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Instantaneous Relativistic Precession of Mercury

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Abstract

The extra precession of Mercury—42.9 seconds of arc per century—explained by General Relativity (GR), is the result of a secular addition of 5.02×10^{-7} rad. at the end of every orbit around the Sun. We will analyze the instantaneous precession and its addition along one orbit, find out the magnitude of oscillations over the mean value, comparing key theoretical proposals. This angular instantaneous precession should be the result reaction of Mercury to the gravitoelectric/magnetic action produced by the geometric curve space-time at each single point of the elliptic orbit. The better we should know about this precession, the better we will determine the external action whatever the perturbing source is. The Doppler tracking of the MESSENGER spacecraft produces only 1 m error, enough precision to deduce the complete geodesic orbit of Mercury as an open free-fall path, isolated from other planets gravitational interference. The aim of this article is also to encourage JPL, IMCCE and other scientific teams to do so because, as far as I know, it has not been yet entirely measured by accurately tracking that motion.

Keywords

Gravitational Interaction, Relativistic Precession, Planetary Ephemerides, Mercury

1. Introduction

As General Relativity has determined, Mercury without taking into account the gravitational actions of other planets, tracks a geodesic as an open free-fall path, embedded in a geometric curved space/time. As an isolated motion, precession could be deduced through the Schwarzschild formulation. It is also deduced with Gauss, Lagrange and Landau equations as a result of the perturbing methods that resolve the effects over the orbit elements. Relativistic precession, is the angle that turns and drags the axes of the regular elliptic orbit. Instantaneous precession should be an almost simultaneous reaction to the external gravitational action, in direct connection with the curvature's intensity of the space-time at each single point of the trajectory.

Gravitation produces gravitoelectric effects in any target, that should be also consistent with the quantization of the gravitational field however, there is not a real verification for that [1] and finding out a solid and accurate theory of quantum gravity, still remains an open task [2].

Classic manuals and scientific research have usually focused on the cumulative secular precession, measured in arc. sec./century and located in one unique point. The main issue of that research has been the "perihelion shift" and new slight contributions just to obtain an accurate measure at this specific point. The perihelion precession was the first crucial contribution of GR, providing a solution to the anomaly observed by Le Verrier in 1859. It is essential to encourage the efforts of qualified scientific teams dedicated to uncover even the most infinitesimal secular advances, due to the Lense-Thirring effect (10^{-5} relative to GR), relativistic "cross-terms" in the post-Newtonian equations (10^{-6}), and 2PPN secular pericenter precession (10^{-7}). Detecting this new infinitesimal secular precession, means a difficult challenge, although its cumulative evolution [3]. Underline also the study of Mercury's perihelion advance looking for the exact geometric definition, with a special focus on the barycenter of the Sun/Mercury system and the method to test the theoretical results related with the experimental proofs [4].

The orbital precession, integral addition of the instantaneous angular one, is a continuous turn of the orbit axes with periodic oscillations either in a forward or backward motion, just as an added output effect of the external action exerted on the target at every instant. These oscillations periodic and not cumulative, located in different true anomaly angles, with two or four peaks and significant dependence on eccentricity, would have tangential amplitudes between 1 km or 22 km according to the different theoretical proposals. Although relativistic precession is not a direct measure, it can be obtained removing the gravitational action of the other planets, modeling the orbit of Mercury as an open free-fall geodesic path.

Scientific works about periodic angular/orbital precession are very few however, highlight the formulations of M. Soffel [5] [6]. A JPL team, published a key article [7], that mentions this periodic effects using PPN parameters. The article asserts that the accuracy of typical spacecraft ranging is about 1 m and then, this periodic effect can be easily measured by accurately tracking the motion of MESSENGER's orbit around Mercury.

As far as I know, it has not yet been full length measured by accurate tracking. Planetary ephemerides and accurate tracking of singular planets like Mercury, not only in one specific point-, is essential to confirm G.R. keystones and also to test any alternative or supplementary theories. The ephemerides give the positions of a planet along the orbit and must agree with the dynamical theories describing their motions [8].

