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Exploring Quantum Coherence and Its Application in Modulated Systems

Junyao Zheng

Department of Electrical Engineering, Duke University, Durham, North Carolina, USA
Email: jzheng6657@gmail.com

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Abstract

This study delves into the intricate dynamics of quantum coherence in modulated systems, particularly focusing on neutrino oscillations and quantum emitters subjected to external driving fields. Utilizing Floquet theory, we examine the modulation of electron wavefunctions and its implications for quantum state preparation, coherence preservation, and applications in quantum computing and astrophysics. Our findings highlight the extended coherence scales in spin-flavor oscillations compared to flavor oscillations, suggesting potential advancements in quantum information technologies.

Keywords

Quantum Coherence, Superposition, Floquet Theory, Neutrinos

1. Introduction

Quantum coherence, which represents the superposition of orthogonal states, is a fundamental concept in quantum mechanics. It can also be precisely defined within quantum resource theory. Exploring quantum coherence in neutrino oscillations can help examine their intrinsic quantum nature and potential applications in quantum information technologies [1]. Previous studies have focused on neutrino flavor oscillations (FO), but FO implies that neutrinos have mass, leading to the generation of a tiny magnetic dipole moment through quantum loop diagrams [1] [2]. This electromagnetic property can induce spin-flavor oscillations (SFO) in the presence of an external magnetic field, enriching the study of coherence. This work investigates quantum coherence in neutrino SFO with three flavors mixing within interstellar and intergalactic magnetic fields, quantified by the l 1 norm and the relative entropy of coherence, expressed in terms of neutrino probabilities.

Quantum coherence, as a resource, is defined within the framework of quantum resource theory, enabling its quantification and analysis across diverse quantum systems [1]-[3]. In this paper, coherence is measured using two widely accepted metrics: the norm of coherence, which sums the absolute values of the off-diagonal elements in the density matrix, and the relative entropy of coherence, which quantifies the "distance" between a given quantum state and its nearest incoherent state. These measures provide a rigorous framework for comparing the coherence properties of different quantum systems. In neutrino oscillations, coherence is interpreted through the cyclic variations in transition probabilities, influenced by factors such as magnetic fields and energy scales. For quantum emitters, coherence is assessed based on the temporal stability of superposition states under external driving fields. By applying these metrics to modulated systems, this paper aims to clarify the role of quantum coherence in maintaining information fidelity, extending coherence times, and enabling advanced applications in quantum computing and astrophysics.

2. Principles of Optical Fibers

Within the Standard Model (SM) of electroweak interactions, neutrinos were initially considered massless and interacting solely through weak interactions. However, the observation of neutrino oscillation indicates otherwise, signifying a fundamental departure from the SM framework and suggesting that neutrinos possess mass. This discovery opens up intriguing possibilities regarding their electromagnetic properties, such as the generation of neutrino magnetic moments through quantum loop corrections. These interactions could extend to charged particles, potentially influencing a diverse array of astrophysical and particle physics phenomena. The value of the neutrino magnetic moment can be enhanced in new physics models, making the exploration of these interactions a powerful avenue in the pursuit of a comprehensive understanding beyond SM physics.

Quantum coherence is a critical aspect of quantum mechanics, representing the ability of quantum systems to exhibit superposition states. It is foundational to various quantum technologies, including quantum computing, quantum cryptography, and quantum communications. The concept of coherence has been rigorously defined within the framework of quantum resource theory, allowing for precise quantification and analysis. Neutrino oscillations, a phenomenon where neutrinos switch between different flavors as they propagate, provide a unique context for studying quantum coherence. The discovery of neutrino mass and oscillations has profound implications for particle physics, indicating physics beyond the Standard Model. Previous studies have primarily focused on flavor oscillations (FO), but recent research suggests that spin-flavor oscillations (SFO), induced by neutrino magnetic moments in the presence of external magnetic fields, offer a richer framework for exploring coherence.

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi(t)\right\rangle = H(t)\left|\psi(t)\right\rangle$$

The mechanism of neutrino oscillations is deeply rooted in the quantum mechanical mixing of mass and flavor eigenstates [4]. This phenomenon arises due to the non-zero mass differences between neutrino species, as confirmed by experiments like those conducted at the Sudbury Neutrino Observatory (SNO) and the Super-Kamiokande detector [3] [4]. The oscillation probabilities are governed by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which encodes the mixing parameters between mass and flavor states. Floquet theory, which deals with the behavior of systems subjected to periodic driving, provides a powerful tool for analyzing modulated quantum systems [4]. This theoretical framework allows for the examination of electron wavefunctions under external driving fields, revealing new insights into quantum state manipulation and coherence preservation. Recent advancements in Floquet engineering have demonstrated its potential for controlling quantum systems, making it a valuable approach for studying quantum coherence in modulated environments. Floquet can be used to solve quantum systems described by a time-periodic Hamiltonian $H(t)$,

