. In contrast to the VMI, we have the CMI or the RMI. In this practice, only the customer or retailer decides the exact amount of product to be received over each period of the planning horizon. This type of inventory management practice is suitable for VRP, given the fact that the quantity of products delivered to a customer is equal to their demand. The achievements resulting from the work identified here [28] allow for further investigation of the question for comparative studies between VMI and CMI.

To facilitate the reading and understanding of this paper, we organize it as follows: in Section 2, a review of the literature on the variants of the 2E-IRP, as well as their methods of resolution, is presented. Section 3 is devoted to the description and proposal of a mathematical model of the problem. In Section 4, we propose a branch and cut (B&C) algorithm for solving the problem. Sections 5 and 6 present the results and the conclusion of the work, respectively.

2. Literature Reviews

IRP is a class of problems encountered in supply chain planning. It combines both the problem of vehicle routing and the problem of stock management. Its basic formulation concerns a supplier (warehouse, retailer) that delivers a type of product to a set of customers, each with a limited storage capacity over a finite planning horizon of several periods. The delivery of products to customers is carried out thanks to a fleet of vehicles with limited capacities. The aim of an IRP is to simultaneously minimize the cost of vehicle routes and the overall cost of storing products at the supplier and customers. Readings will find an excellent literature review in [10] [29] [30]. A very interesting variant from the perspective of its practicality in the real-world of supply chain planning is the two-echelon inventory routing problem (2E-IRP). The 2E-IRP is a generalization of the classic IRP by extending it to the multi-depots case and considering a third actor playing the role of supplier and a special case of the multi-level IRP by limiting the levels to two (suppliers-depots and deposits-customers). Beyond satisfying deterministic or dynamic customer demands, a 2E-IRP aims to simultaneously minimize the costs related to the storage and transportation of products throughout the supply chain. To achieve this goal, the practice of VMI remains the best alternative in terms of integrated inventory management strategy in distribution centers (central depots), at customers and/or at the supplier. This practice is also accompanied by the implementation of an appropriate procurement or replenishment policy aimed at using storage capacity efficiently. The policies applicable in a VMI context are most often related to inventory management at customers' sites. While ML and OU are the most used, a good deal of work focuses on policies such as zero-inventory-ordering policy [31] [32] and politics (R, Q) [33]. The zero-inventory-ordering is a policy in which a customer is delivered when their product inventory level is zero. And the (R, Q) policy is a policy in which a customer is delivered when their inventory level reaches a certain threshold R. At this point, a delivery is made to raise the stock level to Q. In addition to policies related to inventory management, policies related to the configuration of vehicle routes such as fixed partitioning (FP) [32] [34] [35], the power of two (POT)) [35] [36] Split delivery [18] [37], multiple tours [38] [39], as well as the cases with time window and pick-ups and deliveries mentioned above, were also studied. In a recent study, researchers looked at a 2E-IRP whose distribution network is made up of several suppliers, several distribution centers (DCs) and several customers [40]. The distribution of products is carried out in a context of VMI practice and the implementation of the ML policy. In this model, where inventory management and vehicle routing decisions are made at suppliers, a homogeneous fleet of high-capacity vehicles is assigned to collect products from suppliers to supply DCs located on the periphery of an urban area at the first level, and a fleet of smaller vehicles is used to satisfy customer demands from DCs at the second level. It is forbidden here to deliver products directly to customers from suppliers. The authors formulated the problem as a Linear Integer Program and proposed a B&P algorithm for its solution. In another study, these authors proposed a matheuristic algorithm for solving the same problem [41]. Some researchers have instead focused their interest on a 2E-IRP in which storage and delivery decisions are centralized in the DCs. In this area of work, others have carried out a study implementing OU and ML replenishment policies in the practice of VMI in the field of gasoline distribution [3]. Their distribution network is made up of several suppliers, several DCs and several customers. A homogeneous fleet of vehicles parked in the DCs is used to collect inputs from suppliers for the supply of the DCs and then these inputs are mixed with refined gasoline to be delivered to customers. They modeled the problem in the form of an integer linear program and proposed B&C algorithm for its solution. They then proposed a matheuristic for the resolution of large instances. A number of authors have proposed a further variation on the previous work in the petrochemical industry [42]. This variant considers the agreements or authorizations for the use of a vehicle depending on its level of contamination. They formulated the problem as a linear integer program and proposed a B&C algorithm for its solution. They also developed a matheuristic and a parallel approach hybridizing matheuristic with exact resolution to solve large instances. Farias, Hadj-Hamou and Yugma have studied a variant of 2E-IRP in which there is a single supplier with limited storage capacity [43]. Stocking and distribution decision-making in this model is the responsibility of the supplier. The distribution network is therefore made up of a supplier, several DCs and several customers. These customers are divided into subsets of customers that are attached to each DC of the distribution network in a predefined way. The objective of this work is to simultaneously optimize the cost of vehicle rounds and the overall cost of storage at the DCs and the customers. The authors formalized the problem into an integer linear program and solved it by a B&C algorithm with the OFQ, OU, and ML policies in VMI practice. Xiao and Rao proposed a distribution network consisting of a supplier, a DC, several depots and several customers in a 2E-IRP [44]. In this multi-product model, the authors considered the time windows and characteristics of the product at different stages of its life cycle in a VMI context. They then proposed an adaptation of the fuzzy genetic algorithm to solve the problem. Daldoul, Boussaa, Nouaouri and Kastouri have studied a 2E-IRP taking into account the financial flow of working capital needs in a distribution network of several suppliers and several depots [45]. In this work, the overall cost to be optimized is therefore the sum of the costs related to inventories, transport and financial costs. The authors then proposed a mathematical model of the problem that they solved with a solver. Several authors have proposed a distribution model with a single DC. Among these models, we have models with several suppliers and models with a single supplier. In the category of models with multiple suppliers, some researchers focused on a multi-product model with a distribution network consisting of multiple suppliers, a DC and multiple warehouses [46]. The goal in this work is to find the most economical way to supply warehouses from DC and DC from suppliers. Their model considers an order cost for each given product, a fixed cost of using each vehicle, inventory costs in the warehouses and the DC and the costs of vehicle routes between the suppliers and the DC on the one hand and between the DC and the warehouses on the other hand. They formulated the problem as an integer linear program and solved it with three variants of a three-phase heuristic CAR (clustering, allocation, routing). They estimate that their heuristics allow to obtain an improvement in results on large instances ranging from 0.94% to 6.08% compared to the upper bound obtained with the CPLEX solver. In the category of work that focuses on distribution models with a supplier and a DC, researchers [35] proposed a model in which a strategy called fixed partition (FP) and power of two (POT) (FP-POT) [36] have been implemented. They used a variable large neighborhood search (VLNS) algorithm to solve the problem. The results of the different simulations were compared to those of a Tabu search algorithm. Li, Chu and Chen have studied a model in which a customer can receive his delivery directly from the supplier without transiting through the warehouse (DC) [47]. They proposed a decomposition of the problem according to the FP policy and considered a genetic algorithm for its solution. Rohmer, Claassen and Laporte submitted a work in the context of last-mile delivery of fresh produce [9]. To solve this problem which considers the perishability of products, the authors suggested an adaptive large neighborhood search (ALNS) algorithm. Hu, Toriello and Dessouky proposed a model for perishable products with a deterministic lifespan [8]. In this model, products are collected from farmers over short distances to supply a consolidation center. After the consolidation of the products in the center, they are then transported over long distances to satisfy the demands of the retailers. They formulated the problem into a mixed integer linear program (MILP) and developed an iterative local search algorithm based on a decomposition strategy for its solution.

3. Description of the Problem and Mathematical Formulation

3.1. Description of the Problem

We present a 2E-IRP whose distribution network is made up of a supplier, several DCs and several retailers. Figure 1 gives a graphical representation of the distribution network as well as the different types of routes. In this problem, the transport of a single type of product is ensured by a homogeneous fleet of vehicles parked in the DCs. Retailer procurement is carried out only from the DCs at the second level (DCs-retailer). At the first level (supplier-DCs), the supply of the DCs is carried out by the same fleet of vehicles parked in the DCs. Three types of routes are considered in the route of a vehicle associated with a given DC. We have the supply round, which consists of a vehicle associated with a DC leaving this DC to collect products from the supplier and return to the DC. The second type of round is a delivery round. This tour consists of a vehicle associated with a DC delivering orders from a set of retailers and returning to the DC. The third type of round is a delivery and collection round. In this round, a vehicle first delivers orders from retailers and then goes to the supplier for product collection to supply the DC with which it is associated. The objective of this work is to minimize the overall cost of vehicle routing and inventories in the DCs and at retailers over a finite and discreet planning horizon.

