

The Cosmic Origin of High Energy Neutrinos, Ultra High Energy Neutrinos and the Mass of the Muon Neutrino

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Abstract

We show how, via gravitational repulsion associated with the Schwarzschild solution to the field equations of General Relativity, the extragalactic neutrinos, IceCube-170922A and KM3-230213A, were accelerated to high energy and ultra high energy respectively. Our theory supports the presumption that a blazar is the source of IceCube-170922A. However, it does not support the presumption that a blazar is the source of KM3-230213A. On the contrary, it leads to the conclusion that the source possesses stellar mass. The muon neutrino mass has not been experimentally determined. Our theory predicts that it is in the range 0.030 to 0.092 eV, if the blazar TXS 0506+056 is the source of IceCube-170922A. If, however, the source is PKS 0502+049, then the mass of muon neutrino is: $m = 0.09136$ eV. We also pinpoint the fallacy in the arguments of the critics of the concept of gravitational repulsion in the Schwarzschild field.

Keywords

General Relativity, Schwarzschild Field, Gravitational Repulsion, Muon Neutrino

1. Introduction

There are two fundamental questions of astroparticle physics that have yet to be answered. 1) What are the sources of high energy cosmic rays [1]-[6]; 2) How are high energy cosmic rays accelerated to such enormous energies. These questions are particularly acute for ultra high energy cosmic rays, >PeV [7]-[12]. Here, we answer these two questions for the neutrino events IceCube-170922A and KM3-

230213A, whereby we narrow the range of the mass of the muon neutrino.

In the second decade of the twentieth century, Droste [13]-[15] and independently Hilbert [16] [17] discovered that particles can experience gravitational repulsion in the Schwarzschild field. See [18] for the details of the history of this discovery. After decades of debate, in 1982, we confirmed the reality of this phenomenon by clarifying the difference between local and distant observers [19]. In 2017 we showed how gravitational repulsion is responsible for the acceleration of cosmic rays to the highest energy observed and we predicted that neutrinos would also be accelerated to ultra high energy [20]. In February 2025 this prediction was verified by the detection of KM3-230213A, an extragalactic ultra high energy neutrino with 2.2×10^{20} eV [21].

Here we show in detail how KM3-230213A was accelerated to such energy via gravitational repulsion in the Schwarzschild field and how IceCube-170922A was accelerated to high energy via the same process. We also determine the sources of these two neutrino events. Finally, we pinpoint the fallacy in the arguments of the critics of the theory of gravitational repulsion.

In the next section, we review the basic theory of the acceleration of particles to high energy, 1 TeV to 1 PeV, and ultra high energy, >PeV, via gravitational repulsion in the Schwarzschild field [20] and add new aspects to the theory. In Section 3, we review the case that high energy cosmic rays are accelerated in blazars. In Section 4, we specifically discuss how a blazar accelerated the IceCube-170922A neutrino to high energy. The implications of our theory on the determination of the mass of the muon neutrino are discussed in Section 5. In section 6, we apply the theory to the acceleration of the ultra high energy neutrino, KM3-230213A. We discuss the origin of ultra high energy neutrinos in Section 7. In section 8, we refute the arguments of those, who oppose the theory of gravitational repulsion in the Schwarzschild field and we also highlight work that embraces the theory and enriches it. Our conclusion is in Section 9.

2. Acceleration of Particles via Gravitational Repulsion

The basic assumption of our theory is that the gravitational field of the source of high energy cosmic rays is described by the Schwarzschild solution to the field equations [16] [22] of General Relativity. It was discovered by [13] and independently by [23]:

$$ds^2 = \frac{dr^2}{1 - \frac{\alpha}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - \left(1 - \frac{\alpha}{r}\right) dt^2 \quad (1)$$

ds is the line element, t Schwarzschild time coordinate, r Schwarzschild radial coordinate, θ colatitude, ϕ longitude and α is the Schwarzschild radius:

$$\alpha = 2GM \quad (2)$$

where M is the mass of the gravitating body, G is the gravitational constant and the speed of light, $c = 1$.

It is important to note that when $r \rightarrow \infty$ Equation (1) becomes the Minkowski metric of special relativity. We also assume that the sources of high energy cosmic rays are so distant that in the terrestrial environment Equation (1) is the Minkowski metric. Thus, terrestrial observers are not local observers rather distant observers as elaborated in [19].

We also assume that a particle is moving radially outward from the emitting source and that its Schwarzschild velocity obeys the inequality:

$$\frac{dr}{dt} > \frac{1}{\sqrt{3}} \left(1 - \frac{\alpha}{r} \right) \quad (3)$$

This assumption is the prerequisite for a particle to experience gravitational repulsion [14] [15] [17]-[19] [24] [25]. The speed of a neutrino is always very close to the speed of light. So a radially moving neutrino only experiences gravitational repulsion never gravitational attraction.

