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On the Application of the Infinitesimal Method to Two Categories of Problems in College Physics

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Abstract

The conventional approaches to solving typical problems in college physics, such as the catenary (an extremum problem) and molecular collision problems, are often abstract, making them difficult for students to comprehend. This paper explores the application of the infinitesimal method to these two categories of problems. For physical extremum problems, the method transforms global optimality conditions into constraints on infinitesimal elements, leading to the corresponding differential equations. For molecular collision problems, it utilizes infinitesimal analysis to clarify collision criteria, thereby facilitating the calculation of collision probabilities. Case studies demonstrate that the infinitesimal method can provide a more straightforward, visually clearer, and physically more robust deductive pathway for such problems. This research not only contributes to a deeper understanding of the underlying physics but also offers valuable insights for restructuring knowledge presentation in teaching practice and strengthening the training of "from-local-to-global" scientific thinking.

Keywords

Infinitesimal Method, College Physics, Brachistochrone, Mean Free Path, Extremum Problem

1. Introduction

In physics, the infinitesimal method is an integral concept and modeling technique based on local linear approximation. Its essence lies in dividing a continuous object of study—such as a physical body, field, or process—into an infinite number of "infinitesimal elements". Within these elements, fundamental physical laws are applied using the "approximating the variable with a constant" technique, and the overall behavior is deduced through integration or summation.

The key steps involve: strategically selecting infinitesimal elements based on the geometric and physical characteristics of the problem (e.g., symmetry, dominant direction of change); establishing simplified equations on these elements that capture the core physical relationships; and finally, integrating to synthesize results from the local to the global scale. This "division-approximation-summation" philosophy is a classic embodiment of applying calculus to solve practical physical problems.

In college physics, however, the conventional solution paths for two types of problems often perplex students and form barriers to comprehension. The first is physical extremum problems, such as determining the shape of a catenary, the trajectory of the brachistochrone, and the isoperimetric problem. Their standard solutions rely on the calculus of variations (Jin, 2020), whose mathematical abstraction can obscure direct physical intuition (Courant & Hilbert, 2011; Fang, Zeng, Fan, Chen, & Ciappina, 2024). The second involves molecular collisions and related statistical problems, such as deriving the mean free path. The traditional derivation depends on the global assumption that "all other molecules are stationary relative to the molecule under consideration" (Li, Zhang, & Qian, 2015: pp. 80-84), a premise that seems physically unrealistic and may hinder understanding of the problem's essence.

To overcome these comprehension barriers and explore a more intuitive, physically transparent deductive path, this work returns to the infinitesimal method as a fundamental analytical tool. Its "division-approximation-summation" approach naturally follows the "from local to global" cognitive sequence, aligning with constructivist learning principles. It leverages local approximation and integration to decompose complex global problems into simple, describable infinitesimal elements with clear geometric or physical relationships, thereby providing an intuitive visual representation.

Critically, compared to methods like the calculus of variations—which require understanding abstract concepts such as "functionals" and "variations"—the infinitesimal method primarily demands a grasp of basic calculus and fundamental physical laws (e.g., Newtonian mechanics, geometric relations), significantly lowering the cognitive barrier. Furthermore, by transforming probabilistic problems into geometric ones via models like the "collision cylinder", it avoids the counter-intuitive global assumption of stationary molecules, directly addressing a key learning obstacle.

This paper will analyze cases including the catenary, brachistochrone, isoperimetric problem, and molecular mean free path to illustrate the intuitiveness and physical insights gained by applying the infinitesimal method to these two problem categories. Specifically, we aim to demonstrate that through appropriate construction of infinitesimal elements and local analysis, the governing equations for extremum problems can be derived directly from mechanical equilibrium or geometric constraints—thereby bypassing the calculus of variations—and that, without invoking the global stationary assumption, one can rigorously derive collision statistics using molecular isotropy and probabilistic models like the "collision cylinder".

