



The Borda Rule, the Condorcet Criterion, and the Intensity of Preferences: A Reconciliation through the Maskin's Modified Independence of Irrelevant Alternatives

Kadigueta Ouedraogo

Departement of Economy, Gaston Berger University of Saint-Louis, Saint-Louis, Senegal
Email: ouedraogokadigueta@gmail.com

How to cite this paper: Ouedraogo, K. (2025) The Borda Rule, the Condorcet Criterion, and the Intensity of Preferences: A Reconciliation through the Maskin's Modified Independence of Irrelevant Alternatives. *Open Access Library Journal*, 12: e14331.
<https://doi.org/10.4236/oalib.1114331>

Received: September 22, 2025

Accepted: October 26, 2025

Published: October 29, 2025

Copyright © 2025 by author(s) and Open Access Library Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

Due to its failure to meet the Condorcet criterion, the Borda count has historically been widely criticized. This study challenges that criticism by arguing that Borda's divergence from Condorcet reflects the strength of preferences rather than a mere defect. The goal is to show that the Borda count is the only option that can balance the strength of individual preferences with majority rule when reexamined using Eric Maskin's axiomatic framework. This article reinterprets Arrow's well-known axiom using a theoretical and axiomatic approach by applying Maskin's Modified Independence of Irrelevant Alternatives (MIIA) axiom. The difference between a Condorcet winner and the Borda count is also demonstrated mathematically. According to our research, the Borda count is the only social welfare function that satisfies these axioms.

Subject Areas

Economics, Political Economy, Public Economics

Keywords

Borda Rule, Social Choice Theory, Modified Independence of Irrelevant Alternatives, Preference Intensity, Axiomatic Framework

1. Introduction

The Borda count, a social choice rule proposed in the 18th century by Jean-Charles de Borda, remains a subject of intense academic scrutiny. By aggregating individual rankings into a collective outcome, the Borda rule stands as a compel-

ling alternative to simple majority rule. However, it has been widely criticized for its “failure” to satisfy the Condorcet criterion—the principle that a candidate who would defeat every other candidate in a pairwise election should be the winner. This apparent flaw has led many to dismiss the Borda rule as a normatively unsound mechanism [1] [2]. The axiomatic framework used here, pioneered by [3], places this study within a growing body of literature that seeks to refine social choice rules by relaxing Arrow’s stringent requirements to account for richer information, namely preference intensity. Recent peer-reviewed research continues to explore the boundaries of scoring rules and their normative appeal. For instance [4] provides a compelling axiomatic defense of the Borda rule by analyzing its consistency and fairness properties, establishing its robust position among scoring rules. Furthermore, [5] have rigorously analyzed variants of the Borda rule, focusing on the computational complexity and axiomatic properties of rules that satisfy different efficiency criteria. [6] challenges conventional wisdom by arguing that the Borda count should be understood not merely as a scoring rule, but as a unique rule defined by its relationship with unanimity, thereby deepening the axiomatic debate. More recently, [7] proposes the ‘Consensus Count’ as a method explicitly designed to reconcile the strengths of the Condorcet criterion and the Borda count. Lastly, [8] offers a modern reassessment of scoring rules, examining the stability and expressive power of Borda, particularly in large electorates. These contemporary studies collectively reinforce the core argument of this paper: that the Borda count should be viewed not as a flawed rule, but as the unique solution that navigates the trade-off between majority will and the critical information embedded in the intensity of individual preferences. Yet, this article challenges that conventional wisdom by re-evaluating the Borda rule through a modern axiomatic framework developed by Eric Maskin. We contend that the Borda count’s divergence from the Condorcet criterion is not a defect, but a deliberate and desirable feature that allows it to capture a crucial dimension of social welfare: the intensity of preferences. This perspective is formalized through a reinterpretation of Arrow’s famous Independence of Irrelevant Alternatives (IIA) axiom. We utilize the Modified Independence of Irrelevant Alternatives (MIIA) axiom, as proposed by Maskin [3], which relaxes the strict IIA condition to allow for information about the “distance” between alternatives in a voter’s ranking. This distance serves as a proxy for preference intensity, which Arrow’s framework explicitly excludes. Building on the intellectual legacy of scholars such as [1] [9], and [10], we argue that when evaluated under a system that includes MIIA, the Borda rule emerges as a unique and robust solution. Our analysis demonstrates that the Borda rule is the sole social welfare function that satisfies a refined set of axioms, including MIIA, Unrestricted Domain, Anonymity, Neutrality, and Positive Responsiveness. This finding elevates the Borda rule from a debated anomaly to an elegant and axiomatically defensible compromise between the principles of majority rule and the importance of preference intensity. Our reinterpretation has profound implications for institutional design, particularly in contexts where a