This future measure of the angular instantaneous and orbital precession linked with the consecutive positions of Mercury, should answer next questions:

Is it right to accept a constant gradual orbital precession?

How large are the magnitude of oscillations, if there are any?

Why those oscillations with that magnitude are located specifically in those true anomaly angles?

Has been confirmed the relation between eccentricity and oscillations?

Which of these theoretic proposals fits on the real angular/orbital precession of Mercury?

Do these oscillations provide more information about the gravitational actions that produce them?

Have we alternatives if none of these formulations works?

It is difficult to understand if nobody knows the continuous motions, magnitude of the oscillations if there are any, transversal shifts and the complete line track of the geodesic as an open free-fall path along the 87.9 days that takes Mercury, our best gravitational laboratory, to complete his orbit.

2. GR Precession; Classic Formulation

The equation of the elliptic orbit is:

$$r = \frac{p}{1 + e\cos\left(\phi - \Delta(\phi)\right)}; \quad \phi = \text{true anomaly; } p = \text{semi latus} \tag{1}$$

where $\Delta(\phi)$, is a small angle that produces the orbit perturbation, from the Newtonian-Kepler ellipse. Misner, Thorne and Wheeler in their well known relativity textbook "Gravitation" [9], advanced a constant angular precession.

$\delta(\phi)$ = angular precession = $K = 3GM/c^2p$ = constant angular precession.

$\Delta(\phi)$ = orbital precession = $K \times \phi$

At the end of the orbit, the relativistic precession is:

$$\Delta(2\pi) = 2\pi K = \frac{6\pi GM}{c^2 p} \text{ rad. orbit} \tag{2}$$

c = speed of light, and for Mercury:

$\Delta(2\pi) = 5.02 \times 10^{-7}$ rad. orbit = 42.9 sec. arc. century.

Perihelion tangential shift ≈ 23.1 km. orbit.

The authors mentioned that there are also periodic non cumulative effects ≈ 1 km amplitude, but in 1973 had not been determined experimentally.

3. Schwarzschild Method

GR accepts also small periodic oscillations of the orbital precession based in Schwarzschild's methodological approach: [10]

$$r = \frac{p}{1 + e\cos\phi + \alpha(\phi)}, \quad \text{where } \alpha(\phi) \text{ is the perturbing function} \tag{3}$$

$$\alpha(\phi)=\frac{3GM}{c^2 p}\left[1+e^2\left(\frac{1}{2}-\frac{1}{6}\cos 2\phi\right)+e\phi\sin\phi\right] \tag{4}$$

Professor Berry [11] obtains a very similar result, although a different formulation.

$$\alpha(\phi)=\frac{GM}{c^2 p}\left[\left(3+2e^2\right)+\left(\frac{1+3e^2}{e}\right)\cos\phi-e^2\cos^2\phi+3e\phi\sin\phi\right] \tag{5}$$

The final precession at the end of a complete orbit Δ(2π) is the same as the expected one, but the intermediate values of the orbital precession $\Delta(\phi)$, are not now in a lineal progression as before. These periodic swing, is a result of the angular precession that involves the terms $\cos^2\phi$ and $\cos 2\phi$, and should produce in my opinion, ≈0.25 km forward/backward shift when $\phi=\pm\pi/2$, referred to the expected lineal precession.

The other terms of the equation, produces slight periodic variations of the radial distance, but not an angle perturbation.

4. The Laplace-Runge-Lenz Method

We can also study the effects of specific perturbing actions, method that should allow alternative results as those obtained solving the second order Schwarzschild's equation.

A solution to the orbital precession is based on the Laplace-Runge-Lenz vector, located in the same plane as the orbit and pointing in the direction of the perihelion. It is interesting to note that the LRL vector is very similar to the Hamilton vector, unfortunately this last one has vanished from textbooks on classical mechanics in twentieth century [12].

