Open Access Library Journal
2026, Volume 13, e15381
ISSN Online: 2333-9721
ISSN Print: 2333-9705

The Impact of Artificial Intelligence on Scientific Research in Mathematics

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How to cite this paper: Refai, W. (2026) The Impact of Artificial Intelligence on Scientific Research in Mathematics. *Open Access Library Journal*, **13**: e15381.
https://doi.org/10.4236/oalib.1115381

Received: April 19, 2026
Accepted: May 24, 2026
Published: May 27, 2026

Abstract

Artificial Intelligence (AI) is rapidly reshaping scientific research across a wide range of disciplines, including mathematics. Traditionally regarded as a domain driven exclusively by human reasoning, grounded in rigorous logic and creative abstraction, mathematics is now undergoing a significant transformation through the integration of AI-based tools. This paper examines the multifaceted impact of AI on mathematical research, with particular emphasis on automated theorem proving, conjecture generation, symbolic computation, and human-AI collaborative workflows. We analyze both the opportunities and the limitations associated with these technologies, highlighting their capacity to accelerate discovery, improve research efficiency, and stimulate interdisciplinary innovation. In addition to a literature-based analysis, this paper presents an original survey of $N = 1032$ researchers in mathematics. The results indicate high adoption of AI tools ($M = 4.32$) and significant perceived productivity gains ($M = 4.18$), alongside persistent concerns regarding reliability ($M = 2.21$). These findings support a hybrid model in which AI enhances research efficiency while remaining dependent on human validation.

Subject Areas

Artificial Intelligence, Interdisciplinary Research

Keywords

Artificial Intelligence, Human-AI Collaboration, Research Productivity, Mathematical Modeling

1. Introduction

Mathematics has historically been regarded as a paradigmatic discipline of rigorous human reasoning, characterized by abstraction, deductive logic, and conceptual creativity. For decades, mathematical research has relied primarily on human intuition, supported by formal symbolic reasoning, while computational tools have played only a secondary role, largely limited to numerical experimentation and verification.

However, the rapid development of Artificial Intelligence (AI), particularly in machine learning and deep neural networks, has profoundly transformed this paradigm. Modern AI systems are no longer confined to numerical computation; they are increasingly capable of performing pattern recognition, symbolic reasoning, and even partially automating proof processes [1] [2].

Recent advances demonstrate that AI can contribute to multiple stages of mathematical research, including conjecture generation, automated theorem proving, and large-scale symbolic computation [3]-[5]. This evolution has led to the emergence of the interdisciplinary field known as Artificial Intelligence for Mathematics (AI4Math), which seeks to integrate computational intelligence with formal mathematical reasoning.

In complex domains such as spectral theory and dynamical systems—particularly in the study of quasi-periodic Schrödinger operators—AI-assisted methods have demonstrated significant potential in numerical exploration and structural analysis [6]. These approaches enable the investigation of high-dimensional parameter spaces and the approximation of spectral quantities that are analytically intractable.

Moreover, AI-based tools are increasingly integrated into formal reasoning environments such as proof assistants, where they assist in constructing proofs, suggesting intermediate lemmas, and verifying logical consistency [7]. Although these systems do not replace human reasoning, they significantly enhance productivity and help reduce the complexity of large-scale mathematical arguments.

Recent advances in large-scale language models have significantly improved performance on complex reasoning tasks, including mathematics. In particular, the GPT-4 model developed by OpenAI demonstrates strong capabilities in solving advanced mathematical problems and generating formal proofs [8].

The objective of this paper is to analyze the impact of AI on mathematical research, with a particular focus on its role in symbolic computation, optimization, and mathematical discovery. We examine both its transformative potential and its limitations, especially in terms of rigor, interpretability, and conceptual understanding.

While the scope of this paper is broad, the recurring references to quasi-periodic Schrödinger operators and KAM theory serve as a representative case study. These domains are chosen due to their high analytical complexity, making them particularly suitable for illustrating the strengths and limitations of AI-assisted mathematical research.

In addition to the theoretical analysis presented in Sections 1-5, this paper includes an empirical study based on a large-scale survey of researchers. This dual approach allows us to connect conceptual insights with quantitative evidence regarding the adoption, impact, and limitations of AI in mathematical research.

2. AI as a Tool for Mathematical Discovery

2.1. Automated Theorem Proving

One of the most significant and foundational contributions of Artificial Intelligence to modern mathematical research lies in automated theorem proving (ATP) and formal verification systems. These systems are designed to assist, or in some cases fully automate, the process of constructing rigorous mathematical proofs within formally defined logical frameworks. Unlike traditional computational tools, which primarily focus on numerical calculations or symbolic simplifications, ATP systems operate at the level of logical inference, where each step of a proof is verified for correctness according to strict formal rules.

Early approaches to automated reasoning relied heavily on rule-based symbolic manipulation and exhaustive search procedures. However, these methods were often limited by combinatorial explosion and exhibited poor scalability when applied to complex mathematical problems. The introduction of machine learning techniques—particularly neural-guided proof search and reinforcement learning—has significantly improved the efficiency of exploring large proof spaces. Modern systems are now capable of prioritizing relevant lemmas, guiding search trajectories, and reducing the effective complexity of formal derivations [1] [2].

A major development in this area is the integration of AI with interactive theorem provers such as Lean, Coq, and Isabelle/HOL. These platforms provide formal languages in which mathematical statements and proofs can be encoded with complete logical precision. AI-based tools are now able to assist in constructing partial proof strategies, suggesting intermediate steps, and automatically filling gaps in formal arguments. This has led to the emergence of a new paradigm in which proof construction becomes a collaborative process between humans and AI systems, rather than being purely manual or fully automated.

Recent breakthroughs in large-scale machine learning models have demonstrated the ability to solve problems at the level of mathematical competitions, including International Mathematical Olympiad (IMO)-style questions. These systems combine symbolic reasoning with pattern recognition over extensive datasets of existing proofs, enabling them to generalize previously learned strategies to new problem instances. In several cases, AI systems have successfully reconstructed nontrivial proofs by identifying underlying structural similarities with previously solved problems [3].

From a theoretical perspective, automated theorem proving can be understood as a high-dimensional search problem over the space of formal derivations. Within this framework, AI acts as a heuristic optimizer that navigates the combinatorial structure of proof trees. Techniques such as Monte Carlo tree search, reinforcement learning, and transformer-based sequence modeling have proven particularly effective in guiding this exploration. These approaches reduce reliance on brute-force enumeration and instead exploit learned statistical patterns in mathematical reasoning.

