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Black Hole as a Topological Hole in the Fabric of Spacetime

Maciej Rybicki

Independent Researcher, Kraków, Poland
Email: maciej.rybicki.pl@gmail.com

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Abstract

The current model of a Schwarzschild black hole is analyzed in terms of consistency of its basic properties. The concept of gravitational singularity involves mathematical and physical problems; however, replacing singularity with a hypothetic ultimate state of matter characterized by extreme yet still finite properties, also leads to significant difficulties. Likewise, the concept of event horizon defined as an immaterial sphere (with the singularity at event horizon considered to be an apparent effect due to "improper" Schwarzschild coordinates) proves to lack internal consistency. The mathematical correctness and physical adequacy of the alternative coordinates applied to the Schwarzschild metric in order to eliminate singularity at the event horizon are called into question. The main focus in this respect is put on the Kruskal-Szekeres coordinates. As a solution to the revealed problems, a new model of black hole is proposed. It defines black hole as a topological hole in the "fabric of spacetime". The main feature of this model is the absence of black hole interior and singularity, interpreted as the absence of spacetime beyond the event horizon (Schwarzschild sphere). The whole mass of black hole is thought to be contained in the Schwarzschild sphere—a material shell of the supposed quantum-mechanical properties, constituting topological boundary—the "edge of spacetime", at which all geodesics terminate. The Schwarzschild metric remains valid in the new model, albeit as limited to the black hole exterior and as associated solely with the Schwarzschild coordinates—the only ones that correctly define the event horizon as a physical singularity.

Keywords

Black Hole, Schwarzschild Metric, Schwarzschild Coordinates, Kruskal-Szekeres Coordinates, Event Horizon, Gravitational Singularity, Coordinate Singularity, Topology of Spacetime

1. Introduction

The modern theory of black holes originates from the Albert Einstein's theory of gravity (general relativity). It is a consequence of the spherical solution to the Einstein's field equations [1], derived by Karl Schwarzschild [2] [3] (called the Schwarzschild metric)—applied to a simplest case of the spherically symmetric non-rotating uncharged mass. The real existence of black holes, for a long time questionable, is nowadays substantiated by the indirect and direct observations, including the first images of supermassive black holes in the center of galaxy M87 and in the center of Milky Way (Sagittarius A) [4] [5]. Despite these imposing achievements, black holes are still perceived as the extremely unusual (though ubiquitous) objects, with some properties escaping the ultimate understanding. The concept of black hole (BH) encapsulates the main problem in modern theoretical physics, unsolved for several decades, which is connecting general relativity (GR) and quantum mechanics (QM) into one unified theory. So far, this goal is still elusive, even though there exist around dozen different working theories collectively referred to as "quantum gravity", none of them however really effective and complete. Black hole is a model example of such unification made by nature itself; therefore, the complete description of black hole and the self-consistent, experimentally confirmed theory of quantum gravity are like two sides of the same coin. For the time being, the generally accepted model of black hole (here referred to as "current BH model") is based exclusively on GR, except for the Hawking radiation—a hypothetical effect of the black body thermal radiation due to quantum fluctuations in the close neighborhood of the event horizon.

The complete model of black hole involves a considerable part of physics and mathematics, whereby topology is one of the relevant topics examined in multiple papers [6]-[10]. According to the Hawking's black hole topology theorem, in the case of 4D stationary black hole immersed in the 4D (3 + 1) stationary spacetime defined as the asymptotically flat Lorentzian manifold, each cross-section of the event horizon is topologically a 2-sphere. In this paper, we focus however on a somewhat different and likely under-researched topic, also connected with topology, namely the local topology of spacetime due to the presence of a black hole.

There is a broad consensus among physicists that the current BH model with the mathematically defined gravitational/curvature singularity in the center is only a makeshift solution that will give place to a future ultimate model devoid of infinities. What we suggest instead is that the current BH model is not so much incomplete, but is basically inadequate, and therefore should be revised (or replaced)—rather than complemented. The particular subject of our analysis is the concept of gravitational singularity, along with the concept of event horizon interpreted as an immaterial boundary in the basically smooth spacetime (at least as perceived by an infalling observer). It should be strongly emphasized right now that we not only question the concept of singularity as interpreted literally in the mathematical sense (*i.e.*, defined by infinite density and curvature, and by zero volume), but we also challenge any solution with the BH singularity interpreted as a central "core" featured by enormous, but still finite density and curvature, and by extremely small yet nonzero volume. These two BH models, that we call respectively "current" and "prospective", constitute a dominant idea about black holes, here called the "current BH paradigm".

We aim to show that the current BH model causes problems that would presumably not vanish in the prospective BH model (with a material core)—even if some quantum-mechanical mechanism is involved. The preliminary analysis of these problems leads us to a hypothesis going beyond the current BH paradigm. It states that the black hole not so much "has a definite topology" (as any clear-cut physical object does), but has essentially a topological origin and nature, determining thus the local topology of spacetime. Using a classical metaphor, black hole is not a "donut", but is a "hole in the donut". The author's intention is to develop (in next papers) this hypothesis to a more exact form—once it passes the preliminary verification in a public discussion. All the researches sharing, at least tentatively, the author's view, are welcome to contribute to this task. The critical opinions are welcome just as much.

