Open Access Library Journal
2026, Volume 13, e14882
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Two Metres Apart: A Rigorous Topological and Metric Framework for Physical Distancing Policies

Samuel O. Adeyemo, Amarachukwu I. O. Ofomata, Prisca Duruojinkeya, Chinwe B. Okereke

Department of Mathematics and Statistics, Federal Polytechnic, Nekede, Owerri, Imo State, Nigeria
Email: oadeyemo@fpno.edu.ng

How to cite this paper: Adeyemo, S.O., Ofomata, A.I.O., Duruojinkeya, P. and Okereke, C.B. (2026) Two Metres Apart: A Rigorous Topological and Metric Framework for Physical Distancing Policies. *Open Access Library Journal*, **13**: e14882.
https://doi.org/10.4236/oalib.1114882

Received: January 15, 2026
Accepted: February 11, 2026
Published: February 14, 2026

Abstract

The feasibility of physical-distancing interventions during the COVID-19 pandemic implicitly relied on structural properties of physical space. We show that minimum-distance policies (usually formulated as "individuals must remain at least δ metres apart") require not only the Hausdorff property but a compatible metric structure that supports uniform separation with a positive lower bound. Using results from metric topology, we prove that the existence of disjoint open neighbourhoods is necessary but insufficient for well-posed quantitative distancing constraints. We demonstrate using quotient and identification topologies arising in epidemiological modelling (like grid aggregation, mean-field limits) that distancing can become ill-defined even when Hausdorffness is preserved. We discuss implications for spatial epidemiology, where metric and topological assumptions are typically implicit but essential for distance-dependent transmission kernels.

Subject Areas

Algebra, Mathematics

Keywords

Hausdorff Space, Physical Distancing, COVID-19, Mathematical Epidemiology, Metric Topology, Uniform Separation, Quotient Topology

1. Introduction

Physical distancing, one of the most widely implemented non-pharmaceutical interventions during the COVID-19 pandemic, requires maintaining a minimum separation between individuals. Meta-analyses (e.g., [1] [2]) consistently show reduced transmission risk at separations of at least 1 - 2 metres. While these findings rest on epidemiological and statistical modelling, the mathematical feasibility of such constraints depends on a deeper structural assumption: that points representing individuals in physical space can be separated by non-overlapping metric neighbourhoods of controlled size.

This assumption is both topological and metric. In the Euclidean metric on $\mathbb{R}^3$, any two distinct points admit disjoint open balls. That is, $\mathbb{R}^3$ is Hausdorff and metrizable. The Hausdorff property ensures qualitative separability, while the metric provides quantitative scale. Without both, neighbourhoods could intersect unavoidably or collapse under identification, rendering fixed-distance policies inconsistent.

This article develops a rigorous topological and metric account of physical distancing. We formalize constraints in terms of uniform metric separation, show that Hausdorffness is necessary but insufficient, and illustrate pathologies in quotient spaces common in epidemic modelling.

2. Preliminaries

2.1. Topological Spaces

A topological space (X,τ) satisfies:

- $\varnothing, X \in \tau$,
- arbitrary unions of members of τ lie in τ,
- finite intersections of members of τ lie in τ.

2.2. Metric Topology and Open Balls

For a metric space (X,d), the open balls

$$B(x,r)=\{y \in X \mid d(x,y)<r\} \tag{1}$$

form a basis for a topology τ_d. In $\mathbb{R}^3$, this yields the standard Euclidean topology modelling physical space at macroscopic scales [3].

2.3. Hausdorff Property

A space is Hausdorff (T_2) if for all $x, y \in X$ there exist open neighbourhoods $U \ni x$ and $V \ni y$ with $U \cap V = \varnothing$

Proposition 2.1

All metric spaces are Hausdorff

Proof:

Let (X,d) be a metric space. If $x \neq y$, set $r=\frac{d(x,y)}{2}$. Then $B(x,r)$ and $B(y,r)$ are disjoint, as any common point z would imply $d(x,y) \le d(x,z)+d(z,y)<d(x,y)$, a contradiction. ■

3. A Metric and Topological Formulation of Physical Distancing

Let individuals be points in a metric space (X,d). A minimum-distance policy with threshold $\delta > 0$ requires $d(x, y) \geq \delta$ for all distinct x, y.

Definition 3.1 (Uniform Separation)

A metric space is uniformly separated (with parameter $\delta > 0$) if $d(x, y) \geq \delta$ for all distinct x, y. This property mathematically encodes the core requirement of physical-distancing policies [4].

