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Estimate of Neutrino Flavor Mass Sum from the Electroweak and Higgs Sectors with Permutational Symmetry

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Abstract

The origin of the minuscule masses of the known neutrino flavors is an important open question in particle physics. The neutrino-family flavor mass sum is bound by several data-based analyses to less than about 0.12 eV/c^2. This sum is roughly 10 orders of magnitude smaller than the sum of the masses of the flavors of other fundamental fermion families. There are no explanations for the small value of this mass sum that are supported by observations. This paper provides an estimate of this sum using properties of the electroweak sector and the minimal Higgs sector with a derived permutational symmetry. The specific masses of the three generations of neutrino flavors can then be fit based on observations, but not fully determined, using properties of the homogeneous Higgs ghost Lagrangian.

Keywords

Neutrino Masses, Higgs Sector, Quantum Field Theory, Electroweak Sector

1. Introduction

In the nominal standard model of particle physics, neutrinos do not have mass [1] [2] at "tree" level because there are no right-handed neutrinos to couple with left-handed neutrinos in the mass terms. Even with the current phenomenological extension of the standard model that accommodates neutrino masses [1] [3], the nominal model assumes but makes no predictions that the masses should be non-zero. Debate continues as to whether neutrinos should obey the Dirac equation or instead the Majorana equation with experimental efforts underway to resolve the debate [1] [4].

As is well known, observations have emerged in the past 30 years in which neutrinos not only have mass but undergo mass oscillations [5]-[11]. The sum of masses of the neutrinos flavors is denoted here by $\sum_i m_{\nu i}$, where $m_{\nu i}$ is the mass of the i^{th} neutrino flavor, $i = 1$ to 3. This sum has been tightly bound from above in recent years by analyses based on cosmic microwave background (CMB) and gravitational lensing data to a very small value of 0.12 to 0.13 eV/c^2 [10]-[12]. The value of $\sum_i m_{\nu i}$ is constrained from below by mass oscillation measurements to be greater than about 0.06 eV/c^2 [1] [10]. Such a small mass sum is roughly 10 orders of magnitude smaller than the sum of the masses of the flavors of other fundamental fermion families. For example, the sum of the masses of the flavors of the electron family, $\sum_i m_{ei}$, is about 1883 MeV/c^2 [13]. There are a number of hypothetical theoretical explanations for such small neutrino masses [1] [14]-[20]. A leading explanation is the Type 1 see-saw mechanism [16]. This see-saw mechanism evidently requires that neutrinos be Majorana particles, which implies that neutrinos are their own antiparticles [1], and this implies violation of total lepton number. This violation of lepton number has not been observed experimentally.

Other recent approaches introduce a sterile vector-like neutrino [14] or use the well-known Froggatt-Nielsen mechanism [15] which requires a symmetry across families. The former adds a massive vector-like particle in each family and a broken SU(3) family symmetry. The latter introduces a $U(1)_{FN}$ horizontal symmetry across families that is broken by a vacuum expectation value (VEV) of a new scalar field ϕ whose $U(1)_{FN}$ charge is −1. In both cases there are: (a) Many free parameters that are tuned to fit the fermion mass spectra; (b) They do not utilize the minimal Higgs mechanism as the sole source of mass; (c) They do not provide an independent explanation for the observed 3 generations and 4 families. In summary, these explanations do not yet provide compelling explanations for fundamental fermion masses, and in particular those of the neutrinos.

There have also been several systematic analyses of interactions which can give rise to mass for Dirac neutrinos, e.g., [17]-[19]. References [17] and [18] show methods to generate the renormalizable Dirac neutrino mass operators at tree and one-loop level. Reference [19] shows methods to generate neutrino masses at the one-loop level at the price of tree-level flavor-changing neutral-current coupling. Reference [20] shows more general diagrams with dimension-six operators. These approaches typically require an additional symmetry to eliminate Majorana terms. The Z_2 and Z_3 symmetries are explicitly mentioned. These approaches also often require one or more additional fields. The approach of this paper has similarities with these approaches, since the underlying minimal Higgs sector has the ghost fields and gauge functions which serve as additional fermionic scalar fields [21]-[23], and the theories utilized in this paper exhibit a direct product of the Z_2 and Z_3 symmetries [21] ([24] Sec 2.5). The related diagram for neutrinos in this paper is given by the bottom part of **Figure A1(d)** of **Appendix**. **Figure A1(d)** bears a resemblance to Figure 5 in [17], Figure 2 in [18], and Table II, E1-1 in [20]. The neutrino legs ν_R and ν_L are truncated out of **Figure A1(d)** but can be associated with the counterpropagating ghost and gauge function loops. The Dirac fermion lines of [17] or [18] are replaced by a charged lepton Dirac fermion line and a charged W boson line, a dimension-four electroweak interaction. The scalar interactions are all dimension-three terms in the standard-model ghost Lagrangian.

