Journal of Modern Physics, 2026, 17(1), 10-21
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

On Mechanics of the Universe Evolution

Valery V. Vasiliev, Leonid V. Fedorov

Institute of Applied Mechanics, Russian Academy of Sciences, Moscow, Russia
Email: vvvas@dol.ru

How to cite this paper: Vasiliev, V.V. and Fedorov, L.V. (2026) On Mechanics of the Universe Evolution. *Journal of Modern Physics*, **17**, 10-21.
https://doi.org/10.4236/jmp.2026.171002

Received: November 3, 2025
Accepted: January 3, 2026
Published: January 6, 2026

Abstract

The paper is devoted to the problem of cosmology, which is formulated as a spherically symmetric problem for a pressure-free model of continuum. The problem is described by equations of general relativity corresponding to the special four-dimensional space, which is Euclidean with respect to space coordinates and Riemannian with respect to the time coordinate only. Within the framework of this model, the analytical solution of the field equations incorporating the cosmological constant is obtained and analyzed. This solution describes the continuous process of the Universe evolution, consisting of the phases—the Universe expansion and subsequent gravitational collapse. The existing data for the observable Universe age and the cosmological constant are used for numerical evaluations. As a result, the obtained solution specifies the dependency of the continuum density on time and allows us to evaluate the durations of the processes of the Universe expansion and gravitational collapse.

Keywords

Universe Expansion, Gravitational Collapse, General Relativity, Cosmological Constant

1. Introduction

Universe evolution consists of two processes-expansion and gravitational collapse, which are traditionally studied separately. Particularly, gravitational collapse is widely discussed in the literature [1]-[4]. History and the state-of-the-art of the problem are presented in a review [5], which contains brief descriptions of the results obtained by 400 authors working in this field. The traditional model is a pressure-free continuum, which is described by the Schwarzschild geometry and is referred to as the so-called co-moving coordinates, with the line element in the following form:

$$\mathrm{d}s^2 = U(r,t)\mathrm{d}r^2 + V(r,t)\mathrm{d}\Omega^2 - c^2\mathrm{d}t^2, \quad \mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2$$

in which r and t are the radial and the time coordinates. The first solution of the collapse problem was obtained in 1939 by Oppenheimer and Snyder [6]. After some assumptions, they arrived at the following asymptotic expressions for the metric coefficients of the Schwarzschild metric form for the sphere surface $r = R$:

$$g_{11}(R) = 1 + e^{tc/r_g}, \quad g_{44} = g_{11} e^{-2tc/r_g} \left(1 + e^{-tc/r_g}\right)$$

in which $r_g = 2mG/c^2$ is the so-called gravitation radius expressed in terms of the sphere mass m, the classical gravitation constant G and the velocity of light c. As can be seen, $g_{11}(t \to \infty) \to \infty$ and $g_{44}(t \to \infty) \to 0$. An alternative solution of the collapse problem was presented by Weinberg [3]. More recent results can be found in [7]. The expansion problem was originally formulated by Friedman [8] and further discussed by Weinberg [4].

In the present paper, the solution of the collapse problem is based on the proposed special model of the four-dimensional space [9]. This space is Euclidean with respect to coordinates r, θ, φ and Riemannian with respect to time. Expansion and collapse problems are solved for this model of space in [10], where the solutions of the field equations without the cosmological constant are obtained. These solutions describe the Universe expansion and gravitational collapse as separate processes. As shown further, the allowance for the gravitational constant leads to the solution, which describes the Universe evolution as a continuous process of the Universe expansion and collapse.

