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Massive Discrepancy between the General Theory of Relativity and the Latest Experiments Measuring the Constancy of the Speed of Light

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Abstract

The theoretical study of optical resonator experiments in terms of the foundation of the theory of relativity gives an important result according to which the existence of a lowest limit to the ratio $\Delta\nu/\nu$ arises, which is of the order of 10^{-12}, and is attributed to the rotational motion of the Earth. Given that the latest experimental results of measuring the ratio $\Delta\nu/\nu$, for the period from 2003 to 2015, are several orders of magnitude lower than the theoretically predicted, the present study highlights a massive discrepancy between the general theory of relativity and highly robust experimental data spanning over a decade that come from the latest experiments measuring the constancy of the speed of light.

Keywords

Constancy of the Speed of Light, Optical Resonator Experiments, Michelson-Morley Experiment, Light Speed with Respect to the Earth's Rotating Frame, Anisotropies of the Speed of Light

1. Introduction

The long history of research into the speed of propagation of light waves dates back mainly to the late nineteenth century, with the most prominent being the experimental implementations by Albert Abraham Michelson in 1881 and the Michelson-Morley experiment in 1887. The experiments of the period 1881-1930 show a highly significant attempt to measure the speed of the Earth with respect to a hypothetical medium of propagation of light, the ether. This view has its origin in the electromagnetic theory of light, formulated by James Maxwell in

1861. Despite the difficulties arising from the inadequate technological means of that time, the attempt continued throughout this period [1]-[14]. Initially, the basic view regarding the motion of the Earth with respect to the hypothetical medium of light propagation, *i.e.* the "preferred" reference system of the ether, was that the placement of this medium is in the solar system, so the speed of the Earth with respect to it is the known orbital speed of the Earth of 30 km/s.

In 1969, an implemented Michelson-Morley type experiment by J. Shamir and R. Fox [15] gave a result of $v \le 6.64\ \text{km/s}$. Several years before the implementation of the experiment by J. Shamir and R. Fox new optical tests of the isotropy of the speed of light, using lasers, masers, cryogenic optical resonators, had been devised and implemented, the results of which had already been announced. The calculated values of the speed of the Earth with respect to the hypothetical "preferred reference system" in these newer tests were much lower, specifically this value was approximately 30 km/s according to the experiments of Cedarholm *et al.* in 1958, 1959 [16] [17], and Jaseja *et al.* in 1964 [18]. This very large difference in the results in these two different experimental methods was partly attributed to the greater accuracy of the optical resonator experimental method. The known optical resonator experiments continued until 2015 [19]-[30], and the speed of light based on the latest results is now considered practically constant in all directions. The present study attempts to analyze this very serious issue in depth.

In this paper is studied, based on the foundation of the general theory of relativity, the contribution of the Earth's rotational motion to the fractional frequency shift, $\Delta\nu/\nu$, of high-precision experiments using optical resonators, and it is shown to be of the order of magnitude of 10^{-12}. This result is several orders of magnitude larger than the newer experimental results, which are of the order of magnitude of 10^{-17} in the year 2009 [29], and of the order of magnitude of 10^{-18} (95% confidence interval) in the year 2015 [30]. Despite the fact that the velocity of the Earth on its surface due to its rotational motion is considered negligible compared to its velocity with respect to the Cosmic Microwave Background, nevertheless, since the metric of space in a rotating frame is non-Euclidean according to the foundation of the general theory of relativity, the aforementioned contribution to the fractional frequency shift arises.

2. Theoretical Evaluations

As already mentioned in the introduction, the history of experimental tests on the validity of this principle begins more than a century ago with the famous Michelson-Morley experiment, which was carried out in order to measure the speed of the Earth with respect to the supposed medium of propagation of a light wave, that is, with respect to the ether. According to the beliefs of that time, if the orbital speed of the Earth around the Sun, v, is in the same or opposite direction to the direction of propagation of a light ray, then the measured speed of light in the Earth's reference system should be equal to $c \mp v$ respectively, while if these two directions are perpendicular to each other, the measured speed of light should be equal to $\sqrt{c^2-v^2}$. However, this supposed anisotropy of the speed of light with respect to the Earth's reference system has never been confirmed despite continuous repetitions of this experiment.

The modern relativistic concept regarding a possible anisotropy of the speed of light is based on the exploration of a possible violation of Lorentz invariance. From the study of the tests of the theory of relativity it follows that most of them focus on kinematic aspects. Some theories, relevant to the subject under consideration here, explore scenarios where Lorentz invariance might be violated, leading to potential anisotropy (direction-dependent speed of light). Two of them are the following.

Robertson-Mansouri-Sexl (RMS) kinematic framework

Reza Mansouri and Roman U. Sexl introduced a generalization of Lorentz transformations [31], as follows

$$\begin{aligned} t &= aT + \boldsymbol{\epsilon}\cdot\boldsymbol{x} \\ \boldsymbol{x} &= d\boldsymbol{X} + (b-d)\frac{\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{X})}{v^2} - b\boldsymbol{v}T \end{aligned} \tag{1}$$

where $\boldsymbol{X}$, T are the vector position and time measured in the postulated preferred frame Σ in which the speed of light is axiomatically isotropic, and $\boldsymbol{x}$, t are the vector position and time measured in frame S. As stated in [31], since only v can be used to construct the transformation coefficient, (1) contains the functions $a(v)$, $b(v)$, $d(v)$ (physics) and $\boldsymbol{\epsilon}(v)=(\epsilon,\epsilon_2,\epsilon_3)$ (synchronization). With Einstein clock synchronization, in the case of no Lorentz violation, the values of the coefficients are $a^{-1}=b=\left(1-v^2/c^2\right)^{-1/2}$, $d=1$, $\boldsymbol{\epsilon}=-\boldsymbol{v}/c^2$, where c is the constant value of measured light velocity in Σ.

Transforming the light cone $\boldsymbol{X}^2-(cT)^2=0$ into S, and assuming, for simplicity, $\epsilon_2=\epsilon_3$, in the case of Einstein synchronization we obtain the velocity of a light ray propagating in an angle θ with respect to the x axis, $c_S(\theta)$, as follows

$$\frac{c_S(\theta)}{c} = \frac{b\left(1-v^2/c^2\right)}{a\left[\cos^2\theta + b^2d^2\left(1-v^2/c^2\right)\sin^2\theta\right]^{1/2}} \tag{2}$$

In this case, for the second-order effects, we obtain

$$\frac{c}{c_S(\theta)} = 1 + \left(\beta+\delta-\frac{1}{2}\right)\frac{v^2}{c^2}\sin^2\theta + (\alpha-\beta+1)\frac{v^2}{c^2} \tag{3}$$

where α, β, δ are the coefficients in the expansions

$$a = 1+\alpha\frac{v^2}{c^2}+\cdots,\ b = 1+\beta\frac{v^2}{c^2}+\cdots,\ d = 1+\delta\frac{v^2}{c^2}+\cdots \tag{4}$$

The term $(\beta+\delta-1/2)\left(v^2/c^2\right)\sin^2\theta$ is tested by the Michelson-Morley and the optical cavity experiment, while the Kennedy-Thorndike experiment [32] measures $(\alpha-\beta+1)$.