We anticipate that this exploration will not only demonstrate that the infinitesimal method provides a clearer and more physically grounded approach but also offers a valuable perspective for understanding the relevant physical laws and provide practical references for instructional design.

2. Application of the Infinitesimal Method to Physical Extremum Problems

Physical extremum problems aim to find the conditions or forms that extremize (minimize or maximize) a given physical quantity. The following three typical cases demonstrate how the infinitesimal method can be used to establish governing equations through local analysis.

2.1. Case Study 1: Derivation of the Catenary Equation (Jin, 2020)

Problem: Determine the curve formed by a uniform, inextensible, flexible rope hanging freely under gravity.

Analysis and Derivation:

Assume the catenary has a uniform linear mass density, λ. Consider an infinitesimal arc element $\mathrm{d}s$. Let θ be the angle between the horizontal axis (x-axis) and the tangent to the arc element. The arc length from the lowest point O to the element $\mathrm{d}s$ is denoted by s. A force analysis on the infinitesimal element $\mathrm{d}s$ (see Figure 1) shows that the horizontal components of tension at both ends are equal in magnitude (denoted as F). As the arc element can be considered to have uniform curvature, it is approximated as a circular arc with a central angle $\mathrm{d}\theta$. The angle between the tangent at the midpoint of the element and the horizontal is θ. The force balance equation in the vertical direction (y-direction) is:

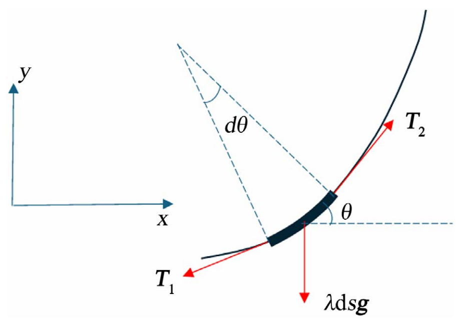

Figure 1. Force diagram of the arc element.

$$F\tan\left(\theta+\frac{\mathrm{d}\theta}{2}\right)-F\tan\left(\theta-\frac{\mathrm{d}\theta}{2}\right)=\lambda g\,\mathrm{d}s. \quad (2.1)$$

Simplifying and neglecting second-order small quantities yields:

$$F\left(1+\tan^2\theta\right)\mathrm{d}\theta=\lambda g\,\mathrm{d}s. \tag{2.2}$$

Integrating both sides, and noting that $s=0$ when $\theta=0$, gives:

$$s=A\tan\theta, \tag{2.3}$$

where $A\equiv F/(\lambda g)$. Since $\tan\theta=\mathrm{d}y/\mathrm{d}x$, Equation (2.3) becomes:

$$s=A\frac{\mathrm{d}y}{\mathrm{d}x}. \tag{2.4}$$

Using the trigonometric identity $\sec^2\theta=1+\tan^2\theta$, we obtain:

$$\frac{\mathrm{d}s}{\mathrm{d}x}=\sqrt{1+\frac{s^2}{A^2}}. \tag{2.5}$$

Separating variables and integrating, with the condition $x=0$ when $s=0$, gives:

$$x=A\operatorname{arcsinh}\frac{s}{A}. \tag{2.6}$$

Combining Equations (2.4) and (2.6) leads to:

$$\sinh\frac{x}{A}=\frac{\mathrm{d}y}{\mathrm{d}x}. \tag{2.7}$$

Separating variables and integrating results in the catenary equation:

$$y=A\left(\cosh\frac{x}{A}-1\right). \tag{2.8}$$

here, the constant A can be determined by the coordinates (s and θ, or x and y) of any point on the catenary.

Case Analysis: This example illustrates the typical approach of the infinitesimal method for static extremum problems. Although the catenary corresponds to a minimum potential energy configuration, the concept of variational calculus is not required. By performing a force analysis directly on the infinitesimal element and applying its local equilibrium condition, the differential equation governing the overall shape is derived. This embodies the profound idea that "global extremum conditions must manifest locally", and the infinitesimal method serves as a precise tool for revealing this manifestation.