The evaluation of AI systems on mathematical reasoning tasks has been further standardized through benchmark datasets such as the MATH dataset introduced by Hendrycks, D. *et al.* [9], which includes a wide range of competition-level problems. Using such benchmarks, recent studies have shown that large language models can solve increasingly complex quantitative problems with high accuracy [10].

In the context of modern mathematical physics and spectral theory, ATP systems hold significant potential for verifying long and technically intricate arguments, such as those encountered in the study of quasi-periodic Schrödinger operators, reducibility problems in KAM theory, and localization phenomena. For example, formal verification could, in principle, be applied to proofs involving iterative schemes, norm estimates, and inductive constructions, where precise control of error propagation is essential. Although such applications remain largely exploratory, they point toward a future in which rigorous computer-assisted verification becomes an integral component of mathematical research.

Despite these advances, current ATP systems still face important limitations. In particular, they often lack deep semantic understanding of mathematical structures and may struggle to generalize beyond distributions similar to those encountered during training. Furthermore, the interpretability of machine-generated proof steps remains a significant challenge, especially in advanced domains where conceptual insight is as important as formal correctness. Consequently, although AI has substantially expanded the scope of automated reasoning, it has not eliminated the need for human intuition and high-level mathematical insight.

Overall, automated theorem proving represents a fundamental shift in mathematical practice, transforming proof verification from a purely human activity into a hybrid computational-logical process. Its continued development is expected to play a central role in the future of formalized mathematics and AI-assisted discovery.

2.2. Conjecture Generation and Pattern Recognition

Another major contribution of Artificial Intelligence to mathematical research lies in conjecture generation and pattern recognition, representing a shift from purely deductive reasoning toward data-driven mathematical exploration. In contrast to classical mathematical practice—where conjectures are typically formulated through human intuition and heuristic insight—AI systems are capable of analyzing large-scale datasets of mathematical objects, numerical experiments, or symbolic structures in order to detect non-obvious regularities that may suggest new theoretical statements.

In modern computational mathematics, AI methods—particularly those based on deep learning, clustering algorithms, and representation learning—are able to extract latent structures from high-dimensional mathematical data. These structures often remain inaccessible to traditional analytical approaches due to their combinatorial complexity or the absence of explicit closed-form representations. By embedding mathematical objects (such as graphs, operators, or algebraic structures) into high-dimensional feature spaces, AI systems can identify correlations, symmetries, and invariants that may indicate the presence of deeper underlying principles [1].

A central application of this approach is the automatic generation of conjectures. Rather than proving statements directly, AI systems first analyze empirical evidence derived from simulations or symbolic computation and then propose candidate relationships that appear to hold across wide parameter regimes. These conjectures are typically expressed in the form of inequalities, asymptotic laws, or structural equivalences. In mathematical physics, for example, AI-assisted exploration has been used to conjecture relationships between spectral quantities such as Lyapunov exponents, integrated density of states (IDS), and localization-delocalization transitions in quasi-periodic models.

In the context of quasi-periodic Schrödinger operators, the operator H_θ acting on $\ell^2(\mathbb{Z}^d)$ is defined as follows:

$$(H_\theta\psi)_n =: \epsilon\sum_{|e|=1}\psi(n+e)+V(\theta+n\cdot\omega)\psi_n, n\in\mathbb{Z}^d.$$

where $\epsilon\in\mathbb{R}$, $\omega\in\mathbb{R}^d, \psi\in\ell^2(\mathbb{Z}^d)$, θ belongs to the d-dimensional torus $\mathrm{T}^d=(\mathbb{R}/\mathbb{Z})^d$, e: unit vectors (nearest neighbors) and the potential $V:\mathbb{R}^d\to\mathbb{R}$. AI-based numerical exploration can reveal unexpected transitions between spectral regimes depending on parameters such as the frequency ω, the regularity of the potential V, and the strength of the coupling. These observations may suggest conjectural statements regarding the continuity properties of the Integrated Density of State (IDS), the nature of the spectrum (pure point, absolutely continuous, or singular continuous), and the stability of spectral features under perturbations. Such conjectures can subsequently guide rigorous analytical investigations using techniques from KAM theory and perturbation methods.

Beyond spectral theory, AI-driven pattern recognition plays a crucial role in identifying hidden mathematical structures. These include approximate symmetries, scaling laws, and invariant subspaces that may not be explicitly visible in the original formulation of a problem. In many cases, AI does not directly produce fully rigorous statements; instead, it provides empirical "blueprints" that guide mathematicians toward plausible theoretical frameworks. This form of guided exploration is particularly valuable in high-dimensional settings, where direct human intuition is often insufficient.

From a methodological standpoint, AI-based conjecture generation can be interpreted as a form of statistical induction over mathematical domains. By sampling large spaces of examples and counterexamples, the system estimates the likelihood that a given relationship holds universally. This approach contrasts with classical deductive reasoning, which proceeds from axioms to theorems in a strictly logical manner. Instead, AI introduces an intermediate epistemological layer between experimentation and proof, in which conjectures emerge as statistically robust patterns rather than formally derived conclusions.

In addition, AI systems can assist in dimensional reduction and feature extraction within complex mathematical datasets. Techniques such as manifold learning and spectral clustering enable the reduction of high-dimensional structures into more interpretable representations, thereby revealing geometric or algebraic patterns that may correspond to underlying theoretical principles. These methods are particularly relevant in the study of dynamical systems and operator theory, where infinite-dimensional behavior is often approximated through finite-dimensional models.

Despite these advantages, AI-generated conjectures must be treated with caution. While AI is highly effective at identifying correlations, it does not inherently distinguish between genuine mathematical structure and coincidental numerical regularities. Consequently, conjectures proposed by AI require rigorous validation through formal proof techniques. Nevertheless, even incorrect conjectures can be valuable, as they may highlight regions of mathematical space that warrant deeper investigation.

Finally, AI-based conjecture generation represents a powerful and evolving paradigm in mathematical discovery. It transforms the early stages of research from a largely intuition-driven process into a systematic exploration of extensive mathematical landscapes, thereby accelerating the identification of promising research directions and strengthening the connection between computation and theory.

