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Mathematical Modelling and Analysis of Measles Control Using Vaccination

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Abstract

In this paper, the analysis of a mathematical model for the control of measles using vaccination is presented. The disease reproductive number (R_0) is computed and used to derive an expression of a reduction factor in the disease spread resulting from the implementation of immunization program in the community. The Routh-Hurwitz stability conditions are adopted to analyze the model equilibria in terms of their local stability. Lyapunov function candidates are used to determine the global stability status of both the measles-free and measles-persistent equilibrium states. Elasticity index of each parameter embedded into R_0 is computed to help ascertain the significance of the model parameter contribution to measles epidemics in the society. Furthermore, population simulations are performed to demonstrate how changes in some parameter values influence the dynamical evolution of our model population sub-classes.

Subject Areas

Mathematical Analysis

Keywords

Measles, Vaccination Campaign, Reproductive Number with/without Vaccination, Local and Global Stability, Sensitivity Analysis

1. Introduction

Measles is globally known as the most fatal and contagious human disease [1] [2]. It is one of the diseases with devastating impact among children under the age of five. Measles is caused by a virus called morbillivirus and is primarily transmitted among humans through aerosolized respiratory droplets [3]-[5]. The initial symptoms of measles include but are not limited to: runny nose, cough, rash all over the body, fever, malaise and conjunctivitis which usually manifest between ten to fourteen days after one's exposure to the disease [3] [6]. If left unattended, severe measles can lead to complications such as: encephalitis, blindness, ear infections, severe diarrhea, pneumonia and death [2] [7]. Even though an effective and safe vaccine for the control of measles exists, this disease continues to reemerge in many countries including the developed nations [4] [6] [8] [9]. Thus, measles resurgence still remains a global public health issue that requires attention [2] [10]. Regular measles vaccination campaign for children and young adults in measles endemic countries is currently the recommended control strategy for reducing measles disease burden [11]. In the last twenty-three years, about 60.3 million measles deaths have been averted by vaccination. However, between 2022 and 2023, there was an estimated 20% increase in measles cases globally with the number of countries/territories that reported large scale outbreaks increased from 36 to 57 [12]. Key contributing factors to these outbreaks especially in Africa include: inadequate measles surveillance systems, low measles immunization coverage and poor health care systems. In Ghana for example, a total of 2282 measles cases were reported in the year 2023 [12]. Several studies have reported that measles surveillance remains suboptimal in many countries [13]. Zumah *et al.* [14] evaluated and rated the measles disease surveillance system's performance in the Bono region of Ghana as suboptimal. Some data from their study is shown in **Table 1**. Mathematical modelling has been used as an effective tool for providing meaningful insights into infectious disease dynamics and control [2] [9] [15] [16]. Thus, some researchers have developed dynamical models for measles epidemic. Opoku *et al.* [4] developed a dynamical model for measles control that took into account maternal antibodies protecting new born babies from measles infections and double-dose vaccination program. Their study results suggested that measles first dose vaccination is a potential means of combating measles epidemics. Garba *et al.* [5] used a nine-compartment model in investigating the significance of simultaneously combining vaccination and treatment in the control of measles epidemics. Wireko *et al.* [17] proposed a fractional order dynamical system for studying measles. Alemneh and Belay [18] analyzed measles transmission model that took into account the presence of measles virus in the environment. James *et al.* [6] formulated a deterministic mathematical model to provide insight into measles dynamics in Nigeria. In another study conducted by James *et al.* [2], it was demonstrated that reducing the contact rate between a susceptible and an

Table 1. Measles surveillance data from the bono region of Ghana.

Year	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010
SMC	184	222	220	166	127	165	77	27	30	19
LCMC	4	0	2	5	0	1	5	0	2	0
% MCV1	-	-	89.4	92.3	93.3	94.0	88.2	99.6	96.1	95.6

SMC = Suspected Measles Cases, LCMC = Laboratory Confirmed Measles Cases and %MCV1 = percentage of children vaccinated against the first measles dose vaccine. Source [14].

infectious person and increasing the vaccination rate of an effective measles vaccine were the most effective control measures against measles epidemics. A mathematical model analysis presented by Kuddus *et al.* [19] in Bangladesh revealed that contact rate is the most significant parameter to the spread of measles. Authors in [20] investigated the infection dynamics and control of measles in Pakistan using a dynamical system based on mass action principle. Their numerical simulation results suggested that improving vaccine efficiency and coverage rate will translate into a drastic reduction in the spread of measles in the country. Researchers in [9] examined the dynamics of measles by extending an SEIR compartmental model to include vaccination and treatment. Their new model was simulated using reported measles data from Indonesia. In this current study, we employ mathematical modeling approach to gain more insights into measles infection dynamics in the presence of vaccination campaign.

2. Construction of Measles Vaccination Model

To formulate a dynamical model for measles control using vaccination, the population under study is stratified into five sub-population classes. Namely: susceptibles (S), vaccinated (V), exposed (E), infected (I) and recovered (R) classes. Following this description, $N(t)$, the population size at any given time t is made up of $S(t), V(t), E(t), I(t)$ and $R(t)$. The susceptible population under consideration is generated at a constant rate π. Some of these susceptible individuals get vaccinated against measles at rate ρ. The model assumes that a portion of those vaccinated attain immunity against measles and move to recovery class at rate τ. Following the waning of the vaccine, some vaccinated individuals return to susceptible sub-class at rate η. Other susceptible individuals get exposed to measles at a force of infection $\frac{\beta I}{N}$, where β is the effective contact rate between a susceptible and an infected individual. The exposed people progress to infected class at rate α. The infected individuals recover at rate γ. The model assumes that recovered people attain permanent immunity against further measles attacks [8], hence, there is no movement from the recovery class back to susceptible class. The parameters denoting the transfer and removal rates from the model compartments are described in **Table 2**.