Vector's angular velocity, measures the precession if there is any external perturbation [13] [14] and following their research, we could obtain the instantaneous precession (angular precession), so that the gradual addition along the orbit (orbital precession), reaches a final value when $\phi=2\pi$.

The results of this method related to the angular/orbital periodic precession, are mentioned in [15].

$$\Delta(\phi)=\int\delta(\phi)\,\mathrm{d}\phi=K\left\{\frac{3/4e^2+1}{e}\sin\phi+\frac{1}{2}\sin 2\phi+\frac{1}{12}e\sin 3\phi+\phi\right\}\text{rad.} \tag{6}$$

Underline that the forward oscillation is located when $\phi<\pi$. However, may this position be consequence of a conventional criteria in the sign of precession. The amplitude of oscillations, involves a shift about 22 km forward and backward. Using a negative sign or a π drag in true anomaly angle, results in a symmetric solution very similar to the one obtained under the PPN formulation.

5. Potential Transmission and Precession Reaction

Consider an external action, linked to a relativistic gravitational potential and transfer action inside the target [16].

This potential, should be continuously emitted and updated from its central focus whatever could be the space/time curvature and/or an hypothetical background quantum transmission field. These should shape the curved space-time framework were gravitational effects, are the outcome of a "geometric" but not "frozen" structure of the universe.

We will analyze a target with a radial speed V_r in the same direction as the potential forward emission. The transit time of the potential crossing inside the target, will increase referred to the transit time when $V_r = 0$, and will decrease if they are moving in opposite directions. The larger or reduced transit time inside the target, is associated with (V_r/c) and the potential transmission is with $(V_r/c)^2$ [16] [17]. Consider also that a motion of particles in an external gravitational field with a Maxwell framework, is in first order equivalent to a dynamic system linked with $(V_r/c)^2$ [18]. We must remember also that energy/unit mass, has $(V)^2$ dimension.

We assume potential's transmission velocity (c), equal to that of light.

Potential $P(\phi)$ is then defined as a slight perturbation to the newtonian gravitational potential:

$$P(\phi) = -\frac{GM}{r}\left[1 \pm \left(\frac{V_r}{c}\right)^2\right] = -\frac{GM}{r} \pm S(\phi);\ S(\phi) = -\frac{GM}{r}\left(\frac{V_r}{c}\right)^2 \tag{7}$$

Perturbing potential $S(\phi)$ has physical basis, linked with the laws of impulse and momentum transfer, and the action/reaction effect of the usual mechanics.

Along the upward branch of the orbit, perturbing potential $S(\phi)$ increases gravity. Mercury will then move inward that means, a "previous" point of the canonical trajectory. The orbit, as a whole ellipse, must then rotate a forward angle. Along the descending branch of the orbit, perturbing potential decreases gravity. Mercury will then move outward, that means a "previous" point of the canonical ellipse. The orbit must also rotate now a forward angle, a positive instantaneous precession.

Underline also that, the effect of potential $S(\phi)$ inside any sphere or any compact three dimension target, has a resultant ratio of three times $(V_r/c)^2$. The transmission distance inside the target, has been enlarge with a new length, producing a light increase of the sphere's active volume and mass, that involves a three times ratio compared with the single particle effect [16].

Historic references of this perturbing potential, are the conventional mathematic fix inductive proposals and Weber-like gravitational interactions of Levy [19] and Assis [20]. Underline the divergences with $S(\phi)$ as this one has a deductive formulation of the potential's transit action inside three dimension targets, the geodesic pattern or/and a quantum field action/reaction framework and the instantaneous angular/orbital precession.

