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About the Possibility of Traveling Faster than the Speed of Light

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Abstract

Using the relativistic equation of motion in 1-D, a study is made to see the possibility for a body to travel faster than the speed of light. It is found that neither by external force nor internal force, it is not possible to have a speed-body faster than the speed of light. In addition, an observation is made to respect the so-called proper time of a body. This observation brings about a new definition of the proper time concept.

Keywords

Special Relativity, Proper Time, Faster than Speed of Light

1. Introduction

Since the birth of the special theory of relativity [1]-[6], which appears due to the invariance of Maxwell's equation under Poincaré-Lorentz transformation [7]-[9] between two inertial reference systems S and S' (one is moving with respect to the other with constant velocity) [10]. This invariance of Maxwell's equations brings about the result that the speed of light is the same under any change of inertial system, that is, it is an absolute quantity under any inertial system. Einstein made the postulation that nobody can move faster than the speed of light ($c \approx 300000\ \text{km/h}$) in order for the causality measured with the light not to be in problems. This fact was used by Minkowsky [11] to define a pseudo metric in the space-time which was invariant under Poincaré-Lorentz transformation, where the time dilation and space contraction can be seen clearly [12] from a rotation on this space-time (change of inertial reference system). This fact helps to define the "proper time τ" associated with the body itself in motion, and it was related with the "laboratory time t" through the relation.

$$\frac{d\tau}{dt}=\frac{1}{\gamma},\quad \gamma=\left(1-\beta^2\right)^{-1/2},\ \text{with } \beta=v/c, \tag{1}$$

where v represents the speed of the body, $v=\left(v_x^2+v_y^2+v_z^2\right)^{1/2}$ as seen in the laboratory system, and c is the speed of light. To get the classical mechanics also invariant under Poincaré-Lorentz transformation, instead of having the differentiation with respect to the laboratory time t, the differentiation is made with respect to the proper time τ and a quadri-vector is defined for the dynamics of the body

$$\left(x_\mu\right)=\left(\mathbf{x},ct\right), \tag{2}$$

where $\mathbf{x}$ is the position of the body measured in the laboratory system. A component of the quadri-vector of velocities is defined as $dx_\mu/d\tau=U_\mu$, where one gets

$$\left(U_\mu\right)=\left(\gamma\mathbf{v},\gamma c\right), \tag{3}$$

being $\mathbf{v}$ the velocity of the body, measured in the laboratory system, $\mathbf{v}=d\mathbf{x}/dt$. The component of quadri-vector of the quantity of motion or linear momentum is $P_\mu=mU_\mu$, where m is the mass of the body at rest and is the vector as

$$\left(P_\mu\right)=\left(\gamma m\mathbf{v},\gamma mc\right). \tag{4}$$

The modified Newton's equation is given for each component as

$$\frac{dP_\mu}{d\tau}=\mathcal{F}_\mu, \tag{5}$$

where the quadri-vector of force is given by

$$\left(\mathcal{F}_\mu\right)=\left(\gamma\mathbf{F},\gamma\mathbf{F}\cdot\mathbf{v}/c\right), \tag{6}$$

If the mass of the body is constant, expression (5) would look like

$$m\frac{d^2x_\mu}{d\tau^2}=\mathcal{F}_\mu. \tag{7}$$

To find the relativistic dynamics of the body, one just needs the spatial component of Equation (5), which can be written using (1) as

$$\frac{d\left(\gamma m\mathbf{v}\right)}{dt}=\mathbf{F}, \tag{8}$$

and this is the equation used normally to find the relativistic dynamics of the body. To make the analysis simpler (without losing generality), one will restrict oneself to 1-D motion of the body. Then, the relativistic dynamics of 1-D motion would be given by the equation

$$\frac{d\left(\gamma mv\right)}{dt}=F\left(z,v,t\right), \tag{9}$$

where the mass of the body could also depend on other variables, $m=m\left(z,v,t\right)$, for mass variable problems. As before, $\gamma^{-1}=\sqrt{1-\beta^2}$ with $\beta=v/c$. This parameter β determines whether or not the body can travel faster than the speed of light, that is, if $\beta > 1$ the body travels faster that the speed of light. This very old topic is not only interesting for particles theory [13], but now the humanity will to travel to other stars and Galaxies [14] [15] which must of them are further away than any human life for space ships traveling less that speed of light. So, it's worth to see again a theoretical possibility for a body to travel faster than the speed of light.

