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LMI-Based Sliding Mode Robust Control for a Class of Multi-Agent Linear Systems

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Abstract

This study deals with the multi-agent linear system, which is a more realistic and accurate discrete model with disturbance terms. Based on linear matrix inequality technology and sliding mode control, we give the forward-feedback control term. Furthermore, sufficient conditions for the closed-loop system are established by Lyapunov stability theory. Results of simulation show that the proposed method is effective.

Subject Areas

Automata

Keywords

Linear Matrix Inequality, Multi-Agent Linear System, Asymptotic Stability

1. Introduction

Variable structure control (VSC) is essentially a special kind of non-linear control, the discontinuity of control is a prominent feature of its nonlinearity. The control strategy of VSC is to force the system to move according to the state trajectory of the predetermined sliding mode according to the current state of the system in the dynamic process. So VSC is also called sliding mode control (SMC).

VSC appeared in the 1950s and has been developed and perfected for more than 50 years. Now it has formed a relatively independent research branch and become a general design method of automatic control system. It is suitable for linear and non-linear systems, continuous and discrete systems, deterministic and uncertain systems, centralized and distributed parameter systems, centralized and decentralized control, etc. And it has been gradually applied in practical engineering, such as motor and power system control, robot control, aircraft control, satellite attitude control and so on [1] [2] [3] [4] [5]. This control method makes the system state slide along the sliding surface by switching the control variables, and makes the system invariable when subject to parameter perturbation and external disturbance. Because of this characteristic, VSC method has attracted wide attention of scholars all over the world [6] [7] [8]. Variable structure systems (VSS) and its main mode of operation SMC are recognized as one of the most efficient tools to deal with uncertain systems due to their robustness and even insensitivity to perturbations [9] [10] [11].

In [2] a terminal SMC strategy with projection operator adaptive law is proposed in a hybrid energy storage system (HESS). The controller can be designed by the constraint condition, combining the projection operator adaptive law.

Linear matrix inequality (LMI) is a powerful design tool in the field of control. Many control theory and analysis and synthesis problems can be simplified to corresponding LMI problems, which can be solved by constructing effective computer algorithms. LMI technology has become an effective tool in control engineering, system identification, structural design and other fields. Using LMI technology to solve some control problems is an important direction in the development of control theory. Recently, the problem of LMI-based sliding mode robust control has received significant attention due to its important applications [12] [13].

A LMI based sliding surface design method for integral sliding-mode control of mismatched uncertain systems has been presented in [14]. Moreover, [15] investigated the chaos control problem for a general class of chaotic systems based on SMC via LMI. The robust stability of uncertain linear neutral systems with time-varying discrete and distributed delays is investigated via a descriptor model transformation and the decomposition technique of the discrete-delay term matrix. In the form of a LMI, they put forward delay-dependent stability criteria in [4]. A SMC method via LMI was presented in [5], which is used for the flutter suppression problem in supersonic airflow.

The optimal sampled-data state feedback control for continuous-time Markov jump linear systems (MJLS) was designed in [16]. Stability and performance robustness against polytopic uncertainty acting on the system parameters including the transition rate matrix are analyzed through differential linear matrix inequalities [16].

Based on LMI technology and SMC, we first consider the multi-agent linear system with disturbance terms to our best knowledge. The main contributions and primary distinctions of this paper with other works can be given as follows.

1) A more realistic and accurate discrete model with disturbance terms is proposed which is relevant for many practical sampled data systems;

2) New forms for forward-feedback control term and sliding mode robust term are proposed in this paper;

3) Sufficient conditions for the closed-loop system are established using Lyapunov stability theory.

The organization of this paper is as follows: Firstly, notations and preliminaries are introduced in Section 2, which are useful throughout this paper. In Section 3, we give the design of controllers and stability analysis of a multi-agent system is carried out. Then in Section 4, a simulation is presented to demonstrate the effectiveness of proposed technique. Finally, the conclusion is made.

