Journal of Quantum Information Science, 2024, 14, 196-233
https://www.scirp.org/journal/jqis
ISSN Online: 2162-576X
ISSN Print: 2162-5751

Towards a Unified Theory of Everything: Integrating Discrete Time Evolution and Classical-Quantum Dynamics in the Advanced Observer Model (AOM)

Joseph H. C. Wong

Department of Computing, The Hong Kong Polytechnic University, Hong Kong, China
Email: hcjosephwong@hotmail.com

How to cite this paper: Wong, J.H.C. (2024) Towards a Unified Theory of Everything: Integrating Discrete Time Evolution and Classical-Quantum Dynamics in the Advanced Observer Model (AOM). *Journal of Quantum Information Science*, **14**, 196-233.
https://doi.org/10.4236/jqis.2024.144009

Received: September 18, 2024
Accepted: November 11, 2024
Published: November 14, 2024

Abstract

This paper introduces the Advanced Observer Model (AOM), a novel framework that integrates classical mechanics, quantum mechanics, and relativity through the observer's role in constructing reality. Central to the AOM is the Static Configuration/Dynamic Configuration (SC/DC) conjugate, which examines physical systems through the interaction between static spatial configurations and dynamic quantum states. The model introduces a Constant Frame Rate (CFR) to quantize time perception, providing a discrete model for time evolution in quantum systems. By modifying the Schrödinger equation with CFR, the AOM bridges quantum and classical physics, offering a unified interpretation where classical determinism and quantum uncertainty coexist. A key feature of the AOM is its energy scaling model, where energy grows exponentially with spatial dimensionality, following the relationship $E \propto \left(\sqrt{\pi}\right)^n$. This dimensional scaling connects the discrete time perception of the observer with both quantum and classical energy distributions, providing insights into the nature of higher-dimensional spaces. Additionally, the AOM posits that spacetime curvature arises from quantum interactions, shaped by the observer's discrete time perception. The model emphasizes the observer's consciousness as a co-creator of reality, offering new approaches to understanding the quantum-classical transition. While speculative, the AOM opens new avenues for addressing foundational questions in quantum mechanics, relativity, dimensionality, and the nature of reality.

Keywords

Quantum Mechanics, Schrödinger Equation, Constant Frame Rate (CFR), Advanced Observer Model (AOM), Relativistic Physics, Classical-Quantum Transition, Wave Function, Discrete Time Evolution, Spacetime Geometry, Unified Theory, Quantum-Classical Unification, Observer-Dependent Reality, Energy Scaling

1. Introduction

The pursuit of a unified theory that harmonizes classical mechanics, relativistic physics, and quantum mechanics remains one of the foremost challenges in modern theoretical physics. Classical mechanics, rooted in Newtonian principles, provides a deterministic framework for describing macroscopic systems. Quantum mechanics, represented by the Schrödinger equation, governs the probabilistic behavior of microscopic systems. In contrast, relativistic mechanics, derived from Einstein's general theory of relativity, elegantly describes spacetime and gravity at large scales and high velocities. Despite the remarkable success of each framework, their mutual incompatibility presents a formidable barrier to unification.

The Advanced Observer Model (AOM) offers a fresh perspective on this longstanding challenge. Wong (2024) explores the AOM's potential to bridge classical, quantum, and relativistic domains, revealing how its principles connect these seemingly disparate frameworks [1]. Building on this, Wong (2024) presents a conceptual and theoretical foundation for the AOM, highlighting its promise in unifying physical theories [2]. Additionally, Wong (2024) demonstrates how the integration of quantum mechanics within the AOM redefines reality construction, aiming to reconcile the probabilistic nature of quantum mechanics with the determinism of classical physics and the curvature effects of relativity [3].

This paper introduces the $\hbar$/CFR-modified Schrödinger equation, which incorporates the Constant Frame Rate (CFR) from the AOM, presenting an innovative approach to unify these domains. By embedding CFR into the Schrödinger equation, the model modifies time evolution in quantum systems and aligns quantum wave dynamics with relativistic spacetime curvature. This synthesis of quantum discreteness, classical determinism, and relativistic geometry offers a novel perspective on the quest for a unified physical theory.

The $\hbar$/CFR-modified Schrödinger equation facilitates the description of quantum systems in terms of the observer's frame rate, enabling the emergence of classical spacetime geometry from quantum processes. By integrating quantum mechanics, relativity, and frame-rate-dependent time evolution, this approach opens new avenues for understanding the fundamental nature of reality and reinterprets the transition between classical and quantum realms.

1.1. Literature Review

Classical mechanics, rooted in Newton's laws of motion, provides a deterministic framework for macroscopic systems. Central concepts such as phase space and Hamiltonian formalism are foundational to understanding these systems, but classical mechanics cannot explain quantum phenomena such as superposition and entanglement [4] [5]. In contrast, quantum mechanics introduces wavefunctions and operators, leading to probabilistic outcomes that diverge from classical determinism [6].

One of the primary challenges in modern physics is reconciling the probabilistic nature of quantum mechanics with the deterministic framework of classical mechanics. Richard Feynman's path integral formulation offers one potential solution, presenting classical and quantum mechanics as specific cases of a more general theory [7]. In this framework, quantum mechanics emerges from path interference, while classical mechanics results from the principle of least action. However, limitations arise when applying this formulation to many-body systems and in contexts involving strong gravitational fields [8].

The Wigner-Weyl formulation, based on the Wigner function, represents a quasi-probability distribution that helps bridge quantum and classical mechanics. While this formulation allows for direct comparison between quantum and classical systems, the Wigner function's negative values present conceptual challenges, as they do not have a classical counterpart [9] [10]. This issue highlights the difficulty in reconciling these two fundamental domains of physics.

Niels Bohr's correspondence principle asserts that quantum mechanics must reduce to classical mechanics in the limit of large quantum numbers [11]. Although this principle has been invaluable in extending quantum theory to classical scales, it struggles to explain systems where quantum and classical behaviors overlap or coexist, such as in mesoscopic systems [12].

Another approach, known as deformation quantization, aims to unify classical and quantum mechanics by deforming the classical Poisson algebra into a noncommutative quantum algebra. This method enables both regimes to be treated within a single algebraic structure, but its lack of intuitive physical interpretation remains a significant challenge [13] [14]. The technique is mathematically rigorous but often difficult to apply to practical problems in quantum mechanics.

In certain areas of physics, coherent states have emerged as a practical bridge between classical and quantum mechanics, particularly in quantum optics and the study of harmonic oscillators. These states exhibit classical-like behavior, allowing for semiclassical approximations that prove useful in understanding quantum systems [15]. However, despite their utility, coherent states do not generalize to all quantum phenomena, limiting their application in broader quantum theories [16] [17].

The integration of classical and quantum mechanics is further complicated when considering general relativity and the curvature of spacetime. Aharonov and Bohm's work on the significance of electromagnetic potentials in quantum theory reveals the non-local effects that quantum mechanics must accommodate [18]. In addition, theories such as string theory propose that higher dimensions might hold the key to unifying these domains [19].

More recently, quantum gravity theories have attempted to reconcile quantum mechanics and general relativity by rethinking the nature of space, time, and energy. In particular, quantum consciousness models, as proposed by Kodukula, emphasize that observer-consciousness plays a critical role in synchronizing quantum mechanics with relativity [20]. These models explore the influence of the observer in shaping reality and how consciousness may bridge the gap between these two frameworks.

Building on this, Christensen Jr. explores the complex interplay between quantum mechanics and general relativity, suggesting that the integration of these two theories requires a deeper understanding of the observer's interaction with spacetime [21]. Similarly, Ko proposes a framework based on in-out duality, which delves into the foundational role of the observer within social quantum mechanics and how this duality might contribute to a broader understanding of quantum phenomena [22].

Together, these efforts underscore the importance of developing a framework that fully integrates the observer's role in both quantum mechanics and relativity, pointing toward a more holistic and unified theory.

1.2. Challenges in Achieving a Unified Theory

Unifying classical and quantum mechanics is challenging due to their fundamental differences. Classical mechanics is deterministic, while quantum mechanics introduces probabilistic outcomes and discrete quantities [15] [16]. Efforts to reconcile these realms, such as string theory and loop quantum gravity, face significant mathematical and experimental hurdles [17].

Complexity of Integration: Classical and quantum mechanics use distinct tools and concepts. Integrating these methods into a unified theory requires synthesizing diverse mathematical frameworks and resolving conflicts between classical determinism and quantum indeterminism. For instance:

- Classical mechanics relies on well-defined trajectories.
- Quantum mechanics uses probabilities and wavefunctions.
- General relativity models gravity as space-time curvature.
- Quantum field theory describes particles as field excitations.

Unifying these frameworks presents a significant challenge:

a) Lack of Experimental Guidance:

A comprehensive theory of everything ideally requires experimental evidence of how current theories might overlap or fail. However, experiments typically confirm either quantum mechanics or general relativity, without clear indications of their unification. The extreme conditions in black holes, the early universe, and quantum gravity make experimental investigation difficult.

b) Specialization in Physics:

Physics has become highly specialized, with distinct subdisciplines focusing on specific areas:

- Quantum optics explores light-matter interactions.

- General relativity examines space-time and gravity.
- Quantum field theory investigates particle interactions.

Integrating these approaches into a unified theory requires cross-disciplinary collaboration, which is challenging in a field that values specialization.

c) Success of Effective Theories:

Many physicists use effective theories that accurately describe phenomena within specific domains:

- Quantum mechanics excels at atomic and subatomic scales.
- Classical mechanics is effective for macroscopic objects.
- General relativity explains large-scale gravitational phenomena.

These theories are successful in their respective areas, making it practical to work within these frameworks rather than seeking a grand unification.

d) Challenges with Quantum Gravity:

A major obstacle is developing a theory of quantum gravity that reconciles quantum mechanics with general relativity. Quantum mechanics involves probabilities and wavefunctions, while general relativity treats gravity as space-time curvature. Reconciling these approaches, especially at scales where both intersect, such as black holes or the Big Bang, is extremely difficult.

e) The Landscape of Theoretical Physics:

String theory and loop quantum gravity are promising candidates for a theory of everything. String theory aims to unify all forces of nature through vibrating strings, but faces challenges in making testable predictions and integrating its multiple versions. Loop quantum gravity seeks to quantize space-time directly but struggles with experimental validation and integration with other theories.

f) Cultural and Historical Momentum:

Distinct frameworks for classical, quantum, and relativistic physics have deep historical roots. Unifying these frameworks requires significant rethinking and integration of foundational principles, which can be challenging in a field that values incremental progress and specialized expertise.

2. The SC/DC Conjugate and the $\hbar$/CFR-Modified Schrödinger Equation

The quest to unify classical and quantum physics remains a central challenge in theoretical physics. The Static Configuration/Dynamic Configuration (SC/DC) framework, augmented by the Constant Frame Rate (CFR) and the $\hbar$/CFR-Modified Schrödinger Equation, provides a sophisticated approach to this issue.

2.1. Discrete Point in Time (DPIT) and Its Contrast with Planck Time (t_p)

To enhance our understanding of temporal dimensions within the SC/DC framework, we introduce the concept of the Discrete Point in Time (DPIT). Unlike t_p, which represents the shortest meaningful continuous duration of time and is fundamental in quantum mechanics, DPIT denotes a distinct, discrete moment in time.

- **Planck Time (t_p)**: This is the smallest unit of continuous time, characterized by its fundamental role in defining the limits of classical and quantum descriptions of temporal evolution. t_p represents the granularity of time beyond which our current understanding of physics is fundamentally challenged.
- **Discrete Point in Time (DPIT)**: In contrast, DPIT refers to a specific, discrete instant within a temporal sequence. It serves as a marker in a sequence of discrete time steps, used to model and analyze time evolution in quantum and classical systems. DPIT is essential for capturing discrete dynamics that cannot be resolved within the continuous framework of t_p.

The Constant Frame Rate (CFR) is defined as the reciprocal of t_p, calculated as $1/t_p$. Given that t_p is approximately 5.39×10^{-44} seconds, CFR is on the order of 10^{43} Hz. This high frame rate reflects the immense temporal resolution required to accurately model and integrate the SC/DC framework with the $\hbar$/CFR-Modified Schrödinger Equation.

By incorporating CFR and distinguishing between continuous durations (represented by t_p) and discrete points in time (DPIT), the SC/DC framework bridges the gap between classical and quantum descriptions, offering a more nuanced perspective on the nature of time and its role in the unification of physical theories.