Theorem 3.1

Let (X, d) be a metric space. A minimum-distance policy with threshold $\delta > 0$ is equivalent to the existence of disjoint open balls $B\left(x, \frac{\delta}{2}\right)$ and $B\left(y, \frac{\delta}{2}\right)$ for all distinct x, y with $d(x, y) \geq \delta$. This condition implies Hausdorffness but is strictly stronger.

Proof

$\Rightarrow$ If $d(x, y) \geq \delta$, then $B\left(x, \frac{\delta}{2}\right) \cap B\left(y, \frac{\delta}{2}\right) = \varnothing$ (triangle inequality).

$\Leftarrow$ If such disjoint balls exist whenever $d(x, y) \geq \delta$, then the centers are separated by at least δ. Since metric balls are open, this gives Hausdorffness. However, there exist Hausdorff spaces (e.g., certain non-metrizable ones or metric spaces without a positive infimum on distances) that lack uniform separation for any $\delta > 0$. Hence the implication is strict. ■

Here is a standard illustration of disjoint personal zones in the Euclidean plane. (See Figure 1)

Here is a standard illustration of disjoint personal zones in the Euclidean plane:

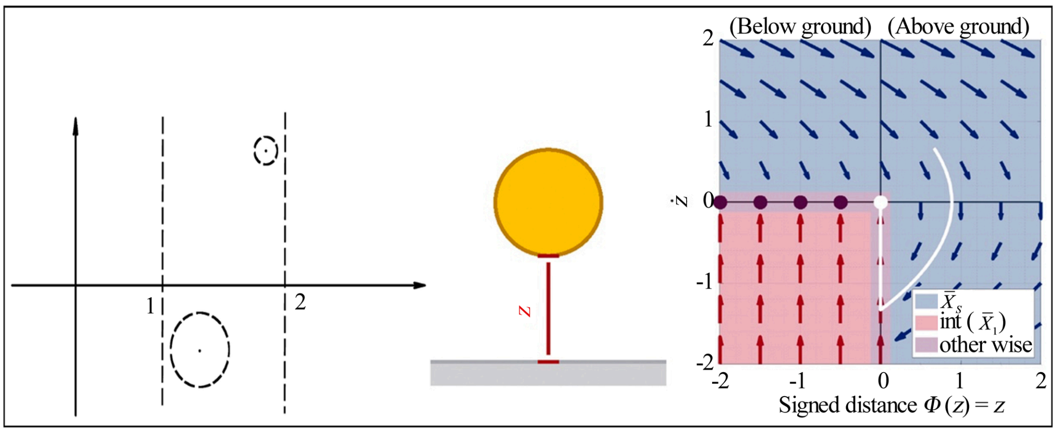

Figure 1. Disjoint open balls of radius $\frac{\delta}{2}$ around distinct points x and y separated by at least δ enforcing a minimum-distance policy (e.g., 2 meters) in Euclidean space.

Epidemiological Interpretation

In spatial SIR/SEIR models, transmission is often governed by a kernel $\beta(d(x, y))$ that decays with distance (e.g., exponential or threshold-based), becoming near-zero beyond δ. This presupposes metric balls of controlled radius exist and do not collapse. However, real models tolerate probabilistic overlaps for crowding and mobility.

Here is an example of such a distance-dependent transmission kernel. (See Figure 2)