2. Systems Definition

Consider the multi-agent linear system:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + d_i,\ i = 1,\cdots,N \tag{1}$$

where $x_i(t) \in R^n$, $u_i \in R^n$ are the state and control input of the "*i*th" agent, $d_i \in R^{n\times 1}$ is disturbance term, *A* and $B \in R^{n\times n}$ are constant matrices and the initial state is defined by $x_i(0)$. The control targets are $x_i \to x_i^{\mathrm{Y}}$, for $i = 1,\cdots,N$. x_i^{Y} is an ideal instruction.

Assumption 1. This study deals with the information exchange among agents is modeled by an undirected graph. We assume that the communication topology is connected.

3. The Design of Controllers

We define the tracking error as $z_i(t) = x_i(t) - x_i^{\mathrm{Y}}(t)$, then

$$\dot{z}_i(t) = \dot{x}_i(t) - \dot{x}_i^{\mathrm{Y}}(t) = Ax_i(t) + Bu_i(t) + d_i - \dot{x}_i^{\mathrm{Y}}(t). \tag{2}$$

Design the tracking error as a sliding mode function, we give the design of control law as

$$u_i(t) = F_i(t)x_i(t) + u_i^{\mathrm{Y}}(t) + u_i^s(t) \tag{3}$$

where $F_i(t)$ is a state feedback gain matrix which can be obtained by designing LMI. Take forward-feedback control item

$$u_i^{\mathrm{Y}}(t) = -F_i(t)x_i^{\mathrm{Y}}(t) - B^{-1}(t)A_i(t)x_i^{\mathrm{Y}}(t) + B^{-1}(t)\dot{x}_i^{\mathrm{Y}}(t), \tag{4}$$

sliding mode robust term $u_i^s = -B^{-1}\left[\eta_i sgn(z_i)\right]$, $\eta_i \in R^{n\times 1}$, $d_i^j - \eta_i^j < 0$ or $d_i^j + \eta_i^j > 0$, and $\eta_i sgn(z_i) = \left[\eta_i^1 sgnz_i^1,\cdots,\eta_i^n sgnz_i^n\right]^{\mathrm{T}}$.

Hence,

$$\begin{aligned} u_i(t) &= F_i(t)x_i(t) - F_i(t)x_i^{\mathrm{Y}}(t) - B^{-1}Ax_i^{\mathrm{Y}}(t) + B^{-1}\dot{x}_i^{\mathrm{Y}}(t) - B^{-1}\left[\eta_i sgn(z_i)\right] \\ &= F_i(t)z_i(t) - B^{-1}Ax_i^{\mathrm{Y}}(t) + B^{-1}\dot{x}_i^{\mathrm{Y}}(t) - B^{-1}\left[\eta_i sgn(z_i)\right] \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{z}_i(t) &= Ax_i(t) + B\left(F_i(t)z_i(t) - B^{-1}Ax_i^{\mathrm{Y}}(t) + B^{-1}\dot{x}_i^{\mathrm{Y}}(t) - B^{-1}\left[\eta_i sgn(z_i)\right]\right) \\ &\quad + d_i - \dot{x}_i^{\mathrm{Y}}(t) \\ &= Ax_i(t) + BF_i(t)z_i(t) - Ax_i^{\mathrm{Y}}(t) + \dot{x}_i^{\mathrm{Y}}(t) - \left[\eta_i sgn(z_i)\right] + d_i - \dot{x}_i^{\mathrm{Y}}(t) \\ &= Az_i(t) + BF_i(t)z_i(t) - \left[\eta_i sgn(z_i)\right] + d_i. \end{aligned} \tag{6}$$

The following theorem holds.

Theorem 1. Assume that $A^{\mathrm{T}}P_i + M_i^{\mathrm{T}} + P_iA + M_i < 0$ is true for any $i = 1,\cdots,N$, where $F_i = (P_iB)^{-1}M_i$. Then the closed-loop system consisting of

(1) and (3) is asymptotic stability.

PROOF. Choose Lyapunov function $V_i = z_i^{\mathrm{T}} P_i z_i$, where $P_i = diag\left\{p_i^j\right\}$ is a diagonal matrix and $p_i^j > 0$.