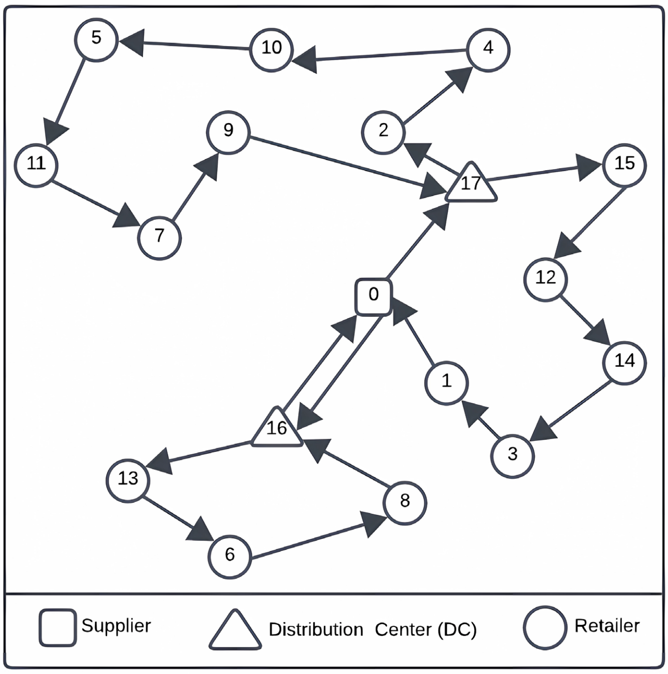

Figure 1. Distribution network graph.

3.2. Mathematical Formulation

$G(N, A)$ is a complete graph in which "N" represents the set of vertices materialized by the supplier, the DCs and the retailers. "A" represents the set of arcs of the graph $A(N)=\{(i,j):i,j\in N,i\neq j\}$. The supplier is identified by the index "0". Retailers and DCs are represented by the sets $\{1,\cdots,n\}$ and $\{n+1,\cdots,N\}$ respectively where n is the number of retailers. Figure 1 represents a distribution network with a supplier (0), two DCs (16 and 17) and 15 retailers (from 1 to 15).

The sets:

$N=\{0,\cdots,n,n+1,\cdots,|N|\}$: The set of vertices representing the supplier, the retailers and the DCs.

$N_R=\{1,\cdots,n\}$: All retailers.

$N_D=\{n+1,\cdots,|N|\}$: All DCs.

$N_{0R}=\{0,\cdots,n\}$: The whole constituted by the supplier and the retailers.

$N_{RD}=\{1,\cdots,|N|\}$: The set made up of retailers and DCs.

$T=\{1,\cdots,l\}$: All periods of the planning horizon (in days).

$K_y=\{1,\cdots,m_y\}$: All the vehicles associated with the DC y.

$K=\bigcup K_y$.

The indexes:

i and j are the indices of all the vertices of graph G.

y is an index associated with the DC.

t represents the index of the periods of the planning horizon.

k is the index of vehicles.

The settings:

h_i is the unit cost of inventories at vertex i.

d_{it} represents the demand of retailer i at period t.

L_i is the maximum storage capacity at vertex i.

I_{i0} is the initial stock level at vertex i.

c_{ij} is the cost of transportation when a vehicle travels directly from vertex i to vertex j.

Q is the maximum load of a vehicle.

$MC_{it}=\min\left\{L_i,Q,\sum_{\tau=t}^{l}d_{i\tau}\right\}$ is the maximum quantity of products that can be delivered to a retailer i during a period t.

Decision variables:

I_{it}: level of stocks at vertex i during period t.

q_{ikt}^{y}: quantity delivered to node i by vehicle k associated with DC y during period t.

Z_{ikt}^{y}: binary variable, equal to 1 if vertex i is visited by vehicle k associated with DC y during the period t or 0 otherwise.

x_{ijkt}^{y}: binary variable, equal to 1 if vehicle k associated with DC y travels directly from the vertex i to vertex j or 0 otherwise.

Mathematics formulation:

$$Z = \min \sum_{i \in N_{RD}} \sum_{t \in T} h_i I_{i,t} + \sum_{(i,j) \in A} \sum_{k \in K_y} \sum_{y \in N_D} \sum_t x_{ijkt}^y \tag{1}$$

S.T

$$\sum_{k \in K} q_{ykt}^y + I_{yt-1} = \sum_{i \in N_R} \sum_{k \in K} q_{ikt}^y + I_{yt}, \forall y \in N_D, \forall t \in T \tag{2}$$

$$\sum_{k \in K} \sum_{i \in N_R} q_{ikt}^y \le I_{y,t-1}, \forall y \in N_D, \forall t \in T \tag{3}$$

$$q_{ykt}^y \le Q z_{0kt}^y, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{4}$$

$$\sum_{y \in N_D} \sum_{k \in K_y} q_{ikt}^y + I_{it-1} = d_{it} + I_{it}, \forall i \in N_R, \forall t \in T \tag{5}$$

$$\sum_{i \in N_R} q_{ikt}^y \le Q z_{ykt}^y, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{6}$$

$$q_{ikt}^y \le MC_{it} z_{ikt}^y, \forall i \in N_R, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{7}$$

$$I_{it} \le L_i, \forall i \in N_{RD} \tag{8}$$

$$q_{0kt}^y = 0, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{9}$$

$$\sum_{i \in N_{0R}} x_{iykt}^y + \sum_{j \in N_{0R}} x_{yjkt}^y = 2Z_{ykt}^y, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{10}$$

$$\sum_{i \in N_{RD}} x_{iakt}^y + \sum_{j \in N} x_{ajkt}^y = 2Z_{akt}^y, \forall a \in N_R, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{11}$$

$$\sum_{i \in N_{RD}} x_{i0kt}^y + x_{0ykt}^y = 2Z_{0kt}^y, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{12}$$

$$\sum_{y \in N_D} \sum_{k \in K_y} z_{ikt}^y \le 1, \forall i \in N_R, \forall t \in T \tag{13}$$

$$\sum_{k \in K} \sum_{y \in N_D} \sum_{t \in T} Z_{ikt}^y \ge 1, \forall i \in N_R \tag{14}$$

$$\sum_{j \in N} x_{ijkt}^y = z_{ikt}^y, \forall i \in N, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{15}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijkt}^y \le |S| - 1, \forall S \subseteq N_{0R}, |S| \ge 2, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{16}$$

$$\sum_{y \in N_D} \sum_{k \in K} Z_{ykt}^y \le m, \forall t \in T \tag{17}$$

$$I_{it}, q_{ikt}^y \ge 0, \forall i \in N, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{18}$$

$$x_{ijkt}^y, z_{ikt}^y \in \{0,1\}, \forall (i,j) \in A(N), \forall i \in N, \forall y \in N_D, \forall k \in K_y, \forall t \in T \tag{19}$$

Equation (1) is the Objective function. It allows us to simultaneously minimize the overall cost of inventories (DCs and retailers) and the cost of the different vehicle routes associated with each DC. Constraints (2) reflect the conservation of the flow of products in each DC. They state that in each DC, the quantity of products received per period and the stock of products from the previous period allow deliveries to be made to retailers and to build up new stock for the current period. Constraints (3) stipulate that in a DC, deliveries for a given period are made with the previous period's stock. Constraints (4) indicate that any vehicle that makes a delivery in a DC must first visit the supplier before making its delivery in the DC. Constraints (5) represent the maintenance of product flows at retailers. They express the fact that the quantity of products received during a period t and the quantity of products stored during period $t-1$ make it possible to satisfy demand and to constitute the stock of products in period t. Inequalities (6) limit the maximum load of a vehicle leaving a DC to a quantity Q. Constraints (7) are used to limit the quantity of products that can be delivered to a retailer for a given period. Inequalities (8) impose a limit on the level of stocks both in retailers and in DCs. Constraints (9) indicate that the load of a vehicle intended for the collection of products from the supplier is zero. **Figure 2** shows the predecessor and successor of each vertex for the formalization of vehicle routing constraints. The constraints associated with vehicle routing are as follows: a vehicle that visits a DC must be directly from the supplier or a retailer, and any vehicle that leaves a DC immediately visits a retailer or the supplier. So, for ease of understanding, we will say that the predecessor of a DC is the supplier or a retailer and the successor of a DC is the supplier or a retailer. This situation is reflected in Constraints (10). Similarly, the predecessor of a retailer is either a DC or a retailer and the successor of a retailer is either a retailer, a supplier or a DC (see Constraint (11)). As far as the supplier is concerned, he accepts a retailer or a DC as his predecessor and is succeeded by a DC (Constraint (12)). Constraints (13) indicate that each retailer must be visited no more than once per period. Constraints (14) require that each retailer be visited at least once over the planning horizon. Constraints (15) indicate the activation of a path to the successor of an *i*-vertex if that vertex is visited by a vehicle during a given period. Inequalities (16) are the subtours elimination constraints (SECs) on the vehicle paths. Constraints (17) limit the total number of vehicles assigned to distribute products in the supply chain. Constraints (18) indicate that the variables representing stock levels in the DCs and at retailers as well as the quantities of products transported by the vehicles must have positive values. And Constraints (19) represent the constraints for defining the binary variables of the mathematical model. In the rest of this work, we present a Branch and Cut (B&C) algorithm to solve the MILP defined in this session.