The Schwarzschild acceleration for a radially outward moving particle is [19]:

$$\frac{d^2r}{dt^2} = g \left[\frac{3}{1 - \frac{\alpha}{r}} \left(\frac{dr}{dt} \right)^2 - \left(1 - \frac{\alpha}{r} \right) \right] \quad (4)$$

where g is:

$$g = \frac{GM}{r^2} \quad (5)$$

In Newtonian gravity the gravitational acceleration is always less than 0, that is $\frac{d^2r}{dt^2} < 0$, which means the gravitational force is attractive and bodies fall. However, in the Schwarzschild field insertion of Inequality 3 into Equation (4) leads to:

$$\frac{d^2r}{dt^2} > 0 \quad (6)$$

Thus, any radially outward moving particle that obeys Inequality 3, will experience gravitational repulsion even out to $r \rightarrow \infty$ [13] [14] [18] [19].

The Schwarzschild acceleration, Equation (4), depends on the Schwarzschild velocity, which is [13]-[15] [17] [18] [24] [25]:

$$\left(\frac{dr}{dt} \right)^2 = \left(1 - \frac{\alpha}{r} \right)^2 + A \left(1 - \frac{\alpha}{r} \right)^3 \quad (7)$$

where A is a constant. For $r \rightarrow \infty$ Equation (7) reduces to:

$$v^2 = \left(\frac{dr}{dt} \right)^2 = 1 + A \quad (8)$$

where v is the velocity according to Special Relativity.

If $v=0$ then we have the case of free fall motion from infinity. Equation (8) then yields $A = -1$. This case has been discussed by [25].

v is related to the Lorentz factor, γ , via:

$$\gamma = \frac{1}{\sqrt{1 - v^2}} \quad (9)$$

Inserting Equation (8) into Equation (9) and solving for A leads to:

$$A = -\frac{1}{\gamma^2} \tag{10}$$

whereby γ is related to the particle energy, E , via:

$$E = (\gamma - 1)m \tag{11}$$

where m is the mass of the particle.

Solving Equation (11) for γ yields:

$$\gamma = 1 + \frac{E}{m} \tag{12}$$

Thus, in general the value of A is obtained from the measured energy of the particle. For example, if we have measured the energy of a cosmic ray proton to be 10^{20} eV, then the above equation yields: $\gamma = 1.06579 \times 10^{11}$. Inserting this value into Equation (10) yields: $A = -8.80354 \times 10^{-23}$. We see the value of A is exceedingly small. Consequently, the second term in the Schwarzschild velocity, Equation (7), can be dropped:

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{\alpha}{r}\right)^2 \tag{13}$$

Figure 1 is a plot of the Schwarzschild radial velocity. Equation (13) and **Figure 1** show an important circumstance. No matter how large the value of r is, the Schwarzschild velocity does not reach the speed of light. We also see the validity of this statement in Equation (8). Since $A < 0$, v never reaches $v = 1$, the speed of light. This circumstance is important in refuting the arguments of the skeptics of our theory, which we present in Section (8).

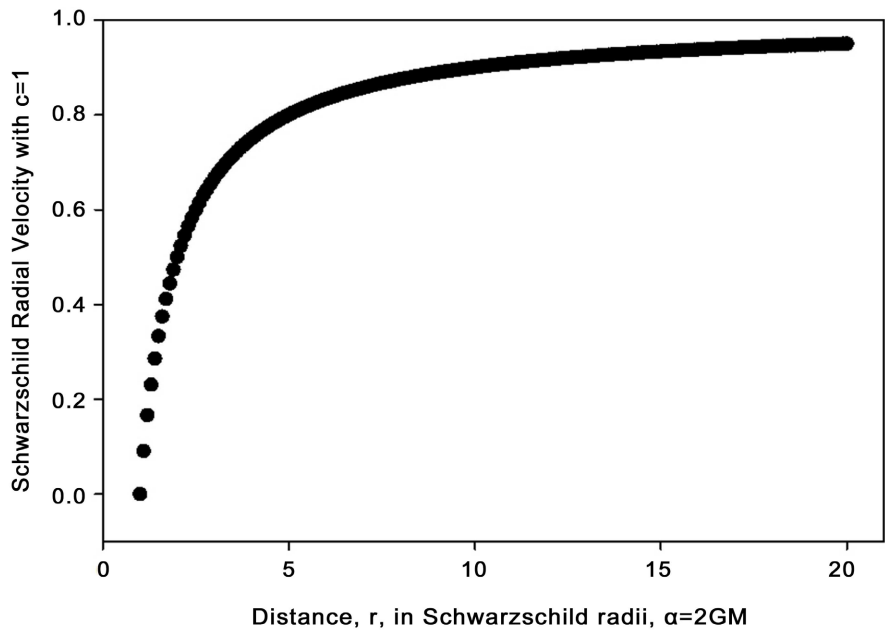


Figure 1. Schwarzschild radial velocity.