Hence,

$$
\begin{aligned}
\dot{V}_i &= \left(z_i^{\mathrm{T}} P_i\right)' z_i + z_i^{\mathrm{T}} P_i \dot{z}_i \\
&= \left(Az_i + BF_i z_i - \eta_i sgn\left(z_i\right) + d_i\right)^{\mathrm{T}} P_i z_i + z_i^{\mathrm{T}} P_i \left(Az_i + BF_i z_i - \eta_i sgn\left(z_i\right) + d_i\right) \\
&= z_i^{\mathrm{T}} A^{\mathrm{T}} P_i z_i + z_i^{\mathrm{T}} F_i^{\mathrm{T}} B^{\mathrm{T}} P_i z_i + \left(-\eta sgn\left(z_i\right) + d_i\right)^{\mathrm{T}} P z_i + z_i^{\mathrm{T}} P_i A z_i \\
&\quad + Z_i^{\mathrm{T}} P_i B F_i Z_i + z_i^{\mathrm{T}} P_i \left(-\eta_i sgn\left(z_i\right) + d_i\right) \\
&< z_i^{\mathrm{T}} \left(A^{\mathrm{T}} P_i + F_i^{\mathrm{T}} B^{\mathrm{T}} P_i + P_i A + P_i B F_i\right) z_i = z_i^{\mathrm{T}} \Omega_i z_i
\end{aligned} \tag{7}
$$

where $\Omega_i = A^{\mathrm{T}} P_i + F_i^{\mathrm{T}} B^{\mathrm{T}} P_i + P_i A + P_i B F_i$, $\left(-\eta sgn\left(z_i\right) + d_i\right)^{\mathrm{T}} P z_i < 0$ and $z_i^{\mathrm{T}} P_i \left(-\eta_i sgn\left(z_i\right) + d_i\right) < 0$.

Remark 1. In order to guarantee that $\dot{V}_i < 0$, $\Omega_i < 0$ is a prerequisite. Therefore,

$$A^{\mathrm{T}} P_i + F_i^{\mathrm{T}} B^{\mathrm{T}} P_i + P_i A + P_i B F_i < 0. \tag{8}$$

In LMI (8), F_i and P_i are both uncertain. We make linearization of (3), let, $M_i = P_i B F_i$ then LMI (8) becomes as

$$A^{\mathrm{T}} P_i + M_i^{\mathrm{T}} + P_i A + M_i < 0. \tag{9}$$

Making use of LMI, we can get M_i and P_i, thus. $F_i = \left(P_i B\right)^{-1} M_i$.

4. Simulation

We focus on the multi-agent linear system in this paper, without loss of generality, we assume that the system has three agents and B is the unit matrix.

According to (1),

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} -2.4 & 9.2 & 0 \\ 1 & -1 & 1 \\ 0 & -16.1 & 3 \end{bmatrix}, \tag{10}$$

the ideal matrix is $\left[\sin(t) \ \cos(t) \ \sin(t)\right]$, the interference matrix is

$$d_1 = \begin{bmatrix} 50\sin(t) \\ 50\cos(t) \\ 50\sin(t) \end{bmatrix}, d_2 = \begin{bmatrix} 40\sin(t) \\ 40\cos(t) \\ 40\sin(t) \end{bmatrix}, d_3 = \begin{bmatrix} 50\sin(t) \\ 50\cos(t) \\ 50\sin(t) \end{bmatrix}, \tag{11}$$

corresponding to the ideal matrix. Solving LMI (9), let

$$P_1 = \begin{bmatrix} 10000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}, \tag{12}$$

$$P_2 = \begin{bmatrix} 100000 & 0 & 0 \\ 0 & 100000 & 0 \\ 0 & 0 & 100000 \end{bmatrix}, \tag{13}$$

$$P_3 = \begin{bmatrix} 5000000 & 0 & 0 \\ 0 & 5000000 & 0 \\ 0 & 0 & 5000000 \end{bmatrix}, \tag{14}$$