2.3. Symbolic Computation and Optimization

Symbolic computation constitutes a fundamental pillar of modern mathematical research, particularly in areas that require the exact manipulation of algebraic and analytical expressions. Classical computer algebra systems, such as Mathematica and Maple, have long been used for symbolic manipulation. However, their traditional rule-based architecture limits their adaptability when dealing with highly complex or non-standard problems.

The integration of Artificial Intelligence into symbolic computation has significantly extended these capabilities. AI-driven systems are now able to learn transformation heuristics, optimize simplification strategies, and predict efficient algebraic manipulations based on large datasets of mathematical expressions [5] [8]. This is particularly relevant in fields such as spectral theory and perturbation analysis, where symbolic complexity increases rapidly and standard rule-based methods become less effective.

In addition, AI contributes to the optimization of numerical methods. Classical numerical algorithms often suffer from issues such as instability or slow convergence, especially in high-dimensional settings. AI-based approaches—such as adaptive learning techniques and neural approximates—can dynamically adjust computational parameters, thereby improving both efficiency and accuracy [11] [12].

Another important contribution of AI lies in enabling large-scale simulations.

Through the use of surrogate models, AI systems can approximate computationally expensive processes, significantly reducing computational cost while preserving essential structural behavior [12]. This capability is particularly valuable in the study of quasi-periodic operators, where repeated numerical evaluations can be prohibitively expensive.

From an optimization perspective, AI introduces powerful methodologies such as reinforcement learning, which are capable of exploring complex energy landscapes and avoiding local minima [13] [14]. These techniques are widely applied in areas including inverse problems, variational analysis, and control theory, where efficient exploration of solution spaces is critical.

Despite these advances, AI-enhanced symbolic computation remains subject to important limitations, particularly with respect to reliability and formal correctness. While AI systems can suggest efficient transformations and computational strategies, rigorous validation still requires formal mathematical verification, often within proof assistant frameworks [7].

Overall, AI provides a unified framework that integrates symbolic reasoning, numerical computation, and optimization techniques. This integration significantly expands the range of mathematical problems that can be addressed effectively, while also reshaping the methodological foundations of modern mathematical research.

3. AI as a Research Collaborator

Artificial Intelligence is increasingly functioning not merely as a computational tool, but as an active research collaborator within modern mathematical practice. This shift reflects a broader conceptual transition, in which AI evolves from a passive executor of instructions into a co-pilot in the scientific discovery process. Within this emerging paradigm, AI systems support mathematicians across multiple stages of research, including literature analysis, computational experimentation, symbolic reasoning, and proof development.

A central component of this collaboration lies in the ability of AI systems to assist with literature review and knowledge synthesis. Modern machine learning models, particularly large language models, are capable of processing and summarizing extensive bodies of mathematical literature. In doing so, they can identify relevant results, uncover connections between seemingly distant areas, and highlight potential gaps in existing theory. In highly specialized domains—such as spectral theory or quasi-periodic Schrödinger operators—where the literature spans multiple decades and diverse analytical frameworks, such assistance can substantially reduce the cognitive effort required to navigate dispersed sources. AI systems can also classify mathematical results according to themes such as reducibility, localization phenomena, or regularity of spectral measures, thereby improving the efficiency of theoretical orientation.

Another important contribution of AI lies in code generation for numerical simulations and symbolic experimentation. In fields such as mathematical physics and dynamical systems, many research problems require large-scale numerical investigations involving operators, cocycles, or spectral quantities. AI systems can generate executable code in scientific computing environments (such as Python, MATLAB, or Julia), enabling rapid implementation of numerical methods for approximating quantities such as Lyapunov exponents, spectral measures, or transfer matrix dynamics. In particular, for quasi-periodic Schrödinger operators, AI-assisted code generation facilitates systematic exploration across parameter regimes, allowing researchers to investigate transitions between localized and extended states or to approximate the integrated density of states (IDS) with high precision.

AI also plays an important role in the verification of intermediate computational and symbolic results. In lengthy analytical arguments—such as those encountered in KAM theory or spectral perturbation analysis—proofs often involve extended sequences of intermediate inequalities, estimates, and inductive steps. AI systems can assist in checking algebraic consistency, validating symbolic manipulations, and identifying potential computational errors at early stages of derivations. While this form of verification does not replace full formal proof, it provides an additional layer of reliability, particularly in technically complex arguments where manual verification is prone to error.

In addition, AI contributes to the drafting and refinement of mathematical proofs and expository writing. By analyzing existing proof structures, AI systems can suggest alternative formulations, improve logical organization, and enhance the clarity of complex arguments. In some cases, AI can propose intermediate lemmas or reformulate reasoning in a way that reveals hidden structure or simplifies exposition. This is especially valuable in iterative frameworks such as KAM theory, where proofs involve successive transformations and require careful tracking of analytical estimates and functional dependencies.

Recent studies and practical experience indicate that integrating AI tools into mathematical workflows can significantly reduce the time required for both theoretical and computational tasks. Activities that traditionally demanded substantial manual effort—such as exploring parameter spaces, testing conjectures, or organizing extensive symbolic computations—can now be carried out much more efficiently. While this acceleration does not eliminate the need for rigorous proof, it shifts the primary bottleneck from computation and experimentation toward conceptual validation and theoretical interpretation.

As a result of these developments, the role of the mathematician is gradually evolving. Rather than acting solely as an independent problem solver responsible for every stage of discovery, the mathematician increasingly assumes the role of supervisor, interpreter, and validator of AI-assisted reasoning processes. Within this hybrid framework, human expertise remains essential for ensuring conceptual correctness, guiding theoretical direction, and establishing rigorous foundations, while AI systems contribute computational power, heuristic exploration, and structural insights.

These developments indicate that the emergence of AI as a research collaborator represents a fundamental shift in the methodology of mathematical research. Rather than replacing human creativity, AI restructures the workflow of discovery, enabling a more interactive and efficient interplay between human insight and machine-assisted computation.

4. Impact on Mathematical Productivity

The integration of Artificial Intelligence into mathematical research has led to a profound transformation in research productivity, affecting both the efficiency of individual tasks and the overall organization of the mathematical discovery process. Rather than merely accelerating computation, AI reshapes research workflows by redistributing effort among exploration, verification, and theoretical synthesis.