2. Equations of General Relativity in a Special Riemannian Space

Consider the traditional model for cosmological problems of continuum, which consists of a system of isolated particles and in which the distance between the particles is much larger than the particle dimensions. For this continuum, the interaction between the particles is restricted to the gravitational forces only and the continuum can be described by the equations of General Relativity Theory (GRT) treated as a phenomenological theory based on the traditional in the mechanics model of space as a homogeneous isotropic continuum whose physical microstructure is ignored. It is important that the density of a pressure-free continuum, which is initially constant, depends only on time [3]. The idea of a special Riemannian space which is Euclidean with respect to spatial coordinates and Riemannian to the time coordinate only has the following reasoning [9]. Consider a static problem for which the GRT field equations link the metric tensor with the stress tensor [11], which means that the Riemannian space is induced not only by gravitation, but by the mechanical stresses as well. Now, assume that we observe in a traditional three-dimensional Euclidean space a solid sphere. The sphere surface is a two-dimensional Riemannian space, whereas the internal sphere space is Euclidean. Further assume that the sphere is loaded with self-balanced forces, inducing inside the sphere a certain stressed state. According to the traditional for general relativity space model, the space inside the sphere becomes Riemannian, whereas the outside space remains Euclidean. Thus, a three-dimensional Riemannian space exists inside of a three-dimensional Euclidean space. However, such situation is not possible in Riemannian geometry-the dimension of the Euclidean space (n_E) in which the Riemannian space with n_R dimensions can exist is $n_E = n_R\left(n_R+1\right)/2$ [12]. Taking $n_R = 2$, we get $n_E = 3$ which is natural for the sphere surface. But for the inside sphere space, taking $n_R = 3$, we have $n_E = 6$, which is not realistic. To satisfy the natural condition, according to which the inside and the outside spaces of the stressed sphere must be Euclidean in the absence of gravitation, we arrive at a new model of a space-time. In this model, the space is Euclidean with respect to coordinates r,θ,φ, whereas the space curvature is induced by gravitation and the space is Riemannian with respect to the time coordinate only.

To support the foregoing reasoning, consider the classical static problem of the theory of elasticity and try to determine the stresses that are induced in a solid sphere (e.g., Earth) by internal gravitation forces. For the Newton gravitation theory, this problem has a well-known solution obtained at the end of the 19th century [13]. In this problem, we need to find two stresses (radial and hoop) from two equations—the equilibrium equation and the compatibility equation. However, in general relativity, we have only one conservation equation for this problem—the equilibrium equation. Compatibility equation, which requires the sphere stressed state to be Euclidean in principle, cannot exist in the Riemannian space. Thus, the problem that can be readily solved in the Newton gravitation theory has no solution in general relativity, which does not look natural. However, for the proposed space model, the set of equations is complete. Since the space is Euclidean in coordinates r,θ,φ, we can supplement the general relativity equations with compatibility equations and solve the gravitation problems for solids. Particularly, a spherically symmetric problem is solved in [14].

The line element for the proposed four-dimensional space in spherical coordinates has the following form [9]:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 + 2g_{14}c\mathrm{d}r\mathrm{d}t - g_{44}c^2\mathrm{d}t^2 \tag{1}$$

where, g_{14} g_{44} depend on *r* and *t*. In contrast to traditional coordinate frames, the space is not "orthogonal" to time in the four-dimensional space corresponding to Equation (1). The metric form in Equation (1) was derived in 1921-1922 by Gullstrand and Painleve [15] [16] from the Schwarzschild form as a result of coordinate transformation. Here, Equation (1) is not associated with the Schwarzschild form and follows from the proposed model of a four-dimensional space. The field equations in General Relativity are

$$E_i^j + \lambda\delta_i^j = \chi T_i^j \tag{2}$$

where δ_i^j is the Kronecker delta, λ is the cosmological constant and

$$E_i^j = R_i^j - \frac{1}{2}R\delta_i^j$$

is the Einstein tensor whose components for the line element in Equation (1) have the following form:

$$E_1^1 = -\frac{1}{r^2 g^2}\left[g\left(rg_{44}' - g_{14}^2\right) + 2rg_{44}\dot{g}_{14} - rg_{14}\dot{g}_{44}\right] \tag{3}$$

$$\begin{aligned} E_2^2 = \frac{1}{4rg^2}[&4g_{14}g_{44}g_{14}' - 4g_{14}^2 g_{44}' - 2g_{14}g_{44}' - 2rg_{44}g_{44}'' + r\left(g_{44}'\right)^2 - 2rg_{14}^2 g_{44}'' \\ &+ 2rg_{14}'g_{44}' - 4g_{44}\dot{g}_{14} + 2g_{14}\dot{g}_{44} + 4rg_{14}g_{14}'\dot{g}_{14} + 2rg_{14}'\dot{g}_{44} - 4rg\dot{g}_{14}'] \end{aligned} \tag{4}$$