2. Basic Facts about Black Holes (Compliant with the Current Model)

Black holes are divided in a twofold way: due to their mass (class) and due to physical properties other than mass (type). The first category, in particular when constrained theoretically, is extremely capacious; it covers the range of approximately 10^{50} orders of magnitude. The upper mass-limit, known as the Eddington limit [11], refers to the maximum rate at which black hole can absorb the infalling matter, in connection with the age of the universe. At the present epoch, the Eddington limit amounts to $2.7\times10^{11} M_{\odot}$, *i.e.*, 5.1×10^{41} kg (comparable to the typical mass of galaxies), while the greatest supermassive BH detected so far (Phoenix A) is estimated at $1.26\times10^{10} M_{\odot}$, *i.e.*, 2.5×10^{40} kg, which fits neatly into this limit. Instead, the lower mass-limit is of the order of Planck mass $\sim2 \times 10^{-8}$ kg. The Schwarzschild radius would then equal the Planck length $\sim10^{-35}$ m, coinciding with the respective Compton wavelength. As long as the Planck mass-limit does not necessarily imply the real existence of Planck black holes, one should rather not expect black holes below this limit, e.g., identified with elementary particles. For example, if the Compton wavelength for electron ($\sim2.4 \times 10^{-12}$ m) were identified with the Schwarzschild radius, then the required mass of respective black hole would be $\sim10^{16}$ kg—around hundredfold mass of Mount Everest, some 10^{46} times bigger than the mass of electron. Therefore, although Planck mass exceeds by many orders of magnitude the masses of elementary particles (22 orders in the case of electron), there are reasonable grounds to believe that Planck mass determined by the fundamental constants $\hbar$, c and G is the smallest theoretically admissible mass of BH.

The second criterion divides black holes according to their (possible) rotation and electric charge. The respective types are: nonrotating, uncharged BH (Schwarzschild), rotating uncharged BH (Kerr), non-rotating charged BH (Reissner-Nordström), rotating charged BH (Kerr-Newman). Our research object is the Schwarzschild BH, connecting simplicity with all paradoxical properties, being thus most suitable for our purpose. This type of BH is like Margherita pizza among other pizzas.

Unlike the degenerate stars whose states of equilibrium are, each time, attained due to the balance between inwardly directed gravity and outwardly directed matter degeneracy pressure restraining gravity, the Schwarzschild black hole is determined by pure gravity. This is because, starting from the mass threshold of about 20 - 25 solar masses (for the main sequence star) or 2 - 3 solar masses (for the remnant core after a supernova), no known quantum mechanism similar to the Pauli exclusion principle and the resultant electron degeneracy pressure, or neutron degeneracy pressure, can prevent a star that has run out of the nuclear fuel from the ultimate gravitational collapse. According to the Penrose singularity theorem [12], any cluster of mass under definite mass-volume conditions, even if not exactly spherical, immutably collapses to the BH *singularity*—a point in space (or a strictly one-dimensional worldline in spacetime) that, despite the null volume, contains the whole collapsed mass "compressed" to the infinite density. In the case of active BH (absorbing mass from external environment), almost the whole mass is contained in the singularity, that is except for the fraction "temporarily" located between the event horizon and singularity. In terms of the spacetime geometry, singularity is the point marked by infinite curvature.

Instead, *event horizon* is the purely theoretical sphere surrounding singularity at a distance of Schwarzschild radius. The Schwarzschild radius r_S is a *scalar quantity* applied to the curved spacetime (*i.e.*, Riemannian geometry)—and not a "path" delineated in Euclidean space. It is defined as the "areal radius"—the circumference of the Schwarzschild sphere divided by 2π. Consequently, any other radius r attached at the singularity is defined in the similar way. Basically, the radius r (including r_S) is referred to a distant observer, so it is a *coordinate quantity*. In that perspective, the curvature associated with radius is not due to "bending" in whatever lateral direction, being instead inwardly directed, so that spacetime is stretched along the radial direction [13].

Although immaterial, the event horizon results however in dramatic consequences for any object (e.g., human observer) entering the BH interior. These are namely: inability to escape, spaghettification due to extreme tidal forces, and the loss of any specific properties at the singularity. Any information about the matter consumed by BH and then evaporated through the Hawking radiation is lost, which is referred to as "information paradox"—a result violating the principle of unitarity in QM. According to the "no-hair theorem" [14], the only physical quantities describing black holes are mass (linearly connected with size/radius), angular momentum and electric charge. In the case of Schwarzschild black hole, this short list is further reduced to mass and size.