This paper provides an explanation for the neutrino mass sum in the context of two related models. The first is the ghost Lagrangian density of the minimal Higgs sector, as given by [21]. The conventional wisdom is that ghosts are artifacts which are mathematical tools in non-Abelian gauge theories. But when the ghost Lagrangian of the electroweak theory is properly included, a different interpretation emerges in which the ghosts are persistent, oscillatory physical constituents [22]. The homogeneous version of this ghost Lagrangian involves only the Higgs, ghost, and gauge phase fields. It was recently shown that solutions of this homogeneous version for the ghost fields yield four persistent, oscillatory states, two of which are charged and two of which are uncharged [22]. In this reference, it is shown that one of the uncharged states and its gauge function couple to the Higgs and so have mass within the context of the standard model. This uncharged state is linked to neutrinos. Moreover, these new solutions show that the nature of the coupling results in precisely three generations of masses for each of the four families of the fundamental fermions [23]. This result also fits the corresponding individual flavor masses. These solutions for the ghost and Higgs fields exhibit permutational symmetry of particles propagating in loops. This result was obtained using classical solutions of the equations. Similar results are obtained using a second related model, a quantum field theory that is anomaly-free and which also has permutational symmetry [24]. In this second model, the masses of the fermion families, including the sum of the masses of family flavors, can be obtained as described in the following paragraph. Both models provide a partial explanation of the masses of the fundamental fermions, explaining both three generations of masses as well as the patterns of the mass spectrums, with two lighter masses and one heavier mass (but for the neutrino family, it makes no explicit prediction). Moreover, if the constituent particles are fermionic, as the ghosts of the Higgs sector are, then there is also an explanation for the four (and only four) families of particles as well, since two identical fermionic quanta cannot occupy the same potential well.

In the Higgs-sector-based model, it was shown that the scalar fermionic classical ghost fields are both the occupants and generators of potential wells in a loop configuration. Three such potential wells in a loop are required for self-consistency (other higher-integer frequencies around such loops are not forbidden but are not stable). On the other hand, this number of potential wells (three) in a loop is an assumption of the quantum field theory of Reference 24. In this second model, the scalar fermionic underlying particles are implicit. If in addition it is assumed that these underlying particles do indeed exist, one can fit the sum of the flavor masses of all four families of fundamental fermions as described in ([24], Ch. 11). This fit involves assigning binding energies to the various "preon" pairs. This fit is then extended to the electroweak bosons, using one justifiable free parameter of order unity, which is the electric repulsion between like-charged particles in different configurations. These fits are based on measurements alone but nonetheless have predictive value because of this last relation between measured electroweak boson masses and measured sums of the flavor masses of the fermion families. The fit includes the neutrino-family flavor mass sum $\sum_i m_{\nu i}$, but the fit is not sensitive to this mass sum, so it is not well-determined. The flavor mass sums of the other three families are better determined, as shown in the reference. As noted above, the mass spectrums within fermion families are partially explained as solutions to a cubic equation. Hence $\sum_i m_{\nu i}$ is the only mass parameter that lacks a credible partial explanation.

This paper provides an independent qualitative explanation for the sum of neutrino-family flavor mass sum $\sum_i m_{\nu i}$, in the context of the minimal Higgs sector, using parameters that are present in that sector's ghost Lagrangian, as well as the electron-family flavor mass sum. As described above, the latter has already been separately derived using the referenced approaches. The approach for the neutrino-family flavor mass sum will be described in Section 2 and the results will be summarized in Section 3.

2. Approach

The overall approach pursued here is similar to that used in ([24], Ch.11), in which the binding energies of the underlying particles ("preons", or now more specifically, Faddeev-Popov ghosts) are related to electroweak parameters. Leading-order diagrams are shown in **Appendix**. Because neutrinos are believed to interact primarily via electroweak forces (also only very weakly to gravity), one may posit that the binding energy is determined by such electroweak forces.