$$\text{such that } H(t+T)=H(t)\text{, where } T=2\pi/\omega \tag{1}$$

T is the driving period and ω is the driving frequency. Time-Dependent Schrödinger Equation: The dynamics of a quantum state $|\psi(t)\rangle$ in a time-dependent Hamiltonian are governed by the Schrödinger equation:

$$ih\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle \tag{2}$$

Floquet Ansatz: Floquet theory asserts that the solution to this equation can be written as:

$$|\psi(t)\rangle = \mathrm{e}^{-i\varepsilon t/h}|\phi_a(t)\rangle \tag{3}$$

Substituting the Floquet ansatz into the Schrödinger equation, we obtain:

$$\left[H(t)-ih\frac{\mathrm{d}}{\mathrm{d}t}\right]|\phi_a(t)\rangle = \varepsilon|\phi_a(t)\rangle \tag{4}$$

This equation indicates that

$$H_F(t)=H(t)-ih\frac{\mathrm{d}}{\mathrm{d}t} \tag{5}$$

This equation allows for the calculation of quasi-energies and Floquet modes.

$$\Sigma(H+mh\omega\delta)|\phi\rangle = \varepsilon|\phi\rangle \tag{6}$$

By leveraging Floquet theory, for neutrino oscillations, the coherence scale is extended through resonance effects, making it possible to study spin-flavor oscillations over astrophysical distances. In quantum emitters, coherence times are enhanced, enabling robust manipulation of quantum states for quantum computing and sensing applications. For example, Floquet theory has been applied to nitrogen-vacancy centers in diamonds to demonstrate quantum frequency mixing. The magnetic spin states of an NV center can be driven by AC magnetic fields that match one of the spin transition resonant frequencies (with a margin of error as large as the detuning) [5]. Quantum frequency mixing in NV centers uses a far detuned AC magnetic field as the signal; a bias signal is applied such that $w_B - w_S = w_{Resonant}$ [5]. The NV's nonlinearity allows for difference frequency generation in which the resonant frequency can be created from two non-resonant MW signals [5]. Floquet theory describes the Hamiltonian of the bias, signal, and resonant MW radiation, analogous to a local oscillator, bias signal, and intermediate frequency in the classical scheme.

3. Design and Optimization Techniques

The effective interaction Lagrangian for the generation of the magnetic dipole moment of a massive neutrino is given by:

$$L_{eff} = -0.5\Sigma\mu F v \sigma + h.c. \tag{7}$$

where μ_{ij} is the magnetic moment matrix in the space of three generations of neutrino states. The massive Dirac neutrino spin eigenstate propagating in the presence of an arbitrarily oriented magnetic field solves the Dirac equation, leading to the energy spectrum for ultrarelativistic neutrinos as:

$$E_i^s \sim p + m_i^2 / 2p + \mu_i s B_{\text{perpendicular}} \tag{8}$$

Coherence refers to the ability to predict a signal's behavior over a given change in one of the observable state variables. Coherence is a fundamental property of light that describes the degree of correlation between its electric field at different times. When dealing with single-photon sources, the concept of coherence becomes particularly intriguing and complex. Unlike classical light sources, where coherence can be measured through interference patterns and temporal correlations, single-photon sources pose unique challenges. A single photon cannot be measured separately to determine coherence since coherence is inherently a measure of correlation between different times. To measure coherence in a single-photon source, one would need at least two photons separated by a time t, to determine their coherence at that time. However, this method does not allow for the characterization of the source itself. For accurate characterization, a single-photon source must be run continuously while measuring coherence at different time delays. Even then, due to the inherent statistical nature of quantum emissions, shot-to-shot variations occur, similar to how a laser cannot be perfectly monochromatic. The coherence time of a photon is analogous to the coherence of a laser's electric field in a vacuum. If the electric field of a laser is measured right at the output, before any propagation, it still exhibits the same coherence time. This indicates that the coherence property is intrinsic to the source rather than the medium through which it propagates.

Consider a single atom oscillating at a frequency ω\omega$\omega$. Regardless of the elapsed time, the atom continues to oscillate at the same frequency, maintaining perfect correlation after an infinite amount of time. For sources with a narrow spectrum, while the frequency varies over time, the general correlation between two non-equivalent times decreases slowly as the time difference increases. Real light sources, however, comprise various mechanisms that reduce coherence. In semiconductor lasers, for instance, multiple factors contribute to decreased coherence: thermal fluctuations in the gain medium, thermal instability of the laser resonator's volume, fluctuating power supply, and non-uniform semiconductor structures with localized defects. Each of these imperfections causes the laser spectrum to vary over time, limiting the coherence time. A non-monochromatic linewidth can be modeled as a sine wave with occasional phase jumps, where the frequency changes at each phase jump. These frequency changes follow a probability distribution described by the spectral profile. For short periods below the noise fluctuations, the observed light wave appears monochromatic. However, real-world spectral measurements integrate over longer times, resulting in a broader, continuous spectrum. Temporal coherence describes how well the relationship between the laser field at different times can be predicted. High coherence time indicates a well-defined and predictable relationship over extended periods. Coherence is primarily a property of the source rather than the propagation medium. While the medium can distort the beam and reduce coherence, it generally does not enhance it. In a vacuum, a coherent beam maintains its coherence, and the coherence time remains unchanged, though it will not increase simply due to the absence of other media.