$$E_4^4 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{rg_{14}^2}{g}\right) \tag{5}$$

$$E_1^4 = \frac{g_{14}}{r}\frac{\partial}{\partial r}\left(\frac{1}{g}\right) \tag{6}$$

$$E_4^1 = \frac{g_{14}^2 g_{44}}{crg^2}\frac{\partial}{\partial t}\ln\frac{g_{44}}{g_{14}^2}, \quad g = g_{44} + g_{14}^2 \tag{7}$$

in which $(\cdot)' = \partial(\cdot)/\partial r$ and $\dot{(\cdot)} = \partial(\cdot)/c\partial t$. Equation (2) includes two constant coefficients—the cosmological constant λ and the gravitation constant

$$\chi = \frac{8\pi G}{c^4}$$

Equation (2) links the Einstein tensor with the energy-momentum tensor [17]

$$T_1^1 = \rho v_1 v^1, \quad T_2^2 = 0, \quad T_4^4 = \rho c^2, \quad T_1^4 = -\rho v_1 c, \quad T_4^1 = \rho v^1 c \tag{8}$$

in which ρ is the continuum density and v is the velocity. Recall that the density does not depend on the radial coordinate, so that $\rho = \rho(t)$. Tensor T_i^j must satisfy the following conservation equations:

$$\left(T_1^1\right)' + \frac{2}{r}\left(T_1^1 - T_2^2\right) + \frac{g_{44}'}{2g}\left(T_1^1 - T_4^4\right) + \frac{g_{14}g_{44}'}{2g}T_1^4 + \frac{g_{14}'}{g}T_4^1 + \dot{T}_1^4 + \frac{\dot{g}}{2g}T_1^4 = 0 \tag{9}$$

$$\begin{aligned} &rg_{44}'\left[g_{14}\left(T_1^1 - T_4^4\right) - g_{44}T_1^4\right] + 2g\left[r\dot{T}_4^4 + r\left(T_4^1\right)' + 2T_4^1\right] \\ &+ 2rg_{14}g_{14}'T_4^1 - 2rg_{44}\dot{g}_{14}T_1^4 + rg_{14}\dot{g}_{44}T_1^4 = 0 \end{aligned} \tag{10}$$

Equation (9) is the motion equation, whereas Equation (10) provides the conservation of mass.

3. The Process of the Universe Evolution

Determine the metric coefficients. Equations (2) and (5) in conjunction with the corresponding Equation (8) give the following equation for g_{14}:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{rg_{14}^2}{g}\right) + \lambda = \chi\rho(t)c^2$$

Integration under the condition that requires the solution to be regular at $r = 0$ yields

$$g_{14}^2 = \frac{1}{3}\lambda g r^2\left(k\rho - 1\right), \quad k = \frac{\chi c^2}{\lambda} = \frac{8\pi G}{\lambda c^2} \tag{11}$$

Using Equation (7) for g, we arrive at

$$g_{44} = g - g_{14}^2 = g\left[1 - \frac{1}{3}\lambda r^2 (k\rho - 1)\right] \tag{12}$$

Thus, the metric coefficients are specified by Equations (11) and (12). To determine the velocities, first apply Equations (2) and (9) in conjunction with the corresponding Equation (8). The resulting equation

$$\frac{g_{14}}{r}\frac{\partial}{\partial r}\left(\frac{1}{g}\right) = -\rho v_1 c$$

allows us to find v_1. Substituting the metric coefficients from Equations (10) and (11), we get

$$v_1 = \mp\frac{1}{c\rho}\frac{\partial}{\partial r}\left(\frac{1}{g}\right)\sqrt{\frac{1}{3}\lambda g (k\rho - 1)} \tag{13}$$

Finally, Equations (2), (7) and (8) yield the following equation for v^1:

$$\frac{g_{14}^2 g_{44}}{g^2 cr}\frac{\partial}{\partial t}\left(\ln\frac{g_{44}}{g_{14}^2}\right) = \chi\rho v^1 c$$

Substituting the metric coefficients from Equations (10) and (11) and taking into account Equation (11) for k, we arrive after some transformations at

$$v^1 = -\frac{r}{3\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} \tag{14}$$