Expressing **Figure 1** in the form of equations gives:

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = \pi + \eta V - \left(\frac{\beta I}{N} + \rho + \mu\right) S \\ \frac{\mathrm{d}V}{\mathrm{d}t} = \rho S - (\eta + \tau + \mu) V \\ \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\beta I S}{N} - (\mu + \alpha) E \\ \frac{\mathrm{d}I}{\mathrm{d}t} = \alpha E - (\mu + \delta + \gamma) I \\ \frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I + \tau V - \mu R \end{cases} \tag{1}$$

Table 2. Measles model parameter description and initial conditions.

Parameter	Description	Value	Source
π	Human recruitment rate	123.7	Computed
μ	Natural human mortality rate	0.015	[17]
ρ	Vaccination rate	0.842	Estimated
η	Vaccine waning rate	0.6	[4]
τ	Immunity rate of vaccinated individuals	0.5	Assumed
β	Effective contact rate	0.000402	Estimated
α	Progression rate of exposed individuals to infected class	0.25	[4]
γ	Immunity rate of infected individuals	0.6	[4]
δ	Measles induced mortality rate	0.125	[4]
State Variable Initial Value			
$S(0)$	Initial susceptible number	1000	Assumed
$V(0)$	Initial vaccinated number	250	Assumed
$E(0)$	Initial exposed number	500	Assumed
$I(0)$	Initial infected number	100	Assumed
$R(0)$	Initial recovered number	0	Assumed

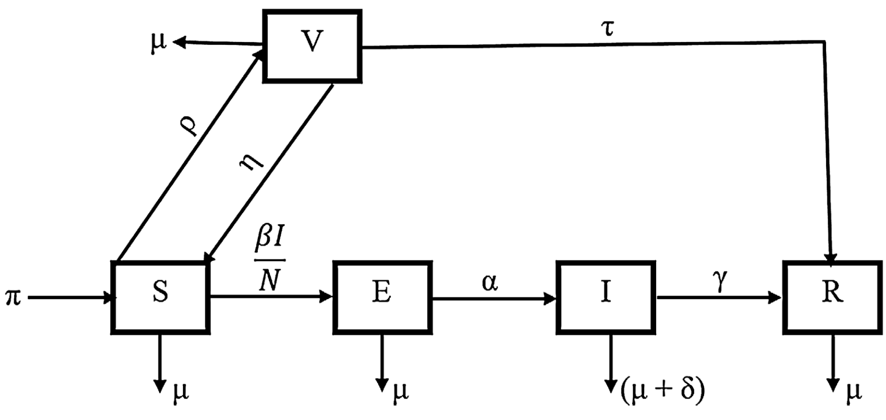

Figure 1. Illustrative diagram for measles transmission dynamics.

To ease our analysis, we use: $g_0 = (\rho + \mu)$, $g_1 = (\eta + \tau + \mu)$, $g_2 = (\alpha + \mu)$, and $g_3 = (\gamma + \mu + \delta)$ in the rest of the study.

2.1. Positivity of the Model Solutions

Here, we verify the basic properties of the system of differential equations representing model (1).

Lemma 1. *The solutions* $S(t), V(t), E(t), I(t)$ *and* $R(t)$ *are non-negative and bounded* $\forall\, t \geq 0$ *whenever the initial value set:* $\{S(0), V(0), E(0), I(0), R(0)\}$ *remains positive.*

Proof. First, we consider:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \pi + \eta V - (\Lambda + g_0)S, \quad \Lambda = \frac{\beta I}{N}$$

$$\Rightarrow \frac{\mathrm{d}S}{\mathrm{d}t} \geq -(\Lambda + g_0)S$$

$$\Rightarrow \int \frac{1}{S}\mathrm{d}S \geq -\int (\Lambda + g_0)\mathrm{d}t$$

$$\Rightarrow S(t) \geq S(0)\mathrm{e}^{-(\Lambda + g_0 t)} \geq 0$$

Similarly, the following results can be derived:

$V(t) \geq V(0)e^{-g_1 t} \geq 0,\; E(t) \geq E(0)\mathrm{e}^{-g_2 t} \geq 0,\; I(t) \geq I(0)\mathrm{e}^{-g_3 t} \geq 0$ and $R(t) \geq R(0)\mathrm{e}^{-\mu t} \geq 0.$

□

Lemma 2. *The positive set* $\mathscr{D} = \left\{(S, V, E, I, R) \in \mathbb{R}_+^5 : S + V + E + I + R \leq \frac{\pi}{\mu}\right\}$ *represents the invariant region of the model system of equations.*

Proof. For any given time $t \geq 0$, N is:

$$N = S + V + E + I + R$$
$$\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{d}{dt}(S + V + E + I + R) \tag{2}$$
$$\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}t} = \pi - \mu N - \delta I$$

Thus, in the absence of measles induced mortality,

$$\frac{\mathrm{d}N}{\mathrm{d}t} \leq \pi - \mu N$$
$$\Rightarrow \int \frac{\mathrm{d}N}{N - \frac{\pi}{\mu}} \leq -\int \mu \mathrm{d}t \tag{3}$$
$$\Rightarrow N \leq \frac{\pi}{\mu} \text{ as } t \to +\infty$$
$$\Rightarrow N = S + V + E + I + R \leq \frac{\pi}{\mu}$$

□

2.2. The Measles-Free Equilibrium Point (DFE)

The DFE point of system (1) denoted by ξ^* is the non-trivial solution of system (4) below with the condition $E^* = I^* = 0$

$$\begin{cases} \pi - \left(\dfrac{\beta I^*}{N^*} + g_0 \right) S^* = 0 \\ \rho S^* - g_1 V^* = 0 \\ \dfrac{\beta I^* S^*}{N^*} - g_2 E^* = 0 \\ \alpha E^* - g_3 I^* = 0 \\ \gamma I^* + \tau V^* - \mu R^* = 0 \end{cases} \tag{4}$$

Thus, $\xi^* = \left(S^*, V^*, E^*, I^*, R^*\right) = \left(\dfrac{g_1 \pi}{g_0 g_1 - \eta\rho}, \dfrac{\rho\pi}{g_0 g_1 - \eta\rho}, 0, 0, \dfrac{\tau\rho\pi}{\mu\left(g_0 g_1 - \eta\rho\right)} \right)$.