Inserting the above equation into the expression for the Schwarzschild acceleration, Equation (4), yields:

$$\frac{d^2r}{dt^2} = 2g \left(1 - \frac{\alpha}{r} \right) \quad (14)$$

Figure 2 is a plot of the Schwarzschild radial acceleration. This equation and **Figure 2** show that as $r \rightarrow \infty$ the Schwarzschild acceleration, $\frac{d^2r}{dt^2} \rightarrow 2g$. This circumstance was discovered by [13]-[15] and it agrees with our earlier work [19]. Thus, according to our theory particles continue to experience gravitational repulsion even as $r \rightarrow \infty$.

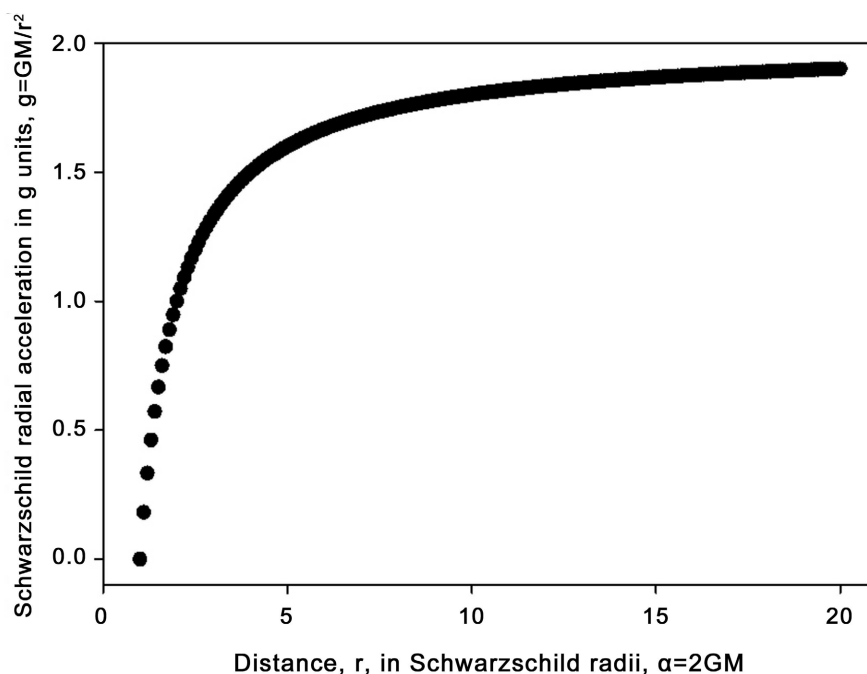


Figure 2. Schwarzschild radial acceleration.

Next we calculate the distance, n , of the source of the cosmic ray particle in units of the Schwarzschild radius, α , from the observed particle energy, E . To accomplish this task, we employ Equation (11). It requires γ , which is given by Equation (9). This equation requires v^2 , which in turn comes from Equation (8). Finally, $\left(\frac{dr}{dt}\right)^2$ in Equation (8) we obtain from Equation (13). So we arrive at the following sequence of equations:

$$v^2 = \left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{\alpha}{r}\right)^2 \quad (15)$$

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-\left(1-\frac{\alpha}{r}\right)^2}} \quad (16)$$

We set:

$$r = n\alpha \tag{17}$$

whereby n is the radial distance of the source in Schwarzschild radii. Inserting Equation (16) into Equation (11) and employing Equation (17) yields:

$$E = (\gamma - 1)m = \left(\frac{1}{\sqrt{1 - \left(1 - \frac{1}{n}\right)^2}} - 1 \right) m \tag{18}$$

This equation tells us: the greater the distance, n , the higher the particle energy, which according to Equation (14) comes about because the particle is constantly being accelerated.

Since we know the value of E , the measured energy of the cosmic ray particle, we can solve the above equation for the distance, n , of the source of the cosmic ray:

$$n = \frac{m^2 + 2mE + E^2 \pm \sqrt{2m^3E + 5m^2E^2 + 4mE^3 + E^4}}{m^2} \tag{19}$$

For instance in our example of a proton of $E = 10^{20}$ eV, the above equation with $m = 938.27208943 \times 10^6$ eV and $+\sqrt{2m^3E + 5m^2E^2 + 4mE^3 + E^4}$ leads to: $n = 2.27181 \times 10^{22}$, which is the distance of the source in Schwarzschild radii. **Figure 3** shows how the proton energy increases as a function of the distance from the source.