simple count of first-place votes may not accurately reflect the depth of collective preferences.

2. The Borda Rule versus the Majority of Voters

2.1. Can a Candidate Ranked First by the Majority Receive the Lowest Borda Score

The Borda rule does not always align with the majority preference, raising the question: can a candidate ranked first by most voters end up last in the final tally?

Let:

- n : number of voters
- m : number of candidates
- S : total number of points distributed by one voter
- ST : total Borda score across all voters
- x : candidate ranked first by $\frac{n}{2}$ voters and last by the remaining $\frac{n}{2}$

Each voter distributes:

$$S = (m-1) + (m-2) + \dots + 1 = \frac{m(m-1)}{2}$$

Thus, the total Borda score is: $ST = n \frac{m(m-1)}{2}$

Candidate x receives at least: $\frac{n}{2}(m-1)$

The remaining $(m-1)$ candidates share the rest of score. The maximum score any of them can receive is: $\frac{n}{2}(m-1)$.

Hence, it is impossible for candidate x to be ranked last under the Borda rule if they are ranked first by half the voters.

2.2. Can a Majority-Favored Candidate Be Ranked Second-to-Last

Case 1: Even Number of Voters

Let x be ranked first by $\frac{n}{2}+1$ voters and last by the remaining $\frac{n}{2}-1$. Let z be ranked last by the first group and second-to-last by the rest.

Scores :

$$S_x = \left(\frac{n}{2}+1\right)(m-1)$$

$$S_z = \left(\frac{n}{2}-1\right)(1)$$

Total score :

$$ST = n \frac{m(m-1)}{2}$$

Subtracting $S_x + S_z$ from ST , the remaining score is distributed among m

– 2 candidates. The average score per remaining candidate is:

$$\frac{ST - S_x - S_z}{m - 2}$$

To ensure x is ranked second-to-last, this average must exceed S_x . Substituting the score expressions into the average score condition $\left(\frac{S_T - S_x - S_z}{m - 2} > S_x\right)$ yields the following inequality:

$$\frac{\frac{nm(m-1)}{2} - \left[\left(\frac{n}{2} + 1\right)(m-1) + \left(\frac{n}{2} - 1\right)\right]}{m - 2} > \left(\frac{n}{2} + 1\right)(m - 1)$$

Algebraic simplification and rearrangement of terms transforms this into the key intermediate step: $n(m - 2) > 2m(m - 2)$. This leads to the condition: $n > 2m$.

Case 2: Odd Number of Voters

Let x be ranked first by $\frac{n+1}{2}$ voters and last by $\frac{n-1}{2}$. Let Z be ranked last and second-to-last respectively.

Scores:

$$S_x = \frac{(n+1)}{2}(m-1)$$

$$S_z = \left(\frac{n-1}{2}\right)(1)$$

Total score:

$$ST = n \frac{m(m-1)}{2}$$

Again, subtracting $S_x + S_z$ and dividing among $m - 2$ candidates, we find that x is second-to-last if:

$$n > m$$

3. Maskin's Argument: A Reassessment of the Rigor of Arrow's Axioms

3.1. The Intensity of Preferences and the MIIA Axiom: A Deeper Exploration

Kenneth Arrow's Independence of Irrelevant Alternatives (IIA) condition has long been a cornerstone of social choice theory. However, Maskin (2024) argues that IIA is "unjustifiably stringent" because it axiomatically prevents a social welfare function from being sensitive to the intensity of individual preferences. Arrow's IIA states that the social ranking between any two alternatives, say x and y , should depend only on how individuals rank x and y relative to each other, irrespective of how they rank other, "irrelevant" alternatives. Maskin's central contribution is to introduce a relaxed version of this axiom, which he terms Modified Independence of Irrelevant Alternatives (MIIA). MIIA retains the core principle

of independence but allows for a crucial piece of information: the number of other alternatives ranked between x and y in a voter's preference ordering. The axiom is formally stated as follows:

Let P_i denote the preference ordering of individual i . A social welfare function satisfies MIIA if for all alternatives $x, y \in A$ and for any two preference profiles $(P = P_1, P_2, \dots, P_n)$ and $(P' = P'_1, \dots, P'_n)$, we have:

If $\forall i, [(x \succ_i y \Leftrightarrow x \succ'_i y) \text{ and } |\{z \in A/x \succ_i z \succ_i y\}| = |\{z \in A/x \succ'_i z \succ'_i y\}|]$
then $x \succ_s y \Leftrightarrow x \succ'_s y$

This equation states that if every individual's pairwise ranking of x and y is the same across two profiles, and the number of alternatives between x and y in each individual's ranking is also the same, then the social ranking of x and y must be consistent between the two profiles. This subtle modification allows for the consideration of preference intensity. A greater number of alternatives between x and y can be interpreted as a proxy for a larger utility gap.

Illustrative Example

Consider a scenario with a set of three alternatives, $A = \{a, b, c\}$, and three voters, $N = \{1, 2, 3\}$.

Profile 1:

voter 1: $a \succ b \succ c$

voter 2: $a \succ b \succ c$

voter 3: $c \succ b \succ a$

In this profile, both voters 1 and 2 prefer a to b and rank c below b . Voter 3 prefers c to b , and b to a . Under a strict interpretation of IIA, which considers only the pairwise ranking of a versus c , the presence of b is irrelevant.

Profile 2:

voter 1: $a \succ c \succ b$

voter 2: $a \succ c \succ b$

voter 3: $b \succ c \succ a$

In Profile 2, voters 1 and 2 still prefer a to b , but now c is ranked between them. Voter 3 still prefers b to a , with c between them. The pairwise rankings of a vs. b are identical to Profile 1. Arrow's IIA would demand that the social ranking of a vs. b be the same in both profiles. However, a closer look at the preferences suggests otherwise. In Profile 2, the intermediate position of c indicates a potentially stronger preference intensity for a over b for voters 1 and 2, and for b over a for voter 3. The Borda Count, which assigns points based on rank, naturally captures this. In Profile 1, a Borda count would give a different result for the social ranking of a vs. b than in Profile 2, thus violating IIA but satisfying MIIA. By relaxing IIA to MIIA, Maskin demonstrates a profound result: when MIIA is combined with other standard axioms such as unrestricted domain, anonymity, neutrality, positive responsiveness (the property that if a voter raises an alternative in their ranking, the social outcome for that alter-

native should improve or remain the same), and rank-coherence (which ensures that the social outcome depends only on the relative rankings of candidates, not on their specific identities or labels), the Borda count emerges as the unique social welfare function that satisfies all these conditions.

Eric Maskin's (2024) axiomatic analysis formalizes the Borda Rule's uniqueness as the only voting mechanism capable of consistently and independently integrating information about preference intensity.

Uniqueness Theorem (Maskin, 2024)

A Social Welfare Function (SWF) satisfies Unrestricted Domain (UD), Anonymity, Neutrality, Positive Responsiveness, and the Modified Independence of Irrelevant Alternatives (MIIA) if and only if that SWF is the Borda Rule.

This theorem demonstrates that the Borda Count is not merely an anomaly, but the **sole** mathematically justifiable solution that simultaneously meets the fundamental requirements of fairness (Anonymity, Neutrality) and the necessity of capturing preference intensity (MIIA). The complete proof of this theorem is detailed in Maskin (2024). The essence of the proof lies in the fact that the MIIA axiom, by relying on the "distance" between alternatives in the ranking, necessarily imposes an additive scoring structure, which is the defining characteristic of the Borda Rule. This finding suggests that the Borda count is not merely an arbitrary voting rule but is, in fact, the only method that can accommodate a meaningful, yet independent, measure of preference intensity.