The Measles Basic Reproductive Number (R_0)

The method of next generating matrix is used to derive R_0. To achieve this, we first expressed the infectious and infected sub-system of system (1) as, $\dfrac{\mathrm{d}X}{\mathrm{d}t} = \left(\mathscr{F} - \mathscr{G}\right) X^{\mathrm{T}}$. Here, X^{T} denotes the transpose of $X = (E, I)$ while $\mathscr{F}$ and $\mathscr{G}$ represent the rates of generation of new infections and transfers respectively. That is,

$$\mathscr{F} = \begin{pmatrix} \dfrac{\beta IS}{N} \\ 0 \end{pmatrix} \text{ and } \mathscr{G} = \begin{pmatrix} g_2 E \\ -\alpha E + g_3 I \end{pmatrix} \tag{5}$$

Evaluating the Jacobian matrices F and G of $\mathscr{F}$ and $\mathscr{G}$ at the DFE gives respectively:

$$F = \begin{pmatrix} 0 & \dfrac{\beta S^*}{N^*} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{\beta\mu g_1}{g_0 g_1 - \eta\rho} \\ 0 & 0 \end{pmatrix} \text{ and } G = \begin{pmatrix} g_2 & 0 \\ -\alpha & g_3 \end{pmatrix} \tag{6}$$

Using F and G from (6) with $S^* = \dfrac{g_1\pi}{g_0 g_1 - \eta\rho}$ and $N^* = \dfrac{\pi}{\mu}$, we obtain FG^{-1} given by:

$$FG^{-1} = \begin{pmatrix} 0 & \dfrac{\beta\mu g_1}{g_0 g_1 - \eta\rho} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{1}{g_2} & 0 \\ \dfrac{\alpha}{g_2 g_3} & \dfrac{1}{g_3} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha\beta\mu g_1}{g_2 g_3\left(g_0 g_1 - \eta\rho\right)} & \dfrac{\beta\mu g_1}{g_3\left(g_0 g_1 - \eta\rho\right)} \\ 0 & 0 \end{pmatrix} \tag{7}$$

Thus, using the equation $\left|FG^{-1} - \lambda I\right| = 0$ where I is a unit matrix and λ an eigenvalue of FG^{-1}, we obtain R_0 as the spectral radius of FG^{-1} given by:

$$R_0 = \frac{\alpha\beta\mu g_1}{g_2 g_3\left(g_0 g_1 - \eta\rho\right)} \tag{8}$$

In epidemiology, the magnitude of R_0 provides an extent of the severity of the epidemics in the community [16]. In other words $R_0 < 1$, gives hope that the disease will die out with time. On the other hand, if $R_0 > 1$, the outbreak will continue to unfold in the population. Using the parameter values presented in Table 2, we obtain $R_0 = 1.903 \times 10^{-5}$.

2.3. Stability Analysis

2.3.1. Local Stability of Measles Disease-Free Equilibrium (DFE) State

Theorem 1. *The point,* $\xi^* = \left(\frac{g_1\pi}{g_0g_1-\eta\rho}, \frac{\rho\pi}{g_0g_1-\eta\rho}, 0, 0, \frac{\tau\rho\pi}{\mu(g_0g_1-\eta\rho)}\right)$ *admits a local asymptotic stability* (LAS) *if* $R_0 < 1$ *but becomes unstable when* $R_0 > 1$

Proof. We consider the matrix of partial derivatives of the model (1), $J(x)$, that is,

$$J(x) = \begin{pmatrix} -\left(\frac{\beta I^*}{N^*} + g_0\right) & \eta & 0 & -\frac{\beta S^*}{N^*} & 0 \\ \rho & -g_1 & 0 & 0 & 0 \\ \frac{\beta I^*}{N^*} & 0 & -g_2 & \frac{\beta S^*}{N^*} & 0 \\ 0 & 0 & \alpha & -g_3 & 0 \\ 0 & \tau & 0 & \gamma & -\mu \end{pmatrix} \quad (9)$$

Now, evaluating $J(x)$ at the DFE (ξ^*) gives:

$$J(\xi^*) = \begin{pmatrix} -g_0 & \eta & 0 & -\frac{\beta g_1\mu}{g_0g_1-\eta\rho} & 0 \\ \rho & -g_1 & 0 & 0 & 0 \\ 0 & 0 & -g_2 & \frac{\beta g_1\mu}{g_0g_1-\eta\rho} & 0 \\ 0 & 0 & \alpha & -g_3 & 0 \\ 0 & \tau & 0 & \gamma & -\mu \end{pmatrix} \quad (10)$$