Figure 1 illustrates the discrete points in time (DPITs) versus the continuous time evolution underpinned by a wave function.

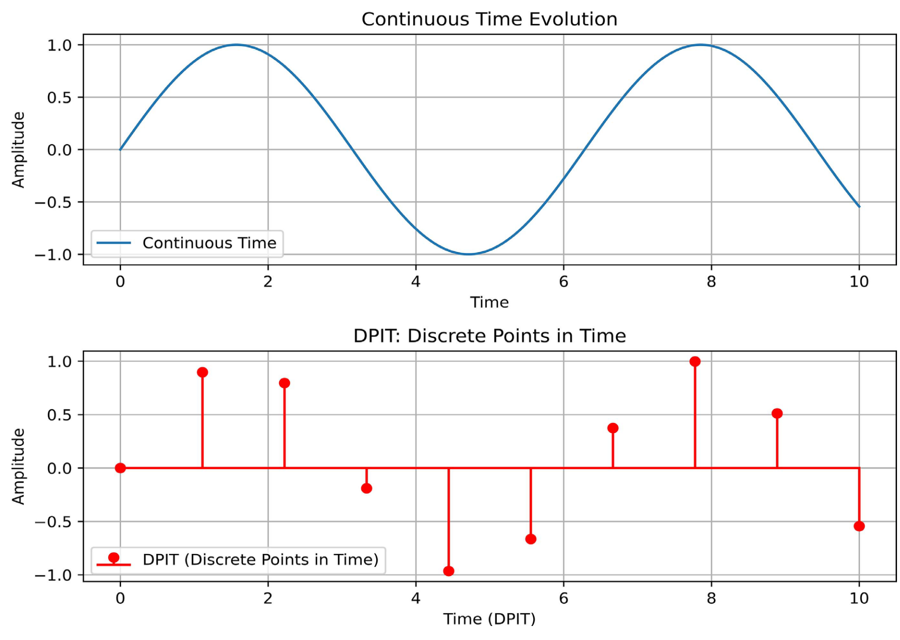


Figure 1. Discrete Points in Time (DPITs) vs. Continuous Time Perception.

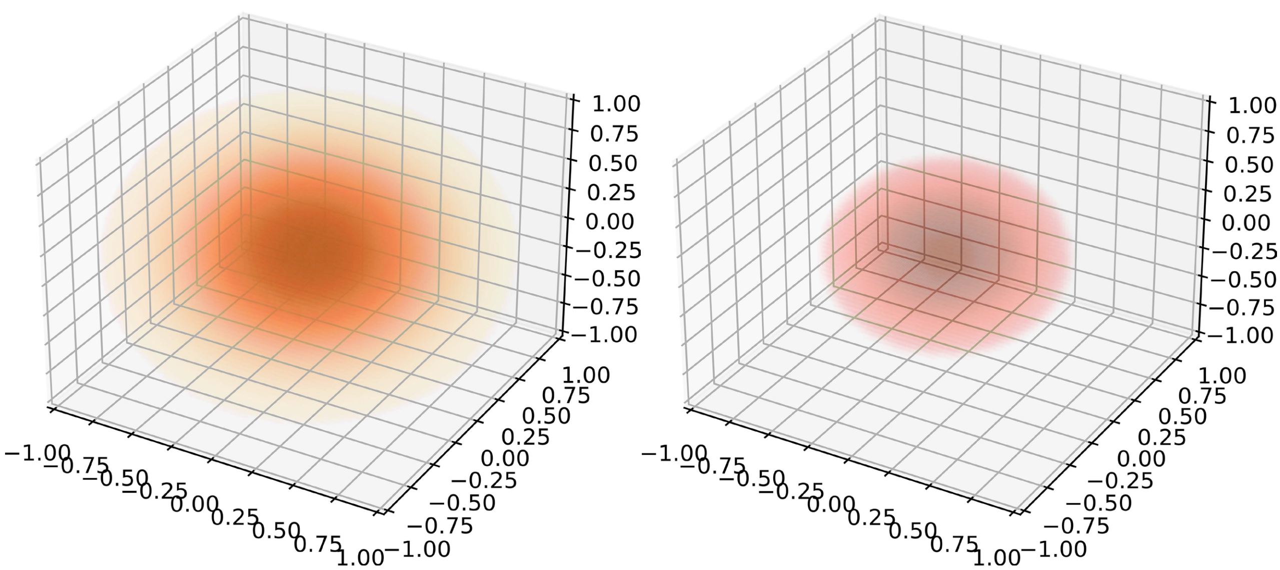
2.2. Unified Description of Scales

The SC/DC model introduces two complementary perspectives for understanding physical systems:

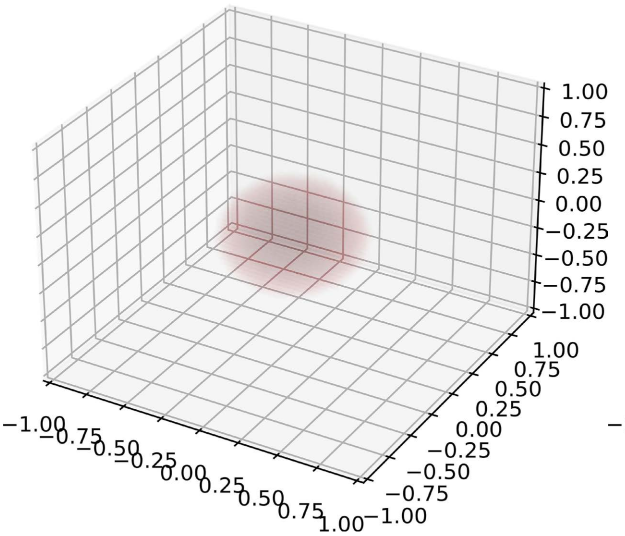
- **Static Configuration (SC)**: This concept encompasses entities across all scales, from macroscopic objects to microscopic particles. At macroscopic scales, SC includes classical phenomena such as gravitational fields and large-scale structures, where classical mechanics and general relativity are applicable. At microscopic scales, SC describes quantum phenomena, including field potentials and wave functions. In the context of the Advanced Observer Model (AOM), SC represents the perceived reality, providing a static snapshot of space curvature.
- **Dynamic Configuration (DC)**: This concept pertains to quantum phenomena and microscopic entities governed by quantum mechanics and described by the Schrödinger Equation. Although typically applied to quantum systems, DC can be extended to macroscopic scales. For example, one can theoretically derive the DC for Earth, describing its wave function. At macroscopic scales, however, the frequency is so low that the kinetic component becomes negligible. Within the AOM, DC represents the intrinsic, dynamic reality of space curvature.

By treating SC and DC as complementary constructs, the model bridges the gap between macroscopic and microscopic descriptions.

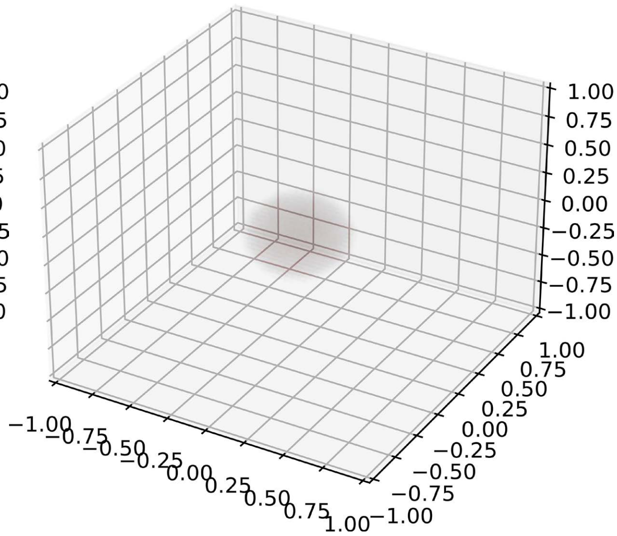
Figure 2 illustrates the evolution of a dynamic configuration (DC) governed by its wave function Ψ, represented across discrete points in time (DPITs). The orange region signifies the DC with positive magnitude, which gradually diminishes as it moves towards the center, where its energy reaches zero. Beyond this point, negative energy emerges and increases until it peaks on the right side, depicting half of the wave function Ψ's cycle. In the framework of the universal quantum system, a DPIT is not a zero-dimensional void; rather, it is a point of completeness, where each moment in the discrete time sequence represents a fully realized state of everything, existing in coherent consistency.



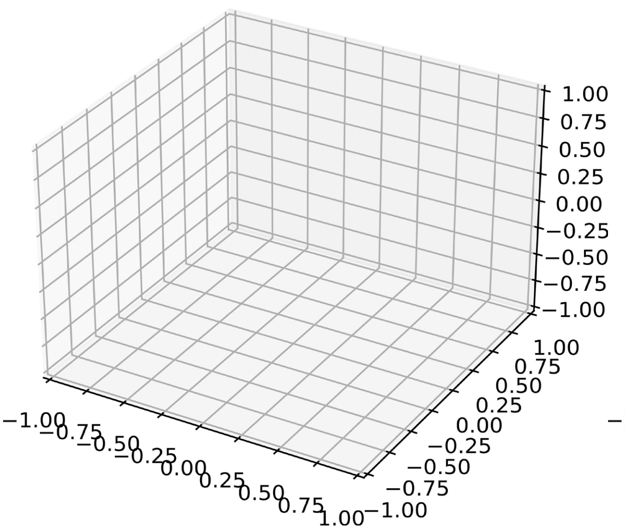
(a) (b)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

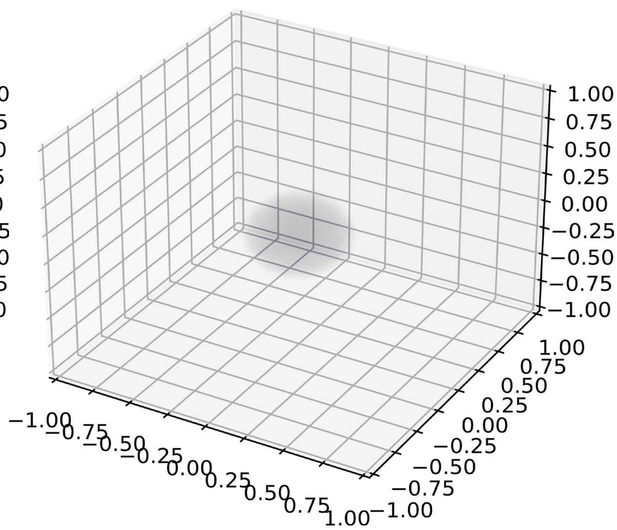
(c)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

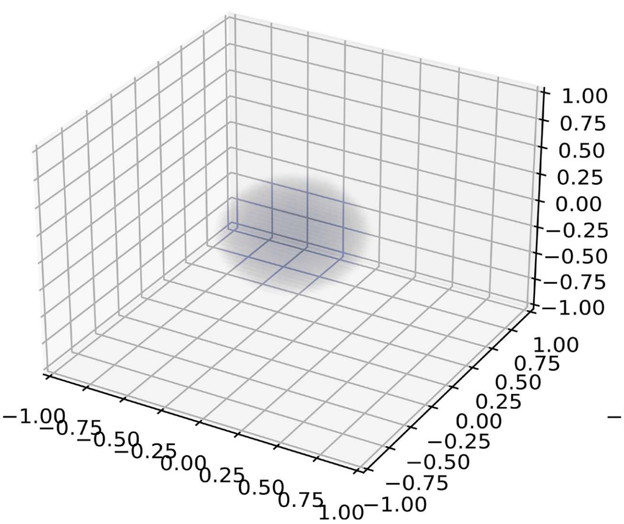
(d)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

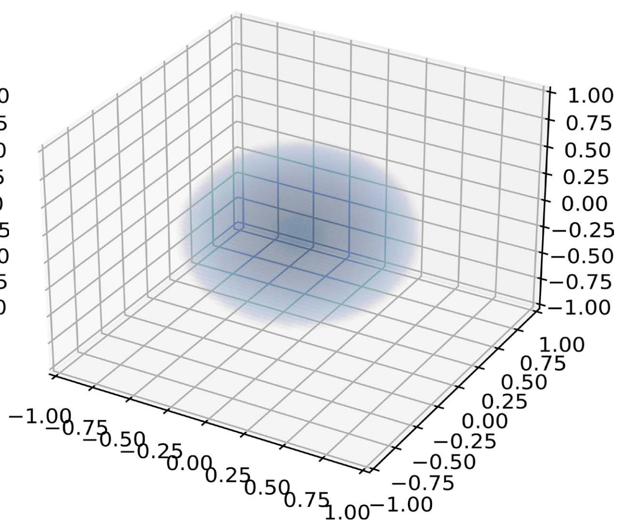
(e)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