Michelson-Morley experiment [33].

Let ℓ_1, ℓ_2 be the lengths of the arms of the interferometer. When the apparatus is rotated by 90˚ the measured displacement of the interference pattern gives us

$$c\delta_\theta(\tau) = c\tau\left(\theta + \frac{\pi}{2}\right) - c\tau(\theta) \tag{5}$$

where $c\delta_\theta(\tau)$ is the difference in the optical path and according to (3), keeping the term $(\beta + \delta - 1/2)(v^2/c^2)\sin^2\theta$, this becomes

$$c\delta_\theta(\tau) = (\ell_1 + \ell_2)(2\beta + 2\delta - 1)\frac{v^2}{c^2}\cos(2\theta) \tag{6}$$

An upper limit for $\beta + \delta - 1/2$ can be derived, depending on the observed displacement of the interference pattern. In the Michelson-Morley experiment of 1887 no fractional shifts of the interference pattern larger than 0.005 were observed. Therefore

$$c\delta_\theta(\tau) < 0.005\lambda \tag{7}$$

where $\lambda = 6\times10^{-7}\ \text{m}$ is the used light wavelength. Finally we obtain

$$\beta + \delta = 0.5 \pm 10^{-3} \tag{8}$$

Local Lorentz Invariance (*LLI*) *Violation Signal According to RMS in optical cavity experiment* [34].

Substituting the expressions (4) into the Equation (2) we obtain

$$\frac{c_S(\theta)}{c} = 1 - \left(\beta + \delta - \frac{1}{2}\right)\frac{v^2}{c^2}\sin^2\theta - (\alpha - \beta + 1)\frac{v^2}{c^2} \tag{9}$$

For $\alpha = -\frac{1}{2}$, $\beta = \frac{1}{2}$ and $\delta = 0$ the generalized (RMS) transformations (1) reduce to Lorentz transformations between Σ and S in their familiar form for the second-order effects. The non-isotropic velocity, $c_S(\theta)$, in S differs from the constant value c measured in Σ by $\Delta c = c - c_S(\theta)$. Therefore, since $\nu \sim c_S(\theta)/L$ for a linear optical Fabry-Perot cavity of length L ($\nu = \nu_0$ for Lorentz invariance), keeping the term $(\beta + \delta - 1/2)(v^2/c^2)\sin^2\theta$ we obtain

$$\frac{\Delta\nu}{\nu_0} = \frac{\Delta c}{c} = \mathcal{B}\frac{v^2}{c^2}\sin^2\theta \tag{10}$$

where $\mathcal{B} = \beta + \delta - \frac{1}{2}$ and θ is the angle between the direction of light propagation and $\boldsymbol{v}$. According to relation (10) the relative frequency change between two resonators oriented relative to $\boldsymbol{v}$ at angles θ_1 and θ_2 is obtained as follows

$$\frac{\Delta(\nu_1 - \nu_2)}{\nu_0} = \mathcal{B}\frac{v^2}{c^2}\left(\sin^2\theta_1 - \sin^2\theta_2\right) \tag{11}$$

The laboratory is considered fixed on the surface of the earth, in the northern hemisphere, so we choose the standard coordinate system in the laboratory frame such that the x axis points south, the y axis points east, and the z axis points vertically upwards. The sun-centered celestial equatorial frame (SCCEF) has the X axis pointing towards vernal equinox, the Z axis pointing towards the celestial north pole and the Y axis is chosen accordingly to complete the right-handed coordinate system. Also, the consideration that the earth's orbit is approximately circular is satisfactory for the requirements of the subject under consideration. The rotation from the sun-centered celestial equatorial frame to the standard laboratory frame is given by

$$R=\begin{pmatrix}\cos\chi\cos\omega_{\oplus}T_{\oplus} & \cos\chi\sin\omega_{\oplus}T_{\oplus} & -\sin\chi\\ -\sin\omega_{\oplus}T_{\oplus} & \cos\omega_{\oplus}T_{\oplus} & 0\\ \sin\chi\cos\omega_{\oplus}T_{\oplus} & \sin\chi\sin\omega_{\oplus}T_{\oplus} & \cos\chi\end{pmatrix} \tag{12}$$

where $\omega_{\oplus}$ is the earth's sidereal angular frequency, approximately equal to $2\pi/(23\,\text{h}\,56\,\text{min})$, and χ is the colatitude of the laboratory. The time $T_{\oplus}$, which is measured in the sun-centered frame, is the time elapsed from one of the times at which the y and Y axes coincide, therefore, for each experiment there is a constant displacement of the axis of time $T_{\oplus}$ with respect to the axis of celestial equatorial time T, equal to ΔT_{exp}.

The supposed preferred isotropic frame Σ is commonly adopted to be the cosmic microwave background (CMB), so, as velocity v is considered the velocity of the laboratory with respect to the CMB. If we consider only the constant leading term of the velocity v neglecting the Earth's orbital and rotational thrusts, its orientation in the Sun-centered celestial equatorial frame is given by the relation

$$\boldsymbol{v}(t)=v\begin{pmatrix}\cos a\cos b\\ \sin a\cos b\\ -\sin b\end{pmatrix} \tag{13}$$

with $a=168^{\circ}$, $b=-6^{\circ}$, and $v=370\,\text{km/s}$. The terms $\sin^2\theta_i$, for $i=1,2$, in Equation (11) can be expressed as follows

$$\sin^2\theta_i(t)=1-\frac{\left(\boldsymbol{v}(t)\cdot\boldsymbol{e}_i(t)\right)^2}{v^2} \tag{14}$$

where $\boldsymbol{e}_1(t)$ and $\boldsymbol{e}_2(t)$ are the unit vectors along the axis of the stationary and the rotating resonator respectively. The expressions of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ in matrix form in the laboratory reference system are

$$(\boldsymbol{e}_1)_{lab}=\begin{pmatrix}1\\0\\0\end{pmatrix},\ (\boldsymbol{e}_2)_{lab}=\begin{pmatrix}\cos\omega_{rot}T\\ \sin\omega_{rot}T\\ 0\end{pmatrix} \tag{15}$$

where ω_{rot} is the angular velocity of each rotating cavity. The forms of these two units vectors with respect to the SCCEF, that is, including the rotation matrix R of the relation (12), are expressed by the following relations

$$(\boldsymbol{e}_1)=(\boldsymbol{e}_1)_{lab}^{\mathrm{T}}R=\begin{pmatrix}\cos\chi\cos\omega_{\oplus}T_{\oplus}\\ \cos\chi\sin\omega_{\oplus}T_{\oplus}\\ -\sin\chi\end{pmatrix} \tag{16}$$