Equations (11) - (14) allow us to express functions $g_{14}(r,t)$, $g_{44}(r,t)$, $v_1(r,t)$, $v^1(r,t)$ in terms of two unknown functions $g(r,t)$ and $\rho(t)$. Note that we still did not use Equations (3) and (4) for the components E_1^1, E_2^2 of the Einstein tensor and the conservation Equations (9) and (10). Substituting the obtained solution in the first two field Equations (2), corresponding to E_1^1, E_2^2, we arrive at the following two equations for functions $g(r,t)$ and $\rho(t)$:

$$\chi\rho c^2 - \frac{g'}{rg}\left[1 - \frac{r^2}{3}\left(\chi\rho c^2 - \lambda\right)\right] \mp \frac{\chi c^2 \dot{\rho}}{\sqrt{3g\left(\chi\rho c^2 - \lambda\right)}} = \pm\frac{rg'\dot{\rho}}{3g\rho}\sqrt{\frac{\chi\rho c^2 - \lambda}{3g}} \tag{15}$$

$$\begin{aligned}
& r\left[(g')^2 - 2gg''\right]\sqrt{g\left(\chi\rho c^2 - \lambda\right)}\left[r^2\left(\chi\rho c^2 - \lambda\right) - 3\right] \\
& - 2gg'\sqrt{g\left(\chi\rho c^2 - \lambda\right)}\left[4r^2\left(\chi\rho c^2 - \lambda\right) - 3\right] \\
& \pm\sqrt{3}gg'\chi c^2 r^2\dot{\rho} - 2\sqrt{3}g\dot{g}\chi\rho c^2 r + 2\sqrt{3}g\dot{g}'\left(\chi\rho c^2 - \lambda\right)r^2 \\
& + 2\sqrt{3}g'\dot{g}\lambda r^2 - 12g^2\chi\rho c^2 r\sqrt{g\left(\chi\rho c^2 - \lambda\right)} \pm 4\sqrt{3}g^2\chi\dot{\rho}c^2 r = 0
\end{aligned} \tag{16}$$

These rather cumbersome equations have a remarkably simple solution. To justify the proposed approach, consider the vacuum solution. For the line element form in Equation (1), the metric coefficients are [9]

$$g_{14}^2 = \frac{r_g}{r}, \quad g_{44} = 1 - \frac{r_g}{r} \tag{17}$$

in which r_g is the gravitation radius. Equations (17) were originally obtained by Gullstrand and Painleve [15] [16]. As follows from these equations, the following condition is valid for the vacuum solution:

$$g = g_{14}^2 + g_{44} = 1 \tag{18}$$

The coefficient g enters the expression for the determinant of the metric tensor, which is $d = gr^4 \sin^2\theta$. Thus, the condition $g = 1$ means that d does not depend on time. Now assume that the condition in Equation (18) is valid for the continuum as well. Taking $g = 1$, we can reduce Equations (15) and (16) to one and the same equation for the density, *i.e.*,

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \pm c\sqrt{3\left(\chi c^2 \rho - \lambda\right)} \tag{19}$$

For $g = 1$, Equations (11) - (14), can be simplified as

$$g_{14}^2 = \frac{1}{3}\lambda r^2\left(k\rho - 1\right),\quad g_{44} = 1 - \frac{1}{3}\lambda r^2\left(k\rho - 1\right),\quad v_1 = 0,\quad v^1 = -\frac{r}{3\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} \tag{20}$$

Calculating the components of the Ricci curvature tensor, we arrive at

$$R_1^1 = R_2^2 = R_3^3 = R_4^4 = -\frac{1}{3}\lambda^2\left(k\rho - 1\right)^2$$

These expressions allow us to conclude that the space is Riemannian and isotropic.

Now, using Equations (8) and (20), determine the components of the energy-momentum tensor

$$T_1^1 = 0,\quad T_2^2 = 0,\quad T_4^4 = \rho c^2,\quad T_1^4 = 0,\quad T_4^1 = -\frac{1}{3}rc\frac{\mathrm{d}\rho}{\mathrm{d}t} \tag{21}$$

Substituting Equations (21) in the conservation equations, Equations (9) and (10), we can prove that these equations are satisfied identically. Thus, all the field equations are satisfied and Equations (19) and (20) specify the solution of the problem.