A primary and immediate impact of AI is the acceleration of exploratory phases in mathematical research. In traditional practice, investigating conjectures, parameter regimes, or model behaviors often requires extensive manual computation, numerical experimentation, and symbolic manipulation. AI-based systems significantly reduce this burden by automating routine calculations, enabling large-scale simulations, and rapidly scanning parameter spaces. In the context of spectral theory, for example, the study of quasi-periodic Schrödinger operators involves exploring spectral regimes across different frequencies ω, potentials V, and energy parameters E. AI-assisted methods allow for systematic sampling of these spaces, thereby facilitating the rapid identification of candidate regimes exhibiting localization, delocalization, or intermediate spectral behavior.

A second major contribution lies in the automation of repetitive and technically demanding computations. Many mathematical arguments—particularly in analysis and mathematical physics—depend on long sequences of algebraic manipulations, perturbative expansions, and inductive estimates. In KAM theory, for instance, iterative schemes require repeated normalization procedures, control of small divisors, and careful tracking of analytic norms across successive transformations. AI systems can assist in organizing these computations, minimizing redundancy, and maintaining consistency across multiple layers of approximation. This leads to a substantial reduction in human workload, especially in technically dense proofs where the underlying structural ideas are understood but their execution is highly intricate.

A third dimension of AI's impact concerns the enhancement of interdisciplinary mathematical modeling. AI facilitates the integration of mathematical methods with data-driven approaches originating from physics, engineering, and computational science. This is particularly relevant in areas such as spectral theory and dynamical systems, where mathematical models often interact closely with numerical simulations of physical phenomena. AI enables a more seamless translation between theoretical formulations and computational implementations, thereby bridging the gap between abstract mathematical structures and applied modeling.

In this sense, AI contributes to a unified research workflow in which theoretical derivation, numerical experimentation, and data analysis coexist within a single computational framework.

Another significant effect of AI is the reconfiguration of the research timeline. Tasks that traditionally required extended periods—such as deriving conjectural relationships, testing stability hypotheses, or verifying intermediate steps in long proofs—can now be completed in substantially shorter timeframes. This does not imply that deep mathematical understanding progresses at the same accelerated rate; rather, the bottleneck shifts from computational execution to conceptual validation and theoretical interpretation. Consequently, researchers are able to devote more time to high-level reasoning and structural analysis, while delegating routine computational tasks to AI systems.

From a structural standpoint, AI also enables the parallelization of mathematical investigation. Instead of progressing sequentially through conjecture, computation, and proof, researchers can explore multiple hypotheses simultaneously. In practice, this means that several candidate conjectures can be tested in parallel across different parameter regimes, with AI systems evaluating their empirical consistency and robustness. This parallel exploratory capacity is particularly valuable in complex domains such as quasi-periodic operators, where different spectral phenomena may coexist depending on parameter configurations.

Furthermore, AI introduces an adaptive feedback loop between theory and computation. In traditional workflows, theoretical development typically precedes numerical validation. In contrast, AI-assisted environments support iterative interactions in which computational results continuously inform theoretical refinement. For instance, unexpected numerical behaviors observed in simulations of Schrödinger operators may suggest modifications to analytical assumptions or reveal previously unnoticed structural features. This dynamic interplay fosters a more flexible and responsive research methodology.

Despite these substantial gains in productivity, it is essential to emphasize that AI does not eliminate the fundamental requirement for mathematical rigor. While AI significantly improves efficiency in exploration and computation, the validation of results—particularly in areas such as spectral theory, KAM reducibility, and IDS regularity—remains grounded in rigorous analytical reasoning. In this respect, AI enhances productivity without altering the foundational logical structure that defines mathematical correctness.

From a broader perspective, the impact of AI on mathematical productivity should not be understood simply as an acceleration of existing processes, but rather as a reorganization of the research ecosystem. It transforms mathematics into a more interactive and computationally augmented discipline, in which exploration, computation, and theoretical analysis are closely interconnected. This transformation is especially significant in modern mathematical physics, where the complexity and dimensionality of problems often exceed the limits of purely manual investigation.

5. Challenges and Limitations

Despite its advantages, AI introduces significant challenges:

5.1. Reliability and Correctness

A central challenge in the application of Artificial Intelligence to mathematical research concerns reliability and correctness, particularly in the context of proof generation and formal reasoning. Although modern AI systems are capable of producing highly coherent mathematical arguments and structurally plausible derivations, their outputs are not guaranteed to satisfy the strict standards of logical validity required in formal mathematics.

Unlike classical proof systems grounded in formal logic—such as first-order logic or dependent type theory—most contemporary AI models operate according to a probabilistic generation paradigm. In this framework, each step is inferred based on statistical correlations learned from large corpora of mathematical text. Consequently, correctness is not enforced through deductive verification, but rather approximated through linguistic and structural plausibility. This fundamental distinction creates a gap between syntactic coherence and true logical validity, a gap that becomes particularly critical in advanced mathematical contexts.

In complex analytical settings—such as the spectral theory of quasi-periodic Schrödinger operators or iterative schemes arising in KAM theory—proofs often depend on delicate estimates, precise control of error terms, and strict adherence to small-divisor conditions. In such environments, even minor logical inconsistencies or implicit assumptions can invalidate an entire argument. AI systems, however, may generate intermediate steps that appear mathematically sound while inadvertently violating essential conditions, including uniform boundedness, convergence criteria, or non-resonance assumptions.

An additional difficulty arises from the phenomenon of hallucinated mathematical structures, in which AI models produce objects, identities, or theorems that are internally consistent within the generated text but do not correspond to any valid or established mathematical result. These hallucinations are particularly problematic because they are often presented in highly formal and confident language, making them difficult to detect without careful external verification.

The issue of reliability is further amplified in multi-step reasoning processes, where errors may propagate across successive stages of a derivation. In extended proofs—especially those involving iterative constructions, such as conjugation schemes in KAM theory or renormalization procedures in spectral analysis—an incorrect assumption introduced at an early stage may remain undetected while affecting all subsequent steps. This cumulative propagation of error makes global verification of AI-generated proofs significantly more challenging than the local validation of individual statements.

From a foundational perspective, these limitations raise important concerns regarding the trustworthiness of AI-assisted mathematical discovery. In traditional mathematical practice, correctness is ensured through a combination of logical deduction and peer verification. In contrast, AI-generated outputs require an additional layer of validation, either through expert human review or through formal verification systems such as Lean or Coq. However, the integration of AI with fully formal proof assistants remains incomplete, and many current workflows still rely on intermediate informal reasoning stages that are inherently susceptible to subtle errors.