$$(\boldsymbol{e}_2)=(\boldsymbol{e}_2)_{lab}^{\mathrm{T}}R=\begin{pmatrix}\cos\chi\cos\omega_{\oplus}T_{\oplus}\cos(\omega_{rot}T_{\oplus}+\phi)-\sin\omega_{\oplus}T_{\oplus}\sin(\omega_{rot}T_{\oplus}+\phi)\\ \cos\chi\sin\omega_{\oplus}T_{\oplus}\cos(\omega_{rot}T_{\oplus}+\phi)+\cos\omega_{\oplus}T_{\oplus}\sin(\omega_{rot}T_{\oplus}+\phi)\\ -\sin\chi\cos(\omega_{rot}T_{\oplus}+\phi)\end{pmatrix} \tag{17}$$

where $\phi=\omega_{rot}\Delta T_{exp}$, and ΔT_{exp} is the time-axis displacement mentioned above. The Equation (11) is evaluated based on the relations (13), (14), (16), and (17), so the final expression gives an oscillation signal with angular frequency $2\omega_{rot}$ and the sidebands at $\omega_{\oplus}$ and $2\omega_{\oplus}$. A relevant experimental study is in [29].

Minimal standard model extension (mSME) framework [35]

Including only the photonic sector extension as a part of Lorentz and CPT violating extension of QED, the relevant Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\left(k_F\right)_{\kappa\lambda\mu\nu} F^{\kappa\lambda}F^{\mu\nu} \tag{18}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. This Lagrangian contains the standard Maxwell term and an additional Lorentz-violating term, whose coefficient $\left(k_F\right)_{\kappa\lambda\mu\nu}$ is dimensionless and has the symmetries of the Riemann tensor and a vanishing double trace, which implies a total of 19 independent components. The equations of motion from Lagrangian (18), which are modified source-free inhomogeneous Maxwell equations, are given by

$$\partial_\alpha F_\mu{}^\alpha + \left(k_F\right)_{\mu\alpha\beta\gamma}\partial^\alpha F^{\beta\gamma} = 0 \tag{19}$$

whereas, defining the electromagnetic tensor $\tilde{F}$ according to the following equivalence equation

$$\partial_\mu \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu F_{\kappa\lambda} \tag{20}$$

the following homogeneous Maxwell equations remain unchanged

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \tag{21}$$

In analogy to the theory of dielectrics, we define two new fields $\boldsymbol{D}$ and $\boldsymbol{H}$ by the six-dimensional matrix equation

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{H} \end{pmatrix} = \begin{pmatrix} 1+\kappa_{DE} & \kappa_{DB} \\ \kappa_{HE} & 1+\kappa_{HB} \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{B} \end{pmatrix} \tag{22}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields which are solutions of the modified Maxwell Equation (19), and κ_{DE}, κ_{DB}, κ_{HE}, κ_{HB} are 3×3 matrices defined by

$$\begin{aligned} \left(\kappa_{DE}\right)^{jk} &= -2\left(k_F\right)^{0j0k} \\ \left(\kappa_{HB}\right)^{jk} &= \frac{1}{2}\epsilon^{jpq}\epsilon^{krs}\left(k_F\right)^{pqrs} \\ \left(\kappa_{DB}\right)^{jk} &= -\left(\kappa_{HE}\right)^{kj} = \left(k_F\right)^{0jpq}\epsilon^{kpq} \end{aligned} \tag{23}$$

Following these definitions, the modified Maxwell Equations (19), (21) take the following form

$$\begin{aligned} &\nabla \times \boldsymbol{H} - \partial_0 \boldsymbol{D} = 0, \quad \nabla \cdot \boldsymbol{D} = 0 \\ &\nabla \times \boldsymbol{E} + \partial_0 \boldsymbol{B} = 0, \quad \nabla \cdot \boldsymbol{B} = 0 \end{aligned} \tag{24}$$

According to this view, the Lorentz-violating background tensor $\left(k_F\right)^{pqrs}$ can be considered as a dielectric medium in a space in which there is no charge or current density. Since $\boldsymbol{H}$ and $\boldsymbol{D}$ depend on the components of $\left(k_F\right)^{pqrs}$ it follows that the properties of this dielectric depend on orientation.

Optical cavity experiments.

Let $\boldsymbol{E}_0$, $\boldsymbol{B}_0$, $\boldsymbol{D}_0$, $\boldsymbol{H}_0$ be the fields in a conventional operation of an optical cavity with resonant frequency ν_0. For a resonant mode in the presence of Lorentz violation, with resonant frequency ν, we consider that the nonzero k_F coefficients perturb these resonance fields. We define by $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{D}$, $\boldsymbol{H}$ these perturbed fields. The case we will consider is that in which the cavity is void of matter, so the Equation (22) yields the following approximate relations

$$\begin{aligned} \boldsymbol{D}-\boldsymbol{E} &\simeq \kappa_{DE}\cdot\boldsymbol{E}_0+\kappa_{DB}\cdot\boldsymbol{B}_0 \\ \boldsymbol{H}-\boldsymbol{B} &\simeq \kappa_{HE}\cdot\boldsymbol{E}_0+\kappa_{HB}\cdot\boldsymbol{B}_0 \end{aligned} \tag{25}$$

Let $\delta\nu=\nu-\nu_0$ be the change in the resonant frequency relative to the conventional operation resonant frequency. The resultant fractional frequency shift is

$$\frac{\delta\nu}{\nu}=-\frac{1}{4\langle U\rangle}\int_V \mathrm{d}^3x\left(\boldsymbol{E}_0^*\cdot\kappa_{DE}\cdot\boldsymbol{E}_0-\boldsymbol{B}_0^*\cdot\kappa_{HB}\cdot\boldsymbol{B}_0+2\,\mathrm{Re}\left(\boldsymbol{E}_0^*\cdot\kappa_{DB}\cdot\boldsymbol{B}_0\right)\right) \tag{26}$$

where $\langle U\rangle=\frac{1}{4}\int_V \mathrm{d}^3x\left(\boldsymbol{E}_0\cdot\boldsymbol{D}_0^*+\boldsymbol{B}_0\cdot\boldsymbol{H}_0^*\right)$ is the time-averaged energy stored in the cavity, and is originating from unperturbed fields. In our case the optical cavity is void of matter, therefore $\epsilon=1$ and $\mu=1$. The unperturbed fields can be expressed as

$$\begin{aligned} \boldsymbol{E}_0(x) &= \boldsymbol{E}_0\cos\left(\omega_0\hat{N}\cdot\boldsymbol{x}+\phi\right)\mathrm{e}^{-\imath\omega_0 t} \\ \boldsymbol{B}_0(x) &= \imath\hat{N}\times\boldsymbol{E}_0\sin\left(\omega_0\hat{N}\cdot\boldsymbol{x}+\phi\right)\mathrm{e}^{-\imath\omega_0 t} \end{aligned} \tag{27}$$

where $\hat{N}$ is a unit vector in the direction of the length of the cavity, ϕ is a phase, and $\boldsymbol{E}_0$ is a vector perpendicular to $\hat{N}$ that specifies the polarization. Substituting the relations (27) into Equation (26) we obtain the fractional frequency shift:

$$\frac{\delta\nu}{\nu}=-\frac{1}{2\left|\boldsymbol{E}_0\right|^2}\left[\boldsymbol{E}_0^*\cdot\left(\kappa_{DE}\right)_{lab}\cdot\boldsymbol{E}_0-\left(\hat{N}\times\boldsymbol{E}_0^*\right)\cdot\left(\kappa_{HB}\right)_{lab}\cdot\left(\hat{N}\times\boldsymbol{E}_0\right)\right] \tag{28}$$