Coherence measures are derived using the l1 norm and the relative entropy of coherence. The l1 norm of coherence sums the absolute values of the off-diagonal elements of the density matrix, while the relative entropy of coherence measures the distance between the quantum state and its nearest incoherent state.

$$C_{l1}(\rho) = \Sigma|\rho| \tag{9}$$

$$C_{RE}(\rho) = S(\rho_D) - S(\rho) \tag{10}$$

For neutrino flavor oscillations (FO), coherence measures can sustain higher values over distances of several kilometers, relevant for terrestrial experiments. For spin-flavor oscillations (SFO), the coherence scale extends to astrophysical distances, spanning from kiloparsecs to gigaparsecs. The study of SFO in the presence of interstellar and intergalactic magnetic fields shows that the l1 norm of coherence and the relative entropy of coherence exhibit cyclic variations, with their values influenced by the combined effect of the neutrino magnetic moment and the external magnetic field strength. Neutrino sources within the Milky Way include supernovae, cosmic ray interactions, neutron stars, and black holes. For these sources, the interstellar magnetic field is approximately micro-Gauss. For extra-galactic sources like the TXS 0506+056 blazar, intergalactic magnetic fields with an upper bound of 10^−9 Gauss are considered. For a neutrino energy of 10 TeV and a magnetic moment of 10^−12 μB, the coherence measures for SFO exhibit extended coherence scales compared to FO, maintaining substantial coherence over kiloparsec distances. For higher neutrino energies and more stringent magnetic moment limits, the coherence measures extend even further, reaching gigaparsec scales.

4. Quantum Applications

Quantum coherence plays a foundational role in quantum computing, as it underpins the ability to maintain and manipulate quantum superposition states within qubits [6]. In practical implementations, coherence is utilized to perform unitary operations that constitute quantum gates, enabling complex algorithms such as Shor's factoring and Grover's search. The preservation of coherence is achieved through careful system design, including error-correction protocols and decoherence mitigation techniques. For example, superconducting qubits leverage coherence by operating at cryogenic temperatures to reduce thermal noise, while trapped-ion systems employ laser-cooling and electromagnetic traps to isolate qubits from environmental interactions. Coherence is also directly measured through techniques like Ramsey interference and Rabi oscillations, which validate the stability and accuracy of quantum state preparation. Recent advances in Floquet engineering provide additional tools for enhancing coherence in quantum systems, allowing for dynamic decoupling schemes that prolong qubit lifetimes [6]. These methods demonstrate how coherence is not only preserved but actively controlled to enable fault-tolerant quantum computing, highlighting the critical interplay between theoretical models of coherence and their implementation in hardware systems.

5. Future Trends and Innovation

The ability to manipulate quantum states with high precision is crucial for the development of quantum information technologies. By leveraging the principles of Floquet theory and the modulation of electron wavefunctions, it is possible to achieve controlled quantum state preparation. This involves using external driving fields to engineer specific quantum states that can be utilized for various applications, such as quantum computing and quantum communication [6]. Maintaining coherence in quantum systems is a significant challenge due to decoherence effects caused by interactions with the environment. However, the extended coherence scales observed in spin-flavor oscillations (SFO) suggest that it is possible to preserve coherence over much longer distances and timescales. This has important implications for the development of robust quantum communication protocols and the implementation of quantum networks [6].

One of the most promising applications of extended coherence in SFO is in long-distance quantum communication [6]. Neutrinos, due to their weak interactions with matter, can travel vast distances with minimal decoherence. This makes them ideal candidates for transmitting quantum information over interstellar and intergalactic distances. The ability to maintain quantum coherence in neutrino oscillations could revolutionize the field of quantum communication, enabling secure and efficient transmission of information across the cosmos. Within the Milky Way, neutrino sources include supernovae, cosmic ray interactions, neutron stars, and black holes. These astrophysical events produce high-energy neutrinos that travel through the interstellar medium, interacting with magnetic fields along the way. The study of quantum coherence in neutrino oscillations provides valuable insights into the behavior of these neutrinos as they traverse the galaxy.