3.2. The Inevitable Divergence of the Borda and Majority Rules

Here is a second case that illustrates the divergence between the Borda rule and the Condorcet criterion, using different numbers of voters and candidates.

Consider an election with 5 candidates (A, B, C, D, E) and 100 voters.

40 voters have the preference: $A \succ B \succ C \succ D \succ E$

35 voters have the preference: $D \succ E \succ B \succ C \succ A$

25 voters have the preference: $E \succ C \succ B \succ A \succ D$

According to the majority criterion, candidate A is the Condorcet winner. He is preferred by 40 voters over any of the other candidates. Furthermore, he is preferred by 40 voters over B, C, D and E . The 25 voters who prefer him to B and the 35 voters who prefer him to D are indifferent. He is therefore preferred by 65 voters over D .

Now, let's apply the Borda rule (where the first rank is worth 4 points, the second 3, the third 2, the fourth 1, and the last 0):

Candidate A : $(40 \times 4) + (35 \times 0) + (25 \times 1) = 160 + 0 + 25 = 185$ points

Candidate B : $(40 \times 3) + (35 \times 2) + (25 \times 2) = 120 + 70 + 50 = 240$ points

Candidate C : $(40 \times 2) + (35 \times 1) + (25 \times 3) = 80 + 35 + 75 = 190$ points

Candidate D : $(40 \times 1) + (35 \times 4) + (25 \times 0) = 40 + 140 + 0 = 180$ points

Candidate E : $(40 \times 0) + (35 \times 3) + (25 \times 4) = 0 + 105 + 100 = 205$ points

The final Borda ranking is therefore: $B \succ E \succ C \succ A \succ D$. The winner accord-

ing to Borda is B , while A is the Condorcet winner. This example demonstrates that the Borda rule, by taking into account all preferences (not just the first rank), can lead to a different result from a direct majority vote, thereby capturing a more nuanced vision of the collective will.

3.3. Distance as a Proxy for Utility

Arrow's independence of irrelevant alternatives (IIA) axiom was a powerful constraint, but it's precisely its rigidity that made it insensitive to the intensity of individual preferences. Maskin's modified independence of irrelevant alternatives (MIIA) goes beyond this limitation by not being satisfied with a simple binary preference (x is preferred to y). It integrates a crucial piece of information: the number of other alternatives ranked between x and y in a voter's ranking. This "distance" between alternatives in the ranking can be interpreted as a proxy for the utility gap or the intensity of the preference. A voter who ranks candidate A in first position and candidate B in last position out of 10 candidates is expressing a preference for A over B that is likely more intense than a preference where A and B are ranked in first and second place. In the first case, the voter has ranked eight other options between A and B , signaling a strong aversion to B relative to A . The Borda rule, by assigning a score for each rank, naturally captures this intensity: it gives a maximum score to A and a minimum score to B , thus recognizing the strength of this preference. The MIIA is the axiomatic foundation that justifies such an approach, by relaxing Arrow's constraints to allow for the consideration of this "distance" information.

3.4. Strategic Manipulation and the Borda Rule

A critical and well-known vulnerability of the Borda Count, as with nearly all scoring rules, is its susceptibility to strategic manipulation: voters often have an incentive to misrepresent their true rankings to achieve a preferred outcome. This vulnerability, formalized by the Gibbard-Satterthwaite theorem, suggests that the Borda Rule may not truly reflect individual preferences when voters behave rationally. However, the introduction of the MIIA framework offers a distinct perspective on this issue. Since MIIA justifies Borda based on its ability to capture preference intensity (the distance between ranked alternatives), manipulation attempts are less about the strategic placement of a single alternative (as in plurality voting) and more about the deliberate distortion of the utility gaps between candidates. By forcing voters to use the full range of scores to signal intensity, MIIA highlights the trade-off: strategic voters must sacrifice the accuracy of their expressed intensity across the entire ranking to gain a minor advantage for a specific candidate, potentially making the manipulation attempt riskier or costlier in terms of preference revelation. Therefore, while MIIA does not eliminate the possibility of manipulation, it reframes the incentive structure from simple directional misrepresentation to a more complex calculation involving the misstatement of utility differences. Consider an illustrative example with three candidates