It is well known that the binding energy of electrons to the nucleus in atoms is proportional to the fourth power of the electromagnetic coupling constant, e, in accord with solutions from Schrodinger's equation ([25], Ch. 5). A further dynamical justification can also be seen in particle physics in the binding energies for positronium, and a similar fourth-power dependence is seen in Mott and Rutherford cross sections [26] [27]. The underlying diagrams are of the same form for both the electromagnetic and electroweak interactions for electrons and neutrinos. This includes the topologies of the Higgs binding for neutrinos and electrons, as shown in **Appendix**. These observations indicate that the ratio of cross-sections is a reasonable proxy for the ratio of binding energies between the two cases.

Here, the preons have repulsive electrical interactions, as shown in diagram A1(c) of **Appendix**. In any given charged particle, the preons will have the same charge, else they will annihilate, so the electrical interaction must be repulsive. From the diagram, the interaction should scale like that of elastic QED cross sections (e.g., [26], Chs. 4, 6),

$$\sigma_{QED} \propto \left[\left(g_{QED}/3 \right)^2 \hbar c / \left(4\pi E_e \right) \right]^2, \quad (1)$$

where g_{QED} is the dimensionless quantum electrodynamic (QED) coupling constant in quantum electrodynamics in the low-energy limit. $\hbar$ is Planck's constant, c is the speed of light, and E_e is the center-of-momentum energy of the charged ghosts. Note that Equation (1) exhibits the fourth power of $g_{QED} \propto e$ as expected. There is an extra factor of 1/3 because the charged constituent states, $\eta^{\pm}$, are assigned a charge of ±*e*/3. The appropriate energies for E_e are the mass-energies of the electron family members, so that an incoherent sum of such cross sections give

$$\sigma_{QED} \propto \left[\left(g_{QED}/3\right)^2 \hbar c/(4\pi)\right]^2 \left[1/\left(m_e c^2\right)^2 + 1/\left(m_\mu c^2\right)^2 + 1/\left(m_\tau c^2\right)^2\right], \tag{2}$$

where m_e is the mass of the electron, m_μ is the mass of the muon, and m_τ is the mass of the tauon.

Using the same approach for the binding of the underlying particles for neutrinos, diagram A1(d) gives (e.g., [27], Ch. 9),

$$\sigma_{EW} \propto G_{F0}^2 s \propto (1/32)\left[\left(g_W/M_W c^2\right)^2 \hbar c E_\nu\right]^2, \tag{3}$$

where G_{F0} is the reduced Fermi coupling constant and s is the usual square of the center-of-mass energy of the interacting particles. Here g_W is the dimensionless electroweak coupling constant (in the appropriate low-energy running limit), M_W is the mass of the *W*-boson, and E_ν is the center-of-momentum energy of the weakly-interacting charged leptons (by conservation of charge). The cross sections should be computed using an incoherent average of the cross sections corresponding to the energies of each of the three electron-family flavor rest masses, based on the diagram. The average is used since only one such interaction is present at any given time to lowest order. In this case, one obtains

$$\sigma_{EW} \propto 1/(32\times 3)\left[g_W/\left(M_W c^2\right)\right]^4 (\hbar c)^2 \left[m_e^2 + m_\mu^2 + m_\tau^2\right] c^4, \tag{4}$$

These electron-family particles should be viewed as virtual particles created in electroweak interactions of the underlying uncharged constituent particles, as shown in the diagram. Equation (4) only involves a simple incoherent average of the squares of the lepton masses. The squares of the mass energies are obviously used in accordance with well-known Equation (3). The use of an average is explained immediately above and corresponds to an approximately equal phase-space weighting (probability 1/3) that should be applied to each of these masses. This is because the energy, *E*, is here much larger than the lepton masses considering the very short range of the interaction. This weighting is used in many electroweak calculations. See, for example, Section 9.1 of [27] (with all of $m_{e,\mu,\tau}/E$ much less than one in this case).

One can form the ratio $R_{\nu e}$ of these cross-sections of the neutrino family members and electron family members as

$$R_{\nu e} = \sigma_{EW}/\sigma_{QED} \cong \left(27\pi^2/2\right)\left[\left(g_W/g_{QED}\right)\left(m_e m_\tau\right)^{1/2}/M_W\right]^4. \tag{5}$$

It remains to show that one may apply this ratio to the sum of the masses of the flavors of the electron family, 1883 MeV/c^2, to obtain a rough estimate of $\sum_i m_{\nu i}$. This is outlined in the following paragraph. Note that three of the five parameters in Equation (5) can be found in both the electroweak and Higgs sectors. The remaining two parameters, m_e and m_τ, can be "fit with explanation" from Higgs sector parameters, as explained in [23] [24].