2. Constant Mass and $F \neq 0$ Analysis

One will analyze the cases where the force F depends on each variable, and the motion is considered along the z-direction to simplify the analysis.

a) Assume that the force is of the form $F = F(z)$. Thus, Equation (9) is of the form

$$\frac{\mathrm{d}(\gamma m v)}{\mathrm{d}t} = F(z), \tag{10}$$

which has a constant of motion given by

$$E = (\gamma - 1) mc^2 + V(z), \quad V(z) = -\int F(z)\mathrm{d}z, \tag{11}$$

and this constant of motion has units of energy. Passing the terms mc^2 and $V(z)$ to the left-hand side. Then, making the square on both sides and making some rearrangements, it is not difficult to see that

$$\beta = \frac{\sqrt{\left(E + mc^2 - V(z)\right)^2 - m^2c^4}}{E + mc^2 - V(z)}. \tag{12}$$

So, if $\beta > 1$ (the body travels faster than the speed of light), it would imply that $-1 > 0$, which is not possible.

If $\beta = 1$ (the body travels at the speed of light), it would imply that $-1 = 0$, which is not possible.

If $\beta < 1$ (the body travels less than the speed of light), it would imply that $-1 < 0$, which is possible.

b) Assume that the force is of the form $F = F(t)$, Then, Equation (9) is of the form

$$\frac{\mathrm{d}(\gamma m v)}{\mathrm{d}t} = F(t), \tag{13}$$

which has the following constant of motion

$$C = \gamma m v - \int F(t)\mathrm{d}t, \tag{14}$$

and the solution can be obtained from this expression as

$$\beta = \frac{\frac{C}{mc} + \frac{1}{mc}\int F(t)\mathrm{d}t}{\sqrt{1 + \frac{1}{(mc)^2}\left[C - \int F(t)\mathrm{d}t\right]^2}}. \tag{15}$$

The constant of motion C is determined by the initial conditions. Assuming that at $t = 0$, one has $v = 0$, the resulting expression is

$$\beta=\frac{\frac{1}{mc}\int_0^t F(t)\,\mathrm{d}t}{\sqrt{1+\frac{1}{(mc)^2}\left[\int_0^t F(t)\,\mathrm{d}t\right]^2}}. \tag{16}$$

Thus, for any type of nonsingular force $F(t)$, it follows that $0\le\beta\le 1$. Note the here the constant of motion has units of linear momentum.

c) Assume that the force is of the form $F=F(v)$, Then, the Equation (9) is of the form

$$\frac{\mathrm{d}(\gamma m v)}{\mathrm{d}t}=F(v), \tag{17}$$

which can be written as

$$mc\frac{\mathrm{d}\beta}{\mathrm{d}t}=F(\beta c)\left(1-\beta^2\right)^{3/2}, \tag{18}$$

having the following constant of motion

$$C=mc\int\frac{\mathrm{d}\beta}{F(\beta c)\left(1-\beta^2\right)^{3/2}}-t. \tag{19}$$

The initial conditions $\beta(0)=\beta_0$ bring about the following expression of the time as a function of β

$$t(\beta)=mc\int_{\beta_0}^{\beta}\frac{\mathrm{d}\beta}{F(\beta c)\left(1-\beta^2\right)^{3/2}}, \tag{20}$$

and one has the following limit

$$\lim_{\beta\to 1} t(\beta)=\infty. \tag{21}$$

Therefore, $\beta>1$ is not possible. Note that the constant of motion has units of time, and as before, $\beta=v/c$.