2.2. Case Study 2: The Brachistochrone Problem (Jin, 2020)

Problem: In a gravitational field, determine the curve connecting two fixed points A and B such that the time required for a particle to slide down this curve without friction is minimized.

Analysis and Derivation:

In this problem, the path is discretized into infinitesimal segments $\Delta s_1, \Delta s_2, \cdots, \Delta s_n$, each spanning a fixed vertical distance d along the direction of gravity. Given the horizontal displacement x_i and vertical position y_i (relative to starting point A) of segment Δs_i, the speed is given by $v_i=\sqrt{2gy_i}$ as a consequence of the conservation of mechanical energy. To minimize the total travel time

$t = \sum \frac{\Delta s_i}{v_i}$, the condition $\mathrm{d}t = 0$ must be satisfied:

$$\mathrm{d}\left(\sum \frac{\sqrt{x_i^2 + d^2}}{v_i}\right) = \sum \left(\frac{x_i}{\sqrt{x_i^2 + d^2}} \cdot \frac{1}{v_i} \mathrm{d}x_i\right) = 0. \tag{2.9}$$

Given the constraint $\sum x_i = L$ (where L is the total horizontal distance), the above expression simplifies to:

$$\mathrm{d}t = \sum_{i=1}^{n-1} \left(\frac{1}{v_i} \frac{x_i}{\sqrt{x_i^2 + d^2}} - \frac{1}{v_n} \frac{x_n}{\sqrt{x_n^2 + d^2}}\right) \mathrm{d}x_i = 0. \tag{2.10}$$

Since the $\mathrm{d}x_i$ are independent for $i = 1, 2, \cdots, n-1$, the coefficient of each $\mathrm{d}x_i$ should vanish. This yields a crucial intermediate result:

$$\frac{1}{v_i} \frac{x_i}{\sqrt{x_i^2 + d^2}} \equiv A, \ \text{for } i = 1, 2, \cdots, n. \tag{2.11}$$

As each element tends to zero, the discrete formulation converges to a continuous one. In this limit, we have $x_i \to \mathrm{d}x$, $d \to \mathrm{d}y$, and $\sqrt{x_i^2 + d^2} \to \mathrm{d}s$, where $\mathrm{d}s$ denotes the differential arc length. Hence, Equation (2.11), together with the geometric identity $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2$, leads to:

$$\frac{\mathrm{d}x}{\mathrm{d}s} = Av = \sqrt{Cy}, \ \frac{\mathrm{d}y}{\mathrm{d}s} = \sqrt{1 - Cy}, \tag{2.12}$$

where $C = 2gA^2$. Eliminating $\mathrm{d}s$ from Equations (2.12) yields the brachistochrone equation:

$$x = \int_0^y \frac{\sqrt{Cy}}{\sqrt{1 - Cy}} \mathrm{d}y = \frac{1}{C}\left(\arcsin\sqrt{Cy} - \sqrt{Cy(1 - Cy)}\right). \tag{2.13}$$

The constant C is determined by the endpoint coordinates (X_f, Y_f). Introducing the dimensionless variables $\xi = Cx$ and $\tau = \sqrt{Cy}$, Equation (2.13) can be written as:

$$\xi = \arcsin\tau - \tau\sqrt{1 - \tau^2}. \tag{2.14}$$

Case Analysis: Here, the infinitesimal method serves as a "cognitive bridge". It transforms the abstract problem of finding the extremum of a continuous path functional into a discrete optimization problem involving multivariable functions—a concept more familiar to students. By analyzing discrete infinitesimal elements, the local conditions for optimality are clearly revealed. Taking the limit then naturally transitions the discrete formulation into a continuous differential equation. This process significantly lowers the cognitive barrier to understanding the underlying optimization principle.

2.3. Case Study 3: The Isoperimetric Problem (Courant & Hilbert, 2011)

Problem: Among all closed planar curves with a given perimeter, find the one that encloses the maximum area.