It can be easily observed that the matrix in (10) admits three negative eigenvalues: $\lambda_5 = -\mu$, $\lambda_2 = -g_1$, $\lambda_1 = -g_0$, while the other eigenvalues λ_3 and λ_4 can be obtained from the matrix in (11):

$$J_1 = \begin{pmatrix} -g_2 & \frac{\beta g_1\mu}{g_0g_1-\eta\rho} \\ \alpha & -g_3 \end{pmatrix} \quad (11)$$

whose characteristic equation is:

$$\lambda^2 + \Delta_1\lambda + \Delta_0 = 0 \quad (12)$$

where:

$$\Delta_1 = g_2 + g_3, \ \Delta_0 = g_2g_3(1-R_0) \quad (13)$$

Since $\Delta_1 > 0$ and $\Delta_0 > 0$ if $R_0 < 1$, matrix J_1 will have negative eigenvalues. Hence, according to the Routh-Hurwitz stability conditions, our proposed model disease-free state (ξ^*) is a local asymptotic stable equilibrium whenever $R_0 < 1$. □

2.3.2. Global Asymptotic Stability

For the purpose of investigating the asymptotic stability of ξ^* within its global neighborhood, we define a positive bi-variate function as:

$$\mathscr{L} = \alpha E + g_2 I \tag{14}$$

Differentiating $\mathscr{L}$ gives

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\mathscr{L}) &= \frac{\mathrm{d}}{\mathrm{d}t}(\alpha E + g_2 I) \\ &= \alpha \frac{\mathrm{d}E}{\mathrm{d}t} + g_2 \frac{\mathrm{d}I}{\mathrm{d}t} \\ &= \alpha\left(\frac{\beta S I}{N} - g_2 E\right) + g_2(\alpha E - g_3 I) \\ &= g_2 g_3 (R_0 - 1) I \\ &= -g_2 g_3 (1 - R_0) I\end{aligned} \tag{15}$$

It follows from (15) that $\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}t} \le 0$ whenever $R_0 \le 1$ and $\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}t} = 0$ only when $I = 0$. This therefore implies that $\mathscr{L}$ is a Lyapunov function. Thus, it follows from the Lyapunov version of the LaSale's Invariant Principle [21] that the measles DFE (ξ^*) has a GAS if $R_0 \le 1$ but unstable otherwise.

2.4. Existence of a Unique Measles Endemic Equilibrium Point (EE)

This section is dedicated to the derivation of a unique measles persistence equilibrium state. Let $\xi^{**} = (S^{**}, V^{**}, E^{**}, I^{**}, R^{**})$ be the measles persistent equilibrium (EE) point for model (1) that is ξ^{**} satisfies the system:

$$\begin{cases}\pi + \eta V^{**} - \left(\dfrac{\beta}{N^{**}} I^{**} + g_0\right) S^{**} = 0 \\ \rho S^{**} - g_1 V^{**} = 0 \\ \dfrac{\beta}{N^{**}} I^{**} S^{**} - g_2 E^{**} = 0 \\ \alpha E^{**} - g_3 I^{**} = 0 \\ \gamma I^{**} + \tau V^{**} - \mu R^{**} = 0\end{cases} \tag{16}$$

Solving for $S^{**}, V^{**}, E^{**}, I^{**}$ and R^{**} yields, the system:

$$\begin{cases}S^{**} = \dfrac{g_1 \pi}{(g_0 g_1 - \eta\rho) R_0} \\ V^{**} = \dfrac{\pi\rho}{(g_0 g_1 - \eta\rho) R_0} \\ E^{**} = \dfrac{\pi}{g_2}\left(1 - \dfrac{1}{R_0}\right) \\ I^{**} = \dfrac{\pi(g_0 g_1 - \eta\rho)}{\beta g_1 \mu}(R_0 - 1) \\ R^{**} = \dfrac{\beta g_1 \pi\mu\tau\rho + \gamma\pi R_0 (g_0 g_1 - \eta\rho)(R_0 - 1)}{\beta g_1 \mu^2 R_0 (g_0 g_1 - \eta\rho)}\end{cases} \tag{17}$$

It is clear from (17), that ξ^{**} only exists if $R_0 > 1$.

2.4.1. Local Stability Analysis of the Measles Persistent Equilibrium State (EE)

To establish the local stability of the model EE, we consider again the jacobian matrix:

$$J(x)=\begin{pmatrix} -\left(\frac{\beta I^{**}}{N^{**}}+g_0\right) & \eta & 0 & -\frac{\beta S^{**}}{N^{**}} & 0 \\ \rho & -g_1 & 0 & 0 & 0 \\ \frac{\beta I^{**}}{N^{**}} & 0 & -g_2 & \frac{\beta S^{**}}{N^{**}} & 0 \\ 0 & 0 & \alpha & -g_3 & 0 \\ 0 & \tau & 0 & \gamma & -\mu \end{pmatrix} \tag{18}$$

Now, evaluating $J(x)$ at (ξ^{**}) gives:

$$J(\xi^{**})=\begin{pmatrix} -(m_0+g_0) & \eta & 0 & -m_1 & 0 \\ \rho & -g_1 & 0 & 0 & 0 \\ m_0 & 0 & -g_2 & m_1 & 0 \\ 0 & 0 & \alpha & -g_3 & 0 \\ 0 & \tau & 0 & \gamma & -\mu \end{pmatrix} \tag{19}$$

where: $m_0=\frac{1}{g_1}(g_0g_1-\eta\rho)(R_0-1)$, $m_1=\frac{\beta\mu g_1}{R_0(g_0g_1-\eta\rho)}$

The equilibrium state ξ^{**} will be stable if the matrix in (19) has negative eigenvalues. Expanding the matrix in (19) along the last column gives one negative eigenvalue $-\mu$. Now, the properties of the remaining eigenvalues can be obtained from the reduced matrix

$$J_2=\begin{pmatrix} -(m_0+g_0) & \eta & 0 & -m_1 \\ \rho & -g_1 & 0 & 0 \\ m_0 & 0 & -g_2 & m_1 \\ 0 & 0 & \alpha & -g_3 \end{pmatrix} \tag{20}$$