3. Singularities Due to the Schwarzschild Coordinates

3.1. Schwarzschild Metric

The equation for Schwarzschild metric is derived from the Einstein field equations (EFE) [1]. Compactified to a single tensor equation, they read:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

$R_{\mu\nu}$ —Ricci curvature tensor,

$g_{\mu\nu}$ —metric tensor,

R —scalar curvature (whereas $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor $G_{\mu\nu}$),

Λ —cosmological constant,

$T_{\mu\nu}$ —stress-energy tensor,

κ —Einstein constant: $\kappa = 8\pi G c^{-4}$ (where G is the Newton's gravitational constant and c is the speed of light in vacuum)

Replacing Cartesian coordinates t, x, y, z with the polar coordinates: t, r, θ, ϕ, assuming $\Lambda = 0$, and assuming that electric charge and angular momentum of the massive object both equal zero, Karl Schwarzschild obtained the spherically symmetric, static vacuum solution to the Einstein field equations, here written using the metric signature convention −, +, +, + (we skip respective derivation obtainable from many sources, e.g., [15] [16]):

$$\mathrm{d}s^2 = -\left(1 - \frac{r_S}{r}\right) c^2 \mathrm{d}t^2 + \left(1 - \frac{r_S}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2 \tag{2}$$

where $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2$ (θ —the angle from "North Pole" around *x*-axis, ϕ —the angle from "Prime Meridian" around *z*-axis, both expressed in radians). Rewriting $r_S = 2GMc^{-2}$ and defining $c = 1$ gives:

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{r}\right) \mathrm{d}t^2 + \left(1 - \frac{2GM}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2 \tag{3}$$

$\mathrm{d}s^2$ —spacetime interval squared,

t —time coordinate for infinitely distant stationary observer,

r —radial coordinate defined such that circumference of the sphere at r is $2\pi r$,

r_S —Schwarzschild radius

In terms of physics, the Schwarzschild metric is, in essence, the *spacetime interval* squared ($\mathrm{d}s^2$) applied to the curved spacetime. Instead, when considered as a mathematical object, Schwarzschild metric is the *function* of the variables t and r (assuming constant θ, ϕ and M). When referred to a particular instant of t, this function is defined with respect to a one variable only, namely the radial coordinate r. It is easy to notice that for $r \to \infty$, the Schwarzschild metric is asymptotic to the Minkowski metric—becoming thus the "usual" spacetime interval, except for being expressed in spherical coordinates. We are particularly interested in the physical meaning of singularities predicted by the Schwarzschild metric, using the Schwarzschild coordinates, *i.e.*, the results obtained for $r=0$ and for $r=r_S$.

3.2. Gravitational Singularity at $r=0$

For $r=0$, the temporal and spatial separation terms are respectively:

$$-\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2 \quad (\text{undefined}) \tag{4}$$

and

$$\left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2 \quad (\text{undefined}) \tag{5}$$

If, instead of $r=0$, we consider the limit $r\to 0$, then we obtain:

$$\lim_{r=x\to 0} f(x)-\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2=\infty \tag{6}$$

and

$$\lim_{r=x\to 0} f(x)\left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2=0 \tag{7}$$

These results signify "true" singularity, otherwise called "physical" or "absolute". It cannot be removed by any alternative coordinates applied to the Schwarzschild metric, different from the Schwarzschild coordinates t, r, θ, ϕ.

3.3. Singularity at $r=r_S$

For $r=r_S=2GM$ (Schwarzschild radius) one obtains:

$$-\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2=0 \tag{8}$$

and

$$\left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2 \quad (\text{undefined}) \tag{9}$$

By expressing the radial coordinates as limits, we can distinguish between two variants, namely: $r\to r_S$: $r>2GM$, and $r\to r_S$: $r<2GM$. As a result, we have:

$$\lim_{r=x\to 2GM} f(x)-\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2=0 \tag{10}$$

and, regarding both variants written above:

$$\lim_{r=x\to 2GM} f(x)\left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2=\pm\infty \tag{11}$$

Same as in the case $r\to 0$, this also signifies singularity. However, unlike the former, this singularity is not considered "true", but "apparent", due to general assumption as to the "smoothness" of spacetime, and (specifically) because it is removable using alternative coordinates, e.g., the Kruskal-Szekeres coordinates.

3.4. Schwarzschild Metric for $0 < r < r_S$

As implemented to the interior of black hole, the Schwarzschild metric reverses the signs of temporal and radial terms, transforming their shape in such a way that time and radial coordinates swap the roles, so that r becomes *time-like* and t becomes *space-like*. This swap is interpreted in purely formal terms—as the exchange of coordinates, but *not* the exchange of time and space themselves. For the observer traveling through the interior of black hole towards the singularity, an ultimate goal (singularity) would not be the point in space, but the point in time, so that location "in the distance" changes to "in the future". However, this change is wholly apparent and illusory: after all, the goal of every travel is a point in spacetime (an event), located both "in the distance" and "in the future".

The fact that Schwarzschild metric behaves "properly" for $r > r_S$, in the range from $r \to \infty$ up to $r \to r_S$ (but not any further, *i.e.*, for $r \leq r_S$) gives rise to suspicion that event horizon might be a true limit of spacetime—and not just an "apparent artifact". Certainly, the question of alternative coordinates has to be examined in that context; we do that in brief in Sec. 5.

4. Problems with "True" Singularity at $r = 0$

4.1. Why Infinities Are Not Welcome in Physics?

Let us return to the "true" singularity at $r = 0$, starting with a brief overview on the concept of infinity in mathematics—due to the fact that mathematics is the language for physics, or even that nature itself is mathematical at its core.