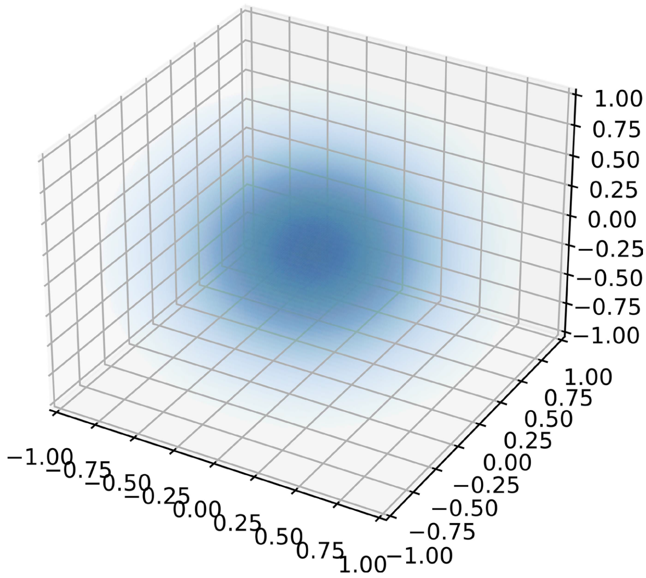
(f)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

(g)


1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
1.00 0.75 0.50 0.25 0.00 −0.25 −0.50 −0.75 −1.00

(h)



(i)

Figure 2. Half a flip-flop of a Dynamic Configuration (DC).

Figure 3 illustrates the energy configurations in the Advanced Observer Model (AOM), comparing the Dynamic Configuration (DC), representing "Reality as is," and the Static Configuration (SC), representing "Reality as perceived." The DC is depicted in a state of timelessness, visualized through the unnormalized squared modulus of its wave function, $|\Psi|^2$, which represents the kinetic energy space. In this state, the DC exists as a concentrated energy cluster, where the wave function's unnormalized squared modulus indicates the energy density distribution, showing regions of equilibrium.

In contrast, the SC, which is embedded within the temporal framework of the AOM, is characterized by the potential energy space. At each spatial position x, the unnormalized squared modulus reflects the kinetic energy level for the DC at that point, while simultaneously determining the corresponding potential energy level for the SC at the same location.

This dual representation highlights how, in the AOM, energy dynamics differ based on the observer's perspective: the DC operates outside of time with energy in its pure kinetic form, while the SC reflects the observer's interpretation of energy evolving within a temporal framework. The figure was generated using numerical solutions of the Schrödinger equation, applying Constant Frame Rate (CFR) discretization to simulate the time evolution of the wave functions for both configurations.

2.3. Reinterpreted Probability Density Function (PDF)

In the SC/DC framework, the squared modulus of a wave function, $|\Psi|^2$, is understood differently than in traditional quantum mechanics. Instead of representing a probabilistic distribution, $|\Psi|^2$ is interpreted as describing the spatial extent and curvature of both Static Configurations (SC) and Dynamic Configurations (DC)

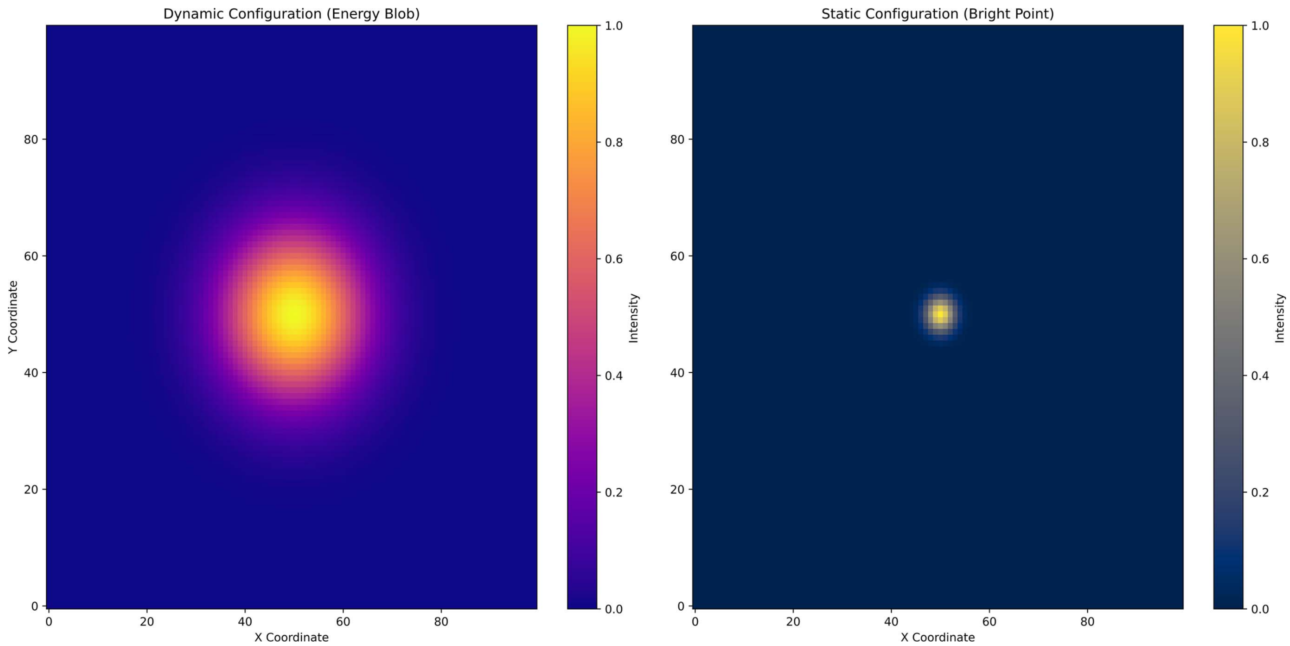


Figure 3. Dynamic Configuration (DC) is the "Reality as is". Static Configuration (SC) is the "Reality as perceived".

at a given Discrete Point in Time (DPIT). Here's how this reinterpretation unfolds:

- **Timeless Context**: In this framework, $|\Psi|^2$ is not employed to compute probabilities but to characterize the spatial distribution of energy and matter. It offers a static depiction of how the SC and DC configurations are organized in space at a given discrete point in time (DPIT). This spatial representation remains invariant and 'does not necessitate normalization' (see **Appendix B**). The unnormalized squared modulus of the wave function, $|\Psi|^2$, illustrates the structure and extent of configurations at a specific moment, where the constant π defines the harmonious balance and optimal state of energy distribution within the SC/DC framework.
- **Temporal Progression**: As time progresses, $|\Psi|^2$ takes on a probabilistic interpretation due to repeated observations and interactions, necessitating normalization to 1. In this sense, while $|\Psi|^2$ provides a static description of spatial arrangements, it acquires probabilistic characteristics when considered over successive DPITs. This aligns with classical probability concepts, where the probabilistic nature emerges from repeated measurements and interactions rather than being an intrinsic property of $|\Psi|^2$ itself.
- **Emergent Properties and π**: In this context, π represents a static property of a balanced SC/DC. It is not a temporal property but rather an emergent constant that describes the coherent balance of the configuration. This π property is related to the spatial extent and curvature of the SC/DC and is preserved across the temporal progression of the Advanced Observer Model (AOM). It appears within the definition of the wave function as a characteristic of the spatial configuration but is not influenced by time.

The unnormalized squared modulus $|\Psi|^2$ defines the spatial extent and curvature of SC and DC configurations at any given DPIT, while the wave function Ψ governs the temporal oscillation or 'flip-flop' (a descriptive way to refer to the complete oscillation or cycle of a wave function in the DC domain) behavior across successive DPITs. The property of π emerges as a static descriptor of balance and coherence in these configurations, representing a fundamental spatial characteristic that does not change over time, although it is evident in the wave function's definition.

In summary, within the SC/DC framework, $|\Psi|^2$ provides a static, spatial description that does not need normalization. The concept of π as a static property reflects the inherent balance and coherence of the SC/DC configurations, preserved across time but not inherently temporal.

2.4. Integration of Constant Frame Rate (CFR)

The Constant Frame Rate (CFR), defined as $\text{CFR} = 1/t_p \approx 1.855 \times 10^{43}$ frame per second, is crucial for linking the temporal aspects of SC and DC:

- **Discrete Point in Time (DPIT)**: DPIT represents distinct points in time within a continuous framework, with t_p being the duration between these points. CFR establishes a relationship between temporal progression and spatial configurations, integrating classical and quantum descriptions.
- **Temporal Dynamics**: The $\hbar$/CFR-Modified Schrödinger Equation incorporates CFR to ensure consistency between the temporal progression and spatial descriptions of SC and DC.

2.5. Addressing the Integration Problem

The SC/DC conjugate framework effectively addresses several challenges:

- **Spatial and Temporal Unity**: Incorporating CFR into the Schrödinger Equation harmonizes spatial and temporal descriptions across SC and DC, providing a unified view of physical systems.
- **Energy Consistency**: The $\hbar$/CFR-modified Schrödinger Equation accounts for energy changes in both SC and DC, ensuring that energy conservation principles are upheld across classical and quantum scales.
- **Probabilistic vs. Deterministic**: The reinterpretation of the $|\Psi|^2$ facilitates a smooth transition between probabilistic quantum mechanics and deterministic classical mechanics, offering a cohesive perspective on physical phenomena.

In summary, the SC/DC conjugate framework, with its definitions of SC and DC, along with CFR and the $\hbar$/CFR-Modified Schrödinger Equation, provides a robust solution to the integration problem between classical and quantum physics. By unifying spatial and temporal descriptions across different scales and incorporating a common temporal progression framework, this model advances our understanding of the fundamental nature of reality and the interplay between classical and quantum domains. The particle-wave duality is more accurately described as the SC/DC duality.

DPIT represents a point without duration, where the quantum state serves as its identity. Thus, DPIT is the natural identifier of a quantum state within the Sequence of Quantum States, or a frame within the Perceptual Sequence of Observations [2]. In this framework, the quantum universe can be viewed as the largest Dynamic Configuration (DC), while our universe is the largest Static Configuration (SC). An SC is static, meaning that at each DPIT—where time has no duration—a quantum state (DC) and a frame (SC) both exist in a timeless state. This is the essence of discreteness. Moving from one DPIT to another represents a transition from one consistent state to the next, where two DPITs at the ends of a Planck time interval t_p describe two consistent Static Configurations related by the duration t_p. At any given DPIT, both SC and DC are timeless, as a DPIT itself has no temporal duration.

2.6. The $\hbar$/CFR-Modified Schrödinger Equation

In standard quantum mechanics, the Schrödinger equation describes how the wave function of a system evolves over continuous time:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{1}$$

where $\hbar$ is Planck's constant, $\hat{H}$ is the Hamiltonian, and Ψ is the wave function. In the SC/DC framework and AOM, time is discrete, and events occur at discrete intervals, or DPITs.

In the discrete version of the Schrödinger equation, CFR (Constant Frame Rate) refers to the frequency of discrete time steps per second. When discretizing time, the time step Δt is inversely proportional to CFR, meaning:

$$\Delta t = \frac{1}{\text{CFR}} \tag{2}$$

This inverse relationship between time step size and CFR is the key idea behind discrete time evolution. When we take the continuous Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{3}$$

and discretize it, we replace the continuous derivative $\partial\Psi/\partial t$ with a discrete difference:

$$\frac{\partial \psi}{\partial t} \to \frac{\Delta \psi}{\Delta t} \tag{4}$$

Now, since $\Delta t = 1/\text{CFR}$, we have:

$$\frac{\Delta \psi}{\Delta t} = \text{CFR} \cdot \Delta \psi \tag{5}$$

Thus, the discrete Schrödinger equation becomes:

$$i\hbar \frac{\Delta \psi}{\Delta \text{DPIT}} = \hat{H}_{SC+DC}\psi, \tag{6}$$

and because $\Delta\text{DPIT} = 1/\text{CFR}$, we don't explicitly multiply by CFR but rather recognize that Δt is small due to large CFR. The key points are:

- The CFR governs the size of time steps (smaller time steps mean a higher CFR).