Analysis and Derivation:

We employ an argument based on analyzing the local characteristics of an extremum via the infinitesimal method, combined with symmetry considerations, to demonstrate that the circle is the solution. To maximize the enclosed area, any chord connecting two points on the curve must lie inside the curve; otherwise, a shorter perimeter enclosing a larger area would exist, contradicting optimality.

Select an arbitrary point P on the curve as a reference. Starting from P, the entire curve is subdivided counterclockwise (taken as the positive direction) into equal-length infinitesimal segments $\overline{PM_1}, \overline{M_1M_2}, \cdots, \overline{M_{2N-1}P}$. For any $i = 1, 2, \cdots, N-1$, suppose the arc PM_i is known. Consider the line element $\overline{M_iM_{i+1}}$ emanating from point M_i. To maximize the area enclosed by the arc PM_{i+1} and the chord $\overline{M_{i+1}P}$, a necessary condition is $\overline{M_iM_{i+1}} \perp \overline{PM_i}$ (see Figure 2(a)). Neglecting higher-order infinitesimals, this also implies $\overline{M_iM_{i+1}} \perp \overline{PM_{i+1}}$. Consequently, for the entire curve to enclose the maximum area, the line element $\overline{M_{N-1}M_N}$ must be orthogonal to the chord $\overline{M_NP}$, as the area enclosed by arc PM_N and chord $\overline{M_NP}$ is at a maximum and the complementary area cannot be larger. Given the arbitrariness of the starting point P, it follows that for any two points dividing the perimeter equally, the line connecting them bisects the area, and the infinitesimal line elements at these points are orthogonal to this connecting line.

From the above reasoning, we deduce that $\overline{M_iM_{i+1}}$ and $\overline{M_{i+N}M_{i+N+1}}$ are parallel to each other and both are orthogonal to $\overline{M_iM_{i+N}}$ (see Figure 2(b)). Since $\overline{M_iM_{i+1}} = \overline{M_{i+N}M_{i+N+1}}$, their intersection point bisects both segments. This implies that the midpoints of all chords connecting points that partition the perimeter equally coincide at a single point, denoted O. Let $\Delta\theta$ be the angle subtended by the line element $\overline{M_iM_{i+1}}$ about point O. Then, $OM_{i+1} - OM_i \sim OM_i(\Delta\theta)^2$, or in the limit $\Delta\theta \to 0$, $\mathrm{d}r \sim r(\mathrm{d}\theta)^2$. This leads to the condition:

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = 0. \tag{2.15}$$

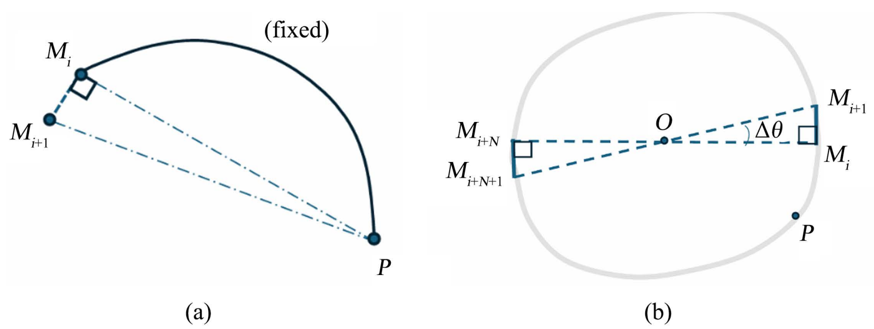

Figure 2. Geometric relations that infinitesimal elements must satisfy under extremum conditions. (a) Geometric relationship between a fixed arc PM_i and the line element $\overline{M_iM_{i+N}}$. (b) Geometric relationship between line element $\overline{M_iM_{i+1}}$ and line element $\overline{M_{i+N}M_{i+N+1}}$ for the assumed optimal curve.

Hence, the distance r from point O remains constant regardless of θ, meaning all points on the curve are equidistant from O. Therefore, the curve is a circle.