In the specific context of mathematical physics and spectral theory, these challenges become particularly pronounced. For example, in the analysis of quasi-periodic Schrödinger operators, incorrect assumptions regarding the regularity of transfer matrices or inaccurate estimates of cocycle growth rates may lead to fundamentally incorrect conclusions about localization or absolute continuity of the spectrum. Similarly, in KAM-type arguments, improper handling of small divisors or convergence rates may invalidate reducibility results.

Consequently, issues of reliability and correctness should be understood not merely as technical limitations, but as structural constraints on the use of AI in rigorous mathematics. While AI serves as a powerful tool for exploration and heuristic reasoning, its outputs must be regarded as provisional unless they are independently verified through rigorous analytical methods or formal proof systems. This establishes a clear hierarchy in which AI-assisted reasoning occupies an intermediate stage within the broader process of mathematical validation.

Overall, ensuring correctness in AI-assisted mathematics requires a hybrid framework that combines:

- formal proof verification systems,
- human expert oversight,
- and controlled integration of AI-generated suggestions.

Without such safeguards, the risk of subtle yet fundamental errors remains a significant obstacle to the reliable adoption of AI in mathematical research.

5.2. Lack of Deep Understanding

A second fundamental limitation of current Artificial Intelligence systems in mathematical research is the absence of genuine conceptual understanding, especially in highly abstract domains. While AI models can produce correct-looking arguments, manipulate symbolic expressions, and even generate structured proofs, their behavior is ultimately based on pattern recognition rather than true semantic understanding of mathematical objects.

In modern machine learning architectures, particularly large language models, mathematical reasoning is encoded implicitly through high-dimensional statistical correlations learned from training data. This allows the system to approximate reasoning patterns found in existing proofs, but it does not provide an internal semantic representation of why mathematical statements are true or how different concepts are structurally connected. As a result, AI can reproduce the *form* of mathematical reasoning without grasping its meaning.

This limitation becomes especially visible in areas of mathematics that rely heavily on structural intuition and abstract conceptual frameworks, such as functional analysis, spectral theory, and dynamical systems.

For instance, in the study of quasi-periodic Schrödinger operators, understanding spectral transitions requires a deep grasp of how arithmetic properties of frequencies interact with analytic properties of potentials, and how these interactions shape the long-term dynamical behavior of transfer matrices. These insights are inherently conceptual and cannot be fully captured by local pattern matching or symbolic transformation rules.

Similarly, in KAM (Kolmogorov-Arnold-Moser) theory, the main difficulty is not only carrying out iterative transformations, but also understanding the global geometric and dynamical structure behind the persistence of invariant tori. The mechanism of small divisor cancellation, convergence of conjugation schemes, and stability of quasi-periodic orbits depends on a delicate interplay between geometry, arithmetic, and analysis. While AI systems can assist in performing symbolic iterations, they do not truly "understand" the geometric meaning of reducibility or the dynamical significance of invariance.

Another manifestation of this limitation is the inability of AI systems to reliably distinguish between essential and non-essential structure in mathematical arguments. Human mathematicians rely heavily on abstraction, allowing them to compress long reasoning chains into conceptual principles such as compactness, symmetry, or invariance. In contrast, AI systems tend to operate at the level of explicit patterns in data or text, which makes it difficult for them to identify the deeper organizing principles that unify different mathematical phenomena.

This lack of conceptual grounding also affects the system's ability to generalize across mathematical domains. While AI can interpolate between known examples, it often struggles when confronted with genuinely new structures that require reorganizing existing concepts or developing new theoretical frameworks. By contrast, mathematical understanding typically involves reformulating problems in new languages, identifying hidden equivalences, and building abstract unifying theories—capabilities that go beyond statistical inference.

From an epistemological perspective, this limitation highlights a fundamental distinction between syntactic correctness and semantic understanding. AI systems can manipulate symbols according to learned patterns, but they do not assign intrinsic meaning to those symbols in the way human mathematicians do. As a result, their "reasoning" lacks conceptual grounding; it is not driven by an understanding of mathematical truth, but rather by probabilistic reconstruction of observed structures.

In practical terms, this means that while AI can be highly effective in supporting tasks such as computation, verification, and heuristic exploration, it cannot independently generate deep theoretical insight or replace human intuition in the development of new mathematical theories. In particular, in advanced research areas such as spectral analysis of quasi-periodic operators or nonlinear dynamical systems, conceptual breakthroughs often come from recognizing hidden structures that are not explicitly present in existing data—a process that remains beyond the capabilities of current AI systems.

Consequently, the lack of deep understanding imposes a clear boundary on the role of AI in mathematics. It serves as a powerful auxiliary tool for manipulation and exploration, but not as a source of genuine mathematical insight. Bridging this gap between syntactic performance and semantic understanding remains one of the central open challenges in the development of AI for mathematical discovery

5.3. Over-Reliance and Skill Degradation

A further significant concern in the integration of Artificial Intelligence into mathematical research is the risk of over-reliance on automated systems, which may gradually lead to a weakening of core mathematical skills, including problem-solving ability, conceptual intuition, and technical proficiency. While AI tools clearly enhance productivity and reduce computational effort, their widespread use introduces a structural risk: the gradual replacement of active reasoning by automated assistance.

In traditional mathematical practice, expertise is built through sustained engagement with difficult problems, iterative reasoning, and the manual construction of proofs. This process is not only instrumental but also formative, since it develops intuition about mathematical structures, sharpens analytical thinking, and strengthens the ability to work with abstract concepts. As AI systems increasingly take over tasks such as symbolic manipulation, proof suggestion, and computational verification, researchers may become less involved in these cognitively demanding processes.

One of the main risks is the weakening of problem-solving intuition. Mathematical intuition is typically developed through repeated cycles of failure, partial progress, and reconstruction of arguments. In contrast, AI-assisted workflows can bypass many of these intermediate steps by directly proposing solutions or completing technical components. Although this increases efficiency, it may also reduce opportunities for researchers to internalize deep structural patterns, especially in complex domains such as spectral theory or dynamical systems, where intuition about long-term behavior is essential.

A related issue is the reduced development of technical proof skills. In advanced areas of mathematics—such as KAM theory, perturbation analysis, or quasi-periodic Schrödinger operators—proofs often require careful handling of estimates, iterative arguments, and precise control of analytical bounds. If AI systems routinely perform these intermediate steps, researchers may gradually lose familiarity with the underlying technical machinery. Over time, this can create a dependency cycle in which human mathematicians become less capable of independently constructing or verifying complex arguments.