As in [22]-[24], all the eigenvalues of matrix J_2 will be negative or have negative real parts if the trace of $(J_2)<0$ and the determinant of ($J_2>0$). It is not hard to see from (20) that

$$\begin{cases} \text{trace}(J_2)=-(g_0+g_1+g_2+g_3)+\frac{1}{g_1}(g_0g_1-\eta\rho)(1-R_0) \\ \text{determinant}(J_2)=(R_0-1) \end{cases} \tag{21}$$

It is clear from (21) that $\text{trace}(J_2)<0$ and $\text{determinant}(J_2)>0$ if and only if $R_0>1$. Thus, matrix J_2 admits negative eigenvalues if $R_0>1$. Consequently the measles persistent equilibrium state is locally asymptotically stable whenever it exists ($R_0>1$).

2.4.2. Global Stability Analysis of the Measles-Persistent Equilibrium State

Theorem 2. *If* $R_0>1$, *there exists a unique measles-persistent equilibrium state*

(ξ^{**}) *that is globally asymptotically stable.*

Proof. We first defined a positive definite function $\mathbb{L}$ as:

$$\mathbb{L}(\xi^{**}) = \left[(S - S^{**}) - S^{**}\ln\left(\frac{S}{S^{**}}\right)\right] + \left[(V - V^{**}) - V^{**}\ln\left(\frac{V}{V^{**}}\right)\right] + \left[(E - E^{**}) - E^{**}\ln\left(\frac{E}{E^{**}}\right)\right] + \left[(I - I^{**}) - I^{**}\ln\left(\frac{I}{I^{**}}\right)\right] + \left[(R - R^{**}) - R^{**}\ln\left(\frac{R}{R^{**}}\right)\right]$$

Taking the time derivative of $\mathbb{L}(\xi^{**})$ gives:

$$\begin{aligned}
\frac{\mathrm{d}\mathbb{L}(\xi^{**})}{\mathrm{d}t} &= \left(\frac{S - S^{**}}{S}\right)\frac{\mathrm{d}S}{\mathrm{d}t} + \left(\frac{V - V^{**}}{V}\right)\frac{\mathrm{d}V}{\mathrm{d}t} + \left(\frac{E - E^{**}}{E}\right)\frac{\mathrm{d}E}{\mathrm{d}t} + \left(\frac{I - I^{**}}{I}\right)\frac{\mathrm{d}I}{\mathrm{d}t} + \left(\frac{R - R^{**}}{R}\right)\frac{\mathrm{d}R}{\mathrm{d}t} \\
&= \left(\frac{S - S^{**}}{S}\right)\left[\pi + \eta V - (\Lambda + g_0)S\right] + \left(\frac{V - V^{**}}{V}\right)(\rho S - g_1 V) \\
&\quad + \left(\frac{E - E^{**}}{E}\right)(\Lambda S - g_2 E) + \left(\frac{I - I^{**}}{I}\right)(\alpha I - g_3 I) + \left(\frac{R - R^{**}}{R}\right)(\gamma I + \tau V - \mu R) \\
&= \pi + \eta V - (\Lambda + g_0)S - (\pi + \eta V)\frac{S^{**}}{S} + (\Lambda + g_0)S^{**} + \rho S - g_1 V - \rho S\frac{V^{**}}{V} \\
&\quad + g_1 V^{**} + \Lambda S - g_2 E - \Lambda S\frac{E^{**}}{E} + g_2 E^{**} + \alpha E - g_3 I - \alpha E\frac{I^{**}}{I} + g_3 I^{**} + \gamma I \\
&\quad + \tau V - \mu R - (\gamma I + \tau V)\frac{R^{**}}{R} + \mu R^{**} \\
&= \mathbb{L}^{+} - \mathbb{L}^{-}
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{L}^{+} &= \pi + \eta V + (\Lambda + g_0)S^{**} + \rho S + g_1 V^{**} + \Lambda S + g_2 E^{**} + \alpha E + g_3 I^{**} + \gamma I + \tau V + \mu R^{**} \\
\mathbb{L}^{-} &= (\Lambda + g_0)S + (\pi + \eta V)\frac{S^{**}}{S} + g_1 V + \rho S\frac{V^{**}}{V} + g_2 E + \lambda S\frac{E^{**}}{E} + g_3 I + \alpha E\frac{I^{**}}{I} + \mu R \\
&\quad + (\gamma I + \tau V)\frac{R^{**}}{R} \quad \text{where} \quad \Lambda = \beta\frac{I}{N}
\end{aligned} \tag{22}$$

If we now assume $\mathbb{L}^{+} \le \mathbb{L}^{-}$

then, owing to the positivity of the model parameters, it follows from (22) that $\frac{\mathrm{d}\mathbb{L}(\xi^{**})}{\mathrm{d}t} \le 0$ with the equality holding only when there lations $S = S^{**}$, $V = V^{**}$, $E = E^{**}$, $I = I^{**}$ and $R = R^{**}$ are satisfied Thus, following [21] [25] [24], the model solution set $\{S(t), V(t), E(t), I(t), R(t)\} \to \{S^{**}, V^{**}, E^{**}, I^{**}, R^{**}\}$ as t approaches infinity. This therefore suggests that $\{\xi^{**}\}$ will be a global asymptotic stable equilibrium whenever it exists ($R_0 > 1$).□

3. Local Sensitivity Analysis

In order to help identify the model parameters with high impact on measles dynamics and if possibly target them during any intervention aimed at curtailing measles outbreaks, we carry out sensitivity analysis on R_0 parameters. Using the normalized forward sensitivity index relation:

$$\Gamma_x^{R_0} = \frac{\partial R_0}{\partial x} \times \frac{x}{R_0} \tag{23}$$

Using the formular in (23), we generate the indices of (R_0) parameters as indicated in Table 3 below:

Table 3. The values of the elasticity/sensitivity indices.