We will use Landau formulation [21] [22] and deduce the precession produced by a perturbing potential. This formula is suitable for any small perturbation and integration is performed over an unperturbed orbit [23].

$$\Delta(2\pi) = \frac{\partial}{\partial h}\left(\frac{2}{h}\int_0^{\pi} r^2 \delta U \mathrm{d}\phi\right) \tag{8}$$

where δU is the perturbing potential/unit mass and h = angular momentum/unit mass. The angular $\delta(\phi)$ and orbital $\Delta(\phi)$ precession are:

$$\Delta(\phi)=\int_0^{\phi}\delta\phi \mathrm{d}\phi=\frac{3GM}{c^2 p}\int_0^{\phi}\frac{\sin^2\phi}{(1+e\cos\phi)^2}\left(2-e^2+e\cos\phi\right)\mathrm{d}\phi \tag{9}$$

$$\Delta(\phi)=\frac{3GM}{c^2 p}\left(\frac{\phi+e\phi\cos\phi-e\sin\phi-\sin\phi\cos\phi}{1+e\cos\phi}\right) \tag{10}$$

This method obtains an orbital precession with four peaks and a shift of ≈2.3 km, forward (**Figure 1**), when ϕ = 5.4 rad./2.5 rad. and backward when ϕ = 0.9 rad./3.8 rad. Angular instantaneous precession $\delta(\phi)$, has a maximum when ϕ = 1.72 and ϕ = 4.57 rad., where radial velocity is very near from its maximum; is null where radial velocity is also null. Using Lagrange planetary perturbing equation, we reach a very similar result however, the maximum angular instantaneous precession comes when ϕ = 1.42 rad. and ϕ = 4.86 rad.

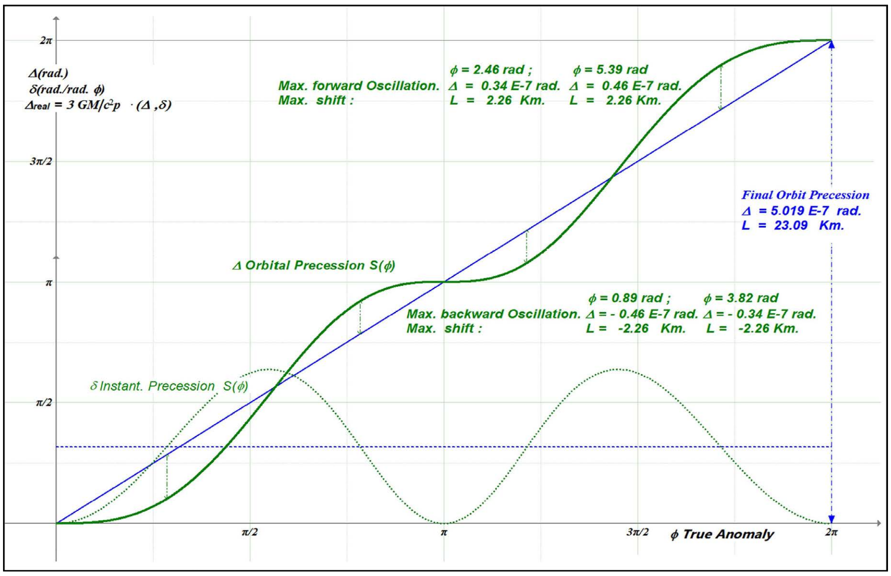

Figure 1. Angular/Orbital precession. Perturbing potential action.

6. Soffel-PPN Formulation

As I mentioned before, a significant research about periodic relativistic precession, was performed by M. H. Soffel [5] [6] using planetary perturbing equations, radial and transversal accelerations related with time, true anomaly and PPN parameters.

Another formulation of the perturbing gravitoelectric vector acceleration, is mentioned in [24], where periodic effects could be also deduced:

$$\boldsymbol{A}^{(1pN)}=-\frac{\mu}{c^2 r^2}\left(v^2-\frac{4\mu}{r}\right)\hat{\boldsymbol{r}}+\frac{4\mu}{c^2 r^2}(\hat{\boldsymbol{r}}\cdot\boldsymbol{v})\boldsymbol{v} \tag{11}$$

Following Soffel proposal, R. S. Park, J. G. Williams and a JPL team in their significant article mentioned before [7], displays the periodic orbital precession and the partial derivative with respect to the PPN parameters.