3. Variable Mass and $F\neq 0$ Analysis

This case can correspond to a body with besides the external force, the mass of the body vary due to the expulsion of gases. So, let assume that the initial mass of the body is m_0 and the mass of the body varies as

$$m(z,v,t)=m_0\left(1-g(z,v,t)\right), \tag{22}$$

where the physical conditions would be $0\le g<1$. The reason to choose this functionality is that one starts with initial mass m_0 (in space ship, for example), one ends up with very small $m_{end}\ll m_0$. The equation of motion (9) is

$$\frac{\mathrm{d}\left(\gamma m(z,v,t)\right)}{\mathrm{d}t}=F(z,v,t), \tag{23}$$

which can be written in terms of the β variable (after some rearrangements) as

$$\frac{\mathrm{d}\beta}{\mathrm{d}t}=\left(\frac{F(z,c\beta,t)}{m(z,c\beta,t)c}+\frac{m_0\beta\dot{g}}{m(z,c\beta,t)\sqrt{1-\beta^2}}\right)\left(1-\beta^2\right)^{3/2}. \tag{24}$$

a) If the mass depends only on the variables z and t, $m = m_0\left(1 - g\left(z,t\right)\right)$, the case is reduced a combination of the previous analyzed cases.

b) If the mass depends only on the variable v, $m = m_0\left(1 - g\left(v\right)\right)$, the case is a little bit more complicated and the equation of motion is given by

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{F\left(z, c\beta, t\right)\left(1-\beta^2\right)^{3/2}}{m_0 c\left[1 - g\left(c\beta\right) - \left(1-\beta^2\right)\beta\frac{\mathrm{d}g}{\mathrm{d}\beta}\right]}. \tag{25}$$

However, the restriction on β comes from the physical restriction $g\left(c\beta\right) < 1$. For example, if $g\left(v\right) = \alpha v = \alpha c\beta$, it would imply that $\beta < 1/c\alpha$ which for any practical proposal, one would have that $\beta < 1$.

4. $F = 0$ Analysis

Since the external force is zero, the possible impulse on the body must come from internal mass of this body, that is, the mass of the body must vary. The equation of motion (9) is now

$$\frac{\mathrm{d}\left(\gamma m v\right)}{\mathrm{d}t} = 0. \tag{26}$$

This implies that the constant of motion of the system is

$$K = mc\gamma\beta. \tag{27}$$

Of course, if initially $\beta\left(0\right) = 0$, one will have $\beta = 0$ at time. Then, one will assume that $\beta\left(0\right) \neq 0$, and the above expression, by taking the square and making some rearrangements, can be written as

$$K^2 = \beta^2\left[\left(mc\right)^2 + K^2\right]. \tag{28}$$

a) If the mass depends only on the variables z and t, $m = m\left(z,t\right)$, from the expression (28), it follows that

$$\beta = \frac{1}{\sqrt{1 + \left(\frac{cm\left(z,t\right)}{K}\right)^2}}, \tag{29}$$

implying that $0 \le \beta \le 1$ for any variation of mass of this type.

b) If the mass depends only on the variable v and is of the form $m\left(\beta\right) = m_0\left(1-\beta\right) - m_f\beta$, where m_0 is the initial mass and m_f is the mass of the body when $\beta = 1$, and one must have that $m_f < m_0$. The constant of motion (27) is given by

$$K = \frac{c\left[m_0\left(1-\beta\right)\beta + m_f\beta^2\right]}{\sqrt{1-\beta^2}}. \tag{30}$$

From this expression, one gets the following polynomial expression for the variable β

$$\beta^4 - \frac{2m_0}{\left(m_0 - m_f\right)}\beta^3 + \frac{\left(m_0^2c^2 + K^2\right)}{c^2\left(m_0 - m_f\right)^2}\beta^2 - \frac{K^2}{c^2\left(m_0 - m_f\right)^2} = 0. \tag{31}$$