Parameter	Elasticity/Sensitivity Index
α	+0.0566
β	+1.0000
η	+0.5181
μ	+0.8709
δ	−0.1689
γ	−0.8108
ρ	−0.9629
τ	−0.5030

4. Evaluating the Impact of Vaccination on Measles Transmission

To evaluate the significance of the measles first dose vaccination campaign, we compare the reproductive number of the disease for the model with and without vaccination. Now, system (1) in the absence of vaccination becomes:

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = \pi - \left(\frac{\beta I}{N} + \mu\right)S \\ \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\beta IS}{N} - g_2 E \\ \frac{\mathrm{d}I}{\mathrm{d}t} = \alpha E - g_3 I \\ \frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I - \mu R \end{cases} \tag{24}$$

The Measles-Free Equilibrium and R_0 of the Model without Vaccination

The disease-free equilibrium of system (24) denoted by

$\xi^0 = \left(S^0, E^0, I^0, R^0\right) = \left(\frac{\pi}{\mu}, 0, 0, 0\right)$. Substituting S^* into (6) by $S^0 = \frac{\pi}{\mu}$ and computing the reproductive ratio of the model (24), we obtain the uncontrolled reproductive number denoted by:

$$R_{0wv} = \frac{\alpha\beta}{g_2 g_3} = \frac{\alpha\beta}{(\alpha+\mu)(\gamma+\mu+\delta)} \tag{25}$$

Now, comparing the expression of R_0 from (8) to that of R_{0wv} from (25), we see that:

$$\begin{aligned} R_0 &= \frac{\alpha\beta\mu g_1}{g_2 g_3 (g_0 g_1 - \eta\rho)} = \frac{\mu g_1}{(g_0 g_1 - \eta\rho)} \frac{\alpha\beta}{g_2 g_3} \\ &= \frac{\mu g_1}{(g_0 g_1 - \eta\rho)} R_{0wv} = \frac{\mu g_1}{\mu g_1 + \rho(\tau+\mu)} R_{0wv} \\ &= \frac{\mu(\eta+\tau+\mu)}{\mu(\eta+\tau+\mu)+\rho(\tau+\mu)} R_{0wv} \end{aligned} \tag{26}$$

Since $\frac{\mu(\eta+\tau+\mu)}{\mu(\eta+\tau+\mu)+\rho(\tau+\mu)} < 1$, it means that R_0 is a decreasing function of R_{0wv}. Hence, an effective first dose vaccination campaign for measles reduces the spread of the disease by $\frac{\mu(\eta+\tau+\mu)}{\mu(\eta+\tau+\mu)+\rho(\tau+\mu)}$ [24].

5. Numerical Simulations Results

To investigate the dynamical evolution of the model population sub-classes, we performed some numerical simulations. To achieve this, the following initial population sizes are considered: $S(0)=1000$, $V(0)=250$, $E(0)=500$, $I(0)=100$ and $R(0)=0$ with the parameter values provided in **Table 2**. Some of these parameters are estimated from the Bono region of Ghana measles surveillance data. The graphical results are shown from **Figures 2-11**. **Figures 2-6** show the behavior of the model sub-classes at different rates of measles vaccination. These results suggest that high vaccination rate is needed to keep measles susceptibility rate down. Also, **Figures 7-11** give the effect of varying contact rate on the disease dynamics in the community. It is clear from these graphs that high contact rates

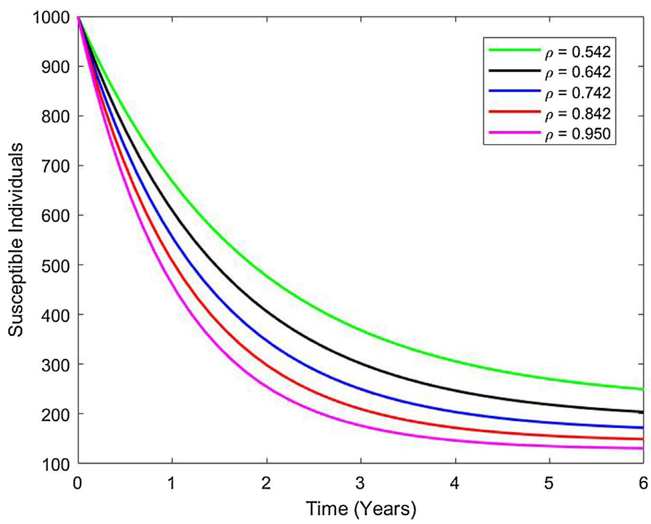

Figure 2. Effect of perturbing ρ on susceptible sub-population.

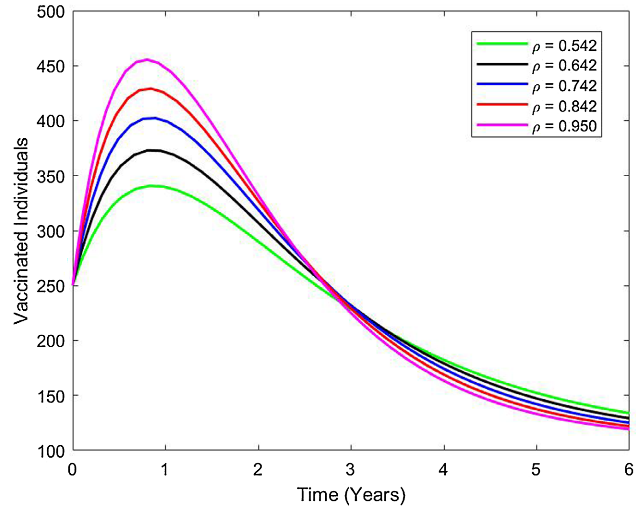

Figure 3. Effect of perturbing ρ on vaccinated sub-population.