Consider the particular case. Neglect the cosmological constant taking $\lambda = 0$. For this case, Equation (19) becomes

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \pm c^2\sqrt{\chi\rho} \tag{22}$$

This equation describes the processes of the Universe contraction (sign +) and expansion (sign −), which are not mutually linked [10]. The solution for the contraction process is [10]

$$\rho(t) = \frac{\rho_0}{(1 - t/t_c)^2},\quad v^1(r,t) = \frac{2r}{t_c\left(1 - t/t_c\right)},\quad R(t) = R_0\sqrt[3]{\left(1 - t/t_c\right)^2}$$

where, $\rho_0 = \rho(t = 0)$, $R_0 = R(t = 0)$, R is the Universe radius which can be found from the conservation condition for the Universe mass and

$$t_c = \frac{1}{\sqrt{6\pi G\rho_0}}$$

is the collapse time. As can be seen, for $t = t_c$ the Universe degenerates into a point with an infinitely high density.

For the expansion process, the solution of Equation (22) yields [10]

$$\rho(t)=\frac{\rho_0}{(1+t\sqrt{6\pi G\rho_0})^2},\quad v^1=\frac{2r\sqrt{6\pi G\rho_0}}{3\left(1+t\sqrt{6\pi G\rho_0}\right)},\quad R(t)=R_0\sqrt[3]{\left(1+t\sqrt{6\pi G\rho_0}\right)^2} \tag{23}$$

In 1934, Lemaitre obtained the following expression for the expansion velocity [18], supporting the Hubble law:

$$v=r\sqrt{\frac{4\pi}{3}G\rho_0}$$

The coordinate acceleration

$$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}=\frac{8\pi}{3}G\rho_0$$

corresponds to the accelerated expansion of the Universe. However, the second equation in Equations (23) contains time in the denominator and corresponds to a different expansion process. To simplify the analysis, assume that $\rho_0\to\infty$.

Then,

$$v^1=\frac{2r}{3t},\quad \frac{\mathrm{d}v^1}{\mathrm{d}t}=\frac{\partial v^1}{\partial t}+v^1\frac{\partial v^1}{\partial r}=-\frac{2r}{9t^2}$$

As can be seen, the acceleration is negative. In looks natural, because gravitation slows down the expansion. Taking $t\to\infty$ in Equations (23), we get $\rho=0$, $v^1=0$, $R\to\infty$ which corresponds to an infinite static Euclidean space.

Return to Equation (19) in which the constant λ is not zero. In contrast to Equation (22), this equation describes the continuous process of the Universe evolution. Present Equation (19) as

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t_e}=-c\sqrt{3\lambda(k\rho-1)},\quad \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t_c}=c\sqrt{3\lambda(k\rho-1)} \tag{24}$$

The first of these equations describes the process of expansion, whereas the second one—the process of gravitational collapse.

Consider the process of expansion and assume that at the initial time $t_e=0$ the Universe density and radius are ρ_0 and R_0. Since the space is Euclidean with respect to spherical coordinates, the Universe mass is

$$m=\frac{4}{3}\pi\rho_0R_0^3 \tag{25}$$

and does not change in the process of evolution, so that

$$\rho(t)R^3(t)=\rho_0R_0^3 \tag{26}$$

Expansion is described by the first equation in Equations (24), which has the following solution:

$$2\tan^{-1}\sqrt{k\rho-1}=-ct_e\sqrt{3\lambda}+C_1 \tag{27}$$

Since $\rho(t_e=0)=\rho_0$,

$$C_1=2\tan^{-1}\sqrt{k\rho_0-1}$$

and the solution in Equation (27) can be reduced to the following form:

$$\frac{\sqrt{k\rho-1}-\sqrt{k\rho_0-1}}{1+\sqrt{(k\rho-1)(k\rho_0-1)}}=-\tan\left(\frac{\sqrt{3\lambda}}{2}ct_c\right)$$

Expressing the density, we arrive at

$$\rho(t_e)=\frac{\rho_0}{\left[\cos\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)+\sqrt{k\rho_0-1}\sin\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)\right]^2} \tag{28}$$