We consider a change in an observable physical quantity O of electrodynamics in a laboratory which is linear in the matrices κ_{DE}, κ_{HB}, and $\kappa_{DB}=-\kappa_{HE}^{\mathrm{T}}$. In the laboratory frame of reference this change can be given as

$$\delta O=\left(\mathcal{M}_{DE}\right)_{lab}^{jk}\left(\kappa_{DE}\right)_{lab}^{jk}+\left(\mathcal{M}_{HB}\right)_{lab}^{jk}\left(\kappa_{HB}\right)_{lab}^{jk}+\left(\mathcal{M}_{DB}\right)_{lab}^{jk}\left(\kappa_{DB}\right)_{lab}^{jk} \tag{29}$$

where $\left(\mathcal{M}_{DE}\right)_{lab}$, $\left(\mathcal{M}_{HB}\right)_{lab}$, and $\left(\mathcal{M}_{DB}\right)_{lab}$ are experiment-specific constant matrices determined by the apparatus. According to the Equation (28) the matrices $\left(\mathcal{M}\right)_{lab}$ are expressed as follows

$$\begin{aligned} \left(\mathcal{M}_{DE}\right)_{lab}^{jk} &= -\frac{\mathrm{Re}\left(\boldsymbol{E}_0^*\right)^j\left(\boldsymbol{E}_0\right)^k}{2\left|\boldsymbol{E}_0\right|^2} \\ \left(\mathcal{M}_{HB}\right)_{lab}^{jk} &= -\frac{\mathrm{Re}\left(\hat{N}\times\boldsymbol{E}_0^*\right)^j\left(\hat{N}\times\boldsymbol{E}_0\right)^k}{2\left|\boldsymbol{E}_0\right|^2} \\ \left(\mathcal{M}_{DB}\right)_{lab}^{jk} &= 0 \end{aligned} \tag{30}$$

When laser light incident on a cavity positioned horizontally in an Earth based laboratory, with the light linearly polarized along the z axis, then, denoting by θ the angle between the x axis and the cavity orientation, $\hat{N} = (\cos\theta, \sin\theta, 0)$, so the obtained fractional frequency shift in the laboratory frame is

$$\begin{aligned} \frac{\delta\nu}{\nu} &= -\frac{1}{4}\left[2(\kappa_{DE})^{33}_{lab} - (\kappa_{HB})^{11}_{lab} - (\kappa_{HB})^{22}_{lab}\right] \\ &\quad -\frac{1}{2}(\kappa_{HB})^{12}_{lab}\sin 2\theta - \frac{1}{4}\left[(\kappa_{HB})^{11}_{lab} - (\kappa_{HB})^{22}_{lab}\right]\cos 2\theta \end{aligned} \tag{31}$$

Transforming the resultant expression of Equation (31) to the Sun-centered celestial equatorial frame the fractional frequency shift is expressed as

$$\frac{\delta\nu}{\nu} = A + B\sin 2\theta + C\cos 2\theta \tag{32}$$

where

$$\begin{aligned} A &= A_0 + A_1\sin\omega_\oplus T_\oplus + A_2\cos\omega_\oplus T_\oplus + A_3\sin 2\omega_\oplus T_\oplus + A_4\cos 2\omega_\oplus T_\oplus \\ B &= B_0 + B_1\sin\omega_\oplus T_\oplus + B_2\cos\omega_\oplus T_\oplus + B_3\sin 2\omega_\oplus T_\oplus + B_4\cos 2\omega_\oplus T_\oplus \\ C &= C_0 + C_1\sin\omega_\oplus T_\oplus + C_2\cos\omega_\oplus T_\oplus + C_3\sin 2\omega_\oplus T_\oplus + C_4\cos 2\omega_\oplus T_\oplus \end{aligned} \tag{33}$$

The quantities $A_{0,1,2,3,4}$, $B_{0,1,2,3,4}$, and $C_{0,1,2,3,4}$ are linear in the coefficients for Lorentz violation and depend on the colatitude χ. A relevant experimental study is in [29].

The theoretically predicted experimental results, regarding the Optical resonator experiments, are the subject of study in the following sections. In the case we study, we denote the speed of the laboratory due to the Earth's rotational motion by u, taking into account the Earth's angular velocity and the geographical location of the laboratory. Therefore the accuracy of the calculations in this study is of the order of u^2/c^2.

Also the difference of the dimensions order of magnitude within the terrestrial laboratory compared to the order of magnitude of the Earth's rotating radius at the geographical location of the laboratory is taken into account.

3. Basic Theory

We define an inertial frame of reference, S', and a uniformly rotating frame S. The angular velocity of rotating frame with respect to the inertial frame is Ω. The coordinate transformations between S and S' are:

$$x' = x\cos\Omega t - y\sin\Omega t \tag{34a}$$

$$y' = x\sin\Omega t + y\cos\Omega t \tag{34b}$$

$$z' = z \tag{34c}$$

$$t' = t \tag{34d}$$

Denoting by x^0, x^1, x^2, x^3, respectively, the coordinates ct, x, y, z, we have for the nonzero components of the metric tensor the expressions

$$\begin{aligned} &g_{00} = 1 - \Omega^2\left(x^2 + y^2\right)/c^2,\ g_{01} = \Omega y/c,\ g_{02} = -\Omega x/c, \\ &g_{11} = -1,\ g_{22} = -1,\ g_{33} = -1. \end{aligned} \tag{35}$$

The expression for $\mathrm{d}s^2$ for the rotating frame is given by the relation:

$$\mathrm{d}s^2 = \left(1-\Omega^2\left(x^2+y^2\right)/c^2\right)c^2\mathrm{d}t^2 - 2\Omega\left(x\mathrm{d}y - y\mathrm{d}x\right)\mathrm{d}t - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \tag{36}$$