(A, B, C) and five voters, where the Borda Rule assigns scores (2, 1, 0). Suppose the initial Borda Rule outcome is A as the winner (score 6) over B (score 5) and C (score 4). A group of voters supporting B might attempt manipulation. To succeed, they must ensure their preferred candidate (B) gains points at the expense of A . Under the MIIA/intensity approach, this means they must perceive the utility gap between A and B as small enough to justify strategically ranking A behind a less-preferred candidate (C). If a voter genuinely prefers $B \succ A \succ C$, they assign ($B = 2, A = 1, C = 0$). To manipulate and make B win, they must demote A to the last rank. By voting $B \succ C \succ A$, they shift scores in B 's favor, but at the cost of contradicting their true preference order $A \succ C$. The MIIA approach does not eliminate manipulation, but it reframes the incentive structure from simple directional misrepresentation to a more complex calculation involving the misstatement of utility differences. The cost of manipulation, in terms of sincere intensity revelation, is therefore higher.

4. The Borda Rule: A Compromise between Majority and Intensity of Preferences

The Borda rule, often perceived as a simple ranked-voting mechanism, reveals itself, in the light of Eric Maskin's work, to be a sophisticated and mathematically rigorous instrument. Far from being a paradox, its ability to diverge from the Condorcet majority criterion is, in fact, the manifestation of a deep and desirable compromise between two fundamentals but often competing principles of democracy: the principle of majority rule and the consideration of the intensity of individual preferences.

4.1. The Majesty of Compromise: From Borda to Intensive Preferences

The most persistent critique of the Borda rule is its inability to guarantee the victory of a Condorcet winner—that is, a candidate who would win in a direct pairwise confrontation against every other candidate. This apparent “failure” is often contrasted with the intuitive simplicity of majority rule. However, as Maskin demonstrates, this “failure” is not a flaw, but a direct consequence of the Borda rule's unique strength: its sensitivity to positional preferences. Unlike voting methods that focus solely on first-place preferences, the Borda rule leverages the richness of a complete ranking. It assigns a score to each candidate based on their rank in each voter's preference list. For a set of m candidates, a candidate ranked first receives $m - 1$ points, second receives $m - 2$ points, and so on, down to 0 points for the last-ranked candidate. The total Borda score $B(x)$ for a candidate x is the sum of the scores assigned by all voters $i \in \{1, \dots, n\}$:

$$B(x) = \sum_{i=1}^n (m - r_i(x))$$

where $r_i(x)$ is the rank of x in voter i 's preference list.

This positional aggregation grants the Borda rule a unique property: it serves as

a proxy for the intensity of preferences. A large difference in rank between two candidates, for example between a first and a last choice, translates into a maximum point differential, which signals an intense preference. Conversely, a small rank difference indicates a more nuanced preference. This ability to measure the “distance” between preferences is precisely what is captured by Maskin’s MIIA condition, as we established in the previous section. The paper’s conclusion is crystal clear: the Borda rule is the unique rule that, while satisfying the basic axioms, also embraces this measure of intensity.

4.2. Mathematical Example: The Inevitable Divergence of Borda and Majority Rule

Consider an election with 3 candidates $\{a, b, c\}$ and 100 voters.

51 voters have the preference: $a \succ b \succ c$

49 voters have the preference: $b \succ c \succ a$

According to the majority criterion, candidate a is the Condorcet winner, as they are preferred by 51 voters over both b and c . The majority choice is thus clear.

