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Stochastic Model of Evolution of Universe

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Abstract

In the paper, on the basis of the notion of entropy of topological spaces, constructed quantum model of evolution of the Universe. In the class of cobordant Riemannian manifolds we built the discrete random process whose probability space in discrete time moment n contains cobordant Riemannian manifolds with entropy less than n. Realizations of this random process are sequences of cobordant Riemannian manifolds. In this work, the entropy of realization (trajectory) of a random process is defined. This made it possible to construct a model of the evolution of the Universe as a quantum system.

Subject Areas

Modern Physics

Keywords

Entropy, Random Process, Cobordant Manifolds, Universe

1. Introduction

In work [1], the notion of topological entropy, which we called the entropy of the compact topological space X. Let a topological space X admit a pseudo convex open covering; such coverings consist of contractible open sets whose intersection is also contractible [1]. There exist topological spaces that admits such a covering, for example: local convex vector topological spaces [2], such as the number

$$H(X)=\frac{n}{m}$$

where n is minimal number of elements of open pseudo-convex coverings among all finite pseudo-convex coverings of X and m is the number of orbits in every n element-containing coverings under the actions by covering preserving corresponding homeomorphisms. For example, the entropy of a circle is 3, and the entropy of a circle with a tail is 4/2 = 2, the entropy of 2-dimensional sphere is 4/1 = 4, the entropy of a 2-dimensional sphere with a tail is 5/2, the entropy of 2-dimensional torus is 18, the entropy of 2 dimensional torus whit tall is 18/2 = 9, In the case this topological spaces number of elements of minimal pseudo convex covering is respectively 3, 4, 4, 5, 18, the number of orbits in each case are 1, 2, 1, 2 1, 2.

The concept of entropy for phase spaces of dynamical systems was used by us to describe the evolution of closed physical systems. By means of the concept of entropy, a random process was constructed that described the evolution of such systems in time [1] [3]. The physical motivation for such a definition of entropy is the increase of the homogeneity of the phase space of the system with the increase of such entropy, as in the case of the increase of thermodynamic and informational entropy.

2. Random Process of Evolution of Phase Spies Closed Dynamic Systems

In the category of n dimensionally closed differentiable manifolds and diffeomorphisms every differentiable manifold admits pseudo-convex cover, and we can define the entropy for elements of this category. Let new on every element of this category Riemannian metric tensor field [4] is given. Denote such a manifold so $\left(M^n, g\right)$ where g Riemannian metric tensor field. The entropy of such manifolds is also determined, but metric tensor fields are preserved additionally by the action of diffeomorphisms, which in this case will be isometries. We say that two n-dimensional Riemannian manifolds are cobordant [5] if there exists an $n+1$ dimensional manifold with Lorentz metric whose some sections by space-like plane [6] are Riemannian manifolds with metrics consistent with the given Lorentz metric, because the restriction of Lorentz metric on such sections is positively determined. The example of such Lorentz manifolds is also the warped product of the real axis R with metric $\left(M^n, g\right)$ and complete Riemannian manifold $\left(M^n, g\right)$. Denote exist by $\left[\left(M^n, g\right)\right]$ class of cobordant to $\left(M^n, g\right)$ manifolds.

Let sequence of Riemannian manifolds,

$$\left(M_1^n, g_1\right), \left(M_2^n, g_2\right), \cdots, \left(M_k^n, g_k\right), \cdots$$

from class of cobordiant manifolds $\left[\left(M^n, g\right)\right]$ represents evolution phase space of closed system in discrete time. If $H\left(M_i^n, g\right), i = 1, 2, \cdots$ the entropy of manifold, $\left(M_i^n, g_i\right)$, then

$$H\left(M_1^n, g_1\right) < H\left(M_2^n, g_2\right) < \cdots < H\left(M_i^n, g_i\right) < \cdots.$$

Every manifold has its entropy. If number of elements of pseudo-convex cover of manifolds from $\left[\left(M^n, g\right)\right]$ is natural number $k \in N$) then the number of numbers representing the entropy of such manifolds is finite.

We denote subclasses of manifolds from $\left[\left(M^n, g\right)\right]$ which pseudo convex covers have $k \in N$ elements, and have the same entropy so $\left[M^n, g\right]_{j_k}^k$, index

$j_k = 1,2,\cdots,q_k$ where q_k number of this classes. Number of this classes is finite. The common entropy we denote so $H\left[M^n,g\right]^k_{j_k}$ $k = 1,2,3,\cdots$.