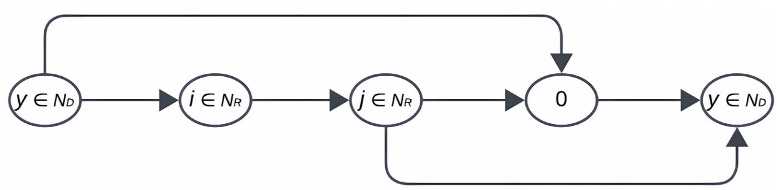

Figure 2. Vehicle routing graph.

4. B&C Algorithm for Solving 2E-IRP with ETR Policy

To carry out our B&C algorithm, we start by defining the following valid inequalities.

4.1. Valid Inequalities

- $$x^{y}_{0,y,kt} \le q^{y}_{ykt}, \forall y \in N_D, \forall k \in K_y, \forall t \in T \quad (20)$$

These inequalities prevent a vehicle from returning empty to the DC after visiting the supplier.

- $$\sum_{t\in T}\sum_{k\in K_y} x^{y}_{0,y,kt} \le \sum_{t\in T}\sum_{k\in K_y}\sum_{i\in N_R} z^{y}_{ikt},\ \forall y\in N_D \quad (21)$$

Under these constraints, any DC that has been supplied at least once over the planning horizon is bound to make product deliveries to retailers.

- $$x^{y}_{ijkt} + x^{y}_{jikt} \le 1,\ \forall i\in N_{RD},\ \forall j\in N_{RD},\ \forall y\in N_D,\ \forall k\in K_y,\ \forall t\in T,\ N_{RD}\in N_{RD} \quad (22)$$

These constraints state that the path of a vehicle between two vertices *i* and *j* is an arc. Thus, if a vehicle travels directly from vertex *i* to vertex *j* then it can no longer travel directly from vertex *j* to vertex *i*.

- $$z^{y}_{y,k+1,t} \le z^{y}_{ykt},\ \forall y\in N_D,\ \forall k\in K_y,\ \forall t\in T \quad (23)$$

These constraints make it possible to ensure that the use of vehicle $k+1$ is conditional on the use of vehicle *k*, so that vehicle $k+1$ can be used only after the use of vehicle *k*.

- $$\sum_{i\in N_D} Z_{ikyt} \le 1,\ \forall t\in T,\ y\in N_D,\ k\in K \quad (24)$$

These constraints prevent a vehicle associated with a DC from visiting another DC.

- $$d_{it} Z_{ikyt} \le q_{ikyt},\ \forall i\in T,\ k\in K,\ y\in N_D \text{ et } i\in N_R \quad (25)$$

These constraints make it possible to ensure that a vehicle visiting a retailer delivers a quantity of products at least equal to its demand.

4.2. B&C Algorithm

To implement our B&C algorithm, we add the valid inequalities expressed by Constraints (20)-(25) in the initial model expressed by the objective function (1) and the constraints ranging from (2) to (19) and we subtract the SECs expressed by Constraints (16). Next, the new linear program (LP) is solved. If the LP solution contains a subtour in the path of a vehicle *k* associated with a DC for a given period t, then all the subtours in the path of this vehicle are identified and each SEC relative to the vertices concerned by each subtour is generated and added to the PL for a new reoptimization. **Table 1** gives the details of our B&C algorithm. In this algorithm, the star symbol (*) represents a component of the PL solution at a node in the B&C search tree. Our B&C algorithm is an improvement of the algorithm developed by [48] for a 2E-PRP with a supplier, a depot and several customers. In this work, the authors attempt to separate all the vertices visited by a vehicle *k* at a time *t* given in two partitions. The first partition consists of the subset of vertices forming a Hamiltonian circuit with the depot, and the second partition is the complement of this first partition in the set of vertices visited by the vehicle *k* at the same time *t*. Thus, when the second partition is empty then the vehicle *k* does not contain any subtour in its period path *t*. However, if the second partition is non-empty then a subtour is detected in the path of vehicle *k* at the period *t*. In the latter case, SECs are generated and added to the model for reoptimization. In the algorithm developed in our work, we exhaustively determine all the subsets of vertices forming a Hamiltonian circuit in the path of vehicle *k* associated with the depot y during the period t. If the number of subsets of vertices forming a Hamiltonian circuit is greater than 1, then a SEC is generated and added to the model for each vertex subset.

Table 1. Branch-and-cut (B&C) algorithm.

```
Initialize the upper bound U* and the incumbent solution.
Initialize the node pool N with the root node.
Generate and insert all known valid inequalities into the program at root node of the
search tree.
    Repeat
    Selection: Select the next node in N to evaluate and remove it from N
    Lower bound: Solve the LP relaxation at the current node,
            let be the obtained lower bound U_l of the current node:
    if current solution is feasible then
            if U_l > then U*
                    Go to the termination check.
            Else
                    U* ← U_l.
                    Update the incumbent solution.
                    Prune nodes with lower bound U > U*
            End
    end
    Cut generation:
    foreach t in T
        foreach y in N_D
            Foreach k in K_y
                    if the current solution of the LP relaxation contains
                    subtours, then
                        Generate and add SEC for each subtour.
                    Endif
            endfor
        endfor
    Endfor
    Branching: if U_l > U*,
                go to the termination check.
  until N = ∅ or time limit is met (termination check).
 Stop with the optimal solution and the corresponding cost U*.
```

5. Experiments and Results

5.1. Experiments

We use the Concert Technology library from IBM ILOG CPLEX Optimization

Studio and the C++ programming language to implement the B&C algorithm exposed in Session 4. The tests are conducted on an AMD Athlon Silver 3050U LAPTOP with Radeon Graphics 2.30 GHz and 8.00 GB of installed RAM. The dataset used for the tests is drawn from the work of [40]. From this work, we have retained the dataset for which the instances are composed of a single supplier. Table 2 gives a description of the dataset used to perform the tests with our B&C algorithm. In this table, the first column indicates the typology of inventory costs. In this typology, we have instances with high unit inventory costs over a 3-period planning horizon (H3) and instances with low unit inventory costs over a 3-period planning horizon (L3). The second column shows the different instance classes from abs1 to abs5. Similarly, going from the third column to the last column, we have respectively the supplier, the DCs, the customers and the homogeneous vehicles used to make the deliveries of the products on the planning horizon. The charge of each Q vehicle in our tests is equal to the average of the charge of a vehicle in the first echelon (Q_1) and a vehicle in the second echelon (Q_2). So, $Q = (Q_1 + Q_2)/2$. By multiplying the cardinal of each instance parameter, we obtain a total of 2 * 5 * 1 * 1 * 5 * 4 = 200 instances (see Table 2).

Table 2. Description of instances.