The conclusion of the isoperimetric problem—that the circle encloses the maximum area—finds intuitive manifestation in nature. For instance, droplets and soap bubbles often assume a spherical shape, a result of surface tension driving the system toward a minimum surface area (i.e., lowest energy) stable state (Li, Zhang, & Qian, 2015: pp. 225-232).

Case Analysis: This example illustrates another application mode of the infinitesimal method in extremum problems. Its core lies in deducing the constraints that a global optimum must satisfy by analyzing the conditions met by local optimal configurations, thereby arriving at the unique solution form. The use of the infinitesimal method in this case highlights its value in logical deduction and aids in cultivating the scientific thinking ability to proceed from local to global considerations.

Through the above case studies applying the infinitesimal method to extremum problems, we can observe that its application follows two core analytical steps. First, translate the characteristics possessed by the system at the global extremum into a specific local constraint that every infinitesimal element must satisfy. Second, reconstruct the global governing equation (or solution form) by integrating these consistent local constraints over the entire domain. This "global-to-local-to-global" procedure encapsulates the method's logical foundation as demonstrated in the case studies.

3. Analysis of the Infinitesimal Method Applied to Molecular Collision Problems

The statistical behavior of a large number of particles is central to the kinetic theory of gases. Using the infinitesimal method to model collision processes provides an intuitive entry point for deriving statistical laws. This section begins with a lemma based on this method and proceeds to perform calculations related to molecular collisions and the mean free path.

Case Study: Molecular Collisions and the Mean Free Path Problem (Li, Zhang, & Qian, 2015: pp. 80-84)

Within the framework of the infinitesimal method, we consider the limiting process where the time interval $\Delta t \to 0$. In this limit, the state of motion of any specified molecule can be considered approximately unchanged. To simplify the analysis, all molecules are treated as identical hard spheres of diameter d and mass m, with no attractive forces. In this model, the probability $P_{A\leftrightarrow B}$ that molecule A collides with molecule B can be calculated as follows.

Assume the container has volume V, and the collision cross-section is $\sigma = \pi d^2$. Let $\boldsymbol{u}_{AB}$ denote the relative velocity of A with respect to B. Assuming B is stationary, A moves with velocity $\boldsymbol{u}_{AB}$. According to the assumption of isotropic molecular distribution, the probability that B collides with A during the infinitesimal process is equivalent to the probability that the center of mass of B lies within a cylinder. This cylinder has the trajectory of A's center as its axis, a height of $u_{AB}\Delta t$, and a base area of σ (see Figure 3). Thus,

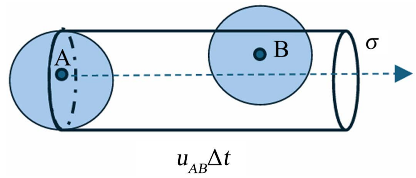

Figure 3. Schematic diagram of the collision condition between molecules A and B during an infinitesimal process.

$$P_{A\leftrightarrow B} = \frac{u_{AB}\Delta t\sigma}{V}. \tag{3.1}$$

If during this infinitesimal process molecule A could potentially collide with other molecules (e.g., B'), the assumption of "unchanged molecular state during Δt" implies that two such collision events are independent. The probability of both occurring is $P_{A\leftrightarrow B}\cdot P_{A\leftrightarrow B'} \propto (\Delta t)^2$, a higher-order infinitesimal. Therefore, the probability of multiple collisions during Δt is negligible.

Consequently, the total probability P_A that molecule A suffers at least one collision during Δt is approximately the sum of its collision probabilities with all other molecules:

$$P_A \approx \sum_B P_{A\leftrightarrow B} = \sum_B \frac{u_{AB}\Delta t\sigma}{V} = n\Delta t\sigma\bar{u}_A. \tag{3.2}$$

here, $\bar{u}_A = \frac{1}{N-1}\sum_B u_{AB}$ is the mean relative speed of A with respect to all other molecules, and $n = N/V$ is the molecular number density.