Beyond the Milky Way, there are numerous extragalactic neutrino sources, such as active galactic nuclei (AGN), gamma-ray bursts (GRBs), and blazars. These sources produce ultra-high-energy neutrinos that travel through the vast expanses of intergalactic space. The presence of intergalactic magnetic fields can induce spin-flavor oscillations, affecting the coherence of these neutrinos. Understanding these interactions is crucial for interpreting the signals detected by neutrino observatories on Earth. The study of quantum coherence in neutrino oscillations also has significant implications for cosmology. Neutrinos are abundant in the universe and play a vital role in various cosmological processes, including the evolution of large-scale structures and the thermal history of the universe. By analyzing the coherence properties of neutrinos, we can gain deeper insights into the fundamental physics governing the universe's evolution and the nature of dark matter and dark energy.

To verify the theoretical predictions of extended coherence in neutrino oscillations, advanced experimental techniques are required. Neutrino observatories, such as IceCube, Super-Kamiokande, and the upcoming Hyper-Kamiokande, are equipped to detect high-energy neutrinos from astrophysical sources. These observatories can provide valuable data on the behavior of neutrinos and their interactions with magnetic fields, allowing researchers to test the coherence measures proposed in this study. Despite the significant advancements in neutrino detection technology, there are still numerous challenges to overcome. One of the primary challenges is the low interaction cross-section of neutrinos, which makes them difficult to detect. Additionally, distinguishing between different types of neutrino interactions and isolating the effects of magnetic fields on coherence requires sophisticated data analysis techniques. However, these challenges also present opportunities for innovation and the development of new experimental methods. Future research in this field should focus on exploring the practical applications of extended coherence in neutrino oscillations. This includes investigating the potential for long-distance quantum communication, developing new quantum information protocols, and studying the implications for astrophysical and cosmological phenomena. Furthermore, advancements in experimental techniques and neutrino observatories will enable more precise measurements and a deeper understanding of the fundamental physics underlying quantum coherence in modulated systems.

6. Conclusions

This study demonstrates that quantum coherence in neutrino spin-flavor oscillations can extend to astrophysical distances, offering new insights into the quantum nature of neutrinos and their potential applications in quantum information technologies. The extended coherence scales observed in SFO compared to FO highlight the importance of considering electromagnetic interactions in the study of neutrino oscillations. Future research could explore the practical applications of these findings in quantum communication and the study of beyond Standard

Model physics. The ability to maintain quantum coherence over vast distances opens up exciting possibilities for long-distance quantum communication and the development of robust quantum networks.

While this paper primarily focuses on theoretical predictions, experimental verification is crucial for validating these findings. For neutrino oscillations, experiments such as those conducted at neutrino observatories like IceCube, Super-Kamiokande, and the upcoming Hyper-Kamiokande offer opportunities to test coherence measures. These facilities can detect high-energy neutrinos and their interactions with interstellar and intergalactic magnetic fields, allowing researchers to observe the predicted extended coherence scales in spin-flavor oscillations. Controlled laboratory experiments using quantum emitters, such as nitrogen-vacancy (NV) centers in diamond, provide a complementary platform for studying coherence under external driving fields. By engineering periodic modulations through microwave or AC magnetic fields, Floquet states can be realized, and coherence properties can be directly measured through fluorescence spectroscopy or Ramsey interference experiments. These experimental setups not only test the theoretical models but also refine the understanding of coherence mechanisms, paving the way for practical applications in quantum information processing and astrophysical studies.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Abad-Arredondo, J. and Fernández-Domínguez, A.I. (2024) Quantum State Preparation and Readout with Modulated Electrons. http://arxiv.org/abs/2407.17885

[2] Alok, A.K., Chall, T.J., Chundawat, N.R.S., Gangal, S. and Lambiase, G. (2024) Quantum Coherence in Neutrino Spin-Flavor Oscillations. http://arxiv.org/abs/2407.16742

[3] Carreño, J.C.L. and Laussy, F.P. (2016) Excitation with Quantum Light. I. Exciting a Harmonic Oscillator. *Physical Review A*, **94**, Article 063825. https://doi.org/10.1103/physreva.94.063825

[4] Engelhardt, G., Luo, J., Bastidas, V.M. and Platero, G. (2024) Photon-Resolved Floquet Theory. I. Full Counting Statistics of the Driving Field in Floquet Systems. *Physical Review A*, **110**, Article 063707. https://doi.org/10.1103/physreva.110.063707

[5] Karlson, S.J., Kehayias, P., Schloss, J.M., Maccabe, A.C., Phillips, D.F., Wang, G., Cappellaro, P. and Braje, D.A. (2024) Quantum Frequency Mixing Using an NV Diamond Microscope. http://arxiv.org/abs/2407.07025

[6] Koudia, S., Oleynik, L., Bayraktar, M., Rehman, J.Ur. and Chatzinotas, S. (2024) Physical Layer Aspects of Quantum Communications: A Survey. http://arxiv.org/abs/2407.09244