The ancient concept of infinity gained a strict form with the introduction of the concept of *limit* in the infinitesimal calculus invented by Newton and Leibniz. This concept was gradually developed and formalized within the mathematical analysis, mainly by Euler, Bolzano, Cauchy and Weierstrass. A fundamental insight into mathematical infinity as a property of sets has been made in the Cantor set theory including such concepts as: (1) *cardinality* – revealing different "sizes" of infinite sets; (2) *countability* and *uncountability* – defined by the criterion of feasibility of the one-to-one correspondence (bijection) with natural numbers; (3) *power set* – defined as the set of all subsets of a given set. Along with related concepts such as Fourier series/integrals, Dedekind cut and Dedekind ring, infinities and limits appear in various branches of mathematics: real analysis (positive and negative infinities), complex analysis (single infinity), non-standard analysis (reintroducing the concept of infinitesimal quantities), projective geometry, vector spaces, fractals, etc.

A specific mathematical object combining finite and infinite terms is the Dirac delta function. The value of this function is zero for all arguments $x \neq 0$, and is infinite at $x = 0$, whereas its integral over the entire real line equals 1. In the smooth (differentiable) form defined by $x = a$, $a \to 0$, the value of Dirac delta function has the *limit* in infinity at $x = 0$, while its integral still equals 1. Due to these properties, Dirac delta function serves for modelling physical objects or phenomena that are spatially or temporally "punctual" (point masses, impulses). In this and other above-mentioned ways, mathematical infinity enters physics.

We should however make a distinction between mathematical infinities used as a tool to describe physical phenomena that are only seemingly infinite (e.g., "infinite" velocity in the scissor paradox in STR) and infinites that *seem to be* inherently present in nature, and consequently in physics. The latter applies in particular to the quantum field theories (QFT). The relevant question is: are the infinities indeed present in physical reality? The dominant answer is negative. Whatever the case may be, infinities are, in general, not welcome in physics, and are usually considered a sign of theory breakdown or incompleteness. Intuitively, the reason to eliminate infinities from physics is the fact that any infinite quantity is not just *slightly greater* than any "very great" yet finite quantity, but is always *infinitely greater*. Referring this problem to black holes: if the matter density in the gravitational singularity is infinitely greater than the otherwise incredibly great density of e.g., neutron star, or even the hypothetic quark star, then the concept of density itself ceases to have any physical sense. Concluding, infinities are basically unphysical, and are usually considered such.

4.2. A Comment to Renormalization

A set of mathematical techniques called *renormalization* comes to the aid in order to remove the unwanted infinities from QFT. A widely acceptable condition for any theory containing infinities to be considered theoretically valid is the formal property of being renormalizable. However, such prominent physicists as Dyson [17], Dirac [18] [19] and Feynman [20], despite their own contribution to different quantum fields theories and/or to the renormalization itself, remained persistently skeptic about the legitimacy of this technique. According to Dirac, "This [renormalization] is just not sensible mathematics. Sensible mathematics involves disregarding a quantity when it is small—not neglecting it just because it is infinitely great and you do not want it!". In turn, Feynman called renormalization a "hocus-pocus", stating decidedly in conclusion: "I suspect that renormalization is not mathematically legitimate".

However, the above objections—arguably legitimate—apply to the quantum field theories that are, after all, "perturbatively renormalizable". Meanwhile, general relativity *is not renormalizable* in whatever natural or artificial manner, which means that infinities produced by GR cannot be absorbed by finite parameters, using standard perturbative techniques. The non-renormalizability of GR makes (beside other reasons) the task of connecting this theory with QM into one whole (quantum gravity) so difficult, if not even impossible, without revision of either of these theories, presumably GR.

In the case of gravitational singularity, the problem is all the more critical. This is because infinities do not "appear" as a theoretical limiting case due to some external conditions, being thus potential but not necessarily implemented. On the contrary, according to the current BH model, gravitational singularity is an unavoidable result of the gravitational collapse, and thus is an actual "physical incarnation" of infinity. Therefore, not only infinitely great, but also infinitely small quantity (volume) cannot be, using Dirac's words, "disregarded" by "sensible mathematics"—because this would mean elimination of the singularity itself.

4.3. Inability of GR to Describe the Gravitational Singularity at $r = 0$

The fundamental problem with the gravitational singularity in the center of BH is that GR *predicts* this singularity, but is unable to *describe* it ("prediction" is not tantamount to "description"). This "inefficiency" of general relativity is usually referred to as "the laws of physics break down at singularity" or: "general relativity breaks down near singularities, where quantum effects are expected to play crucial role". It is therefore generally (though rather tacitly) accepted that singularity is only a temporary solution that will vanish once the long-sought quantum gravity is nicely completed. This so far elusive theory is thought to replace singularity with some finite state of matter, presumably the most fundamental one. However, the physical properties of such hypothetic state of matter are still unknown and in fact are theoretically doubtful. Namely, apart from the arguments against the gravitational singularity understood literally (infinite curvature, infinite density, zero volume) there can also be provided arguments (discussed below) against "physical" singularity, here referred to as "BH core"—understood as the state of matter featured by extreme, but still finite parameters.