Typology of inventory costs	Class of instances	supplier	DC	Retailers (customers)	Vehicles
{H3; L3}	abs {1; 2; 3; 4; 5}	{1S}	{2M}	{5; 10; 15; 20; 25} C	{2; 3; 4; 5} V

5.2. Results

Considering that a solution is optimal when its integrality gap is less than 0.05% [40], our algorithm resolved 168 instances to the optimal (83%), 29 instances to non-optimal (14.5%) and 3 unresolved instances (1.5%). Table 3 illustrates the distribution of the solutions obtained in relation to their integrality gap. To show the effectiveness of the results of our B&C algorithm on the 2E-IRP with ETR policy, we present the details of the result obtained on the H3_abs1_1S_2M_5C_2V-M instance. The instance designates a set of data with a high inventory cost over a planning horizon of three days (H3) in the abs1 type instance register. The instance considers one supplier (1S), two DCs (2M) and two medium-sized vehicles (2V-M). According to the details of the results contained in Table A1, the cost of inventories is equal to 222.32 and the cost of vehicle routes is equal to 2187. This gives a total cost of 2411.32. To verify these costs, we start by presenting the vehicle route file for the H3_abs1_1S_2M_5C_2V-M instance represented in Figure 3. The Readers will find the file for Table A1 and the rest of the files for Figure 3 in link A of **Appendix**. In this figure, the vehicle $k = 1$ associated with the DC of index 7 (second DC) leaves the DC in the first period and successively visits retailers (customers) 5 and 2, giving them respectively the quantities of products 22 and 35. After the visit to the last retailer (2), the vehicle goes to the supplier (0) and collects a quantity of products equal to 194 for the supply of the DC. In period two, the vehicle $k = 1$ leaves the second DC of index 7 and then successively visits retailers 3, 4, 1 delivering them respectively quantities 116, 24, 65 before returning empty to the DC. In the third period, there is no delivery of products. Let's start with the determination of the cost of inventory over the three periods of the planning horizon. Tables 4-6 summarize the calculations of inventory costs over these three periods.

Table 3. Distribution of solutions according to the completeness.

Solutions	Number	Rate_%
Optimal	168	84
Non-optimal	29	14.5
Infinite	3	1.5
Total	200	100

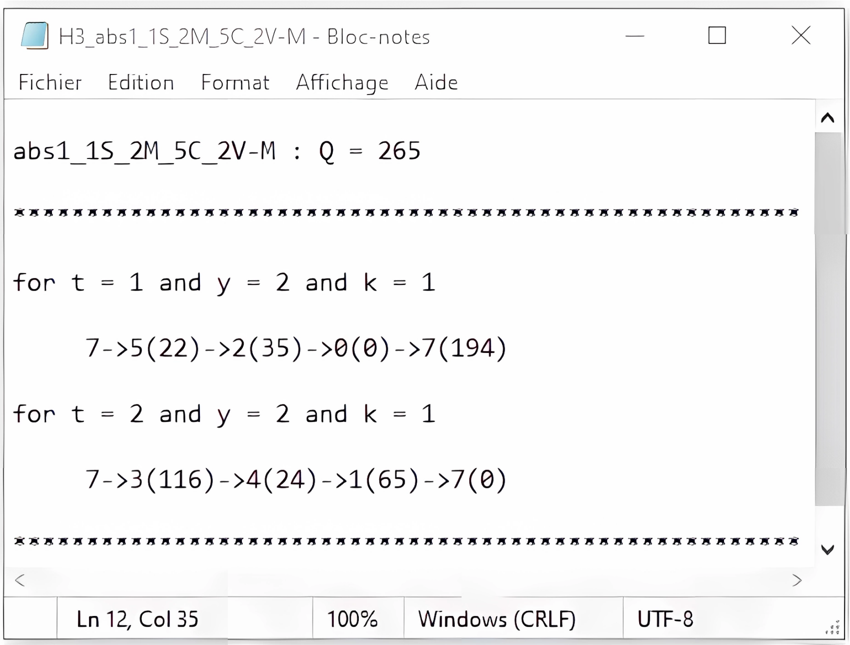

Figure 3. Vehicle routes over three periods.

By postponing vehicle routes and the quantities received in Figure 3 represented respectively by line (i) and line (q_i) of Table 4, we determine the level of inventories at the end of the period (I_i) as follows: at vertex i, we have I_i = level of inventories of the previous period + quantity of products received during the current period − quantity of products consumed for the same period. And we determine the costs of inventories by multiplying the inventory level of the period by the unit cost of storage.

Table 4. Summary of inventory cost calculation in period 1 (t_1).

	t_1					Retailers not visited			
i	7	5	2	0	7	6	1	3	4	
q_i	0	22	35	0	194	0	0	0	0	
of	0	11	35	0	0	0	65	58	24	
$I_i(t=0)$	68	11	70	ND	68	68	130	58	48	
h_i	0.3	0.18	0.32	0	0.3	0.3	0.23	0.33	0.23	
L_i	703	22	105	ND	703	703	195	116	72	
$I_i(t=1)$		22	70		205	68	65	0	24	
IC_i		3.96	22.4		61.5	20.4	14.95	0	5.52	128.73

Table 5. Summary of inventory cost calculation in period 2 (t_2).

	t_2				Retailers not visited				
i	7	3	4	1	7	6	0	2	5	
q_i	0	116	24	65	0	0	0	0	0	
of	0	58	24	65	0	0	0	35	11	
$I_i(t=1)$	205	0	24	65	205	68	ND	70	22	
h_i	0.3	0.33	0.23	0.23	0.3	0.3	0	0.32	0.18	
L_i	703	116	72	195	703	703	ND	105	22	
$I_i(t=2)$		58	24	65	0	68		35	11	
IC_i		19.14	5.52	14.95	0	20.4		11.2	1.98	73.19

This is translated by the equation $IC_i = I_i * h_i$. Let's calculate these values for DC 7, visited customer 5, and unvisited customer 1 for period $t = 1$. We have $I_7 = 68 + 194 - (22 + 35) = 205$ and $IC_7 = 205 * 0.3 = 61.5$. For retailer 5, we have $I_5 = 11 + 22 - 11 = 22$ and $IC_5 = 22 * 0.18 = 3.96$. for the unvisited retailer (1), we have $I_1 = 130 + 0 - 65 = 65$ and $IC_1 = 65 * 0.23 = 14.95$. Similarly, we calculate the inventory costs for each DC and each retailer over the three periods of the planning horizon. The total cost of inventory is therefore equal to the sum of the periodic inventory costs, *i.e.* $IC = 128.73 + 73.19 + 20.4 = 222.32$. For determination of the overall cost of vehicle routes, we organize the calculations in Table 7. In this table, 7 → 5 indicates that the vehicle travels directly from vertex 7 to vertex 5. Vertex 7 has coordinates (1, 360) and vertex 5 has coordinates (38, 152). This allows us to calculate the distance from vertex 7 to vertex 5 ($d(7, 5)$). Thus, $d(7,5) = \text{int}\left(\sqrt{(1-38)^2 + (360-152)^2} + 0.5\right) = \sqrt{(1-38)^2 + (360-152)^2} + 0.5$, so $d(7, 5) = 211$. We can therefore determine the overall cost of vehicle routes by summing the costs of routes per period. Hence the overall cost of vehicle rounds Trans_Cost = 1431 + 758 = 2189. Finally, the overall cost is equal to the sum of the cost of transportation and the cost of inventory. Thus, we have Total_Cost =

Inv_Cost + Trans_Cost = 222.32 + 2189 = 2411.32 monetary unit. In the continuation of this work, we compare the results of our B&C algorithm with the results of the branch and price (B&P) developed by [40]. In both jobs, it is forbidden to deliver to a customer (or retailer) from the supplier. However, the two models differ in the strategy adopted in the procurement of DCs (or Satellites). In the procurement strategy of these authors, the vehicle dedicated to the supply of DCs starts its run at the supplier and visits one or more DCs before ending its run at the supplier. Whereas in our model, the same fleet of vehicles based at the DC is used to make the delivery to the customers and if necessary, one or more vehicles from this fleet go directly to the supplier or after the last retailer's visit to collect products that will be used to supply the DC (ETR policy). In **Tables 8-12**, the rows represent the average of the results obtained with the two, three, four and five vehicle instances. The time (B&P) and time (B&C) columns represent the time taken by the B&P algorithm and the B&C algorithm to find the respective upper bounds (UB) respectively. The Time Gap % column determines the rate of increase (positive value) or rate of reduction (negative value) of the time associated with the execution of the B&C algorithm relative to the B&P algorithm. The values in this column are calculated as follows: Time GAP % = (time (B&C) − time (B&P))/time (B&P). Similarly, UB GAP % = (UB (B&C) − UB (B&P))/UB(B&P). In **Table 8** representing instances of 5 retailers, we see that the average B&C GAP % for each line is equal to 0. This means that each different fleet instance associated with each instance line of five retailers is resolved to the optimum. This is also reflected in the average of the average GAPs of the B&C at the AVERAGE line. In addition, the values of the numbers in the UB GAP % column are all negative. This means that cost reductions are made in favor of the B&C algorithm. For example, we have a discount of 23.93% on average on type H3_abs1_1S_2M_5C instances (4 instances) and overall, we get an average discount of 16.69% on instances of 5 retailers (AVERAGE line). As far as the calculation time is concerned, we also get an overall average decrease of 15.55% (Time GAP%) in favor of the B&C algorithm. Similarly, **Table 9** shows that all instances of ten retailers were resolved to the optimal (GAP B&C % column) with an overall average cost (UB) decrease of 23.45% (AVERAGE UB GAP%) and an overall decrease in computation time of 90.06%. **Table 10** summarizes the results for the instances of fifteen retailers. This table also shows the same trends as **Table 8** and **Table 9**. It also shows that all instances of fifteen retailers are resolved to the optimal with a decrease in average overall cost by 20.66%. We also see an overall average decrease in computation time of 67.90%. **Table 11** shows the results obtained on the instances of 20 retailers. Unlike the previous tables, this table has a non-zero overall average of the GAP B&C % (2.76%). This implies that some instances of twenty retailers have not been resolved to the optimal level. **Table A1** of the details in **Appendix** does show that the instances for twenty retailers H3_abs1_1S_2M_20C_3V, H3_abs1_1S_2M_20C_5V, H3_abs2_1S_2M_20C_5V, and instance H3_abs3_1S_2M_20C _5V were not resolved to the optimal level.