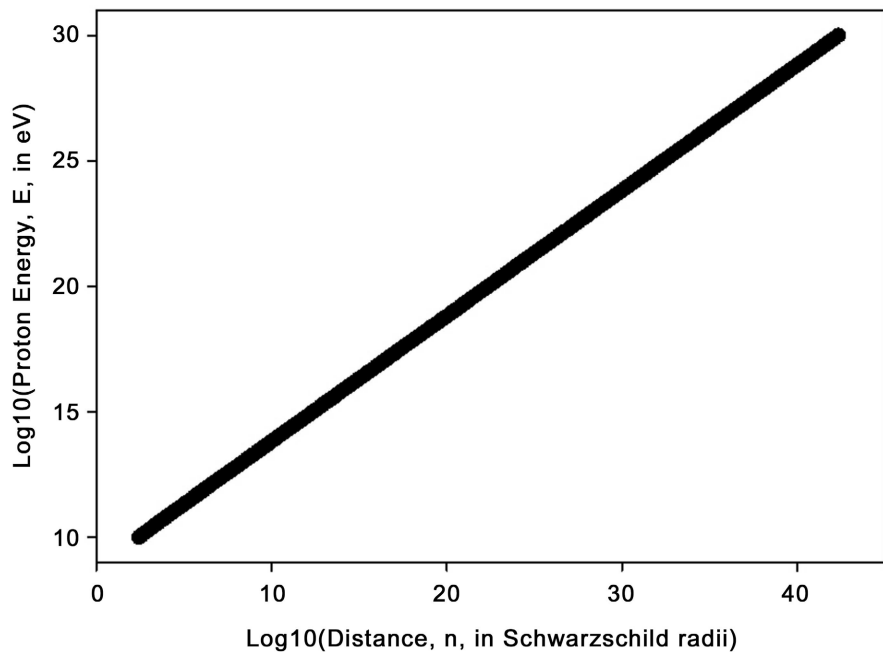


Figure 3. Proton energy vs. distance.

The next step is to convert n into the normal astronomical unit of distance

(parsec). To achieve this we must determine the value of α , the Schwarzschild radius, which we obtain via its redshift.

The redshift, z , leads to the distance, d , of the source:

$$r = n\alpha = d(z) \quad (20)$$

From this equation we can derive the value of α :

$$\alpha = \frac{r}{n} = \frac{d(z)}{n} \quad (21)$$

We obtain the distance of the source from the [26] data set, which consists of 1829 Type Ia supernova. Their table contains the observed relationship between spectral shift, z , and distance modulus, μ . We employ:

$$d = 10^{\left(\frac{\mu}{5} + 1\right) - 9} \quad (22)$$

to convert μ to d , the distance of a supernova in Gpc.

Figure 4 is a plot of distance vs. redshift. The red line in the figure is the curve fitted equation:

$$d(z) = a + bz^c \quad (23)$$

with $a = 0.059720085$, $b = 6.479918172$ and $c = 1.191043989$. The standard error of the fit is: 0.36 Gpc.

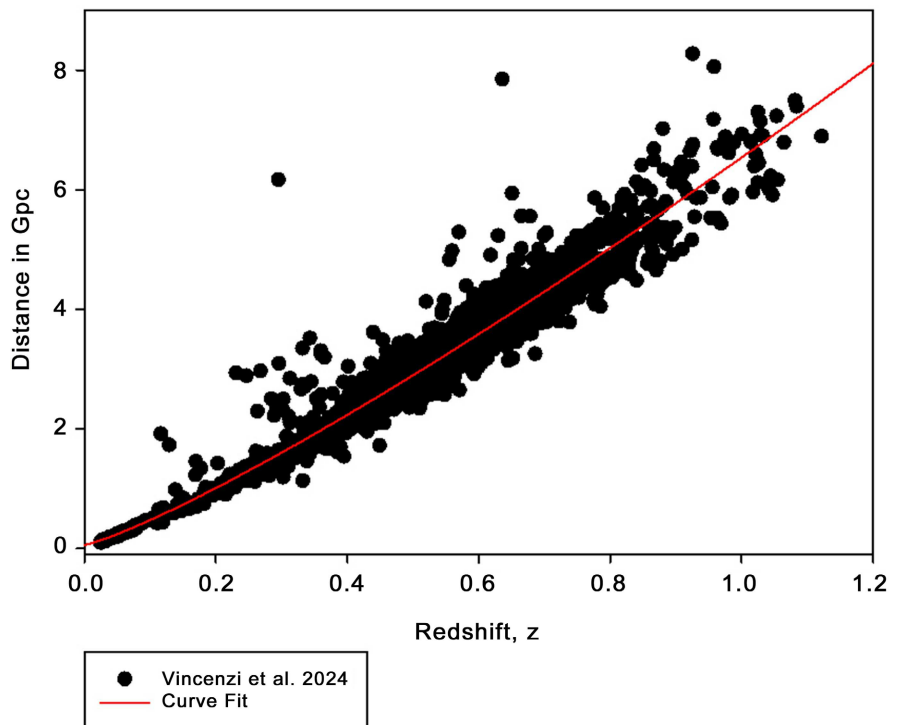


Figure 4. Distance vs. redshift.

Next we employ Equation (2) to obtain the mass of the source, whereby we can no longer let $c = 1$. So the Schwarzschild radius, Equation (2), is now:

$$\alpha = \frac{2GM}{c^2} \quad (24)$$

which leads to the mass, M , of the source of the cosmic ray particle via the above equation and Equation (21):

$$M = \frac{\alpha c^2}{2G} = \frac{c^2}{2G} \frac{d(z)}{n} \quad (25)$$

If the cosmic ray source is a distant source our theory allows us to determine the following:

- 1) From the redshift of the source, particle mass and energy we can calculate not only the distance of the source but also its mass.
- 2) From the mass of the source, the particle mass and energy, we can calculate the distance and redshift of the source.
- 3) From the mass and redshift of the source, we can predict the cosmic ray particle energy measured in the terrestrial environment, if we know the particle mass.
- 4) From the mass and redshift of a neutrino source and the neutrino energy, we can calculate the mass of the neutrino.