Let us suppose that a ray of light propagate along a differential straight line segment of its path between two adjacent points, and that the time coordinates of the passage of the ray of light through these points are defined as x^0, $x^0 + \mathrm{d}x^0$ and that $\mathrm{d}x^0 > 0$. The differential $\mathrm{d}x^0$ is obtained by solving the Equation (36) for $\mathrm{d}s^2 = 0$ [36], and is given by the equation

$$\mathrm{d}x^0 = \frac{1}{g_{00}}\left(-g_{0\alpha}\mathrm{d}x^\alpha + \sqrt{\left(g_{0\alpha}g_{0\beta} - g_{\alpha\beta}g_{00}\right)\mathrm{d}x^\alpha\mathrm{d}x^\beta}\right) \tag{37}$$

The following description refers to the case in which the propagation of the examined light ray takes place in a laboratory on the surface of the Earth, as illustrated in **Figure 1**. The non-inertial motions of the Earth, which are the subject of the general theory of relativity in terms of considering the metric of space as non-Euclidean, are rotational motion and tidal motion, since its orbital motion around the Sun can be considered as approximately inertial during the observation for the case we are studying. However, the tidal effect on the Earth's rotational motion is negligible, because the Earth's day is currently lengthening at a rate of about 0.002 seconds per century due to this phenomenon [37]. We assume that the inertial frame S' is defined by the instantaneously inertial reference frame of the Earth's orbital motion around the Sun and that the axes Z and Z' coincide with the axis of rotation of the Earth. We also assume that the axes X', Y' of inertial frame and X, Y of Earth's rotating frame lie in the plane defined by the Earth's parallel (circle of latitude) and coincide respectively at the initial time of the observation $t = t' = 0$. The axes Y', Y pass through the laboratory space during the observation. The light signal propagates from the point defined by the position $\left(x, y, z\right)$ to a neighboring point defined by the position $\left(x+\mathrm{d}x, y+\mathrm{d}y, z+\mathrm{d}z\right)$. For a typical latitude of $\varphi = 45^\circ$ the distance of the laboratory from the Earth's rotation axis is approximately equal to 4.5×10^6 m. So for observations inside the laboratory the absolute value of the component g_{02} of the metric tensor is much smaller than the absolute value of the g_{01} component, and therefore the component g_{02} is considered negligible. From the Equations (35), (37) we obtain

$$\mathrm{d}x^0 = c\mathrm{d}t \simeq \frac{-\Omega y\mathrm{d}x/c + \sqrt{\mathrm{d}x^2 + \left(\mathrm{d}y^2 + \mathrm{d}z^2\right)\left(1-\Omega^2y^2/c^2\right)}}{1-\Omega^2y^2/c^2} \tag{38}$$

The three-dimensional vector, $\mathrm{d}\ell$, of distance between the two previous mentioned points in terms of the space coordinate elements, is derived from the equation

$$\mathrm{d}\ell^2 = \gamma_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta \tag{39}$$

where $\alpha, \beta = 1, 2, 3$, and

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \tag{40}$$

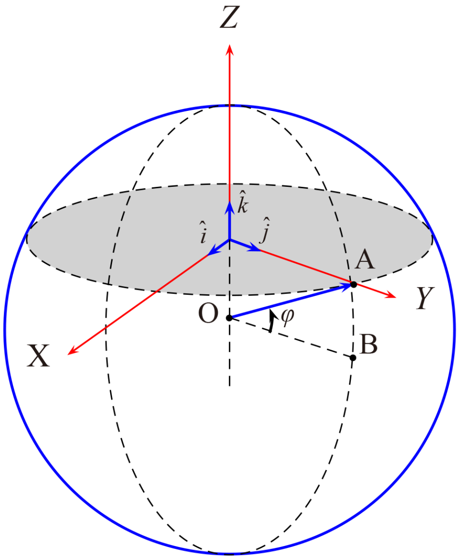

Figure 1. O is the center of the depicted Earth sphere, defined on a spherical model. The location of the laboratory is marked with A. The imaginary circular surface colored in gray is the Earth's parallel (circle of latitude) and lies in the XY-plane. The dashed line OB is parallel to the Y axis. The angle φ is the geocentric latitude of the geographical location of the laboratory.

The non zero, and non negligible, components of the metric tensor $\gamma_{\alpha\beta}$ are

$$\gamma_{11} \simeq 1+\frac{\Omega^2 y^2/c^2}{1-\Omega^2 y^2/c^2}=\frac{1}{1-\Omega^2 y^2/c^2} \tag{41a}$$

$$\gamma_{22} \simeq 1 \tag{41b}$$

$$\gamma_{33} = 1 \tag{41c}$$

The observation place is located at a distance R from the Earth's rotation axis. The velocity of experimental arrangement with respect to the inertial frame S', is given by the equation $\boldsymbol{u}=\boldsymbol{\Omega}\times\boldsymbol{R}$, where $R=\sqrt{x'^2+y'^2}=\sqrt{x^2+y^2}\simeq y$, so $u\simeq -\Omega y$. We also define $\gamma=\left(1-u^2/c^2\right)^{-1/2}$. The components of $d\ell$ are related to the components of dx^α as follows

$$\begin{pmatrix} \mathrm{d}\ell_x^2 \\ \mathrm{d}\ell_y^2 \\ \mathrm{d}\ell_z^2 \end{pmatrix} \simeq \begin{pmatrix} \gamma^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}x^2 \\ \mathrm{d}y^2 \\ \mathrm{d}z^2 \end{pmatrix} = \begin{pmatrix} \gamma^2\mathrm{d}x^2 \\ \mathrm{d}y^2 \\ \mathrm{d}z^2 \end{pmatrix} \tag{42}$$

therefore the three-dimensional vector $\mathrm{d}\boldsymbol{\ell}$ is expressed by the equation

$$\mathrm{d}\boldsymbol{\ell} \simeq \gamma\mathrm{d}x\hat{\boldsymbol{i}}+\mathrm{d}y\hat{\boldsymbol{j}}+\mathrm{d}z\hat{\boldsymbol{k}} \tag{43}$$

where $\hat{\boldsymbol{i}},\hat{\boldsymbol{j}},\hat{\boldsymbol{k}}$ are the unit vectors of the Cartesian coordinate system in the rotating frame of the Earth. It therefore becomes clear that the space geometry defined by the line element $\mathrm{d}\boldsymbol{\ell}$ is non-Euclidean [38]. The Equation (38) is formulated as follows

$$c\mathrm{d}t \simeq \gamma\left(\mathrm{d}\ell+\frac{u\gamma\mathrm{d}x}{c}\right) \tag{44}$$

Integrating the left and right-hand side of Equation (43) we obtain

$$\ell = \ell_x \hat{\boldsymbol{i}} + \ell_y \hat{\boldsymbol{j}} + \ell_z \hat{\boldsymbol{k}} \simeq \gamma x \hat{\boldsymbol{i}} + y \hat{\boldsymbol{j}} + z \hat{\boldsymbol{k}} \tag{45}$$