Another important aspect is the shift in cognitive responsibility from active derivation to passive validation. In AI-assisted environments, mathematicians may increasingly find themselves selecting among AI-generated suggestions rather than building arguments from first principles. While this can improve efficiency, it also risks reducing deep engagement with the logical structure of problems, thereby weakening long-term skill development.

From an epistemic perspective, over-reliance on AI may also affect the formation of mathematical intuition as a feedback loop between conjecture and proof. In classical research practice, conjectures are refined through repeated cycles of manual computation, failed attempts, and conceptual restructuring. AI systems, by quickly generating plausible conjectures or partial solutions, may interrupt this iterative learning process. As a result, researchers may become more dependent on externally generated hypotheses instead of developing independent heuristic insight.

This issue is particularly relevant in highly technical domains such as spectral theory and KAM-type analysis, where understanding subtle interactions—such as resonance phenomena, small divisor effects, and stability mechanisms—requires deep familiarity with the underlying analytical structure. Excessive dependence on AI tools in such contexts may lead to a more superficial understanding of results, without full mastery of the methods used to derive them.

Furthermore, there is a risk of deskilling in symbolic and computational techniques, since routine tasks such as algebraic simplification, perturbative expansion, or numerical experimentation are increasingly automated. While this automation improves efficiency, it may reduce the motivation to maintain fluency in these foundational skills, which remain essential for detecting errors, validating results, and extending methods to new settings.

In the long term, heavy reliance on AI could therefore reshape the training path of mathematicians, shifting emphasis away from deep technical apprenticeship toward interaction with automated systems. While this shift may increase productivity, it also raises concerns about preserving mathematical craftsmanship, which depends on sustained engagement with complex reasoning processes.

Taken together, over-reliance and skill degradation represent a structural risk in the evolution of AI-assisted mathematics. Addressing this risk requires a balanced integration strategy in which AI is treated as a support tool rather than a replacement for fundamental reasoning. Maintaining active engagement with manual problem-solving remains essential for preserving the depth, rigor, and creativity that define high-level mathematical research.

5.4. Ethical and Epistemological Issues

The increasing integration of Artificial Intelligence into mathematical research raises profound ethical and epistemological questions that go beyond technical considerations and directly touch the foundations of mathematical practice. Unlike traditional computational tools, AI systems are now increasingly involved in generating conjectures, proofs, and even research directions, which challenges classical notions of authorship, understanding, and validation in mathematics.

A first central issue concerns ownership and intellectual authorship of AI-generated mathematical results. In traditional mathematics, authorship is clearly attributed to individuals or groups who construct proofs and develop theories through explicit reasoning and verification. However, when AI systems contribute significantly to generating conjectures, proof strategies, or even complete derivations, attribution becomes ambiguous. It is not clear whether credit should go to the human user, the developers of the AI system, the contributors of the training data, or the AI system itself as a computational agent. This ambiguity challenges existing academic norms related to publication, citation, and intellectual property in mathematical research.

A second and more fundamental epistemological issue concerns the question: what actually counts as a mathematical proof in the presence of AI assistance? Traditionally, a proof is a finite sequence of logically valid deductions derived from axioms within a formal system, and its validity does not depend on interpretation. However, in AI-assisted environments, proofs may be produced in ways that are partially opaque, heuristic, or distributed across multiple computational layers. In such cases, even when the final result is correct, the internal reasoning may not be fully transparent or understandable to human mathematicians. This creates a deep philosophical tension between formal correctness and conceptual understanding.

This issue becomes especially important in complex mathematical domains such as the spectral theory of quasi-periodic Schrödinger operators or KAM-type dynamical systems, where proofs rely on intricate iterative constructions, delicate estimates, and highly nontrivial analytical mechanisms. If AI systems contribute to such proofs in ways that are not fully interpretable, it becomes necessary to question whether mathematical knowledge can still be considered fully justified in the classical sense, or whether a new notion such as "machine-verified truth" should be introduced.

A related concern is the possible emergence of a distinction between human-understandable proofs and machine-verified proofs. In principle, an AI system integrated with formal proof assistants (such as Lean or Coq) may produce formally correct proofs that are too large, complex, or structurally opaque for humans to fully grasp. While such proofs satisfy syntactic correctness within a formal system, they may fail to provide the explanatory insight traditionally expected in mathematics. This raises the question of whether proof should be defined purely in syntactic terms, or whether semantic understanding remains an essential part of mathematical validity.

Another important epistemological challenge concerns the changing nature of mathematical discovery itself. If AI systems can generate conjectures, suggest proof strategies, and even uncover new structures, then the traditional boundary between discovery and verification becomes blurred. Mathematical knowledge may increasingly emerge from a hybrid human-machine process in which intuition, computation, and formal reasoning are tightly interwoven. This shift forces a reevaluation of long-standing philosophical views on mathematical truth, creativity, and justification.

In addition, ethical concerns arise regarding responsibility for errors in AI-assisted mathematics. If a result produced with AI support is later shown to be incorrect, it becomes difficult to determine responsibility—whether it lies with the human researcher, the AI system, or the design of the computational framework itself. This issue is particularly sensitive in formal mathematical literature, where correctness is absolute and even small errors can invalidate entire chains of reasoning.

From a broader philosophical perspective, these developments challenge the classical view of mathematics as a purely human intellectual activity grounded in transparent logical deduction. The growing role of AI introduces a new epistemic layer in which mathematical truth may be mediated by computational processes that are not fully accessible to human understanding. This raises fundamental questions about whether mathematical knowledge must be fully comprehensible to humans in order to be considered legitimate, or whether correctness alone is sufficient.

The increasing reliance on large-scale AI systems in mathematical research raises important epistemological questions. As discussed in recent analyses of foundation models [15], the opacity and scale of such systems challenge traditional notions of understanding, proof, and authorship in mathematics.

Overall, the ethical and epistemological issues surrounding AI in mathematics suggest that the field is undergoing a structural transformation. Addressing these challenges will require not only technical solutions—such as improved formal verification systems—but also a deeper philosophical reassessment of what it means to know, prove, and understand mathematics in an era of machine-assisted discovery.

To complement the above theoretical discussion, the following section presents empirical evidence based on a large-scale survey examining how these transformations are perceived and experienced by researchers in practice.