If sequence of Riemannian manifolds

$$\left(M_1^n,g_1\right),\left(M_2^n,g_2\right),\cdots,\left(M_k^n,g_k\right),\cdots$$

where

$$H\left(M_1^n,g_1\right) < H\left(M_2^n,g_2\right) < \cdots < H\left(M_i^n,g_i\right) < \cdots.$$

describes evolution of a closed system. Some members of first sequence may belong to the same subclass $\left[M^n,g\right]^k_{j_k}$.

On the set of subclass $\left\{\left[M^n,g\right]^k_{j_k}\right\}$, $j_k = 1,2,\cdots,q_k$, we can determine probability measure [7] [8] μ_k in this way:

$$\mu_k\left(\left[M^n,g\right]^k_{j'_k}\right) = \frac{H\left[M^n,g\right]^k_{j'_k}}{\sum_{j_k=1}^{q_k} H\left(\left[M^n,g\right]^k_{j_k}\right)},$$

where q_k number of classes, the measure of subsets of the set $\left\{\left[M^n,g\right]^k_{j_k}\right\}$, $j_k = 1,2,\cdots,q_k$

$$\mu_k\left(\left\{\left[M^n,g\right]^k_{j_k}\right\}, j_k = l_1,l_2,\cdots,l_t \le q_k\right) = \sum_{i=1}^{i \le q_k} \frac{H\left[M^n,g\right]^k_{l_i}}{\sum_{j_k=1}^{q_k} H\left[M^n,g\right]^k_{j_k}}.$$

Let $\Re_k$ is the set of $\left\{\left[M^n,g\right]^k_{j_k}\right\}$, $j_k = 1,2,\cdots,q_k$ of ever possible subclasses of manifolds with pseudo-convex covers that contains $k \in N$ elements. For every such set for $k = 1,2,\cdots$ probability measure is defined as above. Denote this measurable space $\Re_k$ with probability measure μ_k so $\left(\Re_k,\sigma\left(\Re_k\right),\mu_k\right)$ where $\sigma\left(\Re_k\right)$ is the σ-algebra in $\Re_k$ which in this case coincides the set of every subsets of $\Re_k$ (the section and union of all members of a sequence of subsets of $\Re_k$ belong to $\Re_k$ eve). The sequence $\Re_1,\Re_2,\cdots,\Re_k,\cdots$ represent the random process [9] with matrix of transitions

$$\mu_k\left(\left[M^n,g\right]^k_{j_k}\right) < \mu_{k+1}\left(\left[M^n,g\right]^{k+1}_{j_{k+1}}\right),$$

where

$$\mu_k\left(\left[M^n,g\right]^k_{j_k}\right) < \mu_{k+1}\left(\left[M^n,g\right]^{k+1}_{j_{k+1}}\right),$$

i.e.,

$$H\left[M^n,g\right]^k_{j_k} < H\left[M^n,g\right]^{k+1}_{j_{k+1}}.$$

Consider the one realization

$$\left[M^n, g\right]_{j_1}^1, \left[M^n, g\right]_{j_2}^2, \cdots, \left[M^n, g\right]_{j_k}^k, \cdots$$

If from all these classes we will take one element, we will receive Sequence

$$\left(M_1^n, g_1\right), \left(M_2^n, g_2\right), \cdots, \left(M_k^n, g_k\right), \cdots$$

which we can consider as a realization of this random process with an accuracy of up to entropy:

$$H_{hm}\left(M_1^n, g_1\right) < H_{hm}\left(M_2^n, g_2\right) < \cdots < H_{hm}\left(M_k^n, g_k\right) < \cdots.$$

The change in the index k describes the passage of discrete time. Also increase k causes increase maximum of entropy in the sets $\left\{\left[M^n, g\right]_{j_k}^k\right\}$.

Because the manifolds

$$\left(M_1^n, g_1\right), \left(M_2^n, g_2\right), \cdots, \left(M_k^n, g_k\right), \cdots,$$

are in the same class of each other cobordisms $\left[\left(M^n, g\right)\right]$, there is $n+1$ dimensional Lorentz manifold, whose sections by space-similar plane are given Riemannian manifolds $\left(M_k^n, g_k\right), k = 1, 2, \cdots$ with metrics consistent with the given Lorentz metric. For example, such is the global hyperbolic space-time, or $n+1$ dimensional Lorentz manifold with n dimensional sub manifolds whose tangent planes consist only of space-like vectors [6].