3. Blazars as a source of Cosmic Ray Neutrinos

A blazar is an extragalactic object that possesses an active galactic nucleus and a relativistic jet that is pointed at or close to the direction of Earth. The nucleus contains a supermassive black hole in the mass range of:

$$10^7 M_{\odot} \lesssim M_{\text{SMBH}} \lesssim 10^{10} M_{\odot} \quad [27]-[32].$$

Blazars are known to produce high energy gamma rays [33]-[37] and therefore it is expected that they also produce high energy particles, that is cosmic rays, which in turn through interactions produce neutrinos [38]-[50].

4. IceCube-170922A

Up until the 1990's it was thought that neutrinos are massless. Massless particles, which travel at the speed of light, can not be accelerated gravitationally. Therefore, our theory requires that both IceCube-170922A and KM3-230213A possess mass. There are three flavors of neutrinos: electron, muon and tau. Both IceCube-170922A and KM3-230213A are muon neutrinos [51] [52]. The three flavors of neutrinos oscillate into each other. We do not take these oscillations in our calculations into account. The muon neutrino mass has not been directly measured. So we do not know the mass of the IceCube-170922A or KM3-230213A neutrinos. However, according to [53] [54] it is not greater than 0.1 eV.

4.1. TXS 0506+056

We employ our theory to see, if it confirms that a blazar is the source of IceCube-170922A. TXS 0506+056 is a blazar. When it was flaring with gamma rays [55] [56], a high energy, 2.9×10^{14} eV, neutrino was observed [51]. TXS 0506+056 is within the positional error of the neutrino. These two facts suggest it was the source of the neutrino. In this section we apply our theory to this source.

First we employ Equation (19) with $-\sqrt{2m^3E + 5m^2E^2 + 4mE^3 + E^4}$ to compute the distance, n , of TXS 0506+056 from the measured energy of the neutrino, $E = 2.9 \times 10^{14}$ eV. If $m = 0.1$ eV, then Equation (19) gives: $n = 5.2 \times 10^{14}$. In order to determine the Schwarzschild radius, α , we need the distance to the source, which we obtain from Equation (23), whereby the redshift is: 0.3365 [57], which leads to: $d = 1.83$ Gpc or 5.65×10^{27} cm. From Equation (21): $\alpha = 1.1 \times 10^{13}$. Equation (25) gives us the mass of the source: $3.7 \times 10^7 M_\odot$, which is above the lowest mass limit of blazars of $10^7 M_\odot$ mentioned in the previous section. The error in the distance determination is: 0.36 Gpc, consequently the source mass is in the range: $2.9 \times 10^7 M_\odot$ to $4.2 \times 10^7 M_\odot$. So our result fits well with the above mentioned mass range of the blazar supermassive black holes of $10^7 M_\odot \lesssim M_{\text{SMBH}} \lesssim 10^{10} M_\odot$.

4.2. PKS 0502+049

Out of the 5 Blazars in the error region of IceCube-170922A apart from TXS 0506+056 only PKS 0502+049 has a measured redshift, a prerequisite for the application of our theory. In this section we follow the procedure employed in the previous section to obtain the mass of the PKS 0502+049 to see, if could possibly be the source of IceCube-170922A. Equation (19): $n = 5.2 \times 10^{14}$. The redshift of this source is: 0.954 [58]. Equation (23): 6.2 Gpc or 1.9×10^{28} cm. Equation (21): $\alpha = 3.7 \times 10^{13}$. Finally, Equation (25): $1.2 \times 10^8 M_\odot$. The range of values with an error of ± 0.36 Gpc is: 1.2×10^8 to $1.3 \times 10^8 M_\odot$. We see that our result fits well with the above mentioned mass range of the blazar supermassive black holes of $10^7 M_\odot \lesssim M_{\text{SMBH}} \lesssim 10^{10} M_\odot$.

We conclude that according to our theory PKS 0502+049 could be the source of IceCube-170922A. However, it was not flaring around the time of the neutrino event [59] unlike TXS 0506+056. Nevertheless, it has been suggested that this Blazar could emit neutrinos [60]-[64].