Next, we calculate the mean number of collisions experienced by A during Δt. For a finite process, the total number of collisions of A equals the sum of collisions with each other molecule:

$$N_A = \sum_B N_{A\leftrightarrow B}. \tag{3.3}$$

For the infinitesimal process Δt, the corresponding incremental relation is:

$$\Delta N_A = \sum_B \Delta N_{A\leftrightarrow B}. \tag{3.4}$$

The key distinction is that during Δt, molecule A cannot collide with the same molecule twice. Statistically, the expectation value is:

$$\overline{\Delta N_{A\leftrightarrow B}} = P_{A\leftrightarrow B}\cdot 1 + \left(1 - P_{A\leftrightarrow B}\right)\cdot 0 = P_{A\leftrightarrow B}. \tag{3.5}$$

Thus, the mean number of collisions for A during Δt is:

$$\overline{\Delta N_A} = \sum_B \overline{\Delta N_{A\leftrightarrow B}} = \sum_B P_{A\leftrightarrow B}. \tag{3.6}$$

Comparing Equations (3.2) and (3.6) shows that $\overline{\Delta N_A} \approx P_A$. **That is, during a sufficiently short time interval, the mean number of collisions a molecule ex-**

periences are approximately equal to the probability of it suffering a collision in that interval. We now use this key result to derive the mean free path.

Again, consider an infinitesimal process with $\Delta t \to 0$. From the derivation above, the probability for molecule A to collide is $n\Delta t \sigma \bar{u}_A$, which, according to our lemma, also equals the mean number of collisions. Therefore, the average number of collisions per molecule per unit time—the collision frequency Z—is obtained by summing over all molecules A and dividing by $N\Delta t$:

$$Z = \frac{1}{N\Delta t}\sum_A n\Delta t \sigma \bar{u}_A = n\sigma\bar{u} = \sqrt{2}\, n\sigma\bar{v}. \tag{3.7}$$

here, $\bar{v}$ is the mean molecular speed, $\bar{u}$ is the mean relative speed averaged over all molecules ($\bar{u} = \frac{1}{N}\sum_A \bar{u}_A$), and the relation $\bar{u} = \sqrt{2}\bar{v}$ has been used. The mean free path λ is then:

$$\lambda = \frac{\bar{v}}{Z} = \frac{1}{\sqrt{2}\, n\sigma}. \tag{3.8}$$

Compared to traditional derivations in standard textbooks, which often rely on the unrealistic assumption that "all molecules are stationary relative to the one under consideration", the derivation presented here fully utilizes the infinitesimal method and avoids that problematic postulate.

As another significant application of the lemma developed in this section, we can employ the infinitesimal method to derive the distribution of molecular free paths. Let $P(x)$ be the probability that a molecule travels a distance greater than x without suffering a collision. Consider a molecule moving first a distance x and then an additional infinitesimal distance $\mathrm{d}x$. The probability of no collision over the entire distance $x + \mathrm{d}x$ is $P(x + \mathrm{d}x)$. This event requires that no collision occurs over the initial distance x and also over the subsequent $\mathrm{d}x$. Assuming independence, the multiplication rule gives:

$$P(x + \mathrm{d}x) = P(x) \cdot P(\mathrm{d}x). \tag{3.9}$$

Taking the logarithm of both sides and defining $F(x) \equiv \ln P(x)$, Equation (3.9) yields:

$$F(x + \mathrm{d}x) = F(x) + F(\mathrm{d}x). \tag{3.10}$$

Note that $1 - P(\mathrm{d}x)$ is the probability of a collision occurring while traversing $\mathrm{d}x$. The average time to cover $\mathrm{d}x$ is $\mathrm{d}t = \mathrm{d}x/\bar{v}$, so this probability equals the mean number of collisions in that interval, $Z\,\mathrm{d}t = (\bar{v}/\lambda)\mathrm{d}t = \mathrm{d}x/\lambda$. Therefore,

