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Polytropic Bulk Viscous Cosmological Model with Variable G and Λ

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Abstract

We consider a Bianchi type-I Polytropic bulk viscous fluid cosmological model with variable G and Λ. To get a deterministic model, it is assumed that $\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{k_1}{t^n}$ and $\frac{\dot{C}}{C}=\frac{k_2}{t^n}$, $\eta=\eta_o\rho^\alpha$, $\bar{p}=k\rho^\gamma$, where $\bar{p}$ is the pressure, ρ is the energy density, η is the coefficient of bulk viscosity, α, k, γ and η_o are constants, H is Hubble constant, $\frac{k_1}{2}=k_2$ where $k_1 > 0$, $k_2 > 0$. The solution obtained lead to inflationary phase and the results obtained match with the observations [31] [32]. The case $n = 1$ for $\alpha = 1$ is also discussed, relating the results with the observations.

Keywords

Bianchi Type-I Space Time, Bulk Viscosity, Shear Scalar, Expansion Scalar

Subject Areas: **Special Theory of Relativity, String Theory**

1. Introduction

Cosmology is the study of largest-scale structures and dynamics of our universe and deals with subjects regarding its origin and evolution. Cosmology involves itself with studying the motions of the celestial bodies. The twentieth century advances made it possible to speculate about the origins of the universe and allowed scientists to establish the Big Bang as the leading cosmological model, which most cosmologists now accept as the basis for their theories and observations. Bianchi type-I space time is the simplest generalizations of Friedmann-Robertson-Walker (FRW) flat models. There is significant observational evidence that the expansion of the

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universe is undergoing late time acceleration [1]-[9].

The cosmological constant problem is very interesting. Recent observations indicate that $\Lambda \sim 10^{-55}$ cm^{-2} while the particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. In modern cosmological theories, the cosmological constant remains a focal point of interest. The bulk viscosity associated with grand unified theory phase transition can lead to the inflationary universe scenario. A wide range of observations now suggests compellingly that the universe possesses a non-zero cosmological constant [10]. H. Zhang *et al.* [11] studied the Friedmann cosmology on codimension 2 brane with time dependent tension. In this model the effective cosmological constant is free of the absolute value of the brane tension. Tiwari and Sonia [12] investigated the non-existence of shear in Bianchi type-III string bulk viscous cosmological model in general relativity. Wang [13]-[16] discussed the solutions of Bianchi type I-IX cosmological models for string clouds. Also several aspects of viscous fluid cosmological models in early universe have been extensively investigated by many authors. Tiwari and Sonia investigated the Bianchi type-I string cosmological model with bulk viscosity and time dependent Λ term [17]. Zeyauddin and Saha [18] investigated the Bianchi type-V bulk viscous cosmological models with particle creation in general relativity. Beesham *et al.* [19] investigated the Bianchi type-I anisotropic bulk viscous fluid cosmological model with variable G and Λ, where bulk viscosity $\eta = \eta_o\rho^n$, $\eta_o \geq 0$, ρ is the energy density and deriving Friedmann type equation $3H^2 = G\rho + \sigma^2 + \Lambda$, with σ being the shear and H the Hubble constant. Tiwari *et al.* [20] investigated the LRS Bianchi type-II cosmological model with a decaying lambda term and Tiwari [21] investigated the Bianchi type-I cosmological models with perfect fluid in general relativity.

In this letter, we consider the Bianchi type-I bulk viscous polytropic cosmological model using Bianchi type-I metric with variable G and Λ. To obtain the solution of the field equations we assume that $\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{k_1}{t^n}$ and $\frac{\dot{C}}{C} = \frac{k_2}{t^n}$, $\eta = \eta_o\rho^\alpha$, $\bar{p} = k\rho^\gamma$ where $\bar{p}$ is the pressure, ρ is the energy density, η is the coefficient of bulk viscosity, α, k, γ and η_o are constants, H is Hubble constant.

2. Field Equations and Their Solutions

We consider the Bianchi type-I space time

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B and C are function of t only.