5. The Mass of the Muon Neutrino

The mass range of TXS 0506+056 derived from host-galaxy scaling relations (bulge luminosity vs. MSBH) [57] [59] is: $(3 - 5) \times 10^8 M_\odot$. Our value compared to their middle value is $\frac{3.7 \times 10^7}{4 \times 10^8}$, which is 9%. But, we must remember that we do not know the muon neutrino mass. We suggest that the discrepancy in mass values comes about because of the muon neutrino mass of $m = 0.1$ eV we employed. If on the contrary we had employed $m = 0.03039$ eV in Equation (19), we would have obtained $5.2 \times 10^8 M_\odot$. If we had employed $m = 0.09192$ eV we would have obtained $3.0 \times 10^8 M_\odot$. We conclude: If TXS 0506+056 is the source of IceCube-170922A, then the muon neutrino mass is in the range: 0.03039 to 0.09192 eV. This range is in agreement with [53] [54], who found that the mass of the muon neutrino is not greater than 0.1 eV. We conclude: Under the assumption that our theory is correct and that TXS 0506+056 is the source, the muon neutrino

mass is constrained to be in the above range. It follows, if the experimental value turns out to be far from this range then our theory is falsified.

If the source of IceCube-170922A is PKS 0502+049, which according to [61] has a mass of: $7.53 \times 10^8 M_{\odot}$. A comparison of these two mass values is:

$\frac{1.2 \times 10^8}{7.53 \times 10^8}$, which is 16%. We suggest that this discrepancy is caused by the fact that we assumed a neutrino mass of $m = 0.1 \text{ eV}$.

According to our equations the mass of the muon neutrino, which best fits $7.53 \times 10^8 M_{\odot}$ is: $m = 0.09136 \text{ eV}$. This result makes clear, if we know the mass of the neutrino source, we can predict the mass of the muon neutrino. Conversely, if experiments determine the mass of the muon neutrino to be: $m = 0.09136 \text{ eV}$, then it means that PKS 0502+049 could be the source of IceCube-170922A.

6. KM3-230213A

In this section we apply our theory to the neutrino, KM3-230213A, to explain how it was accelerated to the ultra high energy of $E = 2.2 \times 10^{20} \text{ eV}$ [21] in the assumed Schwarzschild field of the source.

If $m = 0.1 \text{ eV}$ then Equation (12) yields: $\gamma = 2.2 \times 10^{21}$. Inserting this value into Equation (10) yields: $A = -2.1 \times 10^{-43}$. This value of A is exceedingly small. Consequently, the second term in the Schwarzschild velocity, Equation (7), can be dropped and Equation (13) is valid for KM3-230213A.

Inserting E and m into Equation (19) we obtain: $n = 4.4 \times 10^{21}$. The next step would be to convert n , which is in units of α , to parsecs. The conversion requires that we must know the source and its redshift. There is, however, no clear source for this event. There are 17 blazars within the positional error [52], but it is not clear, which blazar is the source of the event. Our theory can only be applied, if the redshift of a possible source is known. Among the 17 sources only three redshifts have been measured. Below we determine according to our theory whether any of these three possible blazar sources could explain the observed energy of KM3-230213A.

6.1. The Blazar: PKS 0605-085

[52] points out that this possible source shows increased gamma-ray activity during the time of the detection of KM3-230213A. In addition [65], argues that PKS 0605-085 is a “viable” source. So we first apply our theory to this possible source of KM3-230213A. The redshift of this blazar is: 0.872 [66]. Equation (23) yields: 5.6 Gpc or $1.7 \times 10^{28} \text{ cm}$. With $m = 0.1 \text{ eV}$ from Equation (21): $\alpha = 3.7 \times 10^6$. Equation (25): $13.1 M_{\odot}$. This value of the mass of the source of KM3-230213A is stellar mass not the mass of a supermassive black hole. Therefore this blazar according to our theory is not the source of KM3-230213A. We also conclude if a neutrino emitting celestial body with a mass of $13.1 M_{\odot}$ has a cosmological redshift of 0.872, it could very well be the source of the neutrino, KM3-230213A.

6.2. The Blazar PMN J0606-0724

This object experienced a large radio flare very close in time of the KM3-230213A event, with a time difference of five days [52]. The redshift of this blazar is: 1.277 [67]. Equation (23) yields: 8.73 Gpc or 2.7×10^{28} cm. with $m = 0.1$ eV Equation (21): $\alpha = 6.1 \times 10^6$. Equation (25): $20.8M_{\odot}$. This is stellar mass and not the mass of a supermassive black hole. Therefore this blazar according to our theory is not the source of KM3-230213A. We also conclude if a neutrino emitting celestial body with a mass of $20.8M_{\odot}$ has a cosmological redshift of 1.277, it could very well be the source of the neutrino, KM3-230213A.

6.3. Blazar PMN J0609-0615

The redshift of this blazar is: 2.219 [28]. Equation (23) yields: 16.8 Gpc or 5.19×10^{28} cm. with $m = 0.1$ eV Equation (21): $\alpha = 1.2 \times 10^7$. Equation (25): $39.9M_{\odot}$. This is a stellar mass and not the mass of a supermassive black hole. Therefore, this blazar according to our theory is not the source of KM3-230213A. We also conclude if a neutrino emitting celestial body with a mass of $39.9M_{\odot}$ has a cosmological redshift of 2.219, it could very well be the source of the neutrino, KM3-230213A.