- The factor 1/CFR is associated with time steps in a discrete-time framework.

In the final equation:

$$i\hbar \frac{1}{\text{CFR}} \frac{\Delta \psi}{\Delta \text{DPIT}} = \hat{H}_{SC+DC}\psi, \tag{7}$$

the term 1/CFR comes from the inverse relationship between CFR and time step size, and not from multiplying directly by CFR. The discrete version of the Schrödinger equation modifies the time derivative to reflect this discrete nature:

$$i \frac{\hbar}{\text{CFR}} \frac{\Delta \psi}{\Delta t} = \hat{H}_{SC+DC}\psi \tag{8}$$

Here:

- **$\hbar$/CFR** replaces the continuous time derivative $\partial\Psi/\partial t$.
- **$\Delta\Psi/\Delta t$** accounts for changes in Ψ occurring at discrete DPIT intervals.
- **1/CFR** scales the rate of time evolution, indicating that higher CFR results in a faster progression of the system.
- **$\hat{H}_{SC+DC}$** represents the Hamiltonian governing both classical and quantum systems, ensuring the equation applies to both domains.

2.7. Understanding Velocity in the Context of Dynamic Configurations (DCs) and Static Configurations (SCs)

The interplay between classical and quantum physics has been a central theme in theoretical research, with one of the key challenges being how microscopic quantum phenomena translate into observable, macroscopic reality. The Static Configuration/Dynamic Configuration (SC/DC) framework offers a fresh perspective by defining physical systems through both static and dynamic states. This subsection explores how the concept of velocity—a fundamental idea in classical mechanics—emerges within this framework, emphasizing its external nature and dependence on interactions between surrounding configurations.

2.7.1. DCs, SCs, and the Role of the Wave Function

In the SC/DC framework, Dynamic Configurations (DCs) are described by wave functions, which define their spatial characteristics and temporal evolution. The wave function holds two essential attributes:

- **Spatial Extent**: The unnormalized squared modulus of the wave function $|\Psi|^2$ represents the spatial properties of the DC, offering a static "snapshot" of its energy distribution at a given moment.
- **Temporal Evolution**: The frequency component of the wave function dictates the temporal evolution, representing the DC's oscillation between regions of positive and negative (dark) energy amplitudes.

2.7.2. The Spatial and Temporal Duality of DCs

The SC/DC framework highlights the dual nature of wave functions:

- **Spatial Extent**: The wave function details energy distribution in space, distinguishing regions dominated by energy from those dominated by dark energy.
- **Temporal Dynamics**: The DC evolves at Discrete Points in Time (DPITs),

flipping between positive and negative energy amplitudes in a cyclical pattern.

Time is seen as a series of snapshots, with each DPIT capturing a static state, analogous to wave cycles in quantum phenomena like interference patterns.

2.7.3. The Nature of Velocity in DCs and SCs

In the SC/DC framework, velocity is not intrinsic to DCs or SCs but emerges from external interactions:

- **Velocity as an External Influence**: A particle (photon, electron) doesn't inherently possess velocity within its wave function. Instead, velocity arises from interactions with surrounding fields or DCs. For example, the velocity of a photon is constant, influenced by the electromagnetic field.
- **Proximity of DCs/SCs**: Velocity becomes meaningful when multiple DCs or SCs are in proximity. The potential energy fields of neighboring SCs affect each other, creating forces that manifest as motion.

2.7.4. The Emergence of Velocity through Interaction

Velocity arises as an observable property when multiple configurations interact. Classical forces (e.g., gravitational, electromagnetic) apply at specific intervals, determined by the surrounding configurations' potential fields.

In this framework, $F = ma$ still holds but only in the context of external interactions. Each DPIT represents a discrete system state, and changes in velocity and acceleration describe the effects of these interactions between states.

2.7.5. Conclusion: Velocity as a Consequence of External Influences

The SC/DC framework offers a novel understanding of velocity as an emergent property, driven by interactions with external configurations. This interpretation bridges classical and quantum mechanics, linking motion to external influences in both realms.

2.8. Conceptual Implication

- **In the quantum realm (DC)**: The wave function in Dynamic Configurations (DCs) evolves in discrete steps at each Discrete Point in Time (DPIT). Governed by the Constant Frame Rate (CFR), the reduced Planck constant divided by CFR determines the speed of these quantized steps. The flip-flop between positive and negative energy amplitudes shapes the evolution of quantum systems.
- **In the classical realm (SC)**: Static Configurations (SCs) experience changes driven by external forces, occurring at DPITs governed by CFR. The classical evolution, represented by forces like gravity, follows a similar temporal structure as quantum systems, though driven by external field interactions.
- **CFR as a Link**: The CFR synchronizes quantum and classical systems by regulating both in the same discrete temporal structure. Velocity, emerging from external interactions, thus becomes a unified concept across both domains.

2.9. Key Questions to Explore

Question 1. What Does Discrete Time Mean for Quantum Mechanics?

Discrete time governed by CFR would modify quantum evolution, requiring adjustments to the Schrödinger equation. How would phenomena like interference patterns behave under this change?

Question 2. How Does CFR Affect the Classical World?

If classical forces operate in discrete intervals, would classical systems exhibit jumps in motion? This could reshape traditional concepts like velocity and acceleration.

Question 3. What Does the $\hbar$/CFR-Modified Schrödinger Equation Predict?

How does introducing CFR alter quantum predictions, particularly in terms of wave function evolution and probabilistic outcomes? Could new behaviors emerge under this discrete time structure?

To explore these questions:

- Numerical Simulations can reveal how quantum systems behave under CFR, particularly in terms of interference patterns and energy conservation.
- Consistency Checks ensure modified equations align with known results.

2.10. Relation to Existing Mathematical Tools

The introduction of CFR into the Schrödinger equation and classical mechanics redefines how mathematical frameworks describe both quantum and classical systems:

- **Path Integral Formulation**: With CFR, continuous trajectories become discrete, potentially altering interference patterns and the classical-quantum transition [3].
- **Wigner-Weyl Formalism**: CFR could introduce new terms in the Liouville equation, clarifying quantum-classical transitions and velocity emergence (please refer to **Appendix A**).
- **Deformation Quantization**: CFR might introduce novel quantum corrections, reshaping how velocity is understood in quantum dynamics.
- **Coherent State Evolution**: CFR may influence semiclassical systems, offering new insights into velocity as an emergent property of system interactions.

By integrating CFR into these tools, the SC/DC framework offers a deeper understanding of how quantum and classical systems evolve together, with velocity emerging from external interactions.

2.11. Conclusion

Unifying classical, relativistic, and quantum physics is one of the greatest challenges in theoretical physics. The introduction of the $\hbar$/CFR-modified Schrödinger equation with $\hbar$/CFR offers a promising framework to bridge these realms, shedding light on the quantum-classical transition and the discrete nature of time evolution. By integrating this approach, we may gain deeper insights into how quantum mechanics connects to spacetime, potentially offering a new pathway to reconcile the foundational principles of these seemingly distinct domains.

3. Numerical Simulation: Quantum Harmonic Oscillator with CFR

To further investigate the predictions of the $\hbar$/CFR-modified Schrödinger equation, we conduct a numerical simulation of a one-dimensional quantum harmonic oscillator. This simulation illustrates how introducing the Constant Frame Rate (CFR) impacts wave function evolution, particularly when classical potentials interact with quantum systems. By applying the SC/DC framework, we explore how classical influences shape quantum behavior under the modified equation, offering deeper insights into the interplay between discrete time steps and quantum mechanics.

3.1. Simulation Setup

We employ the split-operator method to solve the time-dependent Schrödinger equation numerically. The simulation evolves an initial Gaussian wave packet in a harmonic potential, incorporating the CFR to simulate relativistic time dilation effects (please refer to **Appendix C** for the Python snippet to simulate the Wave Packet Evolution of a Gaussian Wave Function).

3.2. Detailed Explanation of the Simulation

3.2.1. Constants and Initialization

- **Planck's Constant ($\hbar$)**: Fundamental in quantum mechanics, representing the reduced Planck constant.
- **Mass (m)**: Set to unity for simplicity, representing the mass of the particle.
- **Angular Frequency (ω)**: Describes how fast the system oscillates over time. In the context of a harmonic oscillator, it is related to the system's stiffness (spring constant) and mass, dictating the rate at which the system oscillates.

Gaussian Wave Packet Parameters B:

- **(σ)**: Controls the spatial spread of the initial wave packet.
- **(x_0) and (p_0)**: Initial position and momentum, set to zero for a centered and stationary wave packet.

Spatial Discretization:

- **(N)**: Number of points in the spatial grid.
- **(L)**: Length of the spatial domain.
- **(dx)**: Spatial step size.

Constant Frame Rate (CFR):

- **(CFR)**: Fundamental frame rate modifying the time evolution.
- **(dt)**: Time step adjusted by CFR to simulate relativistic time dilation.
- **Time Steps**: Total number of iterations to evolve the wave function.

3.2.2. Gaussian Wave Packet Initialization

The initial wave function $\Psi(x, 0)$ is a Gaussian wave packet:

$$\psi(x)=\frac{1}{\sqrt{\sigma\sqrt{\pi}}}\exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)\exp\left(\frac{ip_0x}{\hbar}\right) \tag{9}$$

This form ensures normalization and introduces an initial momentum p_0.

3.2.3. Harmonic Potential

The harmonic potential $V(x)$ is defined as:

$$V(x)=\frac{1}{2}m\omega^2x^2 \tag{10}$$

This quadratic potential confines the particle, leading to oscillatory behavior characteristic of harmonic oscillators.

3.2.4. Time Evolution via Split-Operator Method

The split-operator method efficiently solves the time-dependent Schrödinger equation by alternating between momentum and position space evolutions:

$$e^{-i\frac{V(x)dt}{\hbar}} \tag{11}$$

- **Momentum Space Evolution**:
  - o The wave function is transformed to momentum space using the Fourier transform.
  - o The kinetic operator $e^{-i\frac{V(x)dt}{\hbar}}$ is applied, simulating free evolution under kinetic energy.
- **Position Space Evolution**:
  - o The wave function is transformed back to position space using the inverse Fourier transform.
  - o The potential operator $e^{-i\frac{V(x)dt}{\hbar}}$ is applied to account for potential energy effects.

This alternating process enables precise and efficient simulation of quantum dynamics.

3.3. Results and Analysis

After evolving the wave packet for 500 time steps, the final probability density $|\Psi(x)|^2$ is plotted. The simulation demonstrates the expected spreading and oscillatory behavior of the wave packet within the harmonic potential, consistent with both quantum and classical predictions. Key observations are:

- **Quantum-Classic**al Integration:
  - o The harmonic potential influences the quantum wave function in alignment with classical predictions.
  - o The $\hbar$/CFR-modified Schrödinger equation effectively integrates classical forces into quantum dynamics, ensuring consistency across scales.
- **Temporal and Frame Rate Consistency**:
  - o The incorporation of CFR aligns the quantum time evolution with the observer's frame of reference.
  - o This alignment strengthens the bridge between classical temporal resolution

and quantum evolution.

- **Broader Implications**:
  - The $\hbar$/CFR-modified Schrödinger equation offers a unified framework for classical and quantum mechanics.
  - It suggests novel approaches for understanding interactions across macroscopic and microscopic scales, potentially extending to relativistic regimes.

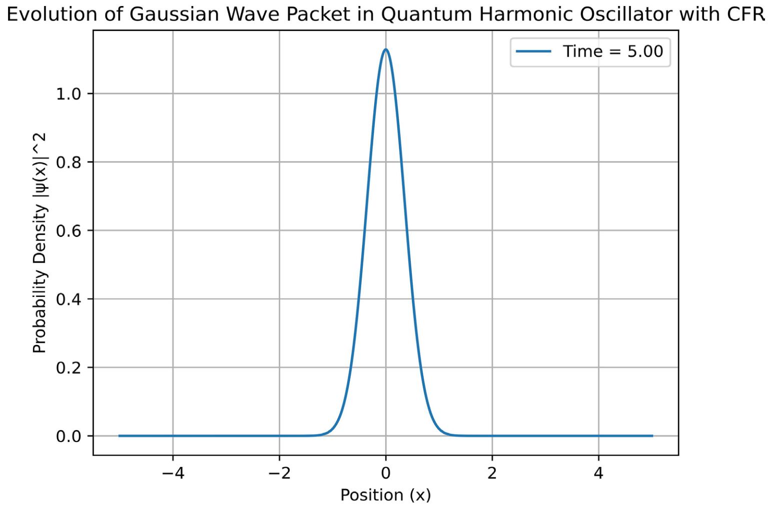


Figure 4. Classical potentials influence quantum wave functions under the modified framework.