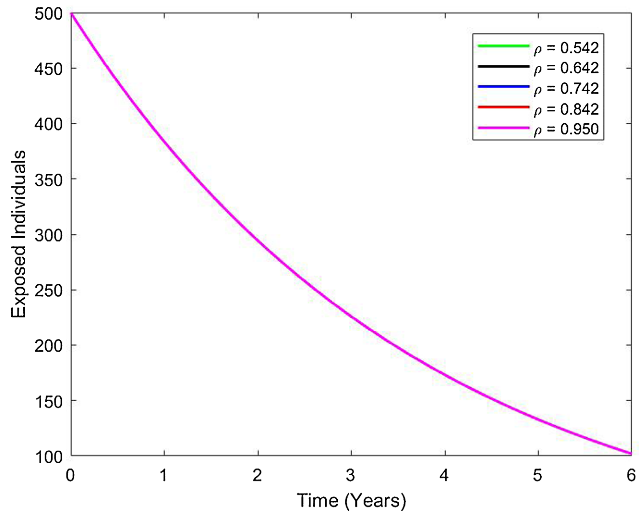

Figure 4. Effect of perturbing ρ on exposed sub-population.

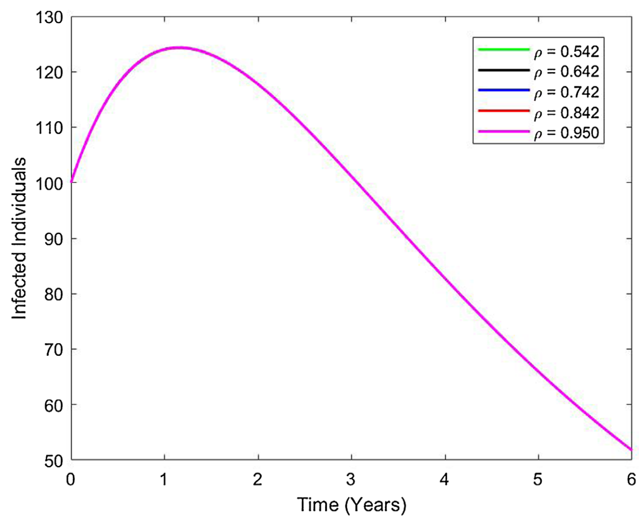

Figure 5. Effect of perturbing ρ on infected sub-population.

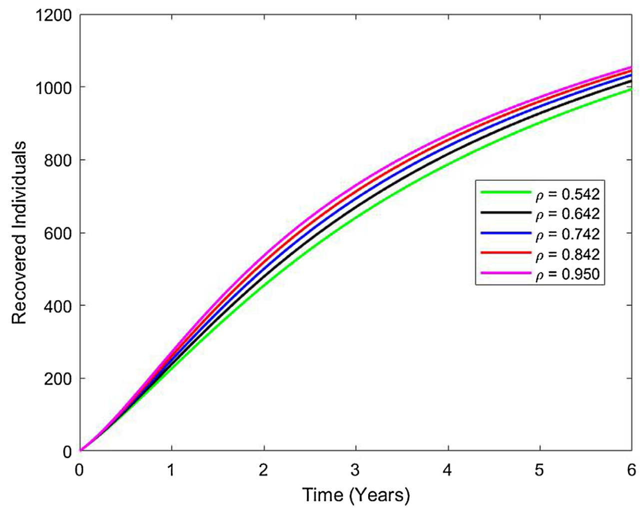

Figure 6. Effect of perturbing ρ on recovered sub-population.

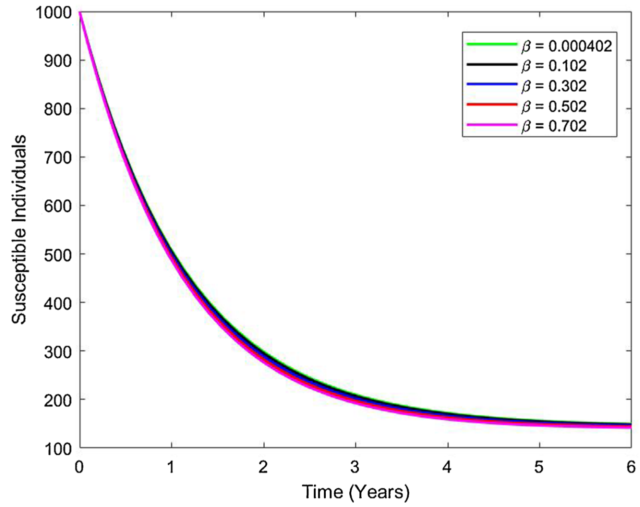

Figure 7. Effect of perturbing β on susceptible sub-population.

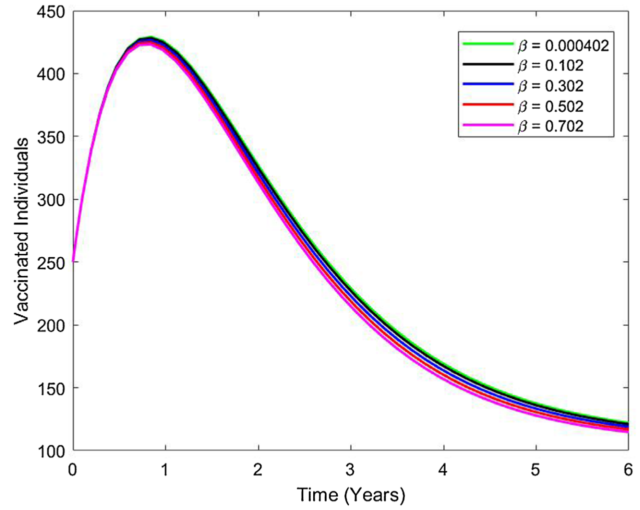

Figure 8. Effect of perturbing ρ on vaccinated sub-population.