It is easy to see that the possible maximal entropy H among elements of $\Re_k$ more than the possible maximal entropy H among elements of $\Re_{k-1}$, in first case it is k, in second case $k-1$.

3. Stochastic Model of Evolution of Universe as Quantum System

Consider now the random process $\Re_1, \Re_2, \cdots, \Re_k, \cdots$ Let $\left(M_1^n, g_1\right), \left(M_2^n, g_2\right), \cdots, \left(M_k^n, g_k\right), \cdots$ a realization of this random process with an accuracy of up to entropy describes evolution of phase spaces of closed dynamic system

On the infinite product $\underset{k \in n}{\times} \Re_k$ represents space with probability measure. There exists the unique probability measure $\mu = \underset{k \in N}{\times} \mu_k$ on the product $\underset{k \in N}{\times} \Re_k$ which coincides with the elements of cylindrical sets [10] [11]

The semi algebra of cylindrical sets consists of subsets A of $\underset{k \in N}{\times} \Re_k$ with form $A = \underset{k \in N}{\times} A_k$ where $A_k \in \sigma\left(\Re_k\right)$ and $A_k \neq \Re_k$ for only finite number of index value k. We denote this semi algebra so $B\left(\underset{k \in N}{\times} \Re_k\right)$. This semi-algebra defines smallest σ-algebra in $\underset{k \in N}{\times} \Re_k$ [10] [11].

In article [1], we define notion entropy of trajectory of evolution of closed system as entropy of one realization of random process that describes this evolution:

If

$$A = \underset{k\in N}{\times} A_k = A_{k_1} \times A_{k_2} \times \cdots \times A_{k_m} \times A_{k_{m+1}} = \Re_{m+1} \times A_{k_{M+2}} = \Re_{m+1} \times \cdots,$$

where

$$A_{k_i} = \left\{ \left[M^n, g \right]_{j_{k_i}}^{k_i} \right\}, j_{k_i} = l_1, l_2, \cdots, l_{t_i} \le q_{k_i}, i = 1, 2, \cdots, m,$$

then

$$\mu(A) = \mu_{k_1}\left(A_{k_1}\right) \cdot \mu_{k_2}\left(A_{k_2}\right) \cdot \ldots \cdot \mu_{k_m}\left(A_{k_m}\right).$$

Consider the set

$$\underset{k\in N}{\cup} A_{s_k}^k = \underset{k\in N}{\times} \Re_k, A_1 = \left[M^n, g\right]_{j_{i_1}}^1, A_2 = \left[M^n, g\right]_{j_{i_2}}^2, \cdots, A_{k-1} = \left[M^n, g\right]_{j_{i_{k-1}}}^k,$$

$$A_k = \Re_k \setminus \left[M^n, g\right]_{j_{i_k}}^k, A_{k+1} = \Re_{k+1}, A_{k+2} = \Re_{k+2}, \cdots$$

$$k \in N, s_k = \left(j_{i_1}, j_{i_2}, \cdots, j_{i_k}\right), j_{i_l} = 1, 2, \cdots, q_k; l = 1, 2, \cdots, k.$$

If k is fixed and $s_k = \left(j_1, j_2, \cdots, j_{l_k}\right)$ changes, then the family $\left\{A_s^k\right\}$ consists of pairwise disjoint elements, is finite and on this family we can determine probability space

$$\left\{A_s^k, \sigma\left(A_s^k\right), \mu^k\right\}, \mu^k = \mu_{j_1} \cdot \mu_{j_2} \cdot \ldots \cdot \mu_{j_k}$$

Consider random variable

$$h^k\left(A_{s_k}^k\right) = H\left[M^n, g\right]_{j_{i_1}}^1 \cdot H\left[M^n, g\right]_{j_{i_2}}^2 \cdot \ldots \cdot H\left[M^n, g\right]_{j_{i_k}}^k,$$

with distribution

$$h^k\left(A_{s_k}^k\right) = H\left[M^n, g\right]_{j_{i_1}}^1 \cdot H\left[M^n, g\right]_{j_{i_2}}^2 \cdot \ldots \cdot H\left[M^n, g\right]_{j_{i_k}}^k \to \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}}.$$

The mathematical expectation of this random variable

$$M\left(h^k\right) = \sum_{s_k} \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} H\left[M^n, g\right]_{j_{i_1}}^1 \cdot H\left[M^n, g\right]_{j_{i_2}}^2 \cdot \ldots \cdot H\left[M^n, g\right]_{j_{i_k}}^k$$
$$= M\left(h_{j_{i_1}}^1\right) \cdot M\left(h_{j_{i_2}}^2\right) \cdot \ldots \cdot M\left(h_{j_{i_k}}^k\right).$$

If we consider the finite trajectory as finite sequence

$$\left[M^n, g\right]_{j_{i_1}}^1 \cdot \left[M^n, g\right]_{j_{i_2}}^2 \cdot \ldots \cdot \left[M^n, g\right]_{j_{i_k}}^k,$$

whose entropies of members are closest to corresponding mathematical expectations in their product, then this trajectory will be more probable.