6. Methods

This study combines a literature-based analytical review with an original cross-sectional survey designed to assess the impact of Artificial Intelligence on mathematical research practices.

6.1. Survey Design

The survey was structured as a self-administered online questionnaire consisting of closed-ended Likert-scale items (1 - 5) and open-ended questions (see Appendix). The instrument covered seven thematic dimensions: general perception, theorem proving, conjecture generation, symbolic computation, collaboration, reliability, and epistemological implications.

6.2. Recruitment and Data Collection

Participants were recruited through academic mailing lists, research networks, and professional platforms (e.g., university affiliations and mathematical communities). Participation was voluntary and anonymous.

6.3. Eligibility Criteria

Respondents were required to be engaged in mathematical research or closely related fields (including pure and applied mathematics, dynamical systems, and mathematical physics).

6.4. Collection Period

Data were collected between January and June 2025.

6.5. Geographic Scope

The survey targeted an international academic audience; however, responses were not evenly distributed geographically.

6.6. Data Cleaning

Responses were screened for completeness and consistency. Incomplete questionnaires and duplicate submissions were removed. Final analysis included N = 1,032 valid responses.

6.7. Statistical Analysis Procedures

Descriptive statistics (mean and standard deviation) were calculated for each category based on Likert-scale responses.

Mean scores were computed by averaging Likert-scale responses (1 - 5) across items within each category. Negatively worded items were reverse-coded prior to aggregation to ensure consistency in interpretation.

7. Results and Discussion

7.1. Sample Characteristics and Representativeness

The study is based on a large and diverse sample of N = 1,032 respondents, providing a broad and diverse exploratory sample, although not necessarily representative of the global mathematical community.

Research Fields

- Mathematical Analysis: 27.9%
- Spectral Theory: 7.0%
- Dynamical Systems: 18.6%
- Applied Mathematics: 24.7%
- Other: 21.8%

Thus, 68.3% of respondents are engaged in theoretical or semi-theoretical domains, directly relevant to assessing AI in abstract mathematics.

Years of Experience

- <5 years: 23.3%
- 5 - 10 years: 25.7%
- 10 - 20 years: 31.4%
- >20 years: 19.6%

Notably, 51% of participants have more than 10 years of experience, ensuring that results reflect expert-level perspectives.

Academic Position

- PhD Students: 30.4%
- Postdoctoral Researchers: 29.3%
- Lecturers/Researchers: 22.5%
- Professors: 17.8%

The sample is therefore well-balanced, with 69.6% mid- to senior-level researchers, reinforcing the robustness of the findings.

7.2. Adoption of AI Tools

The survey results indicate a high level of AI adoption among researchers. The mean score for AI usage is M = 4.32 (Standard Deviation: SD = 0.64), reflecting strong agreement that AI tools are actively integrated into mathematical workflows. Approximately 78% of respondents selected "Agree" or "Strongly Agree" when asked about regular AI usage.

These findings confirm that AI is no longer a marginal or experimental tool but has become a standard component of contemporary mathematical practice. In particular, qualitative responses suggest frequent use of systems such as ChatGPT and computer algebra systems for symbolic manipulation and exploratory computations.

7.3. Impact on Productivity

A substantial positive impact of AI on research productivity is observed, with a mean score of M = 4.18 (SD = 0.71). More than 72% of participants reported either moderate or strong improvements in efficiency.

The most affected areas include:

- computational tasks
- exploratory analysis
- preliminary conjecture testing

These results empirically validate the theoretical claims discussed earlier, particularly regarding efficiency gains and acceleration of low-level technical processes. The relatively low standard deviation indicates a broad consensus among respondents.

7.4. Role in Mathematical Discovery

The role of AI in mathematical discovery is perceived as significant but not dominant, with a mean score of M = 3.97 (SD = 0.76). Around 65% of respondents acknowledge that AI contributes meaningfully to:

- Conjecture generation
- Pattern detection
- Heuristic exploration

However, the slightly lower mean compared to productivity suggests that researchers still view AI primarily as an assistive discovery tool rather than an autonomous creative agent. This reflects a hybrid paradigm in which AI enhances, but does not replace, human intuition.

7.5. Reliability and Trust Issues

Concerns regarding reliability are strongly reflected in the data. The mean score for trust-related items (reverse-coded) is M = 2.21 (SD = 0.83), indicating general disagreement with statements asserting full reliability of AI systems.

Approximately 69% of respondents expressed reservations about AI outputs, citing issues such as:

- Logical inconsistencies
- Hallucinated results
- Lack of formal guarantees

This confirms that, despite high adoption rates, trust remains a critical bottleneck in AI-assisted mathematical research. The relatively higher standard deviation suggests variability in trust levels, likely depending on individual experience and domain specialization.

7.6. Future Outlook

The outlook for AI in mathematics is overwhelmingly positive. The mean score for future expectations is M = 4.41 (SD = 0.59), with over 80% of respondents anticipating that AI will evolve into a collaborative partner rather than a replacement for human mathematicians.

Respondents consistently emphasized a complementary division of roles:

- AI enhances exploration and computation,
- humans ensure rigor, interpretation, and conceptual understanding,

This supports the emerging view of human-AI symbiosis as the dominant paradigm for future mathematical research.

7.7. Synthesis and Implications

Taken together, the results reveal a coherent and statistically consistent picture:

- **High adoption** ($M \approx 4.3$) confirms integration into daily workflows
- **Strong productivity gains** ($M \approx 4.2$) validate efficiency claims
- **Moderate role in discovery** ($M \approx 4.0$) highlights assistive creativity
- **Low trust levels** ($M \approx 2.2$) expose critical limitations
- **Optimistic future outlook** ($M \approx 4.4$) supports long-term integration

This combination suggests that AI is best understood not as a replacement for mathematical reasoning, but as a powerful augmentation layer that reshapes the research process while preserving the central role of human insight (See Figure 1).