The formulation of Soffel, uses the Gauss planetary equation referred to the argument of the pericentre:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t}=\frac{\sqrt{1-e^2}}{nae}\left[-S\cos\phi+T\left(1+\frac{r}{p}\right)\sin\phi\right] \tag{12}$$

where S and T are the radial and tangential components of the perturbing acceleration and ϕ is true anomaly angle.

This equation results in the angular precession:

$$\delta(\phi)=-\frac{3GM}{c^2p}\left(\frac{3-e^2}{3e}\cos\phi+\frac{5}{3}\cos2\phi-1\right)\text{rad./rad.}\quad(<>\mathrm{d}\omega/\mathrm{d}\phi) \tag{13}$$

and then, orbital precession is:

$$\Delta(\phi)=\int_0^{\phi}\delta\phi\mathrm{d}\phi=\frac{3GM}{c^2p}\left[\phi-\frac{5}{6}\sin2\phi-\frac{3-e^2}{3e}\right] \tag{14}$$

This method obtains an orbital precession with two peaks and a shift of ≈20.3 km, forward and backward motions in relation with a constant angular precession. Angular precession has positive maximums between ϕ = 2.37 and 3.91 rad. and negative when ϕ = 0 and ϕ = 2π. This forward maximums are located when Mercury is at the farthest point from the Sun and the backward in the nearest, results that should need a reasonable explanation (Figure 2).

Semiaxis has also oscillations along the orbit when ϕ = π, with amplitudes of 9 km and eccentricity with 1.7×10^{-7} [25].

The equation has a term with an inverse proportion on eccentricity.

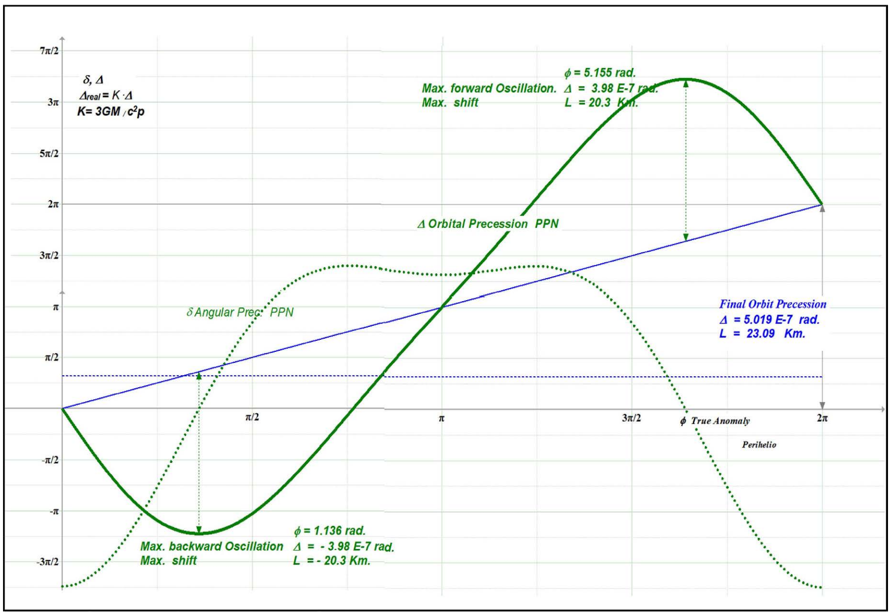

Figure 2. Angular/Orbital precession. PPN formulation.

Underline that the Soffel-PPN formulation is nowadays, the better scientific based and accurate proposal about the periodic oscillations of the orbital precession of Mercury.