The energy momentum tensor for viscous distribution is given by

$$T_i^j = (\rho + p)v_iv^j - pg_i^j \tag{2}$$

$$\bar{p} = p - 3\eta H \tag{3}$$

where p is the equilibrium pressure, η the coefficient of bulk viscosity and ρ is the energy density with $v_iv^i = -1$.

The Einstein field equations are given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi GT_i^j + \Lambda g_i^j \tag{4}$$

where G is the gravitational constant and Λ the cosmological constant, which are time dependent.

The expressions for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\right)$$

Einstein's field Equation (4) for the metric (1) leads to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G\bar{p} + \Lambda \tag{5}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi G\overline{p}+\Lambda \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-8\pi G\overline{p}+\Lambda \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\rho+\Lambda \tag{8}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \tag{9}$$

An additional equation for time changes of G and Λ is obtained by the divergence of Einstein tensor *i.e.*

$$\left(R_i^j-\frac{1}{2}Rg_i^j\right)_{;j}=0$$

which leads to

$$\left(8\pi GT_i^j-\Lambda g_i^j\right)_{;j}=0$$

yielding

$$8\pi\dot{G}\rho+\dot{\Lambda}+8\pi G\left\{\dot{\rho}+\left(\rho+\overline{p}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right\}=0 \tag{10}$$

The conservation of energy Equation (10) after using Equation (4) breaks into two equations:

$$\dot{\rho}+\left(\rho+\overline{p}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{11}$$

$$\dot{\Lambda}+8\pi\dot{G}\rho=8\pi G\eta\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)^2=0 \tag{12}$$

3. Solution of Field Equations

Now, we take

$$\eta=\eta_o\rho^\alpha \tag{13}$$

where $\eta_o>0$, α = constant.

To obtain the complete solution, we assume the polytropic relation

$$\overline{p}=k\rho^\gamma \tag{14}$$

where k and γ are constants, density ρ is a function of pressure p. [For simplicity we set the constant γ to be unit].

We assume the solution of the system of the equations in the form

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{k_1}{t^n},\quad \frac{\dot{C}}{C}=\frac{k_2}{t^n} \tag{15}$$

k_1 and k_2 are constants [18].

Integrating Equation (15), we get

$$A=B=a\exp\left(\frac{k_1t^{1-n}}{1-n}\right),\quad C=b\exp\left(\frac{k_2t^{1-n}}{1-n}\right) \tag{16}$$

where "a" and "b" are constants of integration.

Using Equation (14), (15) in Equation (11), we get

$$\frac{\dot{\rho}}{\rho} = -\frac{(1+k)(2k_1+k_2)}{t^n} \tag{17}$$

Integrating, we get

$$\rho = k_3 \exp\left\{-(1+k)(2k_1+k_2)\frac{t^{1-n}}{1-n}\right\} \tag{18}$$

Differentiating Equation (18), we get

$$\dot{\rho} = -k_3\frac{(k+1)(2k_1+k_2)}{t^n}\exp\left\{-\frac{(k+1)(2k_1+k_2)}{1-n}t^{1-n}\right\} \tag{19}$$

Using Equations (8), (15), (16) and differentiating, we get

$$-\frac{2n\left(k_1^2+2k_1k_2\right)}{t^{1+2n}} = 8\pi G\dot{\rho} + 8\pi G\eta\left(\frac{2k_1+k_2}{t^n}\right)^2 \tag{20}$$