One can use Ferrari's method [16] to find the roots of this polynomial (see Appendix). Let us make the following definitions

$$\Delta = m_0 - m_f,\ a = \frac{2K^2 - m_0^2c^2}{2c^2\Delta^2},\ b = \frac{m_0K^2}{c^2\Delta^3},\ C = \frac{61m_0^4}{16\Delta^4} + \frac{\left(4m_0^2 - \Delta^2\right)K^2}{c^2\Delta^4}, \quad (32)$$

$$P = -\frac{a^2}{12} - C,\ Q = -\frac{a^2}{108} + \frac{aC}{3} - \frac{b^2}{8},\ R = -\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}, \quad (33)$$

$$U = R^{2/3},\ y = -\frac{5a}{6} + U \text{ and } W = \sqrt{a + 2y}. \quad (34)$$

Then, the non-negative solution of (31) can be written as

$$\beta = \frac{m_0}{\Delta} + \frac{W}{2} + \frac{1}{2}\sqrt{-(2a + 2y) + \frac{2b}{W}}. \quad (35)$$

Now, if $\beta > 1$ would mean that

$$\frac{m_0}{\Delta} > 1 - \frac{W}{2} - \frac{1}{2}\sqrt{-(2a + 2y) + \frac{2b}{W}}, \quad (36)$$

which is not possible since m_0/Δ must be a non-negative number. Therefore, in this case the body can not travel faster than the speed of light.

5. Observation on the Proper Time τ

As we know, the proper time τ appears due to the invariance of the pseudo metric

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - c^2\mathrm{d}t^2 \quad (37)$$

under Poincaré-Lorentz transformation, between two inertial reference systems S and S' which are moving with constant velocity $\mathbf{v}$ each other. This invariance is represented by $\mathrm{d}s^2 = \mathrm{d}s'^2 = \mathrm{d}x'^2 + \mathrm{d}y'^2 + \mathrm{d}z'^2 - c^2\mathrm{d}t'^2$, where (x, y, z, ct) are the coordinates on the inertial system S and $(x'y'z', ct')$ are the coordinates on the inertial system S'. If the relative velocity of the reference systems is on the z-direction $\mathbf{v} = (0, 0, v)$, and the inertial reference system S' represents the motion of a body, this invariance $\mathrm{d}s^2 = \mathrm{d}s'^2$ takes the form

$$\mathrm{d}z^2 - c^2\mathrm{d}t^2 = -c^2\mathrm{d}\tau^2, \quad (38)$$

where z, t, v represent the position, time, and velocity of the body measured in the inertial system S, and τ represents the time measures in the body itself (inertial reference system S', $\mathrm{d}z' = 0$), getting the following relation between these times

$$\mathrm{d}\tau = \sqrt{1 - \beta^2}\,\mathrm{d}t \text{ or } \frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{\gamma}, \quad (39)$$

with $\gamma = \left(1 - \beta^2\right)^{-1/2}$, $\beta = v/c$. One needs to point out that this expression is valid if and only if S and S' are two inertial systems of reference. However, if there is a force $\mathbf{F}$ acting on the body, the reference system S' (the body) is no longer an inertial system of reference since there will be acceleration of some kind on the body. Thus, the expression (8) can only be taken as an approximation to the relativistic dynamics of the system.

Now, assume there is a force applied on the body in the z-direction (1-D motion)

$$\mathbf{F} = (0,0,F). \tag{40}$$

Thus, since the body is moving on the z-direction with an instant speed v, and there must be a force dependence, F, the metric on the reference system S would be of the form (anzat)

$$\mathrm{d}s^2 = \left(1+g_{11}(F,v)\right)\mathrm{d}z^2 + g_{12}(F,v)\,\mathrm{d}z\mathrm{d}t - g_{22}(F,v)c^2\mathrm{d}t^2, \tag{41}$$

where the following limits must be satisfied

$$\lim_{F\to 0} g_{11}(F,v) = 0, \tag{42}$$

$$\lim_{F\to 0} g_{12}(F,v) = 0, \tag{43}$$

and

$$\lim_{F\to 0} g_{22}(F,v) = 1 \tag{44}$$

The metric in the noninertial system S' would be given by

$$\mathrm{d}s'^2 = -g_{22}(F,v)c^2\mathrm{d}\tau'^2, \tag{45}$$

and the assuming invariance $\mathrm{d}s^2 = \mathrm{d}s'^2$, now looks like the following expression