7. Special Relativity. Relativistic Gravitational Force

Theory of Special Relativity (SR) and gravity in flat space, was left aside since the GR success as a new science, a geometric space/time framework that gives an exact explanation of the anomalous perihelion shift of Mercury. However, there have been many authors trying to analyze and resolve this precession under SR method. In [26], there is a complete summary beginning with the early works in 1898, 1906, new research until 2022, and also mentions proposals about gravitational fields and rotational time dilation.

There is also a recent and interesting article "Relativistic Gravitational Force" [27] that suggests to give up the principle of equivalence and focus again on SR. This useful proposal, predicts the correct perihelion shift of Mercury and all of the classic experimental tests of GR. This method, developed within the same flat space/time as quantum field theory, could be helpful for a possible transition to the framework of quantum gravity.

R.G.F. describes a precession equation based on Hamilton vector (**Figure 3**), as next equivalent formulation.

$$\Delta\phi = \frac{A}{e^2}\int_0^{2\pi}\left[\Sigma\left(B_i * r^{-i}\right) + B_4\right]\mathrm{d}\phi;\ \ i = 1,2,3 \qquad (15)$$

where A, B_i are constants. Trying by my side to follow and apply that formula, we should obtain the next orbital precession along the orbit, although may be different results related with B_i parameters, and display one alternative under a potential equation.

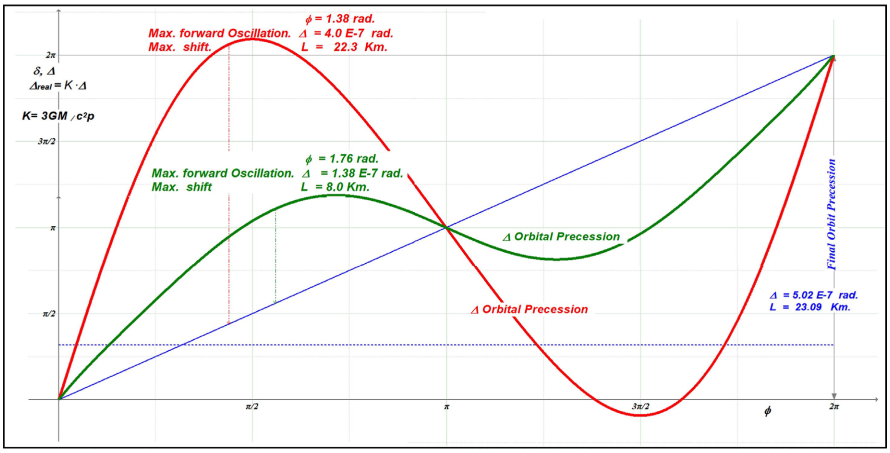

Figure 3. Orbital precession methods: R.G.F - Hamilton vector: (green). Laplace-Runge-Lenz vector: (red).

R.G.F's orbital precession, - with forward motion peak when ϕ = 1.76 rad., backward when ϕ = 4.52 rad. and ≈8 km of transversal shift-, has a similar formulation as the Laplace-Runge-Lenz method, but these last involves a shift about 22 km forward and backward. However as before, a different criteria in the meaning of the ± sign, should produce a symmetric function, similar as PPN equation, but a reduced magnitude.

8. Yukawa Gravitational Potential

The Yukawa potential is characterized by a power-law formulation related with the dimensionless -α- (strength referred to gravity) and -λ- (range). The Yukawa potential should be the static limit of an interaction due to the exchange of virtual bosons or the possibility that a nonzero graviton mass could lead to this potential [28]. The Yukawa potential is usually referred as a component of the fifth force.

$$Y_{Pot.} = -\frac{GM\alpha}{r}\exp\left(-\frac{r}{\lambda}\right) \tag{16}$$

Following Landau/Lifshitz formulation, we will obtain the angular/orbital precession [21] [23].