7. The Origin of Ultra High Energy Neutrinos

Our results show that all three blazars, that are within the positional error of KM3-230213A and have measured redshifts are not the source of this ultra high energy neutrino. On the contrary, in all three cases we concluded the source is a stellar mass body, if it is at the redshift of the blazar. In this section we generalize these results by answering the questions: 1) Are any of the other 14 blazars, which are within the positional error and whose redshifts have not been measured, possible sources KM3-230213A? 2) Is the source of this extragalactic neutrino most likely a distant stellar mass body?

To answer these questions, we employ Equation (25). We start by rearranging this equation:

$$d = \frac{2GM}{c^2} n \quad (26)$$

Since we know the minimum, 10^7M_{\odot} , and maximum, $10^{10}M_{\odot}$, mass of blazars we can employ these mass values in Equation (26) to obtain the minimum and maximum distance these masses correspond to.

We know for KM3-230213A $n = 4.4 \times 10^{21}$. Plugging in the minimum mass into the above equation we obtain: $d = 4.21 \times 10^6$ Gpc and with maximum mass $d = 4.21 \times 10^9$ Gpc. Why are these distances so large? The above equation is directly proportional to both the mass and distance, n , of the source. $n = f(E)$, whereby for KM3-230213A n is large because E is ultra high. M is large for blazars. So two large numbers means the calculated distance is also large.

So far the most distant blazar ever detected is: VLASS J041009.05-013919.88 (J0410-0139) with a redshift of 7.0 at about 9 Gpc [68] [69]. Even the most distance

object ever detected, JADES-GS-z14-0, is at $z = 14.1793$ or at not more than 11 Gpc. These observed distances pale compared to our distance range for a possible blazar source of KM3-230213A. We conclude: It is highly unlikely that blazars are the source of the ultra high energy neutrinos.

Next we apply our theory to stellar mass bodies to see, if they could be sources of ultra high energy neutrinos. We base our analysis on the following: 1) The exterior gravitational field of a spherically symmetric non-rotating star is the Schwarzschild metric. 2) All stars produce neutrinos. 3) The minimum mass of stars is: $(0.72 - 0.75)M_{\odot}$ [70]-[72]. 4) The maximum mass of stars is: $(150 - 200)M_{\odot}$ [73]-[75].

Figure 5 is a plot of stellar mass vs. distance for KM3-230213A. The distances in the figure are many orders of magnitude below the values for blazars. We suggest a neutrino from a stellar mass body is likely the source of KM3-230213A.

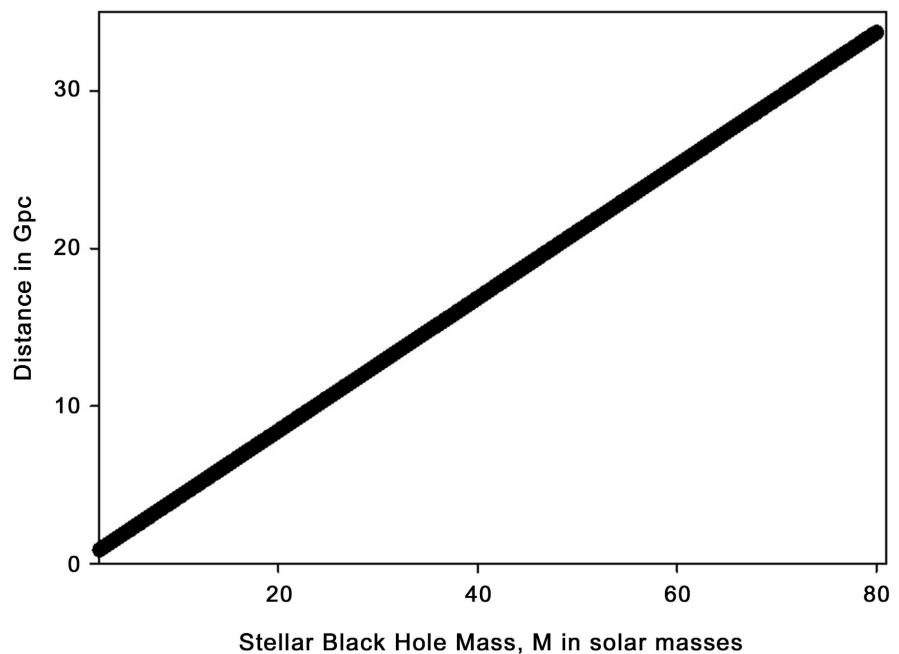


Figure 5. Distance of stellar black holes.

Our answers to our questions lead us to the following conclusions. None of the blazars within the positional error of KM3-230213A are the source of this neutrino. According to our theory stellar mass bodies can be the source of ultra high energy neutrinos, if they are far enough. In the universe there are many more stars than supermassive black holes and they may very well be the source of most ultra high energy neutrinos.

8. Discussion

The history of gravitational repulsion has been characterized by intense controversy [19] [20]. Throughout this long history of over 100 years its very existence has been repeatedly questioned. As an example, responding to [20], [76]-[79] all

concluded that gravitational repulsion in the Schwarzschild field does not exist.