As discussed in ([24], Chs. 2, 11), the family masses for the electron family, Equation (11.1) of that reference, can be written in terms of binding energies of the constituent particles:

$$3E_{c-c} - 2E_{c,rep} = \sum_i m_{ei}/3, \tag{6}$$

where E_{c-c} is the net Higgs bonding energy between 2 charged preons and $2E_{c\,rep}$ is the repulsive potential electromagnetic energy between one preon and two charged preons. Inspection of the constituent content of neutrinos yields a similar equation,

$$3E_{o-o} - 2E_{o,rep} = \sum_i m_{\nu i}/3, \tag{7}$$

where the subscript "o" refers to uncharged preons. Next, assume that the repulsive energy between preons in the two cases are related by

$$E_{o,rep}/E_{c,rep} = \beta\sigma_{EW}/\sigma_{QED} = \beta R_{\nu e} \tag{8}$$

where β is an additional factor of proportionality. Next, detailed inspection of the Higgs ghost Lagrangian indicates that the coupling between charged ghosts to photons is linear in $g_{QED}A_\mu$, where A_μ is the well-known electromagnetic 4-vector field. Further, the coupling between charged and uncharged ghosts is linear in $2\sin\theta_w g_W W_\mu^\pm$ [21] [22], where θ_w is the weak mixing (Weinberg) angle. Hence one should expect that there is an additional factor of $(2\sin\theta_w)^4$ that should be included for σ_{EW} based on the diagram of **Appendix**. One can include this correction factor as follows:

$$E_{o,rep}/E_{c,rep} = \beta'(2\sin\theta_w)^4\sigma_{EW}/\sigma_{QED} = \beta' R'_{\nu e}, \tag{9}$$

i.e., the correct ratio is $R'_{\nu e} = (2\sin\theta_w)^4 R_{\nu e}$. One expects that this ratio should also apply to the ratio E_{o-o}/E_{c-c} of Higgs bonding of these combinations of ghosts, because the bonding is driven by these forcing terms based on **Appendix**. Thus, this ratio should be proportional to $R'_{\nu e}$ as well. It then follows from Equations (6) to (9) that

$$\begin{aligned}\sum_i m_{\nu i} &= R'_{\nu e}\left(3\alpha' E_{c-c} - 2\beta' E_{c,rep}\right)\\ &= R'_{\nu e}\left[\sum_i m_{ei} + 3(\alpha'-1)E_{c-c} - 2(\beta'-1)E_{c,rep}\right],\end{aligned} \tag{10}$$

where α' is an additional factor of proportionality. The expectation based on the above arguments is that α' and β' should be approximately equal to 1, so that the last two terms of Equation (10) are negligible or at least small compared to the first term. Further validation of this expectation can be pursued using the formalism of [22], but such a significant analysis is beyond the scope of this paper.

3. Results and Summary

Current values of g_W, g_{QED}, and $M_W c^2$ are 0.6298, 0.3029, [28] and 80.4 GeV [29], respectively. Similarly, one can use the latest electron-family masses. The latest value of $\sin^2\theta_w$ at low energies is given in Table 10.2 of [30] and is equal to 0.23873. Then, estimating the ratio $R_{\nu e}$ as described above, one finds

$$R'_{\nu e} \cong 4.47\times 10^{-11}\text{, and} \tag{11a}$$

$$\sum_i m_{\nu i} \approx R'_{\nu e} \sum_i m_{ei} = 8.44\times 10^{-8}\ \text{MeV}/c^2 = 0.0844\ \text{eV}/c^2\ . \tag{11b}$$

This is within the currently accepted range for $\sum_i m_{\nu i}$, which is between 0.06 and 0.13 eV/c^2, as discussed in the Introduction. An additional factor of order unity will undoubtedly be present in a more detailed calculation, which should include radiative corrections.