Again substituting Equations (13) and (19) in Equation (20), we get

$$\begin{aligned} G = &-\frac{n\left(k_1^2+2k_1k_2\right)}{4\pi k_3 t^{2n+1}}\exp\left\{\frac{(k+1)(2k_1+k_2)}{1-n}t^{1-n}\right\} \\ &\cdot\left[-\frac{(k+1)(2k_1+k_2)}{t^n}+\eta_o k_3^{\alpha-1}\exp\left\{-\frac{(\alpha-1)(1+k)(2k_1+k_2)}{1-n}t^{1-n}\right\}\right]^{-1} \end{aligned} \tag{21}$$

$$\begin{aligned} \Lambda = &\frac{k_1^2+2k_1k_2}{t^{2n}} + 2n\frac{\left(k_1^2+2k_1k_2\right)}{t^{2n+1}} \\ &\cdot\left[-\frac{(k+1)(2k_1+k_2)}{t^n}+\eta_o k_3^{\alpha-1}\exp\left\{-\frac{(\alpha-1)(k+1)(2k_1+k_2)}{1-n}t^{1-n}\right\}\right]^{-1} \end{aligned} \tag{22}$$

From Equations (13) and (18), we get

$$\eta = \eta_o k_3^{\alpha}\exp\left\{-\alpha(1+k)(2k_1+k_2)\frac{t^{1-n}}{1-n}\right\} \tag{23}$$

where $n \neq 1$.

Thus the metric (1) reduces to

$$ds^2 = -dt^2 + a^2\exp\left(\frac{2k_1t^{1-n}}{1-n}\right)\left(dx^2+dy^2\right) + b^2\exp\left(\frac{2k_2t^{1-n}}{1-n}\right)dz^2 \tag{24}$$

The density ρ, Hubble constant H_i, coefficient of bulk viscosity η, for the model (24)

$$\rho = k_3\exp\left\{\frac{(1+k)(2k_1+k_2)}{(n-1)t^{n-1}}\right\} \tag{25}$$

$$H_1 = H_2 = \frac{k_1}{t^n},\ H_3 = \frac{k_2}{t^n} \tag{26}$$

where $n > 1$.

$$\eta = \eta_o k_3^{\alpha}\exp\left\{-\alpha(1+k)(2k_1+k_2)\frac{t^{1-n}}{1-n}\right\}$$

$$\overline{p} = kk_3\exp\left\{\frac{(1+k)(2k_1+k_2)}{(n-1)t^{n-1}}\right\}$$

4. Special Cases

When $\frac{k_1}{2}=k_2$ and $\alpha=1$, the Gravitational constant G, Cosmological constant Λ, Hubble constant H_i, the shear scalar σ and the expansion scalar θ for the model (24) are given by

$$G=-\frac{2nk_2^2}{\pi k_3 t^{2n+1}}\exp\left\{\frac{5k_2(k+1)}{1-n}t^{1-n}\right\}\left\{\eta_o-\frac{5k_2(k+1)}{t^n}\right\}^{-1}$$

$$\Lambda=\frac{8k_2^2}{t^{2n}}+\frac{16nk_2^2}{t^{2n+1}}\left\{-\frac{5k_2(k+1)}{t^n}+\eta_o\right\}^{-1}$$

$$H_1=H_2=\frac{2k_2}{t^n},\ H_3=\frac{k_2}{t^n}$$

$$\theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=\frac{2k_1+k_2}{t^n}$$

$$\frac{\dot{G}}{G}\propto H,\ \Lambda\propto\frac{1}{t^2},\ H\propto\frac{1}{t},\ G>0,\ \rho>0.$$

The model (24) starts with a big-bang at $t = 0$ when $n > 0$ and the expansion scalar θ decreases as time t increases. However, when $n < 0$, the expansion in the model increases as the time increases. Also at $t\to 0$, Hubble parameter H and shear scalar σ tends to infinity and when $t\to\infty$, Hubble parameter H_i and shear scalar σ tends to zero. Since $\lim_{t\to\infty}\frac{\sigma}{\theta}\neq 0$, the model does not approach isotropy for large value of t. As the time t increases, the spacial volume V decreases. The rate of expansion slows down with the increase in time. Since $\eta=\eta_o\rho^\alpha$ and $\alpha > 1$, the model leads to the inflationary phases [22]-[25].