The Python simulation of the quantum harmonic oscillator in the provided code can be interpreted through the lens of the $\hbar$/CFR-modified Schrödinger equation, which incorporates the Constant Frame Rate (CFR) from the Advanced Observer Model (AOM) to modify time evolution. Let's explain **Figure 4** in the Context of the $\hbar$/CFR-Modified Schrödinger Equation:

- **Wave Packet and Time Evolution**: The initial Gaussian wave packet represents a quantum particle's state, which evolves in time according to the Schrödinger equation. However, in this case, the evolution is influenced by the CFR, which modifies how time is perceived in the system. This time modification arises from the Advanced Observer Model (AOM), where time is treated as a discrete sequence of frames, rather than a continuous flow. The CFR introduces a discrete step-by-step progression, affecting the dynamics of the quantum system.
- **$\hbar$/CFR-Modified Schrödinger Equation**: The traditional Schrödinger equation is modified to account for the discrete time evolution imposed by the CFR. Here, the code's time step, $dt = 0.01/\text{CFR}$, reflects the influence of the CFR on time progression. This modification mirrors the idea that the observer's frame rate influences the evolution of quantum states—effectively slowing or controlling the rate at which the wave function evolves, depending on the CFR.
- **Kinetic and Potential Energy in a Discretized Framework**: The evolution of the wave packet in the harmonic potential still adheres to the principles of quantum mechanics but now aligns with the modified dynamics dictated by

the observer's frame rate. The split-operator method used in the code evolves the wave packet by applying the kinetic and potential energy operators separately. In the $\hbar$/CFR-modified equation, the potential energy and kinetic energy still drive the evolution, but they are now broken into discrete time steps, controlled by the CFR.

- **Oscillatory Behavior**: In the figure, the oscillatory nature of the wave packet is still present, but the CFR introduces subtle adjustments in the time evolution. Since the wave packet evolves with discrete time steps, these oscillations are effectively sampled at intervals dictated by the observer's frame rate. This aligns with the $\hbar$/CFR-modified Schrödinger equation, where the evolution of the quantum state is tied to the observer's perception of time, affecting how the particle moves in its potential energy space.
- **Probability Density Function (PDF)**: The probability density function $|\Psi(x)|^2$, plotted in the figure, represents the likelihood of finding the particle at a specific position x at a given moment. Due to the inclusion of CFR, this PDF reflects the behavior of the wave packet after 500 discrete time steps, offering insight into how the AOM framework governs quantum evolution. The CFR-modified evolution can explain the smooth yet discrete changes in the wave packet's probability density over time.
- **Interpretation in AOM**: In the AOM framework, the observer's frame rate plays a critical role in the perception of quantum systems. By incorporating CFR, the Schrödinger equation becomes a tool for examining how reality is constructed based on discrete snapshots of time. The figure illustrates how quantum systems (in this case, a wave packet in a harmonic potential) evolve in this frame-by-frame view, which provides a new interpretation of quantum mechanics, where time discreteness emerges naturally.

In summary, the figure generated by the Python code demonstrates the evolution of a Gaussian wave packet in a harmonic potential using the $\hbar$/CFR-modified Schrödinger equation. The inclusion of CFR modifies the time evolution by discretizing it, aligning with the AOM's concept of discrete time progression. This offers a new perspective on the behavior of quantum systems, particularly in the context of how an observer's frame rate impacts the emergence of classical spacetime geometry from quantum processes.

3.4. Classical Forces in Two Static Configurations in Proximity

A Python code (see **Appendix D**) simulates and visualizes the interaction between two Static Configurations (SCs) under the influence of a force, where the kinetic component of the Hamiltonian is considered negligible due to their large masses and low frequencies. The key components of the python snippet are:

Component 1. Initialization:

- **Parameters**: The code sets parameters for the simulation, including the standard deviation of the Gaussian functions ('sigma'), the size of the grid ('L'), the number of frames ('total_frames'), and the time step ('dt').

- **Initial Positions**: Two SC centers are initialized at specific positions on the grid.

Component 2. Gradient Calculation: 0.07, 0.01, 0.005, 0.004

- **'compute_gradient()' Function**: Computes the gradient of the potential energy field based on Gaussian functions centered at the SC positions. This function calculates the gradients 'dV_dx' and 'dV_dy', representing the change in potential energy with respect to the x and y coordinates.

Component 3. Position Update:

- **'update_positions()' Function**: Uses the computed gradients to determine the forces acting on the SCs. The forces are then used to update the positions of the SCs. The updated positions are constrained to stay within the bounds of the grid.

Component 4. Plotting:

- **'plot_frame()' Function**: Creates a contour plot showing the potential energy field with the positions of the SCs marked. This function is used to visualize the field and the SC positions at different frames.
- **Frames Visualization**: A series of frames is generated and displayed, illustrating the changes in the potential energy field and SC positions over time.

Component 5. Execution:

- **Simulation Loop**: Runs through the total number of frames, updating positions and plotting selected frames. The final result is saved as a high-resolution image showing the evolution of the SCs within the potential energy field.

The snippet effectively demonstrates how the positions of two massive SCs evolve under an attractive force, with kinetic energy being negligible. The visualization helps in understanding the dynamic interaction and potential energy distribution of these SCs over time.

Figure 5 is the Python generated series of plots that visually represent the interaction between two Static Configurations (SCs) influenced by an attractive force (please refer to **Appendix D** for the Python Snippet). Here is a detailed description of the output figure:

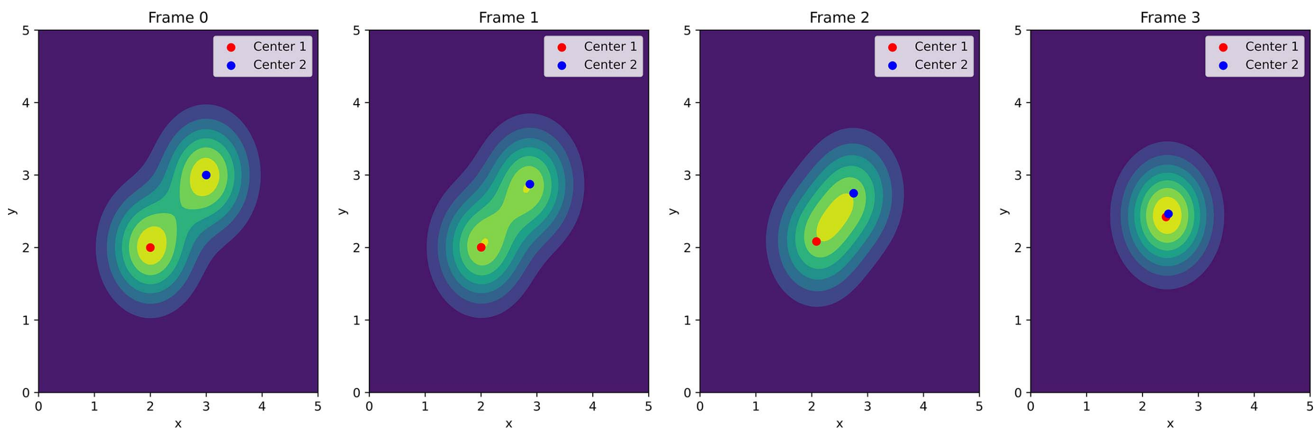


Figure 5. The temporal progression of two Static Configurations (SCs) in proximity.

a) Overall Layout:

- The figure consists of a sequence of contour plots, each representing a different frame of the simulation. The number of frames displayed is specified by 'frames_to_display' (e.g., frames 0, 1, 2, 3).

b) Contour Plots:

- Each subplot visualizes the potential energy landscape as a contour plot, where the energy values are represented by varying shades of color.
- The plot uses the 'viridis' colormap, with different colors indicating different levels of potential energy. Brighter areas correspond to higher potential values, while darker areas represent lower values.

c) Potential Energy Representation:

- The potential energy landscape is modeled as the sum of two Gaussian functions centered at the positions of the two SCs. Each Gaussian represents the potential energy contribution from one SC.
- The resulting plot shows how these Gaussian potentials combine to form the overall potential energy distribution in the grid.

d) Static Configuration Positions:

- The centers of the two SCs are marked with distinct colors: red for Center 1 and blue for Center 2. These markers indicate the locations of the SCs in the energy landscape.
- The positions of these centers update over time, reflecting their movement due to the applied forces.

e) Frame Titles and Labels:

- Each subplot is labeled with its frame number, providing a temporal context for the visualized data.
- The x-axis and y-axis are labeled 'x' and 'y,' respectively, indicating the spatial coordinates on the grid.

f) Forces and Dynamics:

- The plots show how the forces between the SCs, governed by their potential energy profiles, affect their positions over time. The motion of the SCs is influenced by the attractive forces derived from the potential landscape.

g) Legend:

- Each plot includes a legend indicating the static configuration (SC) centers with their corresponding colors (red for Center 1 and blue for Center 2).

In summary, the output figure effectively illustrates the interaction between two static configurations (SCs) within a potential energy landscape. It captures how their positions change over time due to the attractive forces between them, as determined by their Gaussian potential profiles. The sequence of plots demonstrates the dynamic evolution of the SCs' positions within the potential energy grid.

When using the $\hbar$/CFR-modified Schrödinger equation without the kinetic component of the Hamiltonian, the equation effectively mirrors the scenario depicted in this simulation. In this context, the absence of the kinetic term simplifies the Hamiltonian, making it similar to focusing solely on the forces between two

Static Configurations (SCs). The contextual implications are:

Implication 1. Modified Schrödinger Equation:

By omitting the kinetic component in the $\hbar$/CFR-Modified Schrödinger Equation, we are left with an equation that describes the system's behavior based solely on potential interactions. This adjustment aligns with cases where the kinetic energy is negligible compared to the potential energy.

Implication 2. Forces Between SCs:

The simulation described earlier illustrates how the positions of two SCs evolve under an attractive force, assuming their kinetic energy is minimal. Here, the focus is solely on the potential energy, which drives the interactions and subsequent motion of the SCs.

Implication 3. Equivalence:

The modified Schrödinger equation without the kinetic term is equivalent to the scenario in the simulation where the forces between SCs determine their motion. Both approaches describe the system using potential energy interactions alone, reflecting the negligible role of kinetic energy in these specific conditions.

In summary, the $\hbar$/CFR-modified Schrödinger equation without the kinetic component provides an equivalent description of the dynamics between two SCs under the influence of potential forces, mirroring the outcomes observed in the force-based simulation.

4. Consistency Check

To verify that the modified Schrödinger equation with Discrete Points in Time (DPIT) and Constant Frame Rate (CFR) retains consistency with both quantum mechanical principles and classical mechanics, several checks are required:

a) Consistency with Quantum Mechanical Principles:

Core principles such as superposition, energy conservation, and probabilistic evolution should hold within this modified framework.

b) Recovery of Classical Mechanics:

The modified equation should recover classical mechanics in the classical limit (where quantum effects are negligible or averaged out). This involves ensuring that principles like energy conservation and smooth motion are preserved.

c) Limiting Cases:

The modified equation must reduce to known results in limiting cases. For instance, as the CFR becomes large and the discrete time steps approach infinitesimally small intervals, the equation should approximate continuous time evolution, consistent with standard quantum mechanics.

Step 1: Recovering the Standard Schrödinger Equation (Continuous Time Limit)

To ensure that the modified Schrödinger equation aligns with standard quantum mechanics, we first check if it can recover the continuous time limit.