The currently-accepted upper limit of about 0.13 eV/c^2 for $\sum_i m_{\nu i}$ is based on model-dependent cosmological bounds. On the other hand, the current experiments using direct mass measurements limit the neutrino flavor mass sum to about 0.8 eV, based on the neutrino mass in tritium decay [1] [31]. There are a number of current and upcoming high-precision cosmological surveys that could potentially improve the cosmological result, such as DESI and LSST [32] [33]. It is also expected that the DUNE and Hyper-Kamiokande experiments will be able to provide bounds on absolute neutrino masses using supernova neutrinos [34].

One might ask whether the approach resolves the issue of the normal versus inverted hierarchy for neutrino masses. Based on Reference 23, there is no mathematical justification at this time for either the normal or inverted mass hierarchy using this approach. It is possible that future calculations based on the ghost Lagrangian might be able to predict the type of hierarchy. That said, the numerical value of 0.084 eV/c^2 for $\sum_i m_{\nu i}$ is inconsistent with the inverted hierarchy, which requires a mass sum of at least 0.1 eV/c^2 based on neutrino oscillations [34].

As mentioned in the Introduction, the model evidently constrains the number of neutrino generations to three, thereby ruling out *some* possible sterile neutrino states. However, it does not rule out other possible exotic sterile neutrino states, e.g., those with a feeble version of the color force. See [35], for example, for a discussion of this possibility as an explanation for dark matter.

The extremely short range of the electroweak force suggests that the particle is extremely small, and this is in fact what is found in this model. See the "Effective radius" column of Table 3 in Reference 22. This is reconciled with the light mass of the bound state in Sections 3.3 and 5 of the same reference. The light mass is the result of a balance between Higgs bonding and electroweak repulsion. The same can be said for the electron. More work could be done on the issue of particle "size".

Overall, this estimate for the sum of neutrino flavor masses completes a program in which all 12 masses of the fundamental fermions can be estimated or "fit with explanation" from the Higgs portion of the electroweak sector [22] [23]. From these fits, the 8 parameters of the CKM and PMNS matrices are also then determined with good accuracy [36]. This is because the CKM and PMNS matrix parameters in this context derive from the eigenstates of ([24], Ch. 2) and from the ratios of radii and spacings of the potential wells given in Table 3 of [22]. The latter in turn derives from the family-dependent Higgs potential modifications given in [23]. Hence this program provides at least partial explanations for 20 of the 26 input parameters of the standard model ([37], Ch. 18). These numerical values can be derived from the three measured electroweak boson masses, along with one free fit parameter of order unity, as found in ([24], Ch. 11). This program also provides an explanation for the overall structure of the fundamental fermions, predicting 3 (and only 3) generations of fermions in each of 4 (and only 4) families, using four fundamental fermionic scalar ghosts as the constituent particles. This is now *all* done in the framework of the electroweak sector and the minimal Higgs sector, using a recently-derived permutational symmetry for the latter [22] [23].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Some Leading-Order Diagrams Relevant to Ghost Fields and Gauge Functions and the Associated Ghost Lagrangian Terms

This **Appendix** shows some leading order Feynman diagrams that are relevant for fermion solutions of the ghost Higgs Lagrangian density [21]-[23] including those that can affect the binding energy of such solutions. Figure A1 shows some leading-order terms involving electroweak interactions.

In these figures, "h" denotes the Higgs boson (charged for the electron, uncharged for the neutrino), η^- denotes a Faddeev-Popov ghost field with charge $-e/3$, η_Z denotes a neutral Faddeev-Popov ghost field, and ω_Z denotes a neutral gauge function (antighost). γ denotes a photon, W^+ denotes the positively-charged electroweak boson, and ℓ^- denotes any negatively-charged electron family member (electron, muon, or tau). Note that the unit charge $\pm e$ of the $W^\pm$-boson along with conservation of charge implies that the corresponding lepton must be oppositely charged. Note that these schematic diagrams are in x-y coordinates, not x-t.