However, we achieve a 19.87% reduction in average overall cost in favor of the B&C algorithm compared to the B&P algorithm. We also have a decrease in computation time of 85.59%. Table 12 highlights the results obtained on the instances of twenty-five retailers with an average overall B&C GAP % of 15.46%, we can say that several instances have not been resolved to the optimal. The UB GAP % column shows that the B&C algorithm allows for an average overall cost reduction of 9.13% across instances of 25 retailers. It should be noted that for instances with twenty-five retailers, our B&C algorithm resulted in a cost increase of 12.31, 4.86 and 4.47 respectively over the average of the H3_abs2_1S_2M_25C, H3_abs3_1S_2M_25C and H3_abs5_1S_2M_25C instances. This represents a total of 9 instances of twenty-five retailers. In addition, we also see an overall average decrease in calculation time of 63.96%. Overall, our ETR policy and B&C algorithm resulted in 185 (93.9%) best solutions out of the 197 instances for which we were able to obtain a solution.

Table 6. Summary of inventory cost calculation in period 3 (t_3).

	t_3							
i	1	2	3	4	5	6	7	
q_i	0	0	0	0	0	0	0	
of	65	35	58	24	11	0	0	
$I_i\,(t=2)$	65	35	58	24	11	68	0	
h_i	0.23	0.32	0.33	0.23	0.18	0.3	0.3	
L_i	195	105	116	72	22	703	703	
$I_i\,(t=3)$	0	0	0	0	0	68	0	
IC_i	0	0	0	0	0	20.4	0	20.4

Table 7. Summary of the calculation of travel costs over the three periods.

		x_1	y_1	x_2	y_2	d
	7 → 5	1	360	38	152	211
t_1	5 → 2	38	152	267	87	238
	2 → 0	267	87	−124	−150	457
	0 → 7	−124	−150	1	360	525
Total						1431
	7 → 3	1	360	148	433	164
t_2	3→ 4	148	433	355	444	207
	4 → 1	355	444	172	334	214
	1 → 7	172	334	1	360	173
Total						758

Table 8. Comparison of average results for five retailers' instances.

Instance name	Time (B&P)	Time (B&C)	Time GAP %	UB (B&P)	UB (B&C)	UB GAP %	Gap B&P %	Gap B&C %
H3_abs1_1S_2M_5C	5.93	1.72	−72.02	3196.45	2413.57	−23.93	0.03	0.00
L3_abs1_1S_2M_5C	6.69	1.72	−73.49	3091.73	2239.01	−26.99	0.03	0.00
H3_abs2_1S_2M_5C	5.47	2.33	−47.56	2482.54	2095.44	−15.09	0.03	0.00
L3_abs2_1S_2M_5C	5.89	2.06	−54.86	2429.20	1971.73	−18.34	0.04	0.00
H3_abs3_1S_2M_5C	11.22	4.02	106.68	3923.50	3709.04	−4.50	0.04	0.00
L3_abs3_1S_2M_5C	13.90	5.04	12.72	3794.28	3460.87	−8.17	0.03	0.00
H3_abs4_1S_2M_5C	5.21	5.90	27.54	3241.76	2569.25	−19.69	0.03	0.00
L3_abs4_1S_2M_5C	6.22	4.70	12.16	3145.41	2372.06	−23.58	0.03	0.00
H3_abs5_1S_2M_5C	4.64	1.68	2.15	1941.14	1774.43	−7.75	0.03	0.00
L3_abs5_1S_2M_5C	11.86	1.55	−68.82	1836.04	1476.96	−18.87	0.04	0.00
AVERAGE	7.70	3.07	−15.55	2908.20	2408.24	−16.69	0.03	0.00

Table 9. Comparison of average results for ten retailers' instances.

Instance name	Time (B&P)	Time (B&C)	Time GAP %	UB (B&P)	UB (B&C)	UB GAP %	Gap B&P %	Gap B&C %
H3_abs1_1S_2M_10C	6490.83	53.14	−98.11	3784.92	3173.25	−15.59	0.95	0.00
L3_abs1_1S_2M_10C	1310.97	25.41	−98.00	3418.07	2579.45	−23.96	0.14	0.00
H3_abs2_1S_2M_10C	3709.20	21.81	−95.96	4823.68	3782.26	−21.03	0.98	0.00
L3_abs2_1S_2M_10C	2518.85	43.98	−85.93	4539.82	3287.41	−26.97	0.78	0.00
H3_abs3_1S_2M_10C	9930.84	20.25	−99.79	4379.41	3352.36	−22.48	2.70	0.00
L3_abs3_1S_2M_10C	10800.00	15.16	−99.86	4207.36	2878.99	−30.54	3.61	0.00
H3_abs4_1S_2M_10C	3069.89	14.01	−98.05	4820.91	3622.75	−23.83	0.77	0.00
L3_abs4_1S_2M_10C	3713.52	15.65	−97.93	4571.12	3242.71	−27.79	1.08	0.00
H3_abs5_1S_2M_10C	679.08	19.78	−67.11	3658.15	3025.20	−16.53	0.05	0.00
L3_abs5_1S_2M_10C	540.46	12.82	−59.86	3303.37	2395.03	−26.68	0.03	0.00
AVERAGE	4276.36	24.20	−90.06	4150.68	3133.94	−23.54	1.11	0.00

Table 10. Comparison of average results for fifteen retailers' instances.

Instance name	Time (B&P)	Time (B&C)	Time GAP %	UB (B&P)	UB (B&C)	UB GAP %	Gap B&P %	Gap B&C %
H3_abs1_1S_2M_15C	10800.00	743.69	−93.11	5100.68	4141.68	−18.02	5.43	0.00
L3_abs1_1S_2M_15C	10800.00	94.45	−99.13	4608.87	3334.53	−26.78	3.95	0.00
H3_abs2_1S_2M_15C	9162.41	340.72	−95.33	5215.97	4236.95	−18.39	5.27	0.00
L3_abs2_1S_2M_15C	10800.00	462.59	−95.72	4750.71	3485.49	−26.02	3.91	0.00
H3_abs3_1S_2M_15C	8169.43	631.81	22.92	5187.01	4566.16	−11.73	1.49	0.00

Continued

L3_abs3_1S_2M_15C	8126.19	478.71	62.96	4753.56	3710.44	−21.75	1.53	0.00
H3_abs4_1S_2M_15C	10800.00	359.35	−96.67	4435.20	3721.89	−15.88	3.95	0.00
L3_abs4_1S_2M_15C	8050.84	171.35	−95.19	4071.25	3129.83	−22.88	3.91	0.00
H3_abs5_1S_2M_15C	7529.75	361.82	−91.82	4520.81	3703.06	−17.58	1.31	0.00
L3_abs5_1S_2M_15C	8913.36	172.09	−97.92	4131.39	2976.96	−27.57	2.44	0.00
AVERAGE	9315.20	381.66	−67.90	4677.54	3700.70	−20.66	3.32	0.00

Table 11. Comparison of average results for twenty retailers' instances.