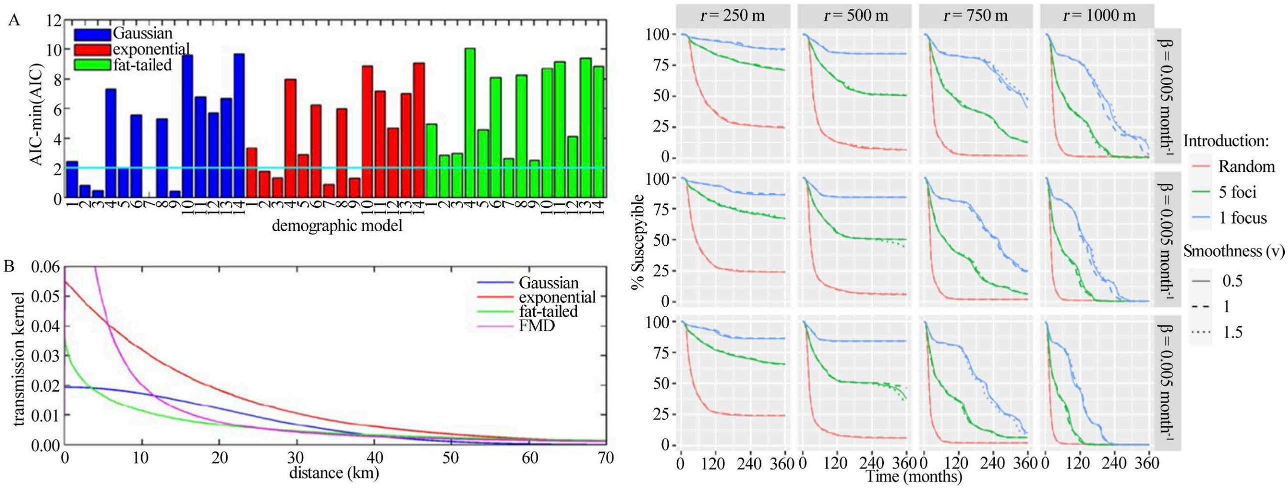

Figure 2. Distance-dependent transmission probability $\beta(d)$ decreasing to near-zero beyond threshold δ, as commonly used in spatial epidemiological models.

4. Distancing in Quotient and Identification Spaces

Many epidemiological models use coarse-grained or aggregated spaces via quotient topologies.

Example 4.1 Grid Aggregation

Divide $\mathbb{R}^2$ into square cells and identify all points within each cell. The resulting quotient space may remain Hausdorff, but distances inside a cell become undefined, destroying uniform separation locally.

Example 4.2 Mean-Field Identification

In mean-field approximations, individuals in the same compartment are identified, collapsing metric structure and preventing minimum-distance enforcement.

Example 4.3 Particle Systems

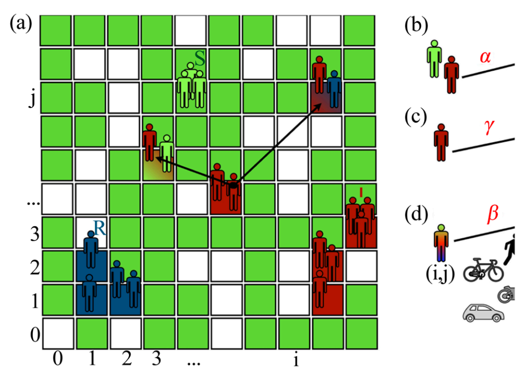

Figure 3. Grid-based coarse-graining: Points within each cell are identified in the quotient space, making local uniform separation ill-defined (relevant to lattice and metapopulation models).

Configuration spaces of identical particles often quotient by permutations, where uniform separation fails even if the space is Hausdorff.

Here is a visual representation of grid aggregation leading to identification. (See Figure 3)

Corollary 4.4

Distance-based physical-distancing models require not only Hausdorffness but a metric that survives aggregation and identification procedures [5].

5. Implications for Mathematical Epidemiology

Distance-dependent kernels $\beta(d(x,y))$ demand a true metric on the agent space. Aggregation must preserve metric separation; otherwise, distancing cannot be meaningfully represented. Future models should consider these structural requirements, especially in coarse-grained or manifold-based simulations [6].

6. Conclusion

Physical distancing is a mathematical constraint on the modelling space. Hausdorffness provides qualitative separability, while a compatible metric with uniform separation ensures quantitative enforcement. Both are essential for well-posed distance-based policies in spatial epidemiology. By clarifying these assumptions, we offer a more rigorous foundation for epidemic modelling, highlighting how implicit topological and metric structures underpin real-world public-health interventions.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Chu, D.K., Duda, S., Solo, K., Yaacoub, S. and Schunemann, H. (2020) Physical Distancing, Face Masks, and Eye Protection to Prevent Person-to-Person Transmission of Sars-Cov-2 and COVID-19: A Systematic Review and Meta-Analysis. *Journal of Vascular Surgery*, **72**, Article 1500-1555. https://doi.org/10.1016/j.jvs.2020.07.040

[2] Qian, H., Miao, T., Liu, L., Zheng, X., Luo, D. and Li, Y. (2020) Indoor Transmission of Sars-Cov-2. *Indoor Air*, **31**, 639-645. https://doi.org/10.1111/ina.12766

[3] Munkres, J.R. (2000) Topology. 2nd Edition, Prentice Hall.

[4] Keeling, M.J. and Rohani, P. (2008) Modeling Infectious Diseases in Humans and Animals. Princeton University Press.

[5] Kiss, I.Z., Miller, J.C. and Simon, P.L. (2017) Mathematics of Epidemics on Networks: From Exact to Approximate Models. Springer.

[6] Bressloff, P.C. (2014) Stochastic Processes in Cell Biology. Springer