If we consider the family of sets $\left\{\overline{A}_{\overline{s}_k}^k\right\}_{k\in N}$ in semi algebra $\underset{k\in N}{\times} \Re_k$, where

$$h^k\left(A_{s_k}^k\right) = H\left[M^n, g\right]_{j_{i_1}}^1 \cdot H\left[M^n, g\right]_{j_{i_2}}^2 \cdot \ldots \cdot H\left[M^n, g\right]_{j_{i_k}}^k$$

$$\overline{A}_{\overline{s}_k}^k = \underset{i\in N}{\times} A_i, A_1 = \left[M^n, g\right]_{j_{i_1}}^1, A_2 = \left[M^n, g\right]_{j_{i_2}}^2, \cdots, A_{k-1} = \left[M^n, g\right]_{j_{i_{k-1}}}^k,$$

$$A_k = \Re_k \setminus \left[M^n, g\right]_{j_{i_k}}^k, A_{k+1} = \Re_{k+1}, A_{k+2} = \Re_{k+2}, \cdots$$

$$k \in N, \overline{s}_k = \left(j_{i_1}, j_{i_2}, \cdots, j_{i_k}\right), j_{i_l} = 1, 2, \cdots, q_k; l = 1, 2, \cdots, k.$$

We have inclusion $\left\{\overline{A}^k_{\overline{s}_k}\right\}_{k\in N} \subset \left\{\overline{A}^k_{s_k}\right\}_{k\in N}$ where $\overline{s}_k = j_{i_1}, j_{i_2}, \cdots, j_{i_k}$ is fixed. Let's assume. In this family $\left\{\overline{A}^k_{\overline{s}_k}\right\}_{k\in N}$ we mean k changes during the time that $\overline{s}_k = j_{i_1}, j_{i_2}, \cdots, j_{i_k}$ is fixed.

$$\bigcup_{k\in N} \overline{A}^k_{\overline{s}_k} = \mathop{\times}_{k\in N} \Re_k ,\ \mu\left(A^k_{\overline{s}_k}\right) = \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} .$$

From the fact that we are dealing with pairwise disjoint sets, we have:

$$\sum_{k\in N} \sum_{\overline{s}_k} \mu\left(A^k_{\overline{s}_k}\right) = \sum_{k\in N} \sum_{\overline{s}_k} \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} = \sum_{k\in N} \sum_{j_{i_1}} \sum_{j_{i_2}} \cdots \sum_{j_{i_k}} \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} = 1$$

The family $\left\{\overline{A}^k_{\overline{s}_k}\right\}_{k\in N}$ is probability space

$$\left(\left\{\overline{A}^k_{\overline{s}_k}\right\}_{k\in N}, \sigma\left(\left\{\overline{A}^k_{\overline{s}_k}\right\}\right), \mu\right), \mu\left(\overline{A}^k_{\overline{s}}\right) = \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} .$$

Consider random variable:

$$h_{\overline{s}_k} : \left\{\overline{A}^k_{\overline{s}_k}\right\} \to \left\{ H\left[M^n, g\right]^1_{j_{i_1}} \cdot H\left[M^n, g\right]^2_{j_{i_2}} \cdot \ldots \cdot H\left[M^n, g\right]^k_{j_{i_k}} , j_{i_l} = 1, 2, \cdots, q_l \right\},$$

where

$$h_{\overline{s}_k}\left(A^k_{s_k}\right) = H\left[M^n, g\right]^1_{j_{i_1}} \cdot H\left[M^n, g\right]^2_{j_{i_2}} \cdot \ldots \cdot H\left[M^n, g\right]^k_{j_{i_k}} ,$$

If $k \neq 1$, and, if $k = 1$, with distribution

$$H\left[M^n, g\right]^1_{j_{i_1}} \cdot H\left[M^n, g\right]^2_{j_{i_2}} \cdot \ldots \cdot H\left[M^n, g\right]^k_{j_{i_k}} \to \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} .$$

The value $H\left[M^n, g\right]^1_{j_{i_1}} \cdot H\left[M^n, g\right]^2_{j_{i_2}} \cdot \ldots \cdot H\left[M^n, g\right]^k_{j_{i_k}}$ increases if increase k, the value $\mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}}$ decreases if increase k.