Instance name	Time (B&P)	Time (B&C)	Time GAP %	UB (B&P)	UB (B&C)	UB GAP %	Gap B&P %	Gap B&C %
H3_abs1_1S_2M_20C	10800.00	1538.25	−85.76	4846.34	4673.53	−3.26	5.68	12.84
L3_abs1_1S_2M_20C	3263.55	2270.15	−30.44	4550.77	3323.05	−26.98	0.05	0.00
H3_abs2_1S_2M_20C	10800.00	2925.79	−72.91	5551.36	5209.41	−6.28	7.21	10.00
L3_abs2_1S_2M_20C	10800.00	895.56	−91.71	4950.17	3819.65	−22.21	5.68	0.00
H3_abs3_1S_2M_20C	10800.00	838.51	−92.24	5682.87	4856.27	−14.48	11.54	4.73
L3_abs3_1S_2M_20C	10800.00	373.51	−96.54	4880.60	3625.03	−25.04	7.04	0.00
H3_abs4_1S_2M_20C	10800.00	372.71	−96.55	6101.01	4599.73	−24.44	8.54	0.00
L3_abs4_1S_2M_20C	10800.00	186.88	−98.27	5598.72	3727.31	−33.11	8.46	0.00
H3_abs5_1S_2M_20C	10800.00	593.14	−94.51	5212.44	4395.94	−15.37	5.67	0.00
L3_abs5_1S_2M_20C	10800.00	325.97	−96.98	4576.28	3295.61	−27.54	5.68	0.00
AVERAGE	10046.36	1032.05	−85.59	5195.06	4152.55	−19.87	6.55	2.76

Table 12. Comparison of average results for twenty-five retailers' instances.

Instance name	Time (B&P)	Time (B&C)	Time GAP %	UB (B&P)	UB (B&C)	UB GAP %	Gap B&P %	Gap B&C %
H3_abs1_1S_2M_25C	10800.00	3538.72	−67.23	5236.87	5088.98	−2.88	6.68	16.60
L3_abs1_1S_2M_25C	10800.00	1415.67	−86.89	4749.41	4213.51	−12.37	4.72	13.73
H3_abs2_1S_2M_25C	10800.00	1107.44	−89.75	6007.65	6746.27	12.31	9.33	32.83
L3_abs2_1S_2M_25C	10800.00	1179.37	−89.08	5380.83	4601.30	−14.86	11.31	20.61
H3_abs3_1S_2M_25C	10800.00	1820.35	−83.14	5730.06	6012.04	4.86	8.48	26.58
L3_abs3_1S_2M_25C	10800.00	3038.45	−71.87	4806.68	4178.90	−13.31	4.71	13.15
H3_abs4_1S_2M_25C	10800.00	6000.90	−44.44	6579.31	5344.12	−18.67	16.57	4.94
L3_abs4_1S_2M_25C	10800.00	1373.48	−87.28	5931.97	4109.20	−30.06	12.90	1.93
H3_abs5_1S_2M_25C	7427.83	2743.54	−14.09	5267.78	5505.96	4.47	5.19	14.55
L3_abs5_1S_2M_25C	8160.68	2957.85	−5.81	4693.13	3695.26	−20.78	5.96	9.64
AVERAGE	10198.85	2517.58	−63.96	5438.37	4949.55	−9.13	8.58	15.46

6. Conclusions

In this paper, we present a B&C algorithm for the resolution of a 2E-IRP with a policy of sourcing DCs at the end of the vehicle journey to reduce transport costs (ETR policy). Thus, we have contributed to the proposal of a review of recent literature, a mathematical model in the form of a mixed integer linear program based on the vehicle index and a B&C algorithm for the resolution of the 2E-IRP-ETR. Testing on 200 instances yielded results on 197 instances. This gives a resolution rate of 98.5%. Comparison of the results obtained with those of a B&P algorithm in the literature yielded better results on 185 instances for a total of 197 resolved instances. Hence, the best solution rate is 93.9%. In addition, our B&C algorithm resulted in cost reductions ranging from 0.26% to 41.44% across the 197 resolved instances and an overall average cost reduction of 18.08%.

Interestingly, our work can be enriched by considering new parameters, such as multiple suppliers, time windows, split deliveries and multiple deliveries. For large instances, the implementation of a metaheuristic such as a genetic algorithm or a memetic algorithm would be wise, given that our algorithm struggles to obtain good quality integrity gaps for these instances.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

Link A:

https://drive.google.com/drive/folders/1J2Y8Qt_Acr_d0Q5xtfc_-fV2q_D0711x?usp=sharing

Table A1. Details of results for each instance.

Instance name	Vehicle load	time	Inv_Cost	Trans_Cost	Total_Cost	GAP_%
H3_abs1_1S_2M_5C_2V-M	265	0.786	222.32	2189	2411.32	0
H3_abs1_1S_2M_5C_3V-M	241	1.291	222.32	2198	2420.32	0
H3_abs1_1S_2M_5C_4V-M	229	1.639	222.32	2189	2411.32	0
H3_abs1_1S_2M_5C_5V-M	222	3.163	222.32	2189	2411.32	0
H3_abs1_1S_2M_10C_2V-M	873	8.015	644.77	2504	3148.77	0
H3_abs1_1S_2M_10C_3V-M	794	76.213	639.73	2566	3205.73	0
H3_abs1_1S_2M_10C_4V-M	754	51.728	639.73	2550	3189.73	0
H3_abs1_1S_2M_10C_5V-M	730	76.608	644.77	2504	3148.77	0
H3_abs1_1S_2M_15C_2V-M	1136	79.205	755.81	3397	4152.81	1.4353E−08
H3_abs1_1S_2M_15C_3V-M	1033	764.298	722.49	3420	4142.49	0
H3_abs1_1S_2M_15C_4V-M	981	217.375	722.49	3420	4142.49	0
H3_abs1_1S_2M_15C_5V-M	950	1913.89	731.93	3397	4128.93	0
H3_abs1_1S_2M_20C_2V-M	1466	500.674	922	3340	4262	0
H3_abs1_1S_2M_20C_3V-M	1333	1950.14	726.54	4886	5612.54	32.9795
H3_abs1_1S_2M_20C_4V-M	1266	2236.32	951.39	3295	4246.39	0
H3_abs1_1S_2M_20C_5V-M	1226	1465.88	949.18	3624	4573.18	18.3888
H3_abs1_1S_2M_25C_2V-M	1728	7941.07	1040.56	3610	4650.56	0
H3_abs1_1S_2M_25C_3V-M	1571	1645.78	1050.8	3876	4926.8	17.1084
H3_abs1_1S_2M_25C_4V-M	1493	1029.3	1102.58	4587	5689.58	32.697
H3_abs1_1S_2M_25C_5V-M	##	##	##	##	##	##
H3_abs2_1S_2M_5C_2V-M	217	1.119	164.04	2004	2168.04	0
H3_abs2_1S_2M_5C_3V-M	198	1.501	165.24	1906	2071.24	0
H3_abs2_1S_2M_5C_4V-M	188	2.56	165.24	1906	2071.24	0
H3_abs2_1S_2M_5C_5V-M	182	4.132	165.24	1906	2071.24	0
H3_abs2_1S_2M_10C_2V-M	749	3.992	509.95	3199	3708.95	0
H3_abs2_1S_2M_10C_3V-M	681	28.638	513.7	3293	3806.7	0
H3_abs2_1S_2M_10C_4V-M	647	16.523	513.7	3293	3806.7	0
H3_abs2_1S_2M_10C_5V-M	627	38.092	513.7	3293	3806.7	0
H3_abs2_1S_2M_15C_2V-M	1086	55.12	725.18	3500	4225.18	0
H3_abs2_1S_2M_15C_3V-M	988	306.551	736.79	3501	4237.79	0
H3_abs2_1S_2M_15C_4V-M	938	576.167	736.79	3501	4237.79	0
H3_abs2_1S_2M_15C_5V-M	909	425.057	760.03	3487	4247.03	0
H3_abs2_1S_2M_20C_2V-M	1434	988.44	946.22	3905	4851.22	0
H3_abs2_1S_2M_20C_3V-M	1304	1782.38	945.06	3877	4822.06	0
H3_abs2_1S_2M_20C_4V-M	1239	8096.3	945.06	3877	4822.06	1.885E−08