4.4. Gravitational Singularity and the Origin of Mass

A specific physical reason to question the gravitational singularity is that black hole is a massive object. The chain of reasoning is in this case the following. Mass is an unquestioned property of any (type of) BH; mass is connected with matter; matter is composed of particles; the particle specifically responsible for mass is the Higgs boson, thought to provide the rest mass to any massive particle via the Higgs mechanism, triggered by the spontaneous symmetry breaking (SSB). Once we accept the Standard Model of particle physics, we have to conclude that mass (including the BH mass) is impossible without elementary particles interpreted as the carriers of mass, and without the Higgs boson interpreted as the source of mass. Admittedly, elementary particles may lose or alter their identity, may also change mass to pure energy carried by photons; however, considering that BH singularity is basically a stationary object, mass should have a material form, which is irreconcilable with zero volume.

Each elementary particle has different "sizes", depending on the experimental criterion applied, e.g., the size of electron can be defined by the Compton wavelength, de Broglie wavelength or the Coulomb's classical radius. This is quite "normal" for the quantum realm, in which the answer is usually not "objective", but depends on the question, identified with a specific act of observation. Because of these different criteria, and because the size is not itself essential in particle physics, it is useful to treat elementary particles as the point-like objects, which is usually referred to as "they don't have size". However, it is merely a useful convention literally suited to a photon (massless particle) only. From the fact that the size of elementary particle is not unambiguously defined in a classical way applicable to macro-objects, in no way follows that the massive particles have zero size strictly required by the concept of singularity. Concluding, if singularity contains mass, it cannot have zero volume.

4.5. Can Gravitational Singularity be Replaced with a Matter Marked by Finite Properties?

A natural solution to the singularity problem, compliant with the current BH paradigm, consists in replacing singularity with some state of matter with finite properties. This, however, suggests that not only general relativity, but also quantum mechanics, along with respective Standard Model of particle physics is incomplete in its present form—being unable to provide an appropriate solution. This is because the most fundamental constituent of the baryonic matter, *i.e.*, quark, cannot be used to replace singularity, since it is already involved (in a form of the *quark degeneracy matter*) in the hypothetical model of quark star, not identifiable with a black hole. So, a new, so far undiscovered state of matter is required. This otherwise logical conclusion does not however seem to be a proper track in solving the BH singularity problem—at least for two reasons. First, because singularity features all classes of black holes: primordial, stellar, intermediate and supermassive—that drastically differ by mass. It would be highly improbable if the same state of matter (with identical quantum mechanism behind) would prevent from singularity in so extremely different conditions. Besides, considering that theoretical model of quark star is fundamentally different from the model of black hole, replacing singularity by a certain highly exotic state of matter would require existence of the constituents of baryonic matter even more elementary than quarks. Although this cannot be completely ruled out, it does not however seem a promising lead. It is rather more reasonable to stay with the current assumption according to which, in the well-defined conditions responsible for creating black holes, nothing can really prevent from the ultimate gravitational collapse. However, unlike what is currently believed, this collapse does not lead to the gravitational singularity, but to some fundamentally different result connecting gravity with quantum effects and curvature with topology—thus affecting the spacetime itself. This "different result" is exactly what we suggest in this paper.

5. Problems with "Apparent" Singularity at the Event Horizon

5.1. Alternative Coordinates as a "Remedy" for the Singularity at $r = r_S$

Several types of coordinates have been invented to eliminate the event horizon singularity predicted by the Schwarzschild polar coordinates t, r, θ, ϕ, applied to the Schwarzschild metric. From among the wide range of "alternative" coordinates: isotropic, Gullstrand-Painlevé, Lemaître, Eddington-Finkelstein, Kruskal-Szekeres, we will focus on the latter [21] as the representative and likely most effective example. Kruskal-Szekeres coordinates smartly change the metric "distorted" due to Schwarzschild coordinates into a smooth highway from infinity to the curvature singularity at $r = 0$. In geometrical terms, the respective idea consists in converting spherical geometry of the Schwarzschild chart into the hyperbolic geometry of the Kruskal-Szekeres chart. The whole construct (including a redefinition of the Schwarzschild coordinates and, consequently, a new formulation of the Schwarzschild metric) is indeed very ingenious and effective. Not questioning these advantages, and indicating the need of further research, let us focus on the basic assumptions on which these coordinates are based.

The intended purpose is achieved by substituting the coordinates t and r with the new coordinates: T and X, defined as follows (using the convention $c = 1$).

For the exterior of the event horizon ($r > 2GM$), T and X are:

$$T = \left(\frac{r}{2GM} - 1\right)^{1/2} \mathrm{e}^{r/4GM} \sinh\left(\frac{t}{4GM}\right) \tag{12a}$$

$$X = \left(\frac{r}{2GM} - 1\right)^{1/2} \mathrm{e}^{r/4GM} \cosh\left(\frac{t}{4GM}\right) \tag{12b}$$

Instead, for the interior of the event horizon ($0 < r < 2GM$), T and X are:

$$T = \left(1 - \frac{r}{2GM}\right)^{1/2} \mathrm{e}^{r/4GM} \cosh\left(\frac{t}{4GM}\right) \tag{13a}$$

$$X = \left(1 - \frac{r}{2GM}\right)^{1/2} \mathrm{e}^{r/4GM} \sinh\left(\frac{t}{4GM}\right) \tag{13b}$$

Despite the complex, highly counter-intuitive form of the Kruskal-Szekeres coordinates, one thing is clear: they are intended and designed to eliminate the coordinate singularity at the event horizon, and to cover the entire spacetime manifold of the maximally extended Schwarzschild metric. The mathematical problem with these coordinates (physical argument is discussed in the next subsection), does not concern their formal efficiency, but lies in the general assumptions behind; therefore, we will focus on latter and skip the former. The relevant question is: what is the *mathematical* difference between the Schwarzschild coordinates and the Kruskal-Szekeres coordinates?