$$P(\mathrm{d}x) = 1 - \frac{\mathrm{d}x}{\lambda}. \tag{3.11}$$

Furthermore, $\mathrm{d}F(x) = F(x + \mathrm{d}x) - F(x)$. Expanding Equation (3.10) to first order and using Equation (3.11) leads to the differential equation:

$$\mathrm{d}F(x) = -\frac{\mathrm{d}x}{\lambda}. \tag{3.12}$$

Integration gives:

$$F(x) = -\frac{x}{\lambda} + C \text{ or } P(x) = C' \exp\left(-\frac{x}{\lambda}\right), \tag{3.13}$$

where C and C' are integration constants. Applying the boundary condition $P(0) = 1$ yields $C' = 1$. Thus,

$$P(x) = \exp\left(-\frac{x}{\lambda}\right). \tag{3.14}$$

This is the exponential distribution law for molecular free paths.

Case Analysis: Here, the infinitesimal method successfully *geometrizes* the problem of random collision probabilities. Under the assumption of molecular isotropy, the "collision cylinder" model transforms a probabilistic event (collision) into a deterministic geometric event (the center of mass lying within a cylinder), thereby establishing a direct link to collision frequency. This provides a solid visual foundation for understanding the abstract concept of the mean free path, serving as a successful example of bridging Newtonian mechanics imagery with statistical physical description.

4. Discussion

The core logic underpinning this work is the fundamental idea of the infinitesimal method: "dividing the whole into parts and integrating the parts into the whole". Through a systematic treatment of physical extremum problems and molecular collision problems, this paper attempts to demonstrate that the infinitesimal method, as a basic analytical approach, can provide a more advantageous deductive pathway for these two classes of problems.

Specifically, the case studies reveal the following methodological characteristics of the infinitesimal method when applied to classical physics problems:

1) In physical extremum problems, this method circumvents advanced mathematical tools such as the calculus of variations. By translating the global optimization objective (e.g., minimal time, minimal potential energy) into specific physical constraints acting on infinitesimal elements (e.g., force equilibrium, geometric relations), it grounds the entire derivation in intuitive mechanical or geometric imagery, thereby directly addressing the heart of the problem.

2) In molecular collision problems, by constructing the probabilistic "collision cylinder" model based on the assumption of isotropic molecular motion, the method unifies the calculation of collision probability and the average collision count. This is achieved without relying on the global assumption that "all other molecules are stationary", thus providing a derivation for the relevant statistical laws that adheres more closely to physical reality.

Building on these findings, we propose a pedagogical framework for integrating the infinitesimal method into the curriculum. First, a dedicated module on extremum problems should be introduced after the core mechanics sequence. Its primary aim is to establish the fundamental logic of "deducing global properties from local constraints" before students encounter the more abstract calculus of variations. Subsequently, in the kinetic theory of gases chapter, the method should be reintroduced as an extended application, highlighting its utility in statistical physics and solidifying its status as a general-purpose analytical tool. Finally, open-ended problems can be designed to guide motivated students toward independent inquiry.

This progressive instructional design not only optimizes the learning sequence but also helps systematically cultivate students' scientific thinking in modeling and analysis.

5. Conclusion

Based on the above, this paper has systematically explored the application of the infinitesimal method to two problem categories, using classical cases such as the catenary, brachistochrone, isoperimetric problem, and the mean free path of molecules. The analysis shows that employing the infinitesimal method for these problems allows for the construction of a deductive approach that is logically more direct and physically more vivid, leveraging fundamental physical laws and intuitive imagery.

This endeavor indicates that the infinitesimal method possesses significant methodological value in bridging intuitive understanding with rigorous theory and in providing a unified framework for handling both deterministic optimization and stochastic processes. It is hoped that this work will not only foster a deeper understanding of the essence of the related physical problems but also offer practical reference for how to utilize fundamental methods to reconstruct analytical pathways and cultivate systematic thinking from local to global perspectives, both in teaching and research.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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