$$\begin{aligned}\Delta(\phi) &= \int \delta(\phi)\,\mathrm{d}\phi \\ &= \alpha\frac{2}{e}\int \frac{\exp\left(-\frac{r}{\lambda}\right)}{(1+e\cos\phi)^2}\left\{\frac{r}{\lambda}\left[2e+\left(1+e^2\right)\cos\phi\right]-(e+\cos\phi)\right\}\mathrm{d}\phi\end{aligned} \tag{17}$$

$\Delta(2\pi)$ is the relativistic precession at the perihelion and every pair of α and λ parameters, have a fixed constraint with it. As result of Equations (16) and (17), each λ is linked with only one α that fits with the final precession, summarized in next **Table 1**:

Table 1. Examples of λ and α parameters.

λ	2.9×10^{10}	5×10^{10}	8×10^{10}	1×10^{11}	1×10^{12}
α	2.5×10^{-7}	9.3×10^{-6}	1.6×10^{-5}	2.1×10^{-5}	1.1×10^{-3}

Orbital precession for $\lambda = 8 \times 10^{10}$, has similar evolution and maximum shifts as the PPN proposal. For other values, there are divergences between them (**Figure 4**).

If we consider the scalar-tensor fourth order gravity type model, then $\lambda \approx 2.82 \times 10^{10}$ and a non commutative spectral gravity, then $\lambda \approx 1.94 \times 10^{9}$ [29].

9. The Effect of Eccentricity

The PPN, Yukawa and L-R-L methods, have an eccentricity inverse proportional term. When $\phi = 2\pi$ or $\phi = \pi$, it has a null result, so final orbital precession in the perihelion, is not disturbed. However, the oscillations along the orbit are direct involved by that term, with increasing magnitudes as eccentricity decreases. Orbital precession under PPN proposal is:

$$\Delta(\phi) = \frac{3GM}{c^2 p}\left[\phi - \frac{5}{6}\sin 2\phi - \left(\frac{3-e^2}{3e}\right)\sin\phi\right]\text{rad.} \tag{18}$$

$\Delta(\phi)$ and maximum shifts in the peak point of each oscillation referred to the planets in the Solar system, is displayed in **Table 2**.

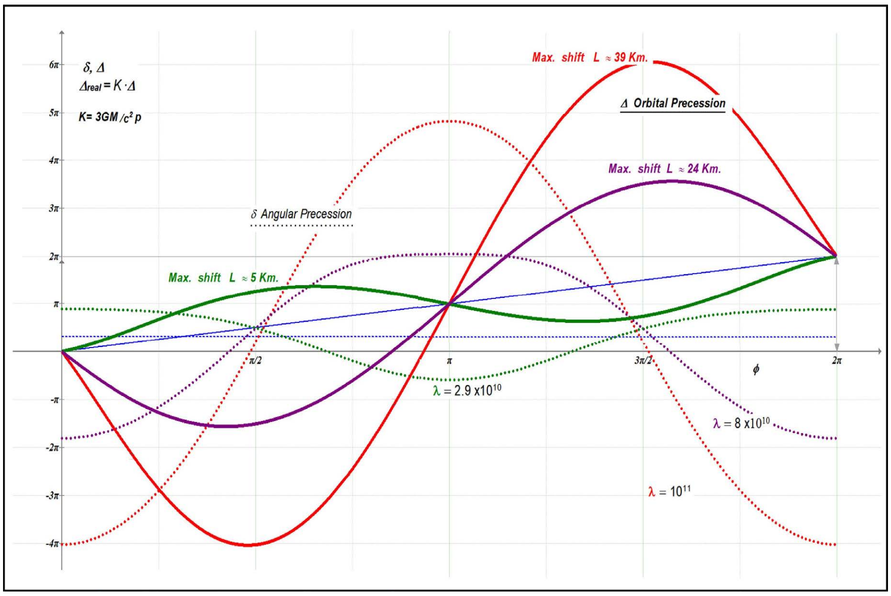

Figure 4. Angular/Orbital precession. Yukawa potential.
$\Delta(\phi) \approx K\left[A\phi^5 + B\phi^4 + C\phi^3 + D\phi^2 + E\phi\right]$

Table 2. PPN Peak orbital precession and shift at peak oscillation.