The answer is as follows. In both cases the respective metric can be defined as a mathematical function, in which the coordinates are the function arguments, (forming together the domain of function), and the resultant spacetime intervals (combinations of space and time separations) are the function values—forming together the codomain of function. However, crucial is the difference: the Schwarzschild coordinates are "uniform", which means that arguments (in particular the value of r are uniformly defined along the whole *x*-axis—thus properly in a mathematical sense. This links with a physical property: the Schwarzschild coordinates t and r are "natural", which means they are determined according to the standards of time and length based on the "clocks" and "rulers" used by a distant observer. This "protocol" is consistent with both special and general relativity, according to which an indispensable condition to detect correctly time dilation is that observer must use the standard clock at rest ticking at "normal" rate. i.e., the *standard clock*. In the case of the curved spacetime, "standard" means also "local". Similar demands apply to the length contraction.

In contrast, the Kruskal-Szekeres coordinates T and X are neither "natural" nor "uniform"; in particular, they are differently defined for the "exterior" and "interior" of black hole—in order to obtain smooth spacetime, devoid of singularity at the event horizon. In other words, the discontinuity and non-uniformity of the Schwarzschild metric outside and inside the event horizon is eliminated by shifting these properties onto the coordinates. However, changing the meaning of arguments depending on their location on x-axis is *mathematically illegitimate*; it's like changing the rules of play during a single chess match. Using the Feynman's words, it looks like a mathematical "hocus-pocus", which in this case consists in *homogenizing* the spacetime by applying the *non-homogenous* coordinates.

Let us notice that similar flaws also characterize the other mentioned above alternative coordinates. The Eddington-Finkelstein coordinates, designed for null geodesics (photon worldlines) are differently defined for the exterior and interior of BH, so they are not "uniform". In turn, the Gullstrand-Painlevé and Lemaître coordinates are not "natural", since they apply the proper time τ (in the latter case, for objects freefalling along the radial geodesic) instead of the coordinate time t appropriate for a distant observer.

5.2. Physical Argument: Are the Results Obtained Using the Kruskal-Szekeres Coordinates Consistent with Observations?

The concept of event horizon interpreted as an immaterial boundary leads to the following paradox. According to GR, an ultimate destiny for any object falling on black hole is the gravitational singularity in the center. However, also according to GR, for any distant observer, and in case of any particular black hole observed (at least potentially) in the universe, the coordinate time connected with an infalling object is increasingly slow-paced, eventually becoming "frozen" at the event horizon, due to asymptotically infinite gravitational time dilation. This manifests (as an observable) in a form of increasing (to infinity) gravitational redshift. Let us notice that the "inability" to observe an object crossing the event horizon cannot depend—and really *does not depend*—on the choice of the *type of* coordinate system, since it is an observational prediction (or just fact) that *will not disappear* if we change the Schwarzschild coordinates for the Kruskal-Szekeres coordinates or any other one designed to remove singularity. Admittedly, we may variously interpret observational data according to different models adopted, but we cannot just assert, *contrary to the facts*, that these data change, or disappear, together with exchange of the coordinate systems. Different types of coordinates are not like different "glasses" through which a given effect (gravitational redshift, in this case) is either observed or not observed. The photon reaching our eyes, or detectors, will not stop being redshifted only because we change one system of coordinates for another one! In this sense, only the Schwarzschild coordinates are "true" (physically adequate), as being consistent with observations—whereas the other ones, including the Kruskal-Szekeres coordinates, *are not adequate.* It is of crucial importance to accept the obvious, namely that gravitational redshift in the neighborhood of the event horizon is a directly measured *observable* and not just one of alternative *interpretations*, each one associated with given arbitrarily chosen coordinates. To put it another way: contrary to what is generally said, the time freezing at the event horizon is not an *apparent artifact* related to the Schwarzschild coordinates, but is a *really observed effect*—independent of the type of coordinates used.

Because of the infinite gravitational time dilation at the very boundary of event horizon, we are forced to assert (in full compliance with the current BH model) that all black holes observed in the universe—actually or potentially—are completely "empty", *i.e.*, do not contain matter and singularity inside the event horizon. Moreover, this "state of affairs" will remain valid "until the end of time". According to general opinion (if it cares at all about this issue), this assertion is only a "relative truth" about black holes, due to "improper" reference frame. However, if the relativity rules (summarized in the principle of relativity) are to be obeyed, the observations performed by a distant stationary observer cannot be considered "worse", "less legal" or "less real" than the observations made by an infalling, or (specifically) freefalling observer; on the contrary, they must be considered equally "real" and "legitimate". What is more, irrespectively of the physics behind, the "distant" observations have an obvious advantage over those performed *in situ.* Namely, unlike in the case of the *really performed* observations on Earth, an observer infalling on black hole is, at least for now, a purely hypothetical entity. Hence, if we appeal to these observations, we appeal in fact to our speculations. In the case of black hole, this remark is particularly justified.