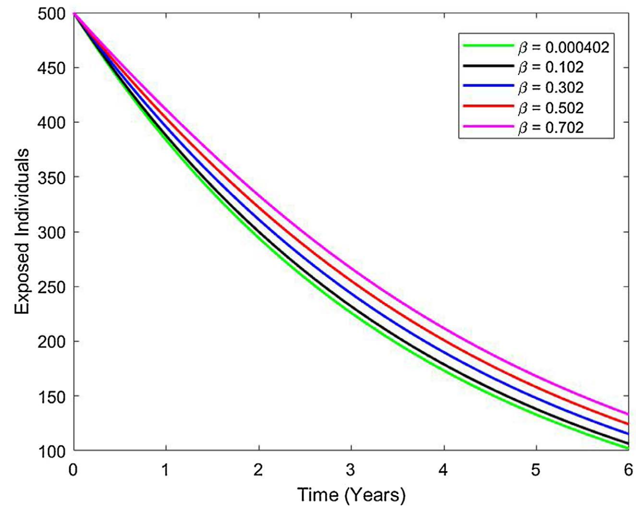

Figure 9. Effect of perturbing β on exposed sub-population.

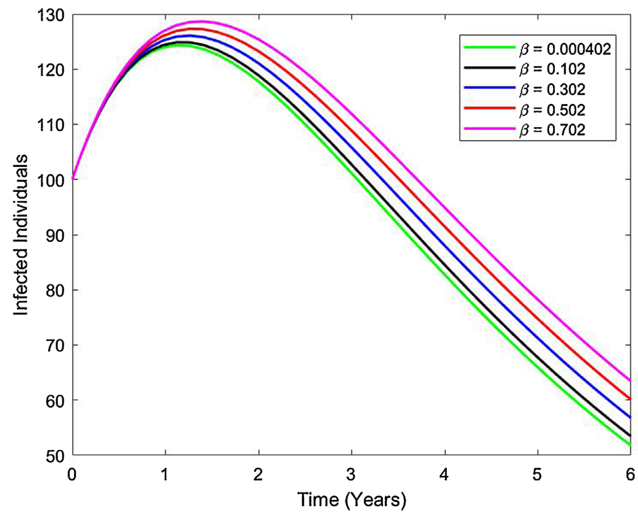

Figure 10. Effect of perturbing ρ on infected sub-population.

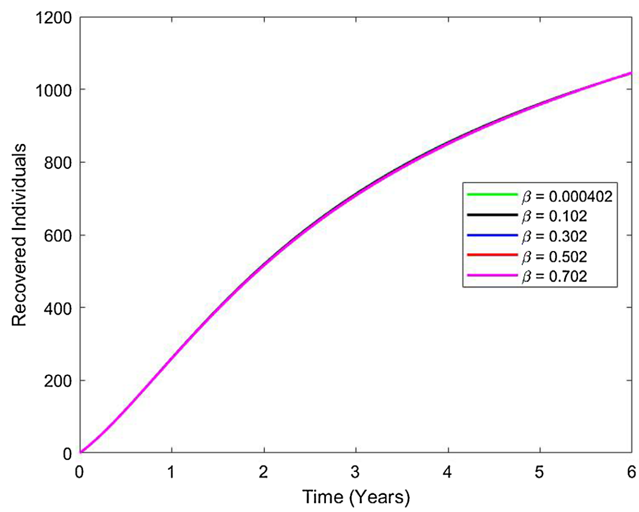

Figure 11. Effect of perturbing β on recovered sub-population.

keep the number of exposed and infected individuals high.

6. Conclusions and Future Research

In this work, a dynamical model for measles control using immunization in a community is formulated and analysed. Concerning the model analysis, we first verified the fundamental qualitative properties of an epidemiological model. We computed the measles infection-free for the model and determined its local and global stability conditions with reference to the magnitude of the model reproductive ratio R_0. We further computed a unique measles-persistent equilibrium in terms of R_0, and proved that this endemic equilibrium state always remained locally and globally stable whenever it exists ($R_0 > 1$). Additionally, we ascertained the significance of using effective vaccination as a control measure for measles by deriving the expression of a reduction factor in the disease spread resulting from the implementation of measles immunization program in a community. The analysis of the sensitivity indices suggested that the contact rate between a susceptible and an infected person followed by measles vaccination rate is most influential in the transmission and control of measles respectively. The local and global asymptotic stability of both the measles-free and persistent equilibria states indicates the feasibility of eradicating measles in the society. Hence, there is a need for combined support from both governments/organizations and the public to strengthen the existing measles surveillance systems and immunization campaign for a measles-free world to be achieved.

For future research, any national data could be used to fit the model. Furthermore, the model could be extended to include the measles vaccine booster, maternally protected babies, and measles treatment.

Author Contribution(s)

MK: Conceptualization, Data Curation, Formal Analysis, Methodology, Resources, Software, Validation; RIMG: Conceptualization, Data Curation, Formal Analysis, Methodology, Resources, Software, Visualization, Writing—Original Draft Preparation, Writing—Review & Editing; AI: Conceptualization, Supervision, Writing—Original Draft Preparation, Writing—Review & Editing

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Data

Data used for our analysis is obtained from measles published works and have been duly referenced.

Conflicts of Interest

The authors declare no conflicts of interest.

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