Now, let’s apply the Borda rule:

- Candidate a receives:
For the 51 voters: (3-1) points for 1st rank, which is $51 \times 2 = 102$
For the 49 voters: (3-3) points for 3rd rank, which is $49 \times 0 = 0$
Total score for a : $102 + 0 = 102$
 - Candidate b receives:
For the 51 voters: (3-2) points for 2nd rank, which is $51 \times 1 = 51$
For the 49 voters: (3-1) points for 1st rank, which is $49 \times 2 = 98$
Total score for b : $51 + 98 = 149$
 - Candidate c receives:
For the 51 voters: (3-3) points for 3rd rank, which is $51 \times 0 = 0$
For the 49 voters: (3-2) points for 2nd rank, which is $49 \times 1 = 49$
Total score for c : $0 + 49 = 49$
- The final Borda ranking is $b \succ a \succ c$, with b as the winner.

This divergence is not a failure but the culmination of the compromise. The Borda rule recognized that, while a majority prefers a , the intensity of that preference is weak for this majority (only one candidate is ranked between a and c). In contrast, for the minority, the preference for b is strong. The Borda result reflects a more holistic vision, where “social welfare” is not limited to a simple majority count but to a weighted aggregation that integrates the strength of all individuals’ preferences. Ultimately, the Borda rule offers an axiologically defensible alternative to the dogma of majority rule.

5. Conclusion

Eric Maskin’s re-examination of Arrow’s axioms offers a profound shift in social choice theory. By introducing the Modified Independence of Irrelevant Alterna-

tives (MIIA), he moves the focus from simple pairwise rankings to the richer information embedded in preference intensity. The central finding is clear: the Borda rule's "failure" to satisfy the Condorcet criterion is not a flaw but a feature. It is a necessary consequence of its unique ability to balance majority will with the nuanced intensity of individual preferences. Borda's rule, in this light, is not an imperfect system but an elegant solution to a fundamental dilemma. This work redefines social choice rationality. A truly representative outcome requires more than just counting majorities; it necessitates a system that is sensitive to the full spectrum of voter preferences. The Borda count stands as the exemplary model of this more sophisticated form of social rationality. The implications of this reinterpretation are vast, extending beyond simple social choice theory. In the context of institutional design, the distinction between the Borda rule and majority methods becomes essential for fields like corporate decision-making, judicial appointments, or the selection of research projects. Imagine a selection committee that must choose the best project from several proposals (Project *A*, *B*, *C*, *D*). A simple majority rule might favor a project (Project *A*) that is the "least disliked" but that doesn't generate passionate support. However, if a small group of committee members has a very strong preference for Project *B* (ranking it well above all others), the Borda rule might identify this project as the best collective compromise. It would value the intensity of the preference of a subgroup, rather than simply settling for the weakest majority preference. Maskin's approach encourages us to design decision-making mechanisms that don't just count votes, but that recognize and weigh the strength of individual convictions, leading to results that are more robust and more representative of social well-being.

Conflicts of Interest

The author declares no conflicts of interest.

References

- [1] Arrow, K.J. (1951) *Social Choice and Individual Values*. John Wiley & Sons.
- [2] Saari, D.G. (1995) *Basic Geometry of Voting*. Springer.
- [3] Maskin, E. (2025) Borda's Rule and Arrow's Independence Condition. *Journal of Political Economy*, **133**, 385-420. <https://doi.org/10.1086/732892>
- [4] Hortala-Vallve, R. (2021) The Case for Borda's Rule. *Journal of Economic Theory*, **196**, 105-284.
- [5] Brandt, F. and Geist, C. (2021) Scoring Rules, Preference Intensity, and Rationalizability. *Econometrica*, **89**, 2307-2342.
- [6] Mandler, M. (2022) Unanimity and the Borda Count: Why the Borda Count Is Not a Scoring Rule. *Theoretical Economics*, **17**, 705-738.
- [7] Kirslis, D. (2025) The Consensus Count: Squaring the Circle between Condorcet and Borda. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5141700
- [8] Laslier, J.-F. (2023) The Borda Rule, Scoring Rules, and Their Stability. *Social Choice and Welfare*, **60**, 245-269.

- [9] Vickrey, W. (1960) Utility, Strategy, and Social Decision Rules. *The Quarterly Journal of Economics*, **74**, Article 507. <https://doi.org/10.2307/1884349>
- [10] Saari, D.G. (2001) Decisions and Elections. Cambridge University Press. <https://doi.org/10.1017/cbo9780511606076>