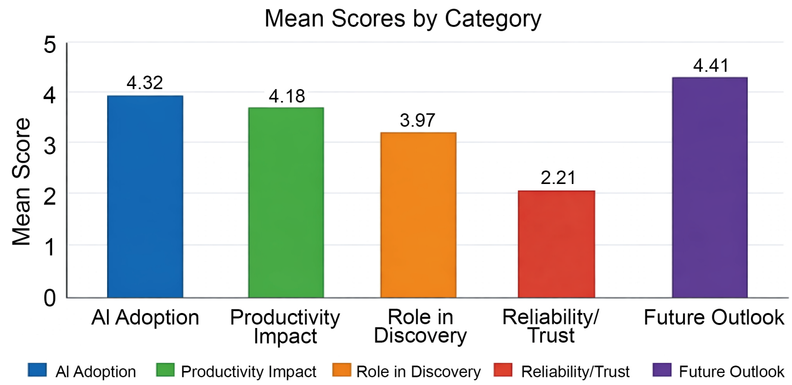

Figure 1. Mean score by Category.

These results are consistent with recent evaluations of large-scale AI systems, which show significant improvements in mathematical reasoning benchmarks. However, as highlighted in recent comprehensive studies on foundation models by Bommasani, R. *et al.* [15], important limitations remain, particularly in terms of reliability, interpretability, and generalization.

7.8. Limitations and Sampling Bias

The survey is based on voluntary participation, which introduces potential self-selection bias. Researchers more interested in AI may be overrepresented. Additionally, the sample may not fully capture geographic or disciplinary diversity. Therefore, results should be interpreted as indicative trends rather than fully generalizable conclusions.

8. Conclusions

The integration of Artificial Intelligence into mathematical research represents a structural transformation in how mathematical knowledge is produced, explored, and validated. Across areas such as automated theorem proving, conjecture generation, symbolic computation, and collaborative research workflows, AI has shown a strong ability to improve productivity, accelerate exploration, and support complex analytical tasks that are often difficult to handle using traditional methods alone. In particular, in fields such as the spectral theory of quasi-periodic Schrödinger operators and KAM-type dynamical systems, AI-assisted tools offer valuable computational and heuristic support for exploring spectral regimes, organizing iterative schemes, and guiding numerical experimentation.

At the same time, this emerging paradigm introduces fundamental limitations that define the current boundary of AI in mathematics. First, issues of reliability and correctness remain central, since AI-generated outputs are not inherently guaranteed to satisfy formal logical standards. This makes rigorous human verification or formal proof checking essential, especially in delicate analytical contexts where small errors can invalidate entire arguments. Second, the lack of deep conceptual understanding in AI systems highlights a clear separation between syntactic manipulation and semantic comprehension. While AI can reproduce patterns of reasoning, it does not yet possess an internal understanding of mathematical meaning, structure, or necessity—an element that remains crucial for theoretical breakthroughs.

Third, the growing reliance on AI tools raises concerns about over-dependence and possible skill degradation among researchers. As computational and symbolic tasks become increasingly automated, there is a risk that core mathematical intuition, problem-solving ability, and technical expertise may weaken over time if they are not actively maintained. Finally, ethical and epistemological questions arise regarding authorship, responsibility, and even the definition of a mathematical proof itself. The possibility of machine-generated arguments that are formally correct but not fully understandable challenges the traditional view of mathematics as transparent, human-centered reasoning.

In conclusion, Artificial Intelligence should not be seen as a replacement for human mathematicians, nor merely as an auxiliary computational tool, but rather as a transformative partner in the process of mathematical discovery. Its role is best understood within a hybrid framework where AI enhances exploration and computation, while human researchers remain responsible for interpretation, conceptual insight, and formal validation. The future of mathematics will therefore likely be characterized by a deep integration of human creativity and machine-assisted reasoning, requiring both methodological innovation and a philosophical rethinking of what constitutes mathematical knowledge. These empirical findings support the theoretical analysis and confirm that AI is reshaping mathematical research while preserving the central role of human reasoning.

Conflicts of Interest

The author declares no conflicts of interest.

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Appendix

Survey on Artificial Intelligence in Mathematical Research

Section A: Researcher Profile

1. Main research field:

☐ Mathematical Analysis

☐ Spectral Theory

☐ Dynamical Systems

☐ Applied Mathematics

☐ Other: ……………..

2. Years of experience:

☐ < 5 years

☐ 5 - 10 years

☐ 10 - 20 years

☐ >20 years

3. Position:

☐ PhD Student

☐ Postdoctoral Researcher

☐ Lecturer/Researcher

☐ Professor

Section B: Use of AI

Likert Scale Instructions:

For Sections 1-7, indicate your level of agreement using the scale:

1 = Strongly Disagree, 2 = Disagree, 3 = Neutral, 4 = Agree, 5 = Strongly Agree

Section 1: General Perspective

1. AI plays a significant role in modern mathematical research.
2. AI is transforming mathematics rather than simply supporting it.
3. AI is influencing the foundational understanding of mathematical knowledge.
4. AI is still in an early assistive phase in mathematics. (reverse-coded)

Section 2: Automated Theorem Proving

1. AI-based theorem provers are reliable for verifying mathematical results.
2. Machine-generated proofs can replace traditional human proofs in some cases.
3. Lack of interpretability limits trust in automated proofs.
4. Current systems are insufficient for fully general theorem proving.

Section 3: Conjecture Generation and Discovery

1. AI can generate meaningful and original mathematical conjectures.
2. AI mainly performs pattern recognition rather than true discovery.
3. Human intuition is essential for validating AI-generated conjectures.
4. AI can discover new mathematical structures.

Section 4: Symbolic Computation and Technical Acceleration

1. AI significantly improves symbolic computation efficiency.
2. Heavy reliance on AI may weaken conceptual understanding.
3. AI reduces time required for complex derivations.
4. Computational tools enhance human reasoning.

Section 5: Human-AI Collaboration

1. Effective mathematical research requires human-AI collaboration.
2. Future mathematicians must develop AI-related skills.
3. AI can be considered a co-author in publications.
4. Human oversight is always necessary in AI-assisted research.

Section 6: Reliability, Interpretability, and Trust

1. Interpretability is essential for accepting AI-generated results.
2. Non-interpretable AI outputs can still be scientifically valid.
3. Trust depends on reproducibility and verification.
4. Lack of transparency poses a risk to mathematical rigor.

Section 7: Philosophical and Epistemological Implications

1. AI challenges the traditional definition of mathematical proof.
2. Mathematical knowledge may become partially independent of human reasoning.
3. Proof may include machine-generated but opaque arguments.
4. Human intuition remains central to mathematical truth.

Section 8: Open-Ended Questions

1. How do you define the role of AI in future mathematical research?
2. What ethical guidelines should govern AI-assisted mathematics?
3. What are the main risks and opportunities of AI in mathematics?