8.1. Response to the Critics of Theory of Gravitational Repulsion in the Schwarzschild Field

Crucial for the understanding of gravitational repulsion in the Schwarzschild field is the distinction between local and distant observers as elaborated in [19]. Among the above mentioned opponents of the theory of gravitational repulsion in the Schwarzschild field only Grøn referenced this work. We showed in [19]: 1) local observers, whose measuring instruments are affected by gravity of the particle emitting source, cannot detect gravitational repulsion. 2) To the distant observer, who uses measuring instruments not affected by the gravity of the particle emitting source, gravitational repulsion can occur anywhere in the Schwarzschild field.

According to [76]-[78] local observers using proper time, orthonormal frames, or horizon-penetrating coordinates never measure a repulsive force. Their results are correct. They correspond to our result from [19], whereby local observers can not detect gravitational repulsion. But, they also conclude that Schwarzschild coordinates do not correspond to actual physical measurements, meaning the gravitational repulsion is simply a coordinate effect without physical reality. However, we assert this is not the case for the distant observer, who is in Minkowski space and detects gravitational repulsion.

[79] thinks that gravitational repulsion only exists if a body at rest experiences gravitational repulsion, which is not the case in the Schwarzschild field [19]. We employ the term gravitational repulsion because it was used by both founders of the theory - Droste and Hilbert. It does not fit Grøn's understanding of the term. We could call the effect gravitational entanglement because like quantum entanglement it is a "spooky action at a distance" [80]. But this term is already in use to describe quantum effects in gravity [81] [82].

Those that oppose the concept of gravitational repulsion in the Schwarzschild field believe that the Schwarzschild coordinate velocity, $\frac{dr}{dt}$, has no direct physical meaning primarily because it may exceed the speed of light. Therefore, the physically meaningful velocity is the local velocity measured by a local inertial (or static) observer, constructed from proper distance and proper time, and this velocity is always less than the speed of light, which is in agreement with [19]. However, in our theory, as **Figure 1** and Equation (13) show, $\frac{dr}{dt} < c$ for material particles. Consequently, the introduction of the different local coordinates mentioned above by those who object to gravitational repulsion is superfluous.

8.2. Application of the Theory of Gravitational Repulsion

Although, as discussed in the previous subsection, there is opposition to the concept of gravitational repulsion, others have applied the concept in various circumstances. Dickau, Kauffmann and Robertson employ it to solve the problem of the accelerating expansion of the universe (in preparation). [83] investigate gravita-

tional repulsion in the Einstein-zero-mass scalar theory. [84] discuss gravitational repulsion in an expanding ball of dust. [85] considered gravitational repulsion in the Kerr-Newman anti-de Sitter spacetime. [86] looked into gravitational repulsion in the Reissner-Nordström and Schwarzschild spacetimes. [87] show that the emission of gravitational waves leads to a repulsive gravitational force that diminishes with time but never disappears. They speculate that the repulsive force may be related to the observed expansion of the Universe. [88] point out that gravitational repulsion could appear in satellite experiments with beams of relativistic particles subject to very precise time measurements. [89] points out that gravitational repulsion occurs in geodesics in a quash-spherical spacetime. [90] investigates a number of aspects of the phenomenon of gravitational repulsion in static sources of the Reissner-Nordström field. [91] study gravitational repulsion in the Kerr and Kerr-Newman fields.

9. Conclusion

Since Newton published his theory of gravitation in 1687, it is believed that gravity is only an attractive force, unlike the electric force, which can be either attractive or repulsive. However, in 1916, first Droste and then Hilbert independently noticed that in Einstein's theory of gravitation, gravity can also act repulsively. The history of this discovery is in [18]. It has been well over a century since this discovery, yet the vast majority of scientists firmly believe that gravity is only an attractive force.

We suggest the history of gravitational repulsion is similar to history of gravitational waves. In 1916, the discovery year of gravitational repulsion, Einstein predicted the existence of gravitational waves [92]. Like gravitational repulsion, there were doubts about their existence. In fact, even Einstein along with Rosen submitted a paper to the Physical Review, in which they argued that gravitational waves do not exist. It was not until 2015 that gravitational waves were detected, almost 100 years after they were first predicted. We suggest the theory of gravitational repulsion has and is experiencing a similar fate.

The history of gravitational repulsion is characterized by intense controversy [19] [20]. Throughout this long history of over 100 years, its very existence has been repeatedly questioned. Up until this year, the theory did not make a prediction that had been confirmed by observations or experiments. Now that the theory's prediction of the existence of ultra high energy cosmic neutrinos [20] has been confirmed by observations [21], the theory has achieved a milestone, that may be the first concrete step to its acceptance.

Finally, we note that our theory suggests that stars are the source of ultra high energy neutrinos, not blazars, as is commonly thought. Since stars are so much more numerous than the supermassive black holes of blazars our results suggest that the detection of very high energy neutrinos like KM3-230213A will not remain a rare occurrence. That is, we predict its discovery is an opening of a new energy window in neutrino astronomy and many more such detections will follow.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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