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H3_abs2_1S_2M_20C_5V-M	1199	836.024	700.31	5642	6342.31	40.0167
H3_abs2_1S_2M_25C_2V-M	1896	926.238	1479.8	5019	6498.8	27.3589
H3_abs2_1S_2M_25C_3V-M	1724	1000.21	1276.02	5678	6954.02	37.1666
H3_abs2_1S_2M_25C_4V-M	1638	1260.94	1243.19	4802	6045.19	26.1597
H3_abs2_1S_2M_25C_5V-M	1586	1242.36	1311.07	6176	7487.07	40.624
H3_abs3_1S_2M_5C_2V-M	418	1.379	313.02	3343	3656.02	0
H3_abs3_1S_2M_5C_3V-M	380	2.727	338.75	3343	3681.75	0
H3_abs3_1S_2M_5C_4V-M	361	4.571	313.02	3343	3656.02	0
H3_abs3_1S_2M_5C_5V-M	350	7.384	368.38	3474	3842.38	0
H3_abs3_1S_2M_10C_2V-M	630	10.653	510.55	2848	3358.55	0
H3_abs3_1S_2M_10C_3V-M	573	16.7	510.55	2848	3358.55	0
H3_abs3_1S_2M_10C_4V-M	544	31.012	504.79	2907	3411.79	0
H3_abs3_1S_2M_10C_5V-M	527	22.641	510.55	2770	3280.55	0
H3_abs3_1S_2M_15C_2V-M	1162	29.762	1084.92	3437	4521.92	0
H3_abs3_1S_2M_15C_3V-M	1056	745.356	827.52	3704	4531.52	0
H3_abs3_1S_2M_15C_4V-M	1003	417.167	850.63	3709	4559.63	0
H3_abs3_1S_2M_15C_5V-M	972	1334.94	804.56	3847	4651.56	0
H3_abs3_1S_2M_20C_2V-M	1408	373.11	1075.99	3730	4805.99	0
H3_abs3_1S_2M_20C_3V-M	1280	726.323	1075.99	3730	4805.99	0
H3_abs3_1S_2M_20C_4V-M	1216	1232.09	1075.99	3730	4805.99	0
H3_abs3_1S_2M_20C_5V-M	1178	1022.52	1085.11	3922	5007.11	18.9219
H3_abs3_1S_2M_25C_2V-M	1943	3442.55	1337.11	3808	5145.11	6.33908
H3_abs3_1S_2M_25C_3V-M	1766	1126.15	1290.92	4331	5621.92	25.8087
H3_abs3_1S_2M_25C_4V-M	1678	884.148	979.48	6125	7104.48	41.4356
H3_abs3_1S_2M_25C_5V-M	1625	1828.56	1281.64	4895	6176.64	32.7405
H3_abs4_1S_2M_5C_2V-M	246	1.45	166.95	2251	2417.95	0
H3_abs4_1S_2M_5C_3V-M	224	5.266	145.95	2489	2634.95	0
H3_abs4_1S_2M_5C_4V-M	213	9.605	150.05	2462	2612.05	0
H3_abs4_1S_2M_5C_5V-M	206	7.284	150.05	2462	2612.05	0
H3_abs4_1S_2M_10C_2V-M	754	2.959	484.75	3138	3622.75	0
H3_abs4_1S_2M_10C_3V-M	685	10.07	484.75	3138	3622.75	0
H3_abs4_1S_2M_10C_4V-M	651	19.853	484.75	3138	3622.75	0
H3_abs4_1S_2M_10C_5V-M	630	23.167	484.75	3138	3622.75	0
H3_abs4_1S_2M_15C_2V-M	987	78.527	669.84	3112	3781.84	0
H3_abs4_1S_2M_15C_3V-M	898	279.363	644.78	3126	3770.78	0
H3_abs4_1S_2M_15C_4V-M	853	331.328	586.22	3099	3685.22	0
H3_abs4_1S_2M_15C_5V-M	826	748.174	672.7	2977	3649.7	0
H3_abs4_1S_2M_20C_2V-M	1374	82.15	878.95	3656	4534.95	0
H3_abs4_1S_2M_20C_3V-M	1249	510.84	855.73	3805	4660.73	0
H3_abs4_1S_2M_20C_4V-M	1186	545.631	864.43	3805	4669.43	0
H3_abs4_1S_2M_20C_5V-M	1149	352.21	877.81	3656	4533.81	0
H3_abs4_1S_2M_25C_2V-M	1860	10803.9	1209.74	4039	5248.74	1.42904

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H3_abs4_1S_2M_25C_3V-M	1691	6186.96	1219.05	4037	5256.05	0
H3_abs4_1S_2M_25C_4V-M	1607	1011.85	1151.57	4376	5527.57	13.385
H3_abs4_1S_2M_25C_5V-M	##	##	##	##	##	##
H3_abs5_1S_2M_5C_2V-M	322	0.857	336.36	1452	1788.36	0
H3_abs5_1S_2M_5C_3V-M	293	1.281	335.79	1434	1769.79	0
H3_abs5_1S_2M_5C_4V-M	278	2.346	335.79	1434	1769.79	0
H3_abs5_1S_2M_5C_5V-M	269	2.219	335.79	1434	1769.79	0
H3_abs5_1S_2M_10C_2V-M	880	7.181	708.69	2378	3086.69	0
H3_abs5_1S_2M_10C_3V-M	800	17.297	677.76	2315	2992.76	0
H3_abs5_1S_2M_10C_4V-M	760	21.34	677.76	2315	2992.76	0
H3_abs5_1S_2M_10C_5V-M	736	33.288	676.59	2352	3028.59	0
H3_abs5_1S_2M_15C_2V-M	936	266.066	711.78	2972	3683.78	0
H3_abs5_1S_2M_15C_3V-M	851	142.929	751.26	2953	3704.26	0
H3_abs5_1S_2M_15C_4V-M	809	315.911	723.22	2994	3717.22	0
H3_abs5_1S_2M_15C_5V-M	783	722.368	780.98	2926	3706.98	0
H3_abs5_1S_2M_20C_2V-M	1590	94.89	1202.8	3194	4396.8	0
H3_abs5_1S_2M_20C_3V-M	1445	344.688	1143.86	3243	4386.86	0
H3_abs5_1S_2M_20C_4V-M	1373	457.15	1143.86	3243	4386.86	0
H3_abs5_1S_2M_20C_5V-M	1329	1475.83	1224.24	3189	4413.24	0
H3_abs5_1S_2M_25C_2V-M	2153	4651.96	1538.74	3555	5093.74	2.55929
H3_abs5_1S_2M_25C_3V-M	1958	2254.2	1474.66	3964	5438.66	13.6585
H3_abs5_1S_2M_25C_4V-M	1860	1324.45	1634.48	4351	5985.48	27.4198
H3_abs5_1S_2M_25C_5V-M	##	##	##	##	##	##
L3_abs1_1S_2M_5C_2V-M	265	0.891	21.2	2198	2219.2	0
L3_abs1_1S_2M_5C_3V-M	241	1.816	21.2	2189	2210.2	0
L3_abs1_1S_2M_5C_4V-M	229	1.548	15.44	2301	2316.44	0
L3_abs1_1S_2M_5C_5V-M	222	2.611	21.2	2189	2210.2	0
L3_abs1_1S_2M_10C_2V-M	873	11.005	77.39	2543	2620.39	0
L3_abs1_1S_2M_10C_3V-M	794	18.219	61.52	2504	2565.52	0
L3_abs1_1S_2M_10C_4V-M	754	30.608	61.95	2504	2565.95	0
L3_abs1_1S_2M_10C_5V-M	730	41.814	61.95	2504	2565.95	0
L3_abs1_1S_2M_15C_2V-M	1136	20.261	98.14	3191	3289.14	0
L3_abs1_1S_2M_15C_3V-M	1033	45.049	98.14	3191	3289.14	0
L3_abs1_1S_2M_15C_4V-M	981	120.466	98.14	3277	3375.14	0
L3_abs1_1S_2M_15C_5V-M	950	192.015	97.71	3287	3384.71	0
L3_abs1_1S_2M_20C_2V-M	1466	155.404	130.57	3132	3262.57	0
L3_abs1_1S_2M_20C_3V-M	1333	467.822	126.81	3164	3290.81	0
L3_abs1_1S_2M_20C_4V-M	1266	1919.85	126.81	3164	3290.81	0
L3_abs1_1S_2M_20C_5V-M	1226	2270.15	93.05	3230	3323.05	0
L3_abs1_1S_2M_25C_2V-M	1728	381.576	145.15	3351	3496.15	0
L3_abs1_1S_2M_25C_3V-M	1571	1168.64	145.37	3347	3492.37	0
L3_abs1_1S_2M_25C_4V-M	1493	3188.73	107.59	3391	3498.59	0