Planets	Excec.	Semi latus (p)	Δ(2π)	Perih.Shift	*Oscillation Max. Shift*		
		m	rad.	km	ϕ rad.	±Δ(ϕ) rad.	±km
Mercurio	0.20563	5.5461E+10	5.02E−07	23.1	1.13 & 5.15	3.98E−07	**20.3**
Venus	0.0068	1.0820E+11	2.57E−07	27.6	1.55 & 4.73	6.02E−06	**652**
Tierra	0.0167	1.4956E+11	1.86E−07	27.4	1.53 & 4.76	1.77E−06	**265**
Marte	0.0934	2.2594E+11	1.23E−07	25.5	1.34 & 4.95	2.12E−07	**48**
Júpiter	0.0484	7.7658E+11	3.58E−08	26.6	1.44 & 4.84	1.18E−07	**92**
Saturno	0.0538	1.4253E+12	1.95E−08	26.4	1.43 & 4.85	5.79E−08	**82**
Urano	0.0472	2.8686E+12	9.70E−09	26.6	1.45 & 4.84	3.28E−08	**94**
Neptuno	0.0086	4.5041E+12	6.18E−09	27.6	1.55 & 4.73	1.14E−07	**515**

Eccentricity has a clear influence in the orbital precession and tangential shift at the oscillation's peak, as it happens with Venus, Neptune and the Earth. As the magnitudes are significant, it is necessary to confirm if those shifts have been really detected and observed by the tracking of the planets mentioned. These effect should be also confirmed with the Moon using the LLR, and also in the near future, with other moons with a very low eccentricity as there are some around Jupiter.

We should admit that any equation with an inverse proportional term, is also valid for very low eccentricities; otherwise, it is difficult to accept an "*ad-hoc*" formulation if the experimental tracking results, does not fit with the expected ones.

10. Conclusions

The geodesic orbit of Mercury is an open line, straight free fall path embedded in a curve space-time. This track is equivalent to the addition of an elliptic orbit, characterized with the usual osculating parameters, and a continuous turn of the orbit as a whole ensemble. This last motion is the precession, which can be displayed by an external perturbing action that, leaving aside the gravitational effect of the other planets of the Solar system, results in the GR precession. Research about this issue, has usually been focused on the angular distance within two consecutive perihelions, cumulative secular effect that represents the final precession at the end of each orbit, summarized as seconds of arc per century. Less interest has deserved the evolution of precession along the 87.9 days that takes Mercury to complete his orbit.

The angular precession and its addition in the orbital precession represent the reaction of the planet to an external perturbing action whatever could be the curve space-time producing an expected geodesic pattern, or/and a quantum field. Underline that gravitational action is a single unique issue, whatever could be cut off in its components, just to identify and study the perturbing actions and linked reactions.

Theoretic proposals mentioned in this work, have different results related with the angular/orbital precession in each point of the orbit. However, there are similarities in their formulations and also clear divergences on the magnitude and peak location of them. It should be appropriate to explain gravitational action, linked with precession reaction, as a complete ensemble that involves theoretic and experimental tracking detection of this motion.

Theoretic research about this issue is unusual and the detailed tracking detection of orbital precession, has not experimentally been determined as far as I know, although a better well-founded information is welcome.

Tracking and modeling the complete orbit of Mercury as a free geodesic trajectory, should be incorporated, in programs as INPOP-IMCC and DE-JPL [30] [31].

The aim of this article is also to encourage these scientific teams to perform, publish it and close an open issue concerning astrophysical research, related with the first and most crucial test of General Relativity.

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Thanks to James G. Williams (JPL) as we debated about secular and periodic relativistic terms of Mercury, that was mentioned in the article [7].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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