The Universe radius can be found from Equations (25) and (26), *i.e.*,

$$R^3(t)=\frac{3mR_0^3}{4\pi\rho(t)}$$

Substituting Equation (28), we get

$$R^3(t_e)=R_0^3\left[\cos\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)+\sqrt{k\rho_0-1}\sin\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)\right]^2 \tag{29}$$

Finally, Equations (22) and (24) allow us to determine the velocity of the expansion

$$v_e^1(r,t_e)=-\frac{r}{3\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t_e}=cr\sqrt{\frac{\lambda}{3}(k\rho-1)} \tag{30}$$

Assume that at $t_e=0$ some forces, the nature of which is beyond the frame of mechanics and is not discussed in this paper, appear and cause the Universe expansion. In the process of expansion two types of forces act inside the continuum. First, inertia forces which drive the expansion and second, gravitation forces which slow down the expansion process. Thus, in some time t^*, the expansion process stops. Using Equation (29) and taking $v^1(r,t^*)=0$, we can determine the Universe density which corresponds to this moment

$$\rho^*=\frac{1}{k}=\frac{\lambda}{\chi c^2}=\frac{\lambda c^2}{8\pi G} \tag{31}$$

As can be seen, the cosmological constant λ is of a principal nature—it specifies the density of the Universe in the state of equilibrium. As follows from Equation (30), this density does not depend on the initial density ρ_0. The condition $v^1(t^*)=0$ imposed above on the coordinate velocity means that at $t=t^*$, the Universe with density ρ^* is in a state of equilibrium. Equations (28) and (31) allow us to derive the following equation for t^*:

$$\cos\left(\frac{\sqrt{3\lambda}}{2}ct^*\right)+\sqrt{k\rho_0-1}\sin\left(\frac{\sqrt{3\lambda}}{2}ct^*\right)=\sqrt{\frac{\rho_0}{k}} \tag{32}$$

Finally, Equation (29) can be used to calculate the Universe radius in the equilibrium state.

$$R^*=R_0\left[\cos\left(\frac{\sqrt{3\lambda}}{2}ct^*\right)+\sqrt{k\rho_0-1}\sin\left(\frac{\sqrt{3\lambda}}{2}ct^*\right)\right]^{2/3} \tag{33}$$

For $t>t^*$ expansion changes to contraction which is described by the second equation in Equations (24). Let us count t_c from the equilibrium time t^*, so that $t^* \le t_c \le t_c^*$ in which t_c^* is the duration of the contraction phase. Then, the solution of the second equation in Equations (24) is

$$2\tan^{-1}\sqrt{k\rho-1} = ct_c\sqrt{3\lambda} + C_2$$

The boundary condition $\rho(t_c = 0) = \rho^* = 1/k$ yields $C_2 = 0$ and

$$\rho(t_c) = \frac{1}{k\cos^2\left(\frac{\sqrt{3\lambda}}{2}ct_c\right)} \tag{34}$$

Naturally, at $t = t_c^*$, *i.e.*, at the end of the collapse, the density reaches its initial value ρ_0 from which the expansion process starts. Taking $\rho(t_c^*) = \rho_0$ in Equation (34), we can determinec the collapse time

$$t_c^* = \frac{2}{c\sqrt{3\lambda}}\cos^{-1}\frac{1}{\sqrt{k\rho_0}} \tag{35}$$

Thus, the total time of the cycle expansion-contraction becomes

$$T = t_e^* + t_c^* \tag{36}$$

The dependence of radius on time follows from the condition, which is similar to Equation (26), *i.e.*,

$$R^3(t_c)\rho(t_c) = \left(R^*\right)^3\rho^*$$

Taking into account Equation (34), we get

$$R^3(t_c) = \left(R^*\right)^3 k\rho^*\cos^2\left(\frac{\sqrt{3\lambda}}{2}ct_c\right) \tag{37}$$

The velocity of the contraction process can be found from Equations (20) and (24), *i.e.*,

$$v_c^1(r,t_c) = -\frac{r}{3\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t_c} = -rc\sqrt{\frac{\lambda}{3}(k\rho-1)} \tag{38}$$