The standard Schrödinger equation assumes continuous time evolution:

$$i\hbar\frac{\partial\Psi}{\partial t}=\hat{H}\Psi \tag{12}$$

In the $\hbar$/CFR-modified Schrödinger equation, time is represented by discrete steps ΔDPIT, and the time derivative $\partial\Psi/\partial t$ is replaced by the discrete derivative ΔΨ/ΔDPIT:

$$i\hbar\frac{\Delta\Psi}{\Delta\text{DPIT}} = \hat{H}\Psi \tag{13}$$

As CFR → ∞ (continuous time), the time steps ΔDPIT become very small, and the discrete derivative ΔΨ/ΔDPIT approximates the continuous derivative $\partial\Psi/\partial t$. In this limit:

$$\lim_{\text{CFR}\to\infty}\frac{\Delta\Psi}{\Delta\text{DPIT}} \approx \frac{\partial\Psi}{\partial t} \tag{14}$$

Thus, the $\hbar$/CFR-modified Schrödinger equation reduces to the standard Schrödinger equation in the continuous time limit:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{15}$$

This ensures that, as time becomes continuous (large CFR), the modified equation recovers the known results from quantum mechanics, verifying consistency in this limit.

Step 2: Energy Conservation in Quantum Systems

In quantum mechanics, energy conservation is governed by the (Hamiltonian operator $\hat{H}$). The expectation value of the Hamiltonian represents the total energy of the system:

$$E = \langle\Psi|\hat{H}|\Psi\rangle \tag{16}$$

To verify energy conservation in the modified framework, we ensure that the time derivative of the energy is zero:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = 0 \tag{17}$$

In the $\hbar$/CFR-modified equation, time is discrete, so the energy conservation condition becomes:

$$\frac{\Delta E}{\Delta\text{DPIT}} = 0 \tag{18}$$

Since the (Hamiltonian $\hat{H}$) is assumed to remain Hermitian (a fundamental requirement in quantum mechanics), the expectation value of energy remains constant between successive discrete time steps. Thus, energy conservation holds in both the modified and standard Schrödinger equations as long as the Hamiltonian remains Hermitian. This consistency with energy conservation principles ensures that the modified equation adheres to one of the core tenets of quantum mechanics.

Step 3: Classical Mechanics Consistency (In the Classical Limit)

In the classical limit, where quantum effects become negligible (e.g., large mass or large action), the modified equation must recover Newtonian mechanics. In quantum mechanics, the classical limit is reached when the action S is large compared to $\hbar$, causing the wave nature of particles to become negligible. In the

$\hbar$/CFR-modified framework:

- CFR introduces discrete time steps that govern both quantum and classical systems.
- As CFR increases, the discrete time steps ΔDPIT become small, and the classical system behaves as though it evolves continuously, reproducing smooth motion consistent with Newton's laws.
- For example, in classical mechanics:
- The motion of particles is described by Newton's laws, with continuous forces acting on the particles.
- In the modified equation, these forces are applied at discrete time steps, but as CFR → ∞, the forces act continuously, ensuring smooth, continuous motion.

Thus, in the large CFR limit, the modified equation reduces to classical mechanics, verifying consistency with known classical behavior.

Step 4: Quantum Probabilities and Superposition

In quantum mechanics, the superposition principle and probabilistic outcomes are essential components of the theory. The wavefunction Ψ evolves according to the Schrödinger equation, preserving superposition, and the probability of finding a particle in a particular state is given by $|\Psi(x, t)|^2$.

In the $\hbar$/CFR-modified Schrödinger equation:

- The wavefunction still evolves according to the Hamiltonian, but in discrete time steps rather than continuously.
- Superposition is preserved because the linear structure of the wavefunction remains unchanged.

As long as the evolution operator in the modified framework remains unitary, the probabilities and superposition principles of quantum mechanics are preserved. This ensures that the modified equation retains consistency with core quantum mechanical principles.

Summary of Consistency Checks

a) Continuous Time Limit:

The modified equation recovers the standard Schrödinger equation when CFR becomes large, ensuring consistency with established quantum mechanics in the continuous limit.

b) Energy Conservation:

The Hermiticity of the Hamiltonian guarantees that energy conservation holds in both the modified and standard equations.

c) Classical Mechanics Consistency:

In the classical limit, where quantum effects diminish, the modified equation recovers Newtonian mechanics, ensuring smooth motion and continuous forces in the large CFR limit.

d) Quantum Probabilities and Superposition:

The modified equation preserves superposition and probabilistic outcomes, consistent with the principles of quantum mechanics.

Overall, the $\hbar$/CFR-modified Schrödinger equation demonstrates consistency with both quantum and classical mechanics, ensuring energy conservation and recovering standard equations in the appropriate limits.

5. Energy Scaling and Dimensionality in the Advanced Observer Model (AOM)

A key insight of the Advanced Observer Model (AOM) is the interplay between dimensionality and energy, reflected in the Static Configuration (SC) and Dynamic Configuration (DC) framework. In this model, the energy of a system is inherently tied to its dimensionality through both static spatial distributions and dynamic quantum interactions. A particularly important result that emerges from this framework is the relationship between energy and spatial dimensionality, encapsulated in the equation:

$$E_n = k_n \cdot \left(\sqrt{\pi}\right)^n \tag{19}$$

where $k_n = A^2 / (2\alpha)^{n/2}$ is a constant that incorporates the amplitude A, the parameter α, and the surface area of the unit sphere in n-dimensional space. The term $\left(\sqrt{\pi}\right)^n$ represents the exponential scaling of energy with spatial dimensionality.

5.1. Dimensionality and Energy in the SC/DC Framework

In the SC/DC framework, energy scaling with $\left(\sqrt{\pi}\right)^n$ represents how the static configuration (SC), which provides a snapshot of the spatial extent of reality, evolves as the number of dimensions increases. The dynamic configuration (DC), driven by quantum interactions, mirrors this scaling, implying that higher-dimensional systems naturally have exponentially higher energy states.

This scaling suggests that as dimensionality increases, the energy of a system grows exponentially, highlighting the profound role dimensionality plays in shaping both static and dynamic configurations. In a three-dimensional system, for example, the energy grows as $\left(\sqrt{\pi}\right)^3$, while in two dimensions it scales as $\left(\sqrt{\pi}\right)^2$, reflecting the inherent geometry of space in AOM.

5.2. Planck's Constant and Energy-Wavelength Relationship

Another significant implication of this result is the link between Planck's constant and dimensionality. Using the relation $E_n = h/\lambda$, we derive:

$$h = \lambda \cdot k_n \cdot \left(\sqrt{\pi}\right)^n \tag{20}$$

This shows that Planck's constant can be expressed as a function of both the wavelength λ and the dimensionality-dependent factor $\left(\sqrt{\pi}\right)^n$. This challenges the notion of Planck's constant as a fixed, universal quantity. Instead, AOM posits that it can vary depending on the observer's dimensional interaction, particularly in higher-dimensional spaces where energy and quantum interactions are more complex.

The equation $h = \lambda \cdot k_n \cdot \left(\sqrt{\pi}\right)^n$ highlights that dimensionality fundamentally alters the quantum mechanics of the system, linking space and energy through a universal scaling factor. In this context, h becomes a dynamic quantity that adapts to the observer's interaction with dimensional configurations.

For a more detailed mathematical derivation of the Gaussian wave function in n-dimensions and the scaling relationship $h = \lambda \cdot k_n \cdot \left(\sqrt{\pi}\right)^n$, please refer to **Appendix B**. The appendix provides the necessary integral steps and normalization techniques used in deriving the dimensional scaling of energy in the context of AOM.

5.3. Verification of the Energy Scaling Equation

To confirm the validity of the Equation (20):

$$E_n = k_n \cdot \left(\sqrt{\pi}\right)^n, \tag{21}$$

where

$$k_n = \frac{A^2}{(2\alpha)^{n/2}} \cdot \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} \tag{22}$$

5.3.1. Normalization of the Gaussian Wave Function in n-Dimensions

Let's analyze its components in relation to quantum mechanics and dimensional consistency. The wave function is a Gaussian:

$$\psi(\boldsymbol{r}) = A\mathrm{e}^{-\alpha r^2}, \quad |\psi(\boldsymbol{r})|^2 = A^2\mathrm{e}^{-2\alpha r^2} \tag{23}$$

The normalization condition is:

$$\int_{\mathbb{R}^n} |\Psi(\boldsymbol{r})|^2 \, \mathrm{d}^n r = 1 \tag{24}$$

Using spherical coordinates and standard Gaussian integrals, we get:

$$A^2 = (2\alpha)^{n/2} \cdot \frac{\Gamma\left(\frac{n}{2}\right)}{2\pi^{n/2}} \tag{25}$$

This confirms the Gaussian wave function is correctly normalized, validating the form of k_n.

5.3.2. Total Energy and $\left(\sqrt{\pi}\right)^n$ Dependence

The total energy in n-dimensions is:

$$E_n = k_n \cdot \left(\sqrt{\pi}\right)^n, \tag{26}$$

where k_n depends on the constants of the wave function and the n-dimensional volume. The factor $\left(\sqrt{\pi}\right)^n$ captures the geometric scaling of energy as dimensionality increases, aligning with the exponential growth of volume in higher-dimensional spaces.

5.3.3. Consistency with the Quantum Harmonic Oscillator

In the quantum harmonic oscillator, the energy levels scale as:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad (27)$$

In higher dimensions, the Gaussian form of the wave function and the dimensional scaling are consistent with the equation. The relationship $h = \lambda \cdot k_n \cdot \left(\sqrt{\pi}\right)^n$ provides a way to express energy in terms of spatial dimensionality.

5.3.4. Dimensional Consistency

The equation is dimensionally consistent, with both E_n and $k_n \cdot \left(\sqrt{\pi}\right)^n$ having the correct energy units, confirming its physical validity.

In summary, the equation $E_n = k_n \cdot \left(\sqrt{\pi}\right)^n$ is mathematically and dimensionally valid, providing key insights into how energy scales with dimensionality in quantum systems. This is significant for exploring higher-dimensional theories like quantum field theory and string theory.

5.4. Significance for a Unified Theory

The dimensional scaling of energy in the Advanced Observer Model (AOM) offers profound insights into the potential unification of classical and quantum mechanics. The equation $E_n = k_n \cdot \left(\sqrt{\pi}\right)^n$ highlights the inherent relationship between dimensionality and energy, suggesting that the geometry of space directly influences the quantum properties of a system. This scaling is not only relevant for quantum mechanics but also has implications for classical systems when viewed through the lens of dimensional interactions.

5.4.1. Dimensionality as a Unifying Principle

In AOM, dimensionality plays a critical role in the interpretation of both Static Configurations (SC) and Dynamic Configurations (DC). As dimensionality increases, the interaction between SC and DC becomes more complex, revealing novel energy dynamics that were previously hidden in lower-dimensional frameworks. This interaction underscores how dimensionality serves as a unifying principle in both classical and quantum realms. Classical systems, traditionally governed by fixed laws, can now be viewed as emergent from the deeper, dimensionally driven quantum interactions modeled by DC.

5.4.2. Implications for Quantum-Gravity Unification

AOM's dimensional scaling equation offers new pathways to address one of the most elusive problems in physics: the unification of quantum mechanics and gravity. The exponential energy growth with dimensionality suggests that higher-dimensional spaces may hold the key to resolving inconsistencies between the two theories. By factoring in dimensionality-dependent factors such as $\left(\sqrt{\pi}\right)^n$, AOM provides a framework where gravitational and quantum forces might emerge from the same underlying structure. This has profound implications for string theory, quantum field theory, and other higher-dimensional theories, where energy scaling naturally accommodates both quantum and gravitational effects.

5.4.3. Observer-Centric Reality and Dimensional Perception

A key feature of the AOM is the role of the observer. In this model, the observer interacts with reality through both SC and DC configurations, and their perception of reality is tied to the dimensionality of the system. As dimensionality increases, the observer perceives higher energy states, which in turn shapes their understanding of the system. This suggests that the observer's reality is inherently dimensional, and the energy scaling described by $E_n = k_n \cdot \left(\sqrt{\pi}\right)^n$ is a reflection of how dimensional space informs the observer's perception of quantum systems.

5.4.4. Pathway to a Unified Theory

The interplay between dimensionality and energy described in AOM is significant for the development of a unified theory of physics. By showing that dimensionality is a fundamental driver of energy distribution and quantum interactions, AOM bridges classical and quantum mechanics under a single framework. The model introduces a dimensionally consistent mechanism that ties together the observer's reality, quantum energy states, and classical laws, paving the way for the unification of spacetime, quantum fields, and gravitational forces.

Ultimately, the scaling principles of AOM provide a robust platform for further exploration into higher-dimensional physics, offering new insights into the structure of the universe and the role of the observer in shaping it. As the theory advances, it may lead to breakthroughs in understanding the fundamental nature of energy, space, and time, contributing to a deeper, unified vision of reality.