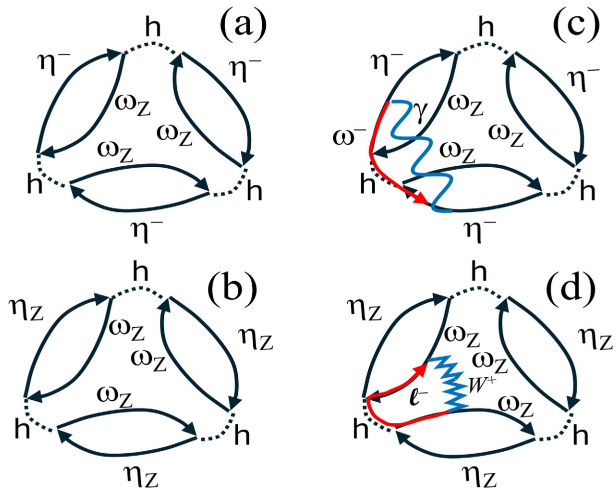

Figure A1. Some leading-order diagrams that contribute to the mass of the leptons in the context of solutions of the ghost Higgs Lagrangian with $Z_3 \otimes Z_2$ permutational symmetry. (a) A bare electron; (b) A bare neutrino; (c) QED interaction between two charged ghosts in an electron; (d) Charged-current interaction with a virtual charged lepton in a neutrino.

The associated ghost Lagrangian terms for Figure A1(a) and Figure A1(b) are given in References [22] and [23], based on Reference [21]. For Figure A1(c) and Figure A1(d), one must add the "inhomogeneous" terms that couple the ghosts and gauge functions to the electroweak bosons. These terms also come from [21] and are given by

$$\begin{aligned}&\left[g_W/(\hbar c)\right]\boldsymbol{\eta}\cdot\partial_\mu\left(\boldsymbol{\omega}\wedge\boldsymbol{W}^\mu\right)\\&=i\left[g_W/(\hbar c)\right]\left[\eta^+\partial_\mu\left(\omega^-W_3^\mu\right)-\eta^-\partial_\mu\left(\omega^+W_3^\mu\right)+\eta^-\partial_\mu\left(\omega_3W^{\mu+}\right)\right.\\&\quad\left.-\eta^+\partial_\mu\left(\omega_3W^{\mu-}\right)+\eta_3\partial_\mu\left(\omega^+W^{\mu-}\right)-\eta_3\partial_\mu\left(\omega^-W^{\mu+}\right)\right].\end{aligned} \tag{A1}$$

Here, $g_W = e/\sin\theta_w$, $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3)$, and similarly for bold-type $\boldsymbol{\omega}$ and $\boldsymbol{W}^\mu$. and $\eta^\pm = (\eta_1 \pm i\eta_2)/\sqrt{2}$, and similarly for $\omega^\pm$ and $W^\pm$. The index μ runs over space-time coordinates in the usual way. Finally, the variables with subscript 3 can be written in terms of the more physical fields as follows, based on [21] and standard textbooks:

$$g_W W_3^\mu = e\left[A^\mu + (g_W/g')Z^\mu\right], \quad \text{(A2a)}$$

$$\eta_3 = -\sin\theta_w \eta_Z + \cos\theta_w \eta_\phi, \text{ and} \quad \text{(A2b)}$$

$$\omega_3 = -\sin\theta_w \omega_Z + \cos\theta_w \omega_\phi. \quad \text{(A2c)}$$

Here $g' = e/\cos\theta_w$, A^μ is the electromagnetic 4-vector potential, and Z^μ is the neutral electroweak boson 4-vector. Also η_Z is the neutral ghost corresponding to the Z-boson, η_ϕ is the neutral ghost corresponding to the photon, and similarly for the gauge functions ω_Z and ω_ϕ. One may read off the electroweak interactions in diagrams (A1c) and (A1d) from Equations (A1) and (A2).

Note for the last two terms of Equation (A1) that $\eta_3\partial_\mu\left(\omega^+ W^{\mu-}\right) - \eta_3\partial_\mu\left(\omega^- W^{\mu+}\right) = -2i\,\mathrm{Im}\left[\eta_3\partial_\mu\left(\omega^- W^{\mu+}\right)\right]$. This also applies to the third and fourth terms on the right-hand side. This and the above definition of η_3 in terms of η_Z and η_ϕ provides the extra factor of $2\sin\theta_w$ discussed in connection with Equation (9). However, this argument should not apply to the first and second terms on the right-hand side of Equation (A1) because only η^- appears in the interactions in the electron, not η^+.

The astute reader of the above will note that Equation (A1) and the fractionally-charged ghosts and gauge functions imply that the charged W-boson states are also fractionally charged, *as they apply to charged ghost interactions*. This is not necessarily inconsistent with the conventional unit charges of the W bosons when they interact with fermions. It seems that the corresponding two types of diagrams are topologically distinct in the context of this approach involving constituent particles. Further work should be done to address this issue, as well as the precise diagrams to be used.