The results of observations, or experiments, are usually not provided in a raw form "straight from the source", but are displayed (on the computer screen) in the already elaborated forms: charts, diagrams, etc., which dulls our sensitivity to the difference between direct and indirect observational data. This could be an additional reason for treating the Schwarzschild coordinates and alternative coordinates on equal footing.

The specifically physical source of confusion is, in this case, an *a priori assumption*, according to which spacetime smoothly extends "beyond" the event horizon. It is taken for granted that spacetime "cannot have edges". This assertion gives rise to interpreting singularity at $r = r_S$ as an "apparent artifact" (visualized on the spacetime diagram as compression of the light cones to the 1D form). It also leads to the acceptance of the "weird" behavior of Schwarzschild coordinates in the "interior" of black hole, namely exchange of time and space coordinates, visualized as 90-degree rotation of the light cones. An a priori assumption according to which spacetime extends to gravitational singularity despite being "distorted" can be compared to treating hypothetic tachyon as the real evidence proving the superluminal velocity—at the "small" expense of accepting the concept of imaginary mass. Concluding, the problem with the Kruskal-Szekeres and other alternative coordinates lies in their physical *inadequacy*, understood as incompatibility with the directly observed data.

5.3. Matter Density Outside and Inside the Event Horizon

The case of matter-density in the neighborhood of event horizon makes an additional, auxiliary argument against the current BH model. The size of black hole is conventionally identified with the region bounded by the event horizon. By virtue of this convention, the density of black hole is defined not as infinite—as it should be due to the claim that entire BH mass is contained in singularity, but as the quotient of mass and volume determined by the Schwarzschild radius. The calculated this way density of a typical stellar BH (e.g., of the mass $\sim 10 M_{\odot}$) would be close to the otherwise enormous density of neutron star ($\sim 10^{17}$ kg·m^{-3}); instead, the averaged density of a typical supermassive BH would be incomparably smaller, roughly equal to the density of water.

However, the so-defined *averaged* density is a purely formal quantity (within the current model), not applying to any real object or region. The BH mass cannot be located both in the singularity—as infinitely compressed, and in the whole volume encompassed by the event horizon—as uniformly spread! So, what is the real density of the "interior" of black hole, experienced by the observer who has the dubious pleasure to enter the black hole? It seems that it should be close to zero, since the (almost) entire mass is located in singularity. But it looks incompatible with density of the really observed accretion disks, in a number of cases estimated for 10^{15} particles per cubic centimeter. This, in turn, implies a paradoxical image: the region adjacent (from outside) to the event horizon is "highly material", while the event horizon itself is totally immaterial. Consequently, the infalling observer would experience the gradually increasing density before crossing the event horizon, then abruptly landing "in a void" after crossing this allegedly immaterial boundary. This hardly corresponds with the claim that an infalling observer shouldn't notice the very moment of passing the event horizon.

5.4. Can an Infalling Observer Miss the Event Horizon?

Another (formally stronger) argument against the "imperceptibility" of the event horizon by an infalling observer is the following. Imagine a long tube-shaped spacecraft, designed as a "laboratory" for this thought experiment. The spacecraft approaches the event horizon while positioned perpendicularly to it. An astronaut (observer) is located at the rear end of the spacecraft, constantly looking in the direction of the front end. From the current BH model it follows that in the proper time interval (measured by an onboard clock) during which the whole spacecraft passes the event horizon, it will gradually fade from the astronaut's view, starting from the front end—until the rear end along with the astronaut find themselves "on the other side" of event horizon. This is an inevitable consequence of the claim that nothing including light can escape a black hole. Hence, contrary to the current belief, it is basically impossible to cross the event horizon without noticing this fact! Or, in other words: the observer has *physical tools* to detect this fact. No matter how big is given black hole (in the case of supermassive BH, the size might be comparable with the size of Solar System), the event horizon cannot be conceptualized as a "smooth" boundary—imperceptible to the observer crossing it. At least, that's exactly what the current model implies.

6. "Global" and "Local" Topology of the Universe: The Concept of Metric Space

The global shape of the universe (interpreted as the spacetime manifold) is described by two interrelated properties: *topology and curvature*—the latter, also referred to as "geometry". According to a brief definition, topology is about these properties that are invariant under continuous reversible deformations such as stretching, twisting, crumpling and bending—but not under discontinuous deformations such as opening or closing holes, tearing, gluing, or passing through itself. All deformations of the first kind, collectively called homeomorphisms, preserve the topology of a spatial object described by such properties as dimension, compactness and connectedness—to mention the ones directly accessible to intuition.