Since the initial density ρ_0 is not known and cannot be found by the methods of mechanics, undertake an asymptotic analysis taking $\rho_0 \to \infty$. Then, Equation (28), which specifies the density in the process of expansion, becomes

$$\rho(t_e) = \frac{1}{k\sin^2\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)} \tag{39}$$

Equation (26) shows that $R_0 = 0$, which means that the expansion process starts from a point. Thus, for $\rho_0 \to \infty$, the solution is singular. To determine the radius, we can use Equation (27). Expressing R_0 from Equation (26) and passing to the limit taking $\rho_0 \to \infty$, we get

$$R^3(t_e) = \frac{3mk}{4\pi}\sin^2\left(\frac{\sqrt{3\lambda}}{2}ct_e\right)$$

For the equilibrium stare, $R(t_e)$ reaches the maximum value

$$\left(R^*\right)^3 = \frac{3mk}{4\pi}$$

at the time moment,

$$t_e^* = \frac{\pi}{c\sqrt{3\lambda}} \tag{40}$$

Substituting this result in Equation (39), we get the density in the state of equilibrium

$$\rho^* = \frac{1}{k} \tag{41}$$

which coincides with Equation (31). The expansion velocity is specified by Equation (30). Substituting ρ from Equation (39), we arrive at

$$v_e^1 = cr\sqrt{\frac{\lambda}{3}}\cot\left(\frac{\sqrt{3\lambda}}{2}ct_e\right) \tag{42}$$

Taking $t_e = 0$, we get $v_e^1 \to \infty$ which means that the expansion process starts with an infinitely high velocity. Naturally, this conclusion is valid if $\rho_0 \to \infty$. For $t = t^*$, we have $v_e^1 = 0$, which corresponds to the equilibrium state.

For gravitational collapse ($t_c \ge 0$), the density is specified by Equation (34), *i.e.*,

$$\rho(t_c) = \frac{1}{k\cos^2\left(\frac{\sqrt{3\lambda}}{2}ct_c\right)} \tag{43}$$

Taking $t_c = 0$, we get $\rho = \rho^*$. For $\rho_0 \to \infty$, Equation (35) yields

$$t_c^* = \frac{\pi}{c\sqrt{3\lambda}} \tag{44}$$

Comparing this result with Equation (40), we can conclude that $t_e^* = t_c^*$. Thus, both phases (extension and compaction) have the same duration if $\rho_0 \to \infty$ and the total time is

$$T = \frac{2\pi}{c\sqrt{3\lambda}}$$

The radius is specified by Equation (37). Taking $t = t_c^*$, we get $R = 0$, which means that at the end of the gravitational collapse, the Universe degenerates to a point. Naturally, it is true if the initial density of the continuum is infinitely high. The velocity of the compaction process is specified by Equation (38). Substituting the density from Equation (43), we arrive at

$$v_c^1 = -cr\sqrt{\frac{\lambda}{3}}\cot\left(\frac{\sqrt{3\lambda}}{2}ct_c\right)$$

Taking $t_c = 0$, we get $v_c^1 = 0$, whereas for $t_c = t_c^*$ we have $v_c^1 \to \infty$, which means that the final velocity of the compaction process is infinitely high.

For calculation, assume that in accordance with the existing data [19], the age of the Universe is $t_0 = 13.8\times10^9$ years $= 435\times10^{15}$ sec, and the cosmological constant is $\lambda = 1.095\times10^{-52}$ m^{-2}. The initial density of the continuum is assumed to be infinitely high. The parameter k, which enters the foregoing equations, is

$$k = \frac{8\pi G}{\lambda c^2} = 17\times 10^{25}\ \mathrm{m}^3/\mathrm{kg}$$

Using Equation (40), we can evaluate the duration of the expansion phase as

$$t^* = \frac{\pi}{c\sqrt{3\lambda}} = 580\times 10^{15}\ \mathrm{sec} = 18.4\times 10^9\ \text{years}$$

Thus, the process of expansion will last for 4.6 × 10^9 years. The total duration of the cycle extension-compaction can be evaluated as $T = 2t^* = 36.8\times 10^9$ years. The current density of the continuum is

$$\rho(t_0) = \frac{1}{k\sin^2\left(\frac{\sqrt{3\lambda}}{2}ct_0\right)} = 6.86\,\mathrm{kg}/\mathrm{m}^3$$