In the case of an optical resonator, an electromagnetic wave propagates along a path of only a few centimeters, so starting the propagation inside the optical cavity at time $t = 0$, the angle Ωt in the Equations (34a), (34b) is of the order of 10^{-14}, and the ratio $(x\Omega t)/y$ is of the order of 10^{-22}. Therefore

$$x' \simeq x - y\Omega t \simeq x + ut \tag{46a}$$

$$y' \simeq x\Omega t + y \simeq y \simeq \ell_y \tag{46b}$$

$$z' = z = \ell_z \tag{46c}$$

From Equations (46a) and (46b) it becomes clear that during the short time of propagation of the light wave inside the optical cavity, practically, the resonator appears to move with respect to an observer of the inertial frame S' rectilinearly in the x'-direction with constant velocity u. Therefore, in this case, the propagation direction of the light wave is considered practically constant. We denote by $\mathrm{d}t_\ell$ the actual differential time of the light wave propagation, so $\mathrm{d}t_\ell = \mathrm{d}\ell / c$. Integrating the left and right-hand side of Equation (44) we obtain

$$t = t' \simeq \gamma \left(t_\ell + \frac{u \ell_x}{c^2} \right) \tag{47}$$

From Equations (45), (47), (46a) we obtain

$$\begin{aligned} x' &\simeq x + ut' \\ &\simeq x + \frac{u^2}{c^2}\gamma^2 x + \gamma u t_\ell \\ &\simeq \gamma \left(\ell_x + u t_\ell \right) \end{aligned} \tag{48}$$

The Equations (47), (46b), (46c), (48), lead to the conclusion that the actual time t_ℓ and the components ℓ_x, ℓ_y, ℓ_z of the three-dimensional vector $\boldsymbol{\ell}$, in the case of the Earth's self-rotating reference frame, are approximated by applying the Lorentz transformations.

The four-dimensional wave vector is transformed according to the following equations

$$k^i = \frac{\partial x^i}{\partial x'^j} k'^j \tag{49a}$$

$$k_i = \frac{\partial x'^j}{\partial x^i} k'_j \tag{49b}$$

where the x', k' are considered as observed in the inertial reference frame S', whereas the x, k are considered as observed in the Earth's rotating frame. Therefore, the matrix of covariant vector transformation from the inertial frame to the Earth's rotating frame is expressed as follows

$$\left(\frac{\partial x'^j}{\partial x^i}\right) = \begin{pmatrix} 1 & -\frac{\Omega x}{c}\sin(\Omega t) - \frac{\Omega y}{c}\cos(\Omega t) & \frac{\Omega x}{c}\cos(\Omega t) - \frac{\Omega y}{c}\sin(\Omega t) & 0 \\ 0 & \cos(\Omega t) & \sin(\Omega t) & 0 \\ 0 & -\sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\frac{\Omega y}{c} & \frac{\Omega x}{c} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Omega t) & \sin(\Omega t) & 0 \\ 0 & -\sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & -\frac{\Omega y}{c} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{u}{c} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

therefore,

$$\begin{pmatrix} k_0 \\ k_1 \\ k_2 \\ k_3 \end{pmatrix} \simeq \begin{pmatrix} 1 & \frac{u}{c} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\omega'}{c} \\ -k'_x \\ -k'_y \\ -k'_z \end{pmatrix} = \begin{pmatrix} \frac{\omega'}{c} - \frac{u}{c}k'_x \\ -k'_x \\ -k'_y \\ -k'_z \end{pmatrix} \tag{50}$$

where $(\omega'/c, -k'_x, -k'_y, -k'_z)$, $(\omega'/c, k'_x, k'_y, k'_z)$ are the four-dimensional vectors in covariant and contravariant form with respect to the inertial reference frame S'. From the relations (50) we obtain

$$k_0 \simeq \frac{\omega'}{c} - \frac{u}{c}k'_x \tag{51a}$$

$$(k_1, k_2, k_3) \simeq (-k'_x, -k'_y, -k'_z) \tag{51b}$$

Following a similar procedure for the transformation of contravariant four-dimensional wave vector we obtain

$$\begin{pmatrix} k^0 \\ k^1 \\ k^2 \\ k^3 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{u}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\omega'}{c} \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} = \begin{pmatrix} \frac{\omega'}{c} \\ k'_x - \frac{\omega' u}{c^2} \\ k'_y \\ k'_z \end{pmatrix} \tag{52}$$

and

$$k^0 \simeq \frac{\omega'}{c} \tag{53a}$$

$$(k^1, k^2, k^3) \simeq \left(k'_x - \frac{\omega' u}{c^2}, k'_y, k'_z\right) \tag{53b}$$

Let f be a quantity describing the field of a wave, which has the form

$$f = a\mathrm{e}^{\imath\psi} \tag{54}$$

In case the wave is not plane, but geometrical optics is applicable, the first order expansion of the eikonal ψ, over a small space region, is expressed as

$$\psi = \psi_0 + \boldsymbol{r}\cdot\frac{\partial\psi}{\partial\boldsymbol{r}} + t\frac{\partial\psi}{\partial t} \tag{55}$$

where the origin for coordinates and time has been chosen within the space region and time interval under consideration, and the derivatives are evaluated at the origin. Since in each small region, and each small interval of time, the wave can be considered as plane, we can write

$$\boldsymbol{k} = \frac{\partial\psi}{\partial\boldsymbol{r}},\quad \omega = -\frac{\partial\psi}{\partial t} \tag{56}$$

and in four dimensional form

$$k_i = -\frac{\partial\psi}{\partial x^i} \tag{57}$$

In the Earth's rotating frame, the angular frequency expressed in terms of the world time x^0/c is $\omega_0 = ck_0 = -c\partial\psi/\partial x^0$ and in our case remains constant during the propagation of the light ray. The angular frequency measured in terms of the proper time is $\omega = -\partial\psi/\partial\tau$. We have

$$\omega = -\frac{\partial\psi}{\partial\tau} = -\frac{\partial\psi}{\partial x^0}\frac{\partial x^0}{\partial\tau} = -\frac{\partial\psi}{\partial x^0}\frac{c}{\sqrt{g_{00}}} = \frac{ck_0}{\sqrt{g_{00}}} \simeq \gamma ck_0 = \gamma\left(\omega' - \boldsymbol{u}\cdot\boldsymbol{k}'\right) \tag{58}$$