For $n = 1$, Equation (15) becomes,

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{k_1}{t},\ \frac{\dot{C}}{C}=\frac{k_2}{t}$$

Integrating, we get

$$A=B=k_4 t^{k_1},\ C=k_5 t^{k_2}$$

Therefore, metric (1) reduces to

$$ds^2=-dt^2+k_4^2 t^{2k_1}\left(dx^2+dy^2\right)+k_5^2 t^{2k_2}dz^2$$

Therefore, the energy density ρ, the Gravitational constant G, Cosmological constant Λ, coefficient of bulk viscosity η, Hubble constant H_i, the shear scalar σ, the expansion scalar θ and spacial volume V are given by

$$\rho=N\exp\left\{\frac{(1+k)(2k_1+k_2)^2}{t}\right\}$$

$$\overline{p}=kN\exp\left\{\frac{(1+k)(2k_1+k_2)^2}{t}\right\}$$

where N is the constant of integration.

$$G=\frac{2k_1(k_1+2k_2)t}{8\pi(1+k)(2k_1+k_2)}\left[\exp\left\{\frac{(1+k)(2k_1+k_2)^2}{t}\right\}\right]^{-1}$$

$$\Lambda=k_1(k_1+2k_2)t\left\{\frac{1}{t^3}-\frac{2N}{(1+k)(2k_1+k_2)}\right\}$$

$$\eta = \eta_o N^{\alpha} \exp\left\{\frac{\alpha(1+k)(2k_1+k_2)^2}{t}\right\}$$

$$H_1 = H_2 = \frac{k_1}{t}, H_3 = \frac{k_2}{t}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{k_1-k_2}{t}\right) \tag{27}$$

$$\theta = \frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} = \frac{2k_1+k_2}{t}$$

$$V = k_4 k_5 t^{(2k_1+k_2)}$$

For $n = 1$, when $\frac{k_1}{2} = k_2$, $k_1 > 0$, $k_2 > 0$ and $\alpha = 1$ then the energy density ρ, the Gravitational constant G, Cosmological constant Λ, coefficient of bulk viscosity η, Hubble constant H_i, the expansion scalar θ and spacial volume V are given by

$$\rho = N \exp\left\{\frac{25k_2^2(1+k)}{t}\right\}$$

$$G = \frac{2k_2 t}{5\pi(1+k)}\left[\exp\left\{25k_2^2(1+k)\frac{1}{t}\right\}\right]^{-1}$$

$$\Lambda = 8k_2^2 t\left\{\frac{1}{t^3} - \frac{2N}{5k_2(1+k)}\right\}$$

$$\eta = \eta_o N \exp\left\{\frac{25k_2^2(1+k)}{t}\right\}$$

$$H_1 = H_2 = \frac{2k_2}{t}, H_3 = \frac{k_2}{t}$$

$$\theta = \frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} = \frac{5k_2}{t}$$

$$V = k_4 k_5 t^{5k_2}$$

For $n = 1$ and $k_2 = 1$, we have

$$H \propto \frac{1}{t}, \frac{\dot{G}}{G} \propto H, \Lambda \propto \frac{1}{t^2}, G > 0, \rho > 0$$

which are considered to be fundamental and match with the observations.

5. Conclusion

In this paper, we have obtained solutions for the Einstein's general relativity equation in Bianchi type-I space time with polytropic bulk viscous fluid. Here it is observed that when time $t \to 0$, then the spacial volume $V \to \infty$. This result shows that the universe starts expanding with zero volume and blows up at infinite past and future. The role of bulk viscosity in the cosmic evolution, especially as its early stages seems to be significant [26]. Also when $t \to 0$, the energy density ρ, cosmological constant Λ, expansion scalar θ, shear scalar σ and coefficient of bulk viscosity η tend to infinite and when $t \to \infty$, ρ, Λ, θ, η and σ tend to zero [22]-[26]. Since $\frac{\sigma}{\theta} = \text{constant}$, therefore model does not approach isotropy for large value of t. The spacial volume V increases as time t increases if $k_2 > 0$ [27]-[32]. Since $\eta = \eta_o \rho^\alpha$, $\alpha > 0$, the model leads to inflationary solution.

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