The mathematical expectation of this random variable will be

$$M\left(h_{\overline{s}_k}\right) = \sum_{k\in N} H\left[M^n, g\right]^1_{j_{i_1}} \cdot H\left[M^n, g\right]^2_{j_{i_2}} \cdot \ldots \cdot H\left[M^n, g\right]^k_{j_{i_k}} \mu_{j_{i_1}} \cdot \mu_{j_{i_2}} \cdot \ldots \cdot \mu_{j_{i_k}} .$$

We call this mathematical expectation $M\left(h_{\overline{s}_k}\right)$ the entropy of trajectory

$$\left[M^n, g\right]^1_{j_1}, \left[M^n, g\right]^2_{j_2}, \cdots, \left[M^n, g\right]^k_{j_k}, \cdots$$

Consider family $\left\{\left\{\overline{A}^k_{\overline{s}^\beta_k}\right\}_{\beta\in L}\right\}_{k\in N}$, where L something countable set. This family contains only pairwise disjoint elements

$$\bigcup_{\overline{s}^\beta_k} \bigcup_{k\in N} \left\{\left\{\overline{A}^k_{\overline{s}^\beta_k}\right\}\right\} = \Re .$$

We have probability space

$$\left(\left\{\left\{\overline{A}^k_{\overline{s}^\beta_k}\right\}_{\beta\in L}\right\}_{k\in N}, \sigma\left(\left\{\left\{\overline{A}^k_{\overline{s}^\beta_k}\right\}_{\beta\in L}\right\}_{k\in N}\right), \mu\right).$$

We consider random variable in this space

$$h : \left\{\left\{\overline{A}^k_{\overline{s}^\beta_k}\right\}_\beta\right\} \to \left\{M\left(h_{\overline{s}^\beta_k}\right)\right\},$$

with distribution

$$M\left(h_{\bar{s}_k^\beta}\right) \to \mu\left(\left\{\bar{A}_{s_k^\beta}^k\right\}\right).$$

Such we for each trajectory (realization)

$$\left[M^n, g\right]_{j_1}^1, \left[M^n, g\right]_{j_2}^2, \cdots, \left[M^n, g\right]_{j_k}^k, \cdots$$

of random process

$$\Re_1, \Re_2, \cdots, \Re_k, \cdots$$

We have determined its entropy end distribution of this entropy as distribution random variable on the set of trajectories.

This random variable shows that each trajectory of random process $\Re_1, \Re_2, \cdots, \Re_k, \cdots$ with what probability can be realized.

If we consider evolution of universe as evolution of quantum system, and describe its evolution as random process constructed as above by means of sequence each other cobordiant Riemannian manifolds $\left(M_1^3, g_1\right), \left(M_2^3, g_2\right), \cdots, \left(M_k^3, g_k\right), \cdots$, which is realization of this random process with an accuracy of up to entropy. Because every member of this sequence are each other, there exists Lorentz 4-dimensional manifold L^4 for which sections by space-like plane in L^4 are the members of the sequence

$$\left(M_1^3, g_1\right), \left(M_2^3, g_2\right), \cdots, \left(M_k^3, g_k\right), \cdots.$$

with the accuracy of diffeomorphism or isometry.

For different realizations corresponding Lorentz manifolds are different. We will attribute to Lorentz 4-dimensional manifold L^4 entropy of trajectory $\left(M_1^3, g_1\right), \left(M_2^3, g_2\right), \cdots, \left(M_k^3, g_k\right), \cdots$ if sections by space-like plane in L^4 are member of this sequence with the accuracy of isometry. If consider the set of Lorentz 4-dimensional manifolds which implement the trajectory and random variable on this set whose value coincides with value of the random variable on the set corresponding to the trajectory. This random variable and its distribution as a mixed quantum state give the model of evolution of the universe as evolution of quantum system, as evolution of parallel worlds.

4. Conclusion

The notion of the entropy of topological space (In this article, for Riemannian manifolds). Allows us to build a stochastic model of the evolution of the universe as evolution of parallel worlds. In the process of developing the theory of quantum gravity, the issue of parallel universes is often discussed [12], so we believe that the model we have constructed is relevant and deserves attention.

Conflicts of Interest

The author declares no conflicts of interest.

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