The three-dimensional vector $\boldsymbol{k}$ in terms of the Earth's rotating frame space metric, obeys the equation

$$\boldsymbol{k}^2 = \gamma_{\alpha\beta}k^\alpha k^\beta \tag{59}$$

therefore

$$\begin{pmatrix} k_x^2 \\ k_y^2 \\ k_z^2 \end{pmatrix} \simeq \begin{pmatrix} \gamma^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \left(k^1\right)^2 \\ \left(k^2\right)^2 \\ \left(k^3\right)^2 \end{pmatrix} = \begin{pmatrix} \gamma^2\left(k^1\right)^2 \\ \left(k^2\right)^2 \\ \left(k^3\right)^2 \end{pmatrix} \tag{60}$$

and

$$\boldsymbol{k} \simeq \boldsymbol{k}' + \left(\gamma - 1\right)\frac{\left(\boldsymbol{k}'\cdot\boldsymbol{u}\right)\boldsymbol{u}}{u^2} - \gamma\frac{\omega'\boldsymbol{u}}{c^2} \tag{61}$$

Equations (58), (61) show that in our case the quantities $\boldsymbol{k}$ and ω are approximated by the Lorentz transformations. They also agree with the hypothesis of locality in the "eikonal limit" as expected [39] [40], since the frequency of the electromagnetic wave for an optical cavity experiment (for example in [29]) is of the order of 10^{14} Hz, while the frequency of rotation of the Earth is of the order of 10^{-5} Hz, so the ratio Ω/ω is of the order of 10^{-19}.

4. Standing Wave

It is useful to present an excerpt from what is stated in [36] (CHAPTER 7, THE PROPAGATION OF LIGHT, at the beginning of the paragraph §53. **Geometrical optics**) regarding the cases in which the electromagnetic wave under consideration can be considered to be plane:

> "A plane wave is characterized by the property that its direction of propagation and amplitude are the same everywhere. Arbitrary electromagnetic waves, of course, do not have this property. Nevertheless, a great many electromagnetic waves, which are not plane, have the property that within each small region of space they can be considered to be plane. For this, it is clearly necessary that the amplitude and direction of the wave remain practically constant over distances of the order of the wavelength. If this condition is satisfied, we can introduce the so-called *wave surface*, *i.e.* a surface at all of whose points the phase of the wave is the same (at a given time)…"

In a rotating reference frame, in general, light does not propagates along straight Euclidean lines. In our case, however, according to the relations (46a), (46b) and the commentary that follows, it is found that the propagation direction of the electromagnetic wave is practically constant within the space of the optical cavity. Therefore, the electromagnetic wave in question can be considered to be plane.

Let F be any quantity describing the field of the wave (any component of $\boldsymbol{E}$ or $\boldsymbol{H}$). For a plane monochromatic wave, F has the form

$$F = \mathrm{Re}\left\{ A e^{i\left(\boldsymbol{k}\cdot\boldsymbol{\ell} - \omega t_\ell + a\right)} \right\} \tag{62}$$

where $\boldsymbol{k}$, ω are the three-dimensional wave vector and the actual angular frequency respectively, with respect to the Earth's rotating frame.

A such plane wave in a resonator cavity, in a laboratory on the Earth's surface, can be expressed in the form

$$F_{ix} = A\sin\left(k_{ix}\ell_x - \omega_{ix}t_\ell\right) \tag{63}$$

for the incident wave, where with the subscript i this wave is declared as incident and the subscript x is used to determine the wave propagation along the X-axis, *i.e.* parallel to the vector velocity $\boldsymbol{u}$ of the laboratory, as it is mentioned in the previous section. The reflected wave can be expressed in the form

$$F_{rx} = A\sin\left(k_{rx}\ell_x + \omega_{rx}t_\ell\right) \tag{64}$$

The derived standing wave is expressed in the form

$$\begin{aligned} F_{sx} &= F_{ix} + F_{rx} \\ &= 2A\sin\left(\frac{1}{2}\left[\left(k_{ix} + k_{rx}\right)\ell_x + \left(-\omega_{ix} + \omega_{rx}\right)t_\ell\right]\right) \\ &\quad \times\cos\left(\frac{1}{2}\left[\left(k_{ix} - k_{rx}\right)\ell_x - \left(\omega_{ix} + \omega_{rx}\right)t_\ell\right]\right) \end{aligned} \tag{65}$$

From Equations (58), (61), we obtain

$$k_{ix} = \gamma k'\left(1 - \frac{u}{c}\right) \tag{66a}$$

$$k_{rx} = \gamma k'\left(1 + \frac{u}{c}\right) \tag{66b}$$

$$\omega_{ix} = \gamma\omega'\left(1 - \frac{u}{c}\right) \tag{66c}$$

$$\omega_{rx} = \gamma\omega'\left(1 + \frac{u}{c}\right) \tag{66d}$$

therefore the Equation (65) takes the form

$$F_{sx} = 2A\sin\left[\gamma k'\left(\ell_x + ut_\ell\right)\right]\cos\left[\gamma\omega'\left(\frac{u\ell_x}{c^2} + t_\ell\right)\right] \tag{67}$$

Finally, according to Equations (47), (48), the obtained expression of standing wave is

$$F_{sx} = 2A\sin\left(k'x'\right)\cos\left(\omega't'\right) \tag{68}$$

If the orientation of the resonator is along the Y-axis, then $k_{iy} = k_{ry} = k'$, $\omega_{iy} = \omega_{ry} = \gamma\omega'$, and according to Equation (47), given that for $\ell_x = 0$ we obtain $t_\ell = t'/\gamma$, the expression for the standing wave is

$$F_{sy} = 2A\sin\left(k'y'\right)\cos\left(\omega't'\right) \tag{69}$$

Following a similar procedure for the orientation of the resonator along the Z-axis, the obtained expression for the standing wave is

$$F_{sz} = 2A\sin\left(k'z'\right)\cos\left(\omega't'\right) \tag{70}$$

5. Fractional Frequency Shift in a Linear Optical Fabry-Perot Cavity

At each observation time, t_{obs}, the coordinate system of the inertial system S' must have rotated by an angle Ωt_{obs} around the z'-axis, so that all the axes of S and S' coincide and $x = x'$, $y = y'$, $z = z'$. We consider that at time t_{obs} the resonator, whose optical cavity has a length L, is oriented parallel to the x-axis and at this time one of the end-plates surface is at position $x = 0$, whereas the other surface is at position $x = x_0$. The Equation (68) takes the form

$$F_{sx} = 2A\sin\left(k'x\right)\cos\left(\omega't_{obs}\right) \tag{71}$$

The Equation (45) which obeys the space metric of the Earth's rotating frame yields the equality $\ell_x = \gamma x$, so in our case, for an orientation parallel to x-axis, $x_0 = L/\gamma$. The value of F_{sx} in Equation (71) must be zero at the two positions $x = 0$, $x = x_0$. Under these conditions the following equality is obtained

$$k'x_0 = k'\frac{L}{\gamma} = n\pi \tag{72}$$

where $n = 1, 2, 3, \cdots$ Therefore the wavelength λ, is given by the relation

$$\lambda = \frac{2L}{n\gamma} \tag{73}$$

and the resonant frequency is obtained from the equation

$$\nu_x = \gamma \frac{nc}{2L} \tag{74}$$

When the orientation of the resonator is in the y-direction, or in the z-direction, then the measured length y_0 or z_0, respectively, of the internal cavity of the resonator is equal to L. Therefore, in this case the wavelength λ is equal to $2L/n$ and the corresponding resonant frequency is

$$\nu_{y,z} = \frac{nc}{2L} \tag{75}$$

Upon rotation of the resonator by 90˚, starting from the x-direction and ending in the y-direction or in the z-direction, the change in the resonant frequency is obtained from the relation

$$\Delta\nu = \nu_x - \nu_{y,z} = (\gamma - 1)\frac{nc}{2L} \simeq \frac{1}{2}\frac{u^2}{c^2}\frac{nc}{2L} \tag{76}$$