6. Conclusions

In this paper, we have introduced the $\hbar$/CFR-modified Schrödinger equation within the Advanced Observer Model (AOM), presenting a novel approach to bridging quantum mechanics and classical physics through discrete time evolution governed by the Constant Frame Rate (CFR). A key insight of this framework is the exponential scaling of energy with spatial dimensionality, captured by the relationship $h = \lambda \cdot k_n \cdot \left(\sqrt{\pi}\right)^n$. This equation highlights the profound role of dimensionality in shaping quantum systems and their correspondence with classical phenomena.

The incorporation of dimensional scaling through $\left(\sqrt{\pi}\right)^n$ suggests that energy in higher-dimensional systems behaves in fundamentally different ways, revealing a deeper interplay between quantum mechanics and classical dynamics. This dimensional scaling also implies that Planck's constant h, typically considered a fixed quantity, may vary depending on the observer's interaction with higher-dimensional spaces. This reinterpretation of Planck's constant provides a critical step towards unifying quantum and classical domains, where the behavior of quantum systems can be understood in terms of both dimensionality and observer-centric reality.

The framework presented here opens new avenues for exploring how higher-dimensional configurations influence both quantum and classical realms. By integrating the observer's role and energy dynamics with dimensionality, this approach offers fresh perspectives on the nature of time, space, and energy. These insights are crucial in the pursuit of a unified theory, as they suggest that classical and quantum systems are not isolated domains but are interwoven through dimensional scaling and discrete time evolution.

Future work will focus on the interaction between higher-dimensional spaces and physical laws, particularly in relation to the dimensional evolution of quantum states. This research lays the groundwork for further investigation into the structure of reality, where dimensional interactions play a central role in the fabric of the universe.

Acknowledgements

The author thanks Professor Qing Li, Head of the Department of Computing at The Hong Kong Polytechnic University, for his acknowledgment of the submission and acceptance of my paper to the Journal of Quantum Information Science as a retired faculty.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Wong, J.H.C. (2024) Quantum Realities and Observer-Dependent Universes: An Advanced Observer Model. *Journal of Quantum Information Science*, **14**, 69-121. https://doi.org/10.4236/jqis.2024.143006

[2] Wong, J.H.C. (2024) Bridging Classical and Quantum Realms: The Conceptual and Theoretical Framework of the Advanced Observer Model. *Journal of Quantum Information Science*, **14**, 123-157. https://doi.org/10.4236/jqis.2024.144007

[3] Wong, H.C. (2024) Integrating Quantum Mechanics and the Advanced Observer Model: A New Paradigm for Reality Construction. *Journal of Quantum Information Science*, **14**, 158-195.

[4] Goldstein, H., Poole, C. and Safko, J. (2002) Classical Mechanics. 3rd Edition, Addison-Wesley.

[5] Feynman, R.P. and Hibbs, A.R. (1965) Quantum Mechanics and Path Integrals. McGraw-Hill.

[6] Wigner, E. (1932) On the Quantum Correction for Thermodynamic Equilibrium. *Physical Review*, **40**, 749-759. https://doi.org/10.1103/physrev.40.749

[7] Bohr, N. (1913) I. on the Constitution of Atoms and Molecules. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **26**, 1-25. https://doi.org/10.1080/14786441308634955

[8] Groenewold, H.J. (1946) On the Principles of Elementary Quantum Mechanics. *Physica*, **12**, 405-460. https://doi.org/10.1016/s0031-8914(46)80059-4

[9] Bayen, F., Flato, M., Fronsdal, C., Lichnerowicz, A. and Sternheimer, D. (1978) Deformation Theory and Quantization. I. Deformations of Symplectic Structures. *Annals of Physics*, **111**, 61-110. https://doi.org/10.1016/0003-4916(78)90224-5

[10] Klauder, J. and Skagerstam, B. (1985) Coherent States: Applications in Physics and Mathematical Physics. World Scientific. https://doi.org/10.1142/0096

[11] Schleich, W.P. (2001) Quantum Optics in Phase Space. Wiley. https://doi.org/10.1002/3527602976

[12] Hall, B.C. (2013) Quantum Theory for Mathematicians. Springer.

[13] Dirac, P.A.M. (1930) The Principles of Quantum Mechanics. Clarendon Press.

[14] Sakurai, J.J. (1967) Advanced Quantum Mechanics. Addison-Wesley.

[15] Weinberg, S. (1995) The Quantum Theory of Fields (Vols. 1-2). Cambridge University Press.

[16] Heisenberg, W. (1930) The Physical Principles of the Quantum Theory. University of Chicago Press.

[17] Aharonov, Y. and Bohm, D. (1959) Significance of Electromagnetic Potentials in the Quantum Theory. *Physical Review*, **115**, 485-491. https://doi.org/10.1103/physrev.115.485

[18] Polchinski, J. (1998) String Theory. Cambridge University Press. https://doi.org/10.1017/cbo9780511618123

[19] Rovelli, C. (2004) Quantum Gravity. Cambridge University Press. https://doi.org/10.1017/cbo9780511755804

[20] Kodukula, S.P. (2021) Mechanism of Quantum Consciousness That Synchronizes Quantum Mechanics with Relativity—Perspective of a New Model of Consciousness. *Journal of Modern Physics*, **12**, 1633-1655. https://doi.org/10.4236/jmp.2021.1212097

[21] Christensen, W.J. (2024) Between Quantum Mechanics and General Relativity. *Journal of Modern Physics*, **15**, 1199-1228. https://doi.org/10.4236/jmp.2024.158049

[22] Ko, K. (2024) From In-Out Duality to the Foundation of Social Quantum Mechanics. *Open Journal of Applied Sciences*, **14**, 893-926. https://doi.org/10.4236/ojapps.2024.144060

Appendix A: Impact of Constant Frame Rate (CFR) on the Liouville Equation and Wigner-Weyl Formalism

In this appendix, we examine the mathematical modifications to the Liouville equation and Wigner-Weyl formalism introduced by the concept of Constant Frame Rate (CFR). Specifically, we explore how discrete time evolution, as dictated by CFR, alters quantum predictions and introduces new terms in the quantum-classical transition.

A.1. Standard Quantum Liouville Equation

The quantum Liouville equation governs the time evolution of the density matrix $\hat{\rho}$ in quantum mechanics:

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}, \hat{\rho} \right], \tag{28}$$

where $\hat{H}$ is the Hamiltonian and $\left[\hat{H}, \hat{\rho} \right]$ represents the commutator. This equation describes how the quantum state evolves over continuous time.

A.2. Wigner-Weyl Formalism

In the Wigner-Weyl formalism, the density matrix $\hat{\rho}$ is expressed as the Wigner function $W(x, p, t)$, which is a quasi-probability distribution function in phase space. The Wigner function evolves according to a Liouville-like equation:

$$\frac{\partial W(x, p, t)}{\partial t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial V(x)}{\partial x}\frac{\partial W}{\partial p} + \mathcal{O}(\hbar^2), \tag{29}$$

where $V(x)$ is the potential, and the $\mathcal{O}(\hbar^2)$ terms represent quantum corrections to the classical Liouville equation.

A.3. Incorporating CFR into the Wigner-Weyl Formalism

When introducing CFR, time becomes discretized into intervals $\Delta t = 1/\text{CFR}$, replacing the continuous time derivative with a finite difference. This modifies the evolution of the Wigner function as follows:

$$\frac{W(x, p, t + \Delta t) - W(x, p, t)}{\Delta t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial V(x)}{\partial x}\frac{\partial W}{\partial p} + \mathcal{O}(\hbar^2) \tag{30}$$

Rearranging this difference equation gives:

$$W(x, p, t + \Delta t) = W(x, p, t) - \Delta t\left(\frac{p}{m}\frac{\partial W}{\partial x} - \frac{\partial V(x)}{\partial x}\frac{\partial W}{\partial p} \right) + \mathcal{O}(\hbar^2) \tag{31}$$

This modified equation reflects the discrete nature of time evolution due to CFR.

A.4. Corrections to the Commutator

In quantum mechanics, corrections enter through the Moyal bracket, which modifies the classical Poisson bracket. The Moyal bracket is given by:

$$\{A, B\}_M = A \star B - B \star A, \tag{32}$$

where $\star$ is the star product, incorporating $\hbar$-dependent quantum terms. With CFR, the time evolution is now discrete, and the commutator between the Hamiltonian Ĥ and the density matrix $\hat{\rho}$ takes the form:

$$\left[\hat{H}, \hat{\rho}(t+\Delta t)\right] = \left[\hat{H}, \hat{\rho}(t)\right] + \Delta t\left[\hat{H}, \frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}\right] + \mathcal{O}\left(\Delta t^2\right) \tag{33}$$

These corrections account for the discrete shifts in time and introduce new terms that depend on Δt.

A.5. New Terms in the Liouville Equation

The introduction of CFR modifies the quantum Liouville equation, introducing additional terms due to the discrete time steps. The modified equation can be expressed as:

$$\frac{W(x,p,t+\Delta t) - W(x,p,t)}{\Delta t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial V(x)}{\partial x}\frac{\partial W}{\partial p} + \mathcal{O}\left(\hbar^2\right) + \mathcal{O}\left(\Delta t^2\right) \tag{34}$$

These new terms, proportional to Δt^2, represent quantum corrections arising from the discrete nature of time evolution. The corrections impact quantum phenomena such as interference patterns and energy conservation, and they could alter the dynamics of systems sensitive to time evolution.

A.6. Quantum-Classical Transition

In the classical limit where $\hbar \to 0$, the Wigner function recovers the classical Liouville equation. However, under CFR, additional terms proportional to Δt^2 persist, modifying the quantum-classical transition. These terms could influence systems where time plays a critical role, such as oscillatory systems or systems with periodic motion, potentially affecting the emergence of classical behavior from quantum systems.

A.7. Conclusion

The introduction of CFR into the Wigner-Weyl formalism and the Liouville equation introduces new terms that arise from discrete time evolution. These terms, proportional to Δt and Δt^2, modify quantum interference patterns, energy conservation, and the quantum-classical transition. Additionally, corrections to velocity emergence and quantum behavior under discrete time structures provide a novel framework for exploring the quantum-classical boundary. This modified approach offers a new perspective on the time evolution of quantum systems and their transition to classical behavior.

Appendix B: Calculation of the Unnormalized Squared Modulus of a Wave Function

This appendix delves into the unnormalized squared modulus of a wave function in one, and n dimensions. It particularly emphasizes how π represents the balanced and optimized static state of the Static Configuration/Dynamic Configuration (SC/DC).

B.1. Wave Function

Consider a Gaussian wave function in one dimension:

$$\psi(x) = A\mathrm{e}^{-\alpha x^2} \tag{35}$$

The squared modulus of this wave function is:

$$|\psi(x)|^2 = A^2\mathrm{e}^{-2\alpha x^2} \tag{36}$$

To determine the area under this curve, we compute the integral:

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 \,\mathrm{d}x \tag{37}$$

This integral evaluates to:

$$\int_{-\infty}^{\infty} A^2\mathrm{e}^{-2\alpha x^2}\,\mathrm{d}x = \frac{A^2\sqrt{\pi}}{\sqrt{2\alpha}} \tag{38}$$

Here, $k_2 = A^2/\sqrt{2\alpha}$ is a constant determined by A and α. The result, $\sqrt{\pi}$, represents the total area under the unnormalized Gaussian curve. In the SC/DC framework, this value signifies the spatial extent of the static and dynamic configurations, illustrating the balanced and optimized nature of the SC/DC in one dimension.

B.2. General Case in n Dimensions

For a Gaussian wave function in n dimensions, we consider:

$$\psi(\boldsymbol{x}) = A\mathrm{e}^{-\alpha|\boldsymbol{x}|^2} \tag{39}$$

The squared modulus is:

$$|\psi(\boldsymbol{x})|^2 = A^2\mathrm{e}^{-2\alpha|\boldsymbol{x}|^2} \tag{40}$$

To find the total volume in n dimensions, we compute the integral:

$$\int_{\mathbb{R}^n} |\Psi(\boldsymbol{x})|^2 \,\mathrm{d}V \tag{41}$$

This integral can be expressed in spherical coordinates, yielding:

$$\int_0^{\infty} A^2\mathrm{e}^{-2\alpha r^2} r^{n-1}\,\mathrm{d}r \tag{42}$$

The radial integral evaluates to:

$$\frac{A^2\pi^{n/2}}{(2\alpha)^{n/2}} \tag{43}$$

Thus, we have:

$$\int_{\mathbb{R}^n} |\Psi(\boldsymbol{x})|^2 \,\mathrm{d}V = k\left(\sqrt{\pi}\right)^n, \tag{44}$$

where $k = A^2/(2\alpha)^{n/2}$ encapsulates contributions from A, α, and the surface area of the unit sphere in n dimensions. Given n, denote k as $k_n = A^2/(2\alpha)^{n/2}$.