Unlike the curvature that can be considered "global" or "local", topology has no concept of "small" and "large". If we consider the surface of a spherical solid (defined topologically as the 2D simply connected space), and make a hole through this solid, e.g., connecting the "poles", so that the inner surface of the hole is smoothly "glued" to the surface of the sphere, then the whole surface will turn into a 2D torus (multiply connected space)—regardless of the width (diameter) of the hole. This is how the pure topology works, that is, how things present themselves if "sphere" and "hole" are interpreted with no regard for their scale. However, it is pretty clear that "in practice" a very thin hole will not change the huge spherical solid into a torus. In terms of physics, the concept of "being drilled through" is not an unequivocal property—the more if we realize that any solid made of a non-degenerate matter is basically made of "emptiness" (especially for neutrinos). Likewise, when we apply topological criteria to cosmology, the concepts of "globality" and "locality" may appear physically justified. In other words, the stringent rules of mathematics may appear not quite adequate in application to the real physical objects, where the scale proves to be decisive for a reasonable classification. The universe is an extreme example. Fortunately, however, we don't need to violate the rules of mathematics in order to reconcile "physical reality" with "theory". Namely, instead of *topological space*, we may use the related, but more "capacious" concept of the *metric space*, i.e., a topological space endowed with metric, which allows to reconcile the notions of "globality" and "locality" with the intrinsically topological properties. Consequently, the spacetime could have definite topology in global scale (this question is still an object of intensive research ([22]-[24]) and represent different homotopy groups on a local scale.

7. Topological Origin and Nature of Black Hole: An Outline of the Hypothesis

The previous discussion leads us to conclusion that the black hole has basically topological nature, being informally defined as a topological hole in the "fabric of spacetime". In this model, the entire mass of a black hole falls on material event horizon (Schwarzschild sphere). This would determine the local topology of spacetime, where the notion of "locality" is interpreted in terms of *metric space*. Accordingly, the black hole would constitute a topological "defect" in spacetime, due to extreme gravitational conditions triggering the phase transition from one topology to another. From among few existing alternatives to the current BH model, the present hypothesis most resembles the model of gravastar [25]. The respective similarities concern the lack of singularity and the concept of Schwarzschild sphere interpreted as a material shell composed of extremely dense matter, while the main difference refers to the BH "interior". In the gravastar model, the "interior" is a justifiable concept defined as the false vacuum with de Sitter metric, filled with the dark energy. Instead, according to the model here proposed, BH has no "interior", which means the absence of spacetime "inside" the Schwarzschild sphere. This concept is justifiable too, and means that all geodesics terminate at (converge with) the Schwarzschild sphere.

The present hypothesis broadly describes black hole as an object at which geometry and topology interact in a specific and unique way. The underlying idea (which also makes an ontological basis for GR) is that spacetime is not a passive background—a "stage" for physical phenomena to appear, but is itself an ultimate physical object endowed with specific properties, in particular the ability to undergo the continuous deformations due to presence of mass and energy. We suggest that "resistance" of the "fabric of spacetime" to these deformations is not unlimited, but is constrained by a certain critical factor of the arguably quantum origin, here called the *maximal curvature of spacetime*. It is likely that maximal curvature can be identified with the event horizon, at which the time-like geodesics are getting similar to the light-like (null) geodesics. This provides fascinating alternative to the concept of material shell: the maximal curvature may play by itself the role of gravitational mass. This resembles the Wheeler's concept of "mass without mass", except that stability is ensured by topology.

After exceeding the curvature limit, the continuous deformation related to curvature transforms into the discontinuous deformation related to topology (phase transition). This transformation consists in opening a *topological hole* in spacetime, which manifests itself from a distance as a black hole. While in mathematical terms this means an interaction between geometry and topology, in physical terms it indicates a "point of contact" between gravitational and quantum-mechanical effects. In that perspective, the creation of black hole can be perceived as the quantum gravity "in action".

8. Conclusion

We have analyzed a few problems connected with the currently accepted model of black hole, focusing on the Schwarzschild BH. In particular, we have revealed significant problems related to the concepts of gravitational singularity and the event horizon singularity, the latter defined as an apparent effect due to the Schwarzschild coordinates. We demonstrated that Kruskal-Szekeres coordinates are incorrect when defined in terms of a mathematical function, and do not comply with observations; consequently, they only seemingly remove the singularity at the event horizon. Other alternative coordinates are also involved in similar problems. As a result, we have proposed a new model defining black hole as a topological hole in spacetime. The Schwarzschild sphere is interpreted as a material shell of the quantum-mechanical properties, containing the entire BH mass. It could be a region where the quantum fluctuations are highly concentrated, or even where spacetime itself is quantized. Accordingly, the spacetime would terminate at the event horizon, with no further extension into the BH "interior". The black hole interpreted this way determines the local topology of spacetime, effectively cutting the region "inside" the event horizon from the rest of the universe. This model opposes the GR prediction, according to which spacetime continues to exist (albeit highly distorted) in the whole area inside the event horizon, except the very singularity in the center. It is worth mentioning that the event horizon defined as a spherical shell "at the edge of spacetime" is not exposed itself to the gravitational collapse—as it would unavoidably happen if the spacetime existed "inside" the sphere. This is because the "force of gravity" does not act across the BH "interior" devoid of spacetime, but rather along the shell itself.

While the new model challenges established physics, it also raises hope for a breakthrough in solving of some of the most fundamental problems in black hole physics and quantum gravity, namely:

- It may provide a way to avoid singularities predicted by GR, which are generally considered unphysical.
- The concept of event horizon as a material shell of the quantum properties (or, alternatively, as a topologically originated curvature generating gravity) may help in reconciling general relativity with quantum mechanics.
- The new model may potentially resolve the information paradox, a long-standing puzzle in the black hole physics.

Further research is necessary to determine if the here proposed hypothesis is theoretically consistent and whether it complies with observations.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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