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L3_abs1_1S_2M_25C_5V-M	1446	923.747	119.92	6247	6366.92	54.9397
L3_abs2_1S_2M_5C_2V-M	217	0.823	16.73	1906	1922.73	0
L3_abs2_1S_2M_5C_3V-M	198	1.753	16.73	2004	2020.73	0
L3_abs2_1S_2M_5C_4V-M	188	2.259	16.73	2004	2020.73	0
L3_abs2_1S_2M_5C_5V-M	182	3.389	16.73	1906	1922.73	0
L3_abs2_1S_2M_10C_2V-M	749	6.416	70.04	3229	3299.04	0
L3_abs2_1S_2M_10C_3V-M	681	16.78	53.21	3200	3253.21	0
L3_abs2_1S_2M_10C_4V-M	647	36.599	52.31	3199	3251.31	0
L3_abs2_1S_2M_10C_5V-M	627	116.126	53.06	3293	3346.06	0
L3_abs2_1S_2M_15C_2V-M	1086	34.393	100.38	3330	3430.38	0
L3_abs2_1S_2M_15C_3V-M	988	176.031	100.14	3383	3483.14	0
L3_abs2_1S_2M_15C_4V-M	938	1209.57	100.14	3383	3483.14	0
L3_abs2_1S_2M_15C_5V-M	909	430.373	74.28	3471	3545.28	0
L3_abs2_1S_2M_20C_2V-M	1434	186.335	130.59	3698	3828.59	0
L3_abs2_1S_2M_20C_3V-M	1304	662.313	130.81	3686	3816.81	0
L3_abs2_1S_2M_20C_4V-M	1239	778.481	130.59	3686	3816.59	0
L3_abs2_1S_2M_20C_5V-M	1199	1955.11	130.59	3686	3816.59	0
L3_abs2_1S_2M_25C_2V-M	1896	219.055	173.4	3628	3801.4	0
L3_abs2_1S_2M_25C_3V-M	1724	2476.12	143.77	4033	4176.77	13.2897
L3_abs2_1S_2M_25C_4V-M	1638	795.592	170.23	4706	4876.23	32.0972
L3_abs2_1S_2M_25C_5V-M	1586	1226.71	140.81	5410	5550.81	37.0594
L3_abs3_1S_2M_5C_2V-M	418	2.099	46.27	3251	3297.27	0
L3_abs3_1S_2M_5C_3V-M	380	5.413	39.04	3470	3509.04	0
L3_abs3_1S_2M_5C_4V-M	361	6.078	36.94	3473	3509.94	0
L3_abs3_1S_2M_5C_5V-M	350	6.588	33.21	3494	3527.21	0
L3_abs3_1S_2M_10C_2V-M	630	7.501	50.55	2770	2820.55	0
L3_abs3_1S_2M_10C_3V-M	573	11.286	59.4	2828	2887.4	0
L3_abs3_1S_2M_10C_4V-M	544	19.006	59.01	2845	2904.01	0
L3_abs3_1S_2M_10C_5V-M	527	22.85	59.01	2845	2904.01	0
L3_abs3_1S_2M_15C_2V-M	1162	17.561	107.45	3437	3544.45	0
L3_abs3_1S_2M_15C_3V-M	1056	420.46	107.48	3649	3756.48	0
L3_abs3_1S_2M_15C_4V-M	1003	806.156	96.74	3669	3765.74	0
L3_abs3_1S_2M_15C_5V-M	972	670.666	108.08	3667	3775.08	0
L3_abs3_1S_2M_20C_2V-M	1408	78.376	134.35	3511	3645.35	0
L3_abs3_1S_2M_20C_3V-M	1280	508.324	135.64	3537	3672.64	0
L3_abs3_1S_2M_20C_4V-M	1216	85.788	134.46	3371	3505.46	0
L3_abs3_1S_2M_20C_5V-M	1178	821.532	133.68	3543	3676.68	0
L3_abs3_1S_2M_25C_2V-M	1943	1947.06	173.38	3638	3811.38	0
L3_abs3_1S_2M_25C_3V-M	1766	3903.35	173.38	3638	3811.38	0
L3_abs3_1S_2M_25C_4V-M	1678	5312.98	137.17	3737	3874.17	11.4085
L3_abs3_1S_2M_25C_5V-M	1625	990.414	157.67	5061	5218.67	41.2094
L3_abs4_1S_2M_5C_2V-M	246	2.969	15.98	2251	2266.98	0

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L3_abs4_1S_2M_5C_3V-M	224	1.667	15.98	2251	2266.98	0
L3_abs4_1S_2M_5C_4V-M	213	6.048	15.14	2462	2477.14	0
L3_abs4_1S_2M_5C_5V-M	206	8.124	15.14	2462	2477.14	0
L3_abs4_1S_2M_10C_2V-M	754	6.376	49.17	3138	3187.17	0
L3_abs4_1S_2M_10C_3V-M	685	30.416	50.42	3357	3407.42	0
L3_abs4_1S_2M_10C_4V-M	651	15.602	49.17	3138	3187.17	0
L3_abs4_1S_2M_10C_5V-M	630	10.208	51.07	3138	3189.07	0
L3_abs4_1S_2M_15C_2V-M	987	76.664	74.47	3077	3151.47	0
L3_abs4_1S_2M_15C_3V-M	898	135.815	85.03	3076	3161.03	0
L3_abs4_1S_2M_15C_4V-M	853	321.218	64.07	3126	3190.07	0
L3_abs4_1S_2M_15C_5V-M	826	151.712	74.73	2942	3016.73	0
L3_abs4_1S_2M_20C_2V-M	1374	39.787	125.38	3538	3663.38	0
L3_abs4_1S_2M_20C_3V-M	1249	81.265	110.44	3579	3689.44	0
L3_abs4_1S_2M_20C_4V-M	1186	233.908	105.22	3589	3694.22	0
L3_abs4_1S_2M_20C_5V-M	1149	392.559	110.18	3752	3862.18	0
L3_abs4_1S_2M_25C_2V-M	1860	210.093	164.61	3826	3990.61	0
L3_abs4_1S_2M_25C_3V-M	1691	868.754	162.61	3830	3992.61	0
L3_abs4_1S_2M_25C_4V-M	1607	2756.42	120.35	4028	4148.35	0
L3_abs4_1S_2M_25C_5V-M	1556	1658.64	135.24	4170	4305.24	7.7053
L3_abs5_1S_2M_5C_2V-M	322	0.787	33.96	1434	1467.96	0
L3_abs5_1S_2M_5C_3V-M	293	1.361	33.96	1452	1485.96	0
L3_abs5_1S_2M_5C_4V-M	278	1.715	33.96	1434	1467.96	0
L3_abs5_1S_2M_5C_5V-M	269	2.324	33.96	1452	1485.96	0
L3_abs5_1S_2M_10C_2V-M	880	2.531	68.61	2352	2420.61	0
L3_abs5_1S_2M_10C_3V-M	800	3.943	68.61	2315	2383.61	0
L3_abs5_1S_2M_10C_4V-M	760	13.862	72.94	2315	2387.94	0
L3_abs5_1S_2M_10C_5V-M	736	30.952	72.94	2315	2387.94	0
L3_abs5_1S_2M_15C_2V-M	936	31.37	74.01	2862	2936.01	0
L3_abs5_1S_2M_15C_3V-M	851	90.196	83.95	2919	3002.95	0
L3_abs5_1S_2M_15C_4V-M	809	285.016	78.81	2871	2949.81	0
L3_abs5_1S_2M_15C_5V-M	783	281.772	79.05	2940	3019.05	0
L3_abs5_1S_2M_20C_2V-M	1590	76.93	155.17	3035	3190.17	0
L3_abs5_1S_2M_20C_3V-M	1445	432.428	154.17	3235	3389.17	0
L3_abs5_1S_2M_20C_4V-M	1373	356.08	132.92	3169	3301.92	0
L3_abs5_1S_2M_20C_5V-M	1329	438.449	132.16	3169	3301.16	0
L3_abs5_1S_2M_25C_2V-M	2153	6018.89	200.63	3432	3632.63	7.54988
L3_abs5_1S_2M_25C_3V-M	1958	2399.58	202.65	3431	3633.65	0
L3_abs5_1S_2M_25C_4V-M	1860	663.448	151.14	3703	3854.14	19.1093
L3_abs5_1S_2M_25C_5V-M	1801	2749.5	199.62	3461	3660.62	11.9093