we can obtain that

$$F_1 = \begin{bmatrix} -37418.0101 & -5.1000 & 0 \\ -5.1000 & -37419.4101 & 7.5500 \\ 0 & 7.5500 & -37423.4101 \end{bmatrix}, \tag{15}$$

$$F_2 = \begin{bmatrix} -3739.6906 & -5.1000 & 0 \\ -5.1000 & -3741.0906 & 7.5500 \\ 0 & 7.5500 & -3745.0906 \end{bmatrix}, \tag{16}$$

$$F_3 = \begin{bmatrix} -72.4035 & -5.1000 & 0 \\ -5.1000 & -73.8035 & 7.5500 \\ 0 & 7.5500 & -77.8035 \end{bmatrix}, \tag{17}$$

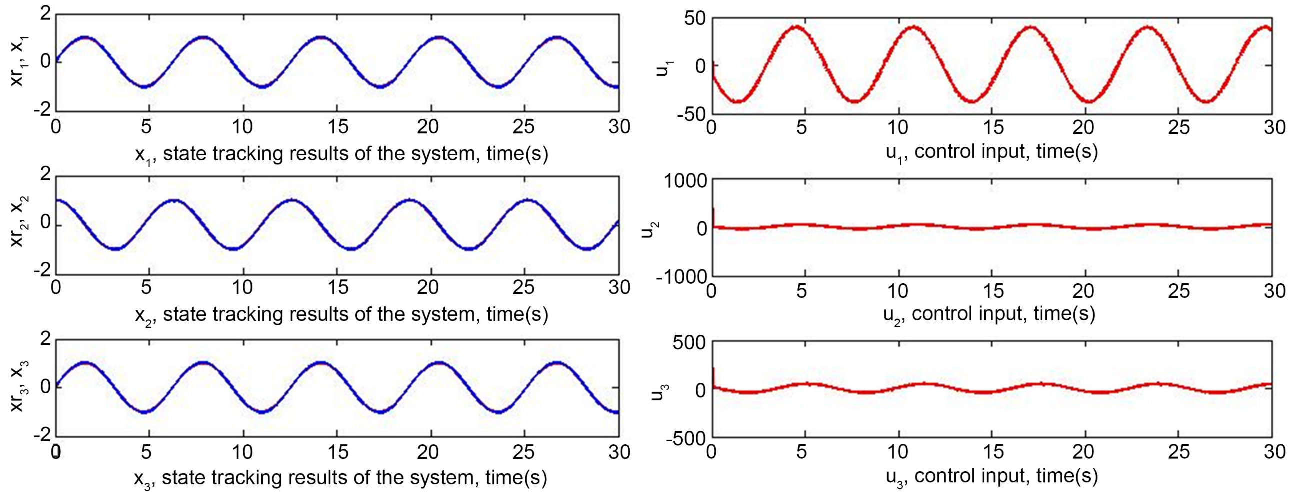

Figure 1. State tracking and control input for agent 1.