4. Conclusion

Within the framework of the proposed model of the four-dimensional space, which is Euclidean with respect to the space coordinates and Riemannian with respect to the time coordinate only, the analytical solution of the general theory of relativity field equations, including the cosmological constant, is found for the pressure-free continuum and used to describe the evolution of the Universe. The obtained solution specifies the time dependencies for the continuum density, the Universe radius and the velocities of the expansion and contraction processes. It is established that in a certain time moment, which depends only on cosmological constant, the expansion process changes to the gravitational collapse. The cosmological model predicting an infinite accelerated expansion of the Universe is not confirmed by the obtained solution. The durations of the expansion process and the gravitational collapse are evaluated. The asymptotic analysis of the solution for an infinitely high initial density of the continuum is undertaken and the current parameters of the Universe are evaluated with the aid of this solution.

Acknowledgements

The study was conducted within the framework of the Russian Science Foundation Project No. 23-1100275 issued to Institute of Applied Mechanics of Russian Academy of Sciences.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this paper.

References

[1] Garrison, B.K., Thorn, K.S., Wakano, M. and Wheeler, J.A. (1965) Gravitation Theory and Gravitation Collapse. The University of Chicago Press.

[2] Misner, C.W., Thorne, K.S. and Wheeler, J.A. (1973) Gravitation. W.H. Freeman and Co.

[3] Weinberg, S. (1972) Gravitation and Cosmology. John Wiley and Sons, Inc.

[4] Weinberg, S. (2008) Cosmology. Oxford University Press.

[5] Burghardt, R. (2022) Gravitation Collapse: An Overview. Austrian Reports on Gravitation.

[6] Oppenheimer, J.R. and Snyder, H. (1939) On Continued Gravitational Contraction. *Physical Review*, **56**, 455-459. https://doi.org/10.1103/physrev.56.455

[7] Batic, D. and Novakovski, M. (2024) Gravitational Collapse via Wheeler-DeWitt Equation. arXiv: 2401.07512.

[8] Friedman, A. (1922) Über die Krümmung des Raumes. *Zeitschrift für Physik*, **10**, 377-386. https://doi.org/10.1007/bf01332580

[9] Vasiliev, V.V. and Fedorov, L.V. (2023) To the Solution of a Spherically Symmetric Problem of General Relativity. *Journal of Modern Physics*, **14**, 147-159. https://doi.org/10.4236/jmp.2023.142010

[10] Vasiliev, V.V. and Fedorov, L.V. (2025) Gravitational Collapse and Expansion in the Newton Theory and General Relativity. *Journal of Modern Physics*, **16**, 294-309. https://doi.org/10.4236/jmp.2025.162015

[11] Singe J.L. (1960) Relativity: The General Theory. North-Holland Publishing Company.

[12] Rashevskii, P.K. (1967) Riemannian Space and Tensor Analysis. Nauka. (In Russian)

[13] Love, A.E.H. (1892) Mathematical Theory of Elasticity. Cambridge University Press.

[14] Vasiliev, V.V. and Fedorov, L.V. (2023) Spherically Symmetric Problem of General Relativity for an Elastic Solid Sphere. *Journal of Modern Physics*, **14**, 818-832. https://doi.org/10.4236/jmp.2023.146047

[15] Painleve, P. (1921) La mechanique classique et la theorie de la relativite. *Comptes Rendus de l'Académie des Sciences* (*Paris*), **173**, 677-680.

[16] Gullstrand, A. (1922) Allgemeine Losung des statischen Einkorperproblems in der Einsteinschen Gravitationstheorie. *Arkiv för Matematik*, *Astronomi och Fysik Archives*, **16**, 1-15.

[17] Vasiliev, V.V. and Fedorov, L.V. (2024) Spherically Symmetric Static Problem of General Relativity for a Continuum. *Physics-Uspekhi* (*Advances in Physical Sciences*), **95**, 203-218.

[18] Lemaitre, G. (1934) Evolution of the Expanding Universe. *Proceedings of the National Academy of Sciences of the United States of America*, **20**, 12-17. https://doi.org/10.1073/pnas.20.1.12

[19] https://en.wikipedia.org/wiki/Observableuniverse