For a typical value of the velocity u equal to 0.4×10^{-3}, the ratio $\Delta\nu/\nu$ is

$$\frac{\Delta\nu}{\nu} \simeq \frac{1}{2}\frac{u^2}{c^2} \simeq 8.88 \times 10^{-13} \tag{77}$$

Therefore, for an approximation of the order of u^2/c^2, there is a low limit to the maximum value of the ratio $\Delta\nu/\nu$ during the rotation of the resonator, as a result of the rotational motion of the Earth.

6. Comparison between Theoretical Result and Experimental Data

The most basic principle of physics is that a theory corresponds to physical reality when its predictions are in agreement with experiment. This principle makes the comparison between theoretical results and experimental data necessary.

In this section we list an experimental result and the corresponding prediction of the general theory of relativity. This theoretical prediction results from the study of the phenomenon of optical resonance observed in a cavity of an optical resonator based on the foundation of the general theory of relativity. The theoretical result of this study, given by the relation (77), is larger than the corresponding experimental one by several orders of magnitude, as shown in **Table 1** in which these results are listed.

A number of experiments that check the stability of the speed of light, using optical resonators, have been carried out since 1955 up to the present day, and of course the accuracy of the experiments of the last decades cannot be doubted. This means that a radical revision of this physical theory generally accepted by the scientific community is required, at the level of basic principles.

Table 1. Theoretical and experimental result. The experimental work entitled "Rotating optical cavity experiment testing Lorentz invariance at the 10^{-17} level" was carried out by Herrmann, S.; Senger, A.; Möhle, K.; Nagel, M.; Kovalchuk, E. V.; Peters, A. in 2009 [29].

Prediction of general relativity	Experimental result
$\frac{\Delta\nu}{\nu}=8.88\times10^{-13}$	$\frac{\Delta\nu}{\nu}\le10^{-17}$

7. Discussion

In the latest optical resonator experiments the results are $\Delta\nu/\nu\le10^{-15}$ during the period 2003-2004 [19]-[22], $\Delta\nu/\nu\le10^{-16}$ during the period 2005-2007 [23]-[27], $\Delta\nu/\nu\le10^{-17}$ in the year 2009 [28] [29], and $\Delta\nu/\nu\le9.2\pm10.7\times10^{-19}$ (95% confidence interval) in the year 2015 [30]. All these experimental results are in complete disagreement with the result of general relativity given by the relation (77).

In the present theoretical study, an anisotropy of the measured fractional frequency shift with respect to the orientation of the resonator's optical cavity is highlighted, which is imposed by the foundation of the theory of relativity in the rotating reference frame of the Earth. This anisotropy is of the order of $(1/2)\left(u^2/c^2\right)$, considering u as the velocity of a laboratory based at the Earth's surface, due to the rotation of the Earth with angular velocity Ω. This order of magnitude is of course much larger than that measured based on the predictions of known models testing the anisotropy of the speed of light. This is understood based on the study of two such models which are presented in section 1, and are developed according to the search for Lorentz violation.

The question arises whether there are external factors that could affect the experimental result. The gravitational field of the Earth for example, from a relativistic point of view creates a curvature of space, but apart from the fact that this effect on an optical beam is weak, it cannot bring about any variation in the speed of light, due to spherical symmetry, since the axis of rotation of the optical resonators during the experiment is vertical, and the optical beams are perpendicular to this axis.

However according to the work in [34], p. 387, regarding the experiment in [29], the following is stated.

> "Substantial effort was spent on minimizing systematic effects associated with turntable rotation (see Fig. 5). Besides good thermal and electromagnetic shielding, prevention of cavity deformations due to external forces is most importantly involved here. Such forces are either of centrifugal or gravitational origin. If not supported in a perfectly symmetric manner the latter causes a deformation when tilted against the horizontal. The observed relative frequency change for our setup is $1.5\times10^{-16}/\mu\text{rad}$. Tilts which vary as a function of the orientation of the turntable enter the analysis of the experiment as a systematic error, and have to be suppressed by keeping the rotation axis vertical and preventing wobble in the setup."

Regarding these systematic effects and their treatment, in [29], p. 105011-2, the following is stated.

> "On the other hand, active rotation potentially causes a systematic modulation of the beat frequency and might thus mimic an anisotropy signal. For example, gravitational or centrifugal forces that act on the resonators may get modulated with the turntable rotation and therefore modulate the length of the resonators. However, most of these effects lead to a modulation at a rate of $\omega_{tt} = 2\pi/T_{tt}$ so that they are in principle distinguishable from the anisotropy signal searched for at $2\omega_{tt}$. Moreover, if the data spans more than one day, systematic effects with a fixed phase in the laboratory average out in the analysis for an anisotropy of c that is fixed relative to a sidereal frame. Although such an analysis helps to discriminate a sidereal anisotropy signal from systematics, a large effort was still made to reduce systematic effects both at $2\omega_{tt}$ and ω_{tt}.
>
> First of all, we use a high precision air bearing turntable specified for <1 μrad rotation axis wobble. Furthermore, we also prevent long-term variations of the rotation axis tilt, caused, for example, by daily fluctuations of the building tilt of several μrad. For this we apply an active stabilization [12] that keeps the rotation axis vertical to better than 1 μrad, which reduces the effect from a periodic deformation of the cavities to frequency variations of less than 0.1 Hz in amplitude. Effects from varying centrifugal forces are also reduced below an amplitude of 0.1 Hz by an active stabilization of the rotation rate. Further measures include balancing the center of mass of the table (<1 mm offset from the rotation axis) and shielding the lasers and optics outside the vacuum chamber against air currents and temperature gradients in the laboratory."

It is obvious that the experimental results derived from the use of optical resonators and concerning tests of the constancy of the speed of light, during the period 2003 to 2015, cannot be questioned. Therefore their massive discrepancy with the theoretical result of relation (77), clearly demonstrates the weakness of the foundation of the general theory of relativity with regard to the interpretation of the phenomenon which is observed in these experiments.

Data Availability

All data generated or analysed during this study are included in this published article.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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