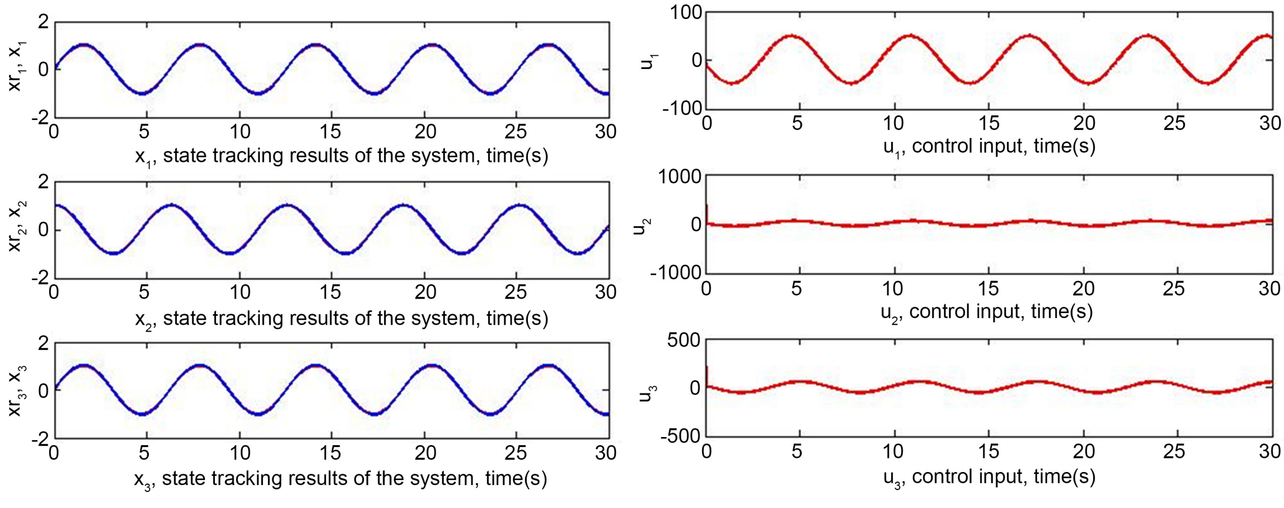

Figure 2. State tracking and control input for agent 2.

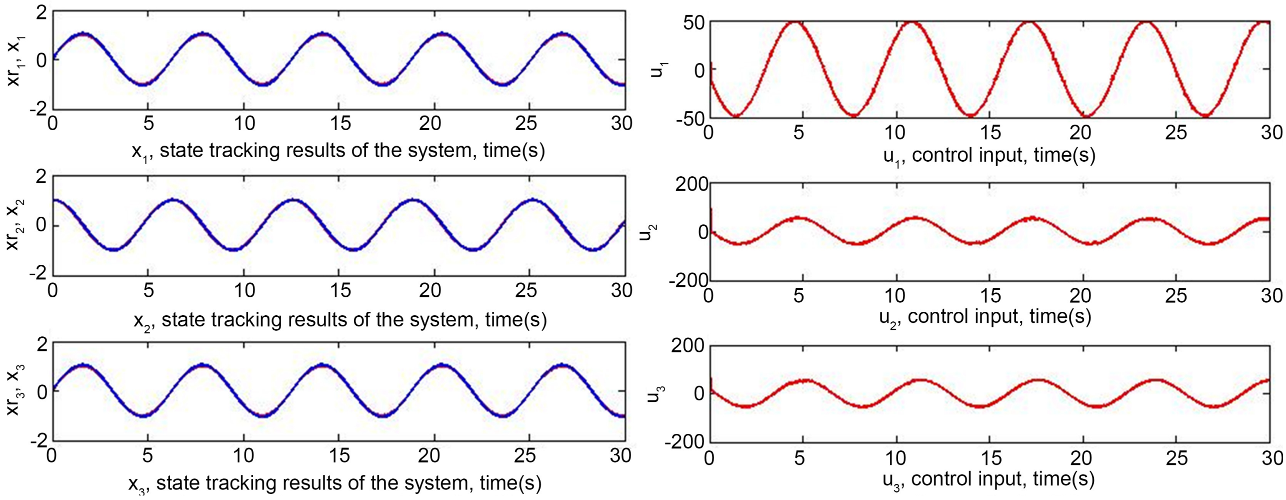

Figure 3. State tracking and control input for agent 3.

respectively. Due to (3), let

$$\eta_1 = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix},\ \eta_2 = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix},\ \eta_3 = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}, \tag{18}$$

replacing switching function with saturation function and choosing the boundary layer as $\Delta = 0.05$. We give simulations are in the following (**Figures 1-3**).

It is clear that from three figures the closed-loop system with disturbance is asymptotic stability, hence, the proposed method is effective.

5. Conclusion

The multi-agent linear system was studied in this paper. Based on linear matrix inequality technology and sliding mode control, the forward-feedback control term was given. Sufficient conditions for the closed-loop system were established by Lyapunov stability theory. Simulations show that the proposed method was effective.

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Conflicts of Interest

The authors declare no conflicts of interest.

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