B.3. Conclusion

- **1-dimension case**: The total area under the unnormalized Gaussian function is $k_1\left(\sqrt{\pi}\right)^1$, reflecting the balanced static state of the wave function in one dimension.
- **2-dimensions case**: Given $n = 2$, the total volume under the unnormalized Gaussian function is represented by $k_2 \cdot \pi$ or $k_2 \cdot \left(\sqrt{\pi}\right)^2$, showcasing the optimal spatial distribution of the Static Configuration (SC) and Dynamic Configuration (DC) in two dimensions.
- **n-dimensions case**: The total volume is $\pi^{n/2}$ or $\left(\sqrt{\pi}\right)^n$, signifying the equilibrium and static nature of the SC/DC in n dimensions.

In all cases, $\sqrt{\pi}$ serves as a fundamental descriptor of spatial dimensions, capturing the essence of the balanced and optimized state of the SC/DC framework. This demonstrates the spatial extent of the wave function's squared modulus and emphasizes $\sqrt{\pi}$'s role as a universal constant across different dimensions, contributing to the structure of space in both quantum systems and the physical world.

Furthermore, with $k \cdot \left(\sqrt{\pi}\right)^n = E_n = h/\lambda$, we arrive at the relationship $h = \lambda \cdot k \cdot \left(\sqrt{\pi}\right)^n$. This shows that Planck's constant can be expressed as a function of λ and k, or more explicitly as $h = f\left(\lambda, k\right)$, highlighting the potential to derive Planck's constant from these fundamental parameters. This formula has significant implications for understanding the relationship between dimensionality, wavelength, and energy in both classical and quantum mechanics.

Appendix C. The Python Code to Simulate the Wave Packet Evolution of a Gaussian Wave Function

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
hbar = 1.0          # Reduced Planck's constant (ħ)
m = 1.0             # Particle mass
omega = 1.0         # Angular frequency of the harmonic oscillator
sigma = 0.5         # Initial Gaussian wave packet spread
x0 = 0.0            # Initial position of the wave packet
p0 = 0.0            # Initial momentum of the wave packet
N = 1024            # Number of spatial points
L = 10.0            # Spatial domain length
dx = L / N          # Spatial step size
CFR = 1.0e43        # Constant Frame Rate (CFR)
dt = 0.01 / CFR * 1.0e-40 # Time step adjusted for scaled-down CFR
time_steps = 500# Number of time steps

# Spatial grid
x = np.linspace(-L / 2, L / 2, N)

# Gaussian wave packet initialization
```

Continued

```
def gaussian_wave_packet(x, x0, sigma, p0):
    norm = 1.0 / (np.sqrt(sigma * np.sqrt(np.pi)))
    return norm * np.exp(-(x - x0)**2 / (2.0 * sigma**2)) * np.exp(1j * p0 * x / hbar)

# Harmonic potential
def harmonic_potential(x, m, omega):
    return 0.5 * m * omega**2 * x**2

# Kinetic operator in momentum space
def kinetic_operator(k, m, dt):
    return np.exp(-1j * hbar * k**2 * dt / (2.0 * m))

# Time evolution using the split-operator method
def time_evolution(psi, V, dx, dt, m):
    psi_k = np.fft.fft(psi)
    k = np.fft.fftfreq(N, dx) * 2.0 * np.pi
    psi_k *= kinetic_operator(k, m, dt)
    psi = np.fft.ifft(psi_k)
    psi *= np.exp(-1j * V * dt / hbar)
    return psi

# Initialize wave packet and potential
psi = gaussian_wave_packet(x, x0, sigma, p0)
V = harmonic_potential(x, m, omega)

# Evolve the wave packet and store results
wave_packets = []
for _ in range(time_steps):
    psi = time_evolution(psi, V, dx, dt, m)
    wave_packets.append(np.abs(psi)**2)

# Plot the final wave packet
plt.plot(x, np.abs(psi)**2, label=f'Time = {time_steps * dt * CFR:.2f}')
plt.title('Evolution of Gaussian Wave Packet in Quantum Harmonic Oscillator with CFR')
plt.xlabel('Position (x)')
plt.ylabel('Probability Density |ψ(x)|^2')
plt.legend()
plt.grid(True)
plt.savefig('A_Oscillotor.png', dpi=1200, bbox_inches='tight')
plt.show()
```

Appendix D. Python Snippet to Generate a Series of Plots That Visually Represent the Interaction between Two Static Configurations (SCs) Influenced by an Attractive Force

```
import numpy as np
import matplotlib.pyplot as plt

# Define parameters
sigma = 0.5    # Standard deviation of the Gaussians
```

Continued

```
L = 5.0    # Size of the grid
total_frames = 10    # Total number of frames
dt = 0.1    # Time step for updating positions
force_strength = 7 # Strength of the attractive force (scaled)

# Initial positions of the two Static Configuration (SC) centers
initial_positions = [(2, 2), (3, 3)]

# Function to calculate the gradient of the potential energy
def compute_gradient(x, y, pos1, pos2, sigma):
        X, Y = np.meshgrid(x, y)
        V1 = np.exp(-((X - pos1[0])**2 + (Y - pos1[1])**2) / (2 * sigma**2))
        V2 = np.exp(-((X - pos2[0])**2 + (Y - pos2[1])**2) / (2 * sigma**2))
        V = V1 + V2

        # Calculate gradients
        dV_dx, dV_dy = np.gradient(V, x, y)
        return dV_dx, dV_dy

# Function to calculate updated positions based on the force
def update_positions(p1, p2, dt, x, y, sigma):
        dV_dx, dV_dy = compute_gradient(x, y, p1, p2, sigma)

        # Convert positions to grid indices
        xi1, yi1 = int(np.clip(p1[0] / L * len(x), 0, len(x) - 1)), int(np.clip(p1[1] / L * len(y),
0, len(y) - 1))
        xi2, yi2 = int(np.clip(p2[0] / L * len(x), 0, len(x) - 1)), int(np.clip(p2[1] / L * len(y),
0, len(y) - 1))

        # Print diagnostic information
        print(f"Positions: p1={p1}, p2={p2}")
        print(f"Indices: xi1={xi1}, yi1={yi1}, xi2={xi2}, yi2={yi2}")
        print(f"Gradients: dV_dx[xi1, yi1]={dV_dx[xi1, yi1]}, dV_dy[xi1,
yi1]={dV_dy[xi1, yi1]}")

        # Calculate forces on the positions (reversed direction for attraction)
        fx1 = dV_dx[xi1, yi1] * force_strength
        fy1 = dV_dy[xi1, yi1] * force_strength
        fx2 = dV_dx[xi2, yi2] * force_strength
        fy2 = dV_dy[xi2, yi2] * force_strength

        # Update positions based on forces
        new_p1 = (p1[0] + fx1 * dt, p1[1] + fy1 * dt)
        new_p2 = (p2[0] + fx2 * dt, p2[1] + fy2 * dt)

        # Ensure new positions are within bounds
        new_p1 = (np.clip(new_p1[0], 0, L), np.clip(new_p1[1], 0, L))
        new_p2 = (np.clip(new_p2[0], 0, L), np.clip(new_p2[1], 0, L))

        return new_p1, new_p2
```

Continued

```
# Plotting function
def plot_frame(ax, x, y, pos1, pos2, frame):
    X, Y = np.meshgrid(x, y)
    Z1 = np.exp(-((X - pos1[0])**2 + (Y - pos1[1])**2) / (2 * sigma**2))
    Z2 = np.exp(-((X - pos2[0])**2 + (Y - pos2[1])**2) / (2 * sigma**2))
    Z = Z1 + Z2

    ax.contourf(X, Y, Z, cmap='viridis')
    ax.set_title(f'Frame {frame}')
    ax.set_xlabel('x')
    ax.set_ylabel('y')
    ax.scatter(*pos1, color='red', label='Center 1')
    ax.scatter(*pos2, color='blue', label='Center 2')
    ax.legend()

# Create a grid for plotting
x = np.linspace(0, L, 100)
y = np.linspace(0, L, 100)

# Frames to display (AOM)
frames_to_display = [0, 1, 2, 3,]

# Prepare the figure
fig, axes = plt.subplots(1, len(frames_to_display), figsize=(15, 5))

# Initialize positions
pos1, pos2 = initial_positions

for frame in range(total_frames + 1):
    # Plot the selected frames
    if frame in frames_to_display:
        idx = frames_to_display.index(frame)
        plot_frame(axes[idx], x, y, pos1, pos2, frame)

    # Update positions for the next frame
    pos1, pos2 = update_positions(pos1, pos2, dt, x, y, sigma)

plt.tight_layout()
plt.savefig('A_SC_Force.png', dpi=1500, bbox_inches='tight')
plt.show()
```

Appendix E. List of Notations for Variables

1) ***A***: Amplitude of the wave function, or amplitude constant
2) ***α***: Parameter affecting energy relations and wave function width
3) **CFR**: Constant Frame Rate (discrete time evolution rate)
4) **Δ*t***: Discretized time interval, defined as $\Delta t = 1/\text{CFR}$
5) **ΔΨ/Δ*t***: Discrete change in wave function over time
6) **ΔΨ/ΔDPIT**: Discrete derivative of the wave function with respect to DPIT
7) **ΔDPIT**: Discrete time step

8) ***E***: Energy (expectation value of the Hamiltonian)

9) E_n: Energy in n dimensions

10) $F = ma$: Force, where F is force, m is mass, and a is acceleration

11) h: Planck's constant

12) $\hbar$: Reduced Planck's constant, where $\hbar = h/2\pi$

13) $\hat{H}$: Hamiltonian (total energy operator)

14) $\left[\hat{H}, \hat{\rho}\right]$: Commutator of Hamiltonian and another operator

15) $\hat{H}_{SC+DC}$: Hamiltonian for Static and Dynamic Configurations

16) k^2: Constant defined as $k^2 = A^2 / \sqrt{2\alpha}$

17) k_n: Dimensionality-dependent constant, $k_n = A^2 / (2\alpha)^{n/2}$

18) λ: Wavelength

19) L: Spatial domain length or grid size

20) $\{A, B\}_M$: Moyal Bracket, quantum modification of the classical Poisson bracket

21) n: Dimensionality of space

22) N: Number of spatial grid points

23) p: Momentum variable in phase space

24) p_0: Initial momentum of the wave packet

25) π: Pi (mathematical constant)

26) $\left(\sqrt{\pi}\right)^n$: Exponential scaling factor with dimensionality

27) $\psi(x)$: Wave function in one dimension

28) $|\psi(x)|^2$: Squared modulus of the wave function

29) S: Action (in the classical limit)

30) S^{n-1}: Surface area of the unit sphere in n-dimensional space

31) **DC**: Dynamic Configuration

32) **SC**: Static Configuration

33) σ: Gaussian standard deviation, controls spatial spread of wave packets

34) t: Time variable

35) t_p: Planck time, the smallest unit of continuous time

36) $V(x)$: Potential energy as a function of position

37) V_n: Volume in n-dimensional space

38) $W(x, p, t)$: Wigner function (quasi-probability distribution in phase space)

39) x: Position variable in one dimension or phase space

40) x_0: Initial position of the wave packet

41) Ψ: Wave function describing quantum states in the Schrödinger equation

42) $|\Psi|^2$: Squared modulus of the wave function, representing spatial energy distribution

43) ω: Angular frequency of oscillation in the harmonic potential

44) Δt^2: Higher-order correction term from discretized time evolution

45) $\mathrm{d}V/\mathrm{d}x$, $\mathrm{d}V/\mathrm{d}y$: Potential energy gradients in x and y coordinates

46) $\star$: Star Product, non-commutative product introducing quantum corrections in phase space

47) **Gaussian wave function**: Mathematical form of the wave function