$$\left(1+g_{11}(f,v)\right)\mathrm{d}z^2 + g_{12}(F,v)\,\mathrm{d}z\mathrm{d}t - g_{22}(F,v)c^2\mathrm{d}t^2 = -g_{22}(F,v)c^2\mathrm{d}\tau'^2, \tag{46}$$

which brings about the following relation between the time t and new proper time τ'

$$\frac{\mathrm{d}\tau'}{\mathrm{d}t} = \frac{1}{\Gamma_F(v)}, \tag{47}$$

where $\Gamma_F(v)$ is defined through the following relation

$$\frac{1}{\Gamma_F(v)} = \left\{\left(1-\frac{\beta^2}{g_{22}}\right) - \frac{g_{11}}{g_{22}}\beta^2 - \frac{g_{12}}{g_{22}}\beta\right\}^{1/2}, \tag{48}$$

with $\beta = v/c$, and one must have the following limit

$$\lim_{F\to 0}\Gamma_F(v) = \gamma. \tag{49}$$

In this way, the relativistic expression (9) is proposed to be written of similar form but with $\Gamma_F(v)$ factor instead of the γ factor,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_F(v)mv\right) = F(z,v,t). \tag{50}$$

This would mean that the above examples with $F \neq 0$ would have to be reconsidered within the new formulation (50). However, for the cases with $F = 0$, the above cases are totally correct.

6. Using the New Proper Time with Constant Force

Assume g_{11}, g_{12}, and g_{22} of the form

$$g_{11} = g_{12} = \alpha F \ \text{ and } \ g_{22} = 1+\alpha F, \tag{51}$$

where F is the constant force and α is a parameter with units inverse of the force units, and both of them are non-negative defined. These expressions satisfy the above limits (42), (43) and (44), and the relation (52) is for this case written as

$$\frac{1}{\Gamma_F(v)} = \left\{1 - \frac{\beta^2}{1+\alpha F} - \frac{\alpha F \beta^2}{1+\alpha F} - \frac{\alpha F \beta}{1+\alpha F}\right\}^{1/2}. \tag{52}$$

Therefore, the Equation (50),

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_F(v) m v\right) = F, \tag{53}$$

has the constant of motion

$$K = \Gamma_F(v) m v - Ft, \tag{54}$$

which can be determined by initial conditions. Squaring this expression and making some rearrangements, it follows that β must satisfy the following quadratic polynomial

$$\left(1 + \alpha F + \frac{m^2 c^2 (1+\alpha F)}{K+Ft}\right)\beta^2 + \alpha F \beta - (1+\alpha F) = 0, \tag{55}$$

where the positive roots is given by

$$\beta = \frac{\sqrt{(\alpha F)^2 + (1+\alpha F)\left[1+\alpha F + m^2c^2(1+\alpha F)/(K+Ft)\right]} - \alpha F}{2\left[1+\alpha F + m^2c^2(1+\alpha F)/(K+Ft)\right]}. \tag{56}$$

For this expression one has the following limit

$$\lim_{F\to 0} \beta = \frac{1}{\sqrt{1 - \frac{m^2c^2}{K^2}}}, \tag{57}$$

which is the expression (29) above for constant mass. Now, assume that $\beta > 1$ on this expression. Then, one has that

$$\begin{aligned} &\sqrt{(\alpha F)^2 + (1+\alpha F)\left[1+\alpha F + \frac{m^2c^2(1+\alpha F)}{K+Ft}\right]} \\ &> 2\left[1+\alpha F + \frac{m^2c^2(1+\alpha F)}{K+Ft}\right] + \alpha F. \end{aligned} \tag{58}$$

Taking the square on both sides and making some rearrangements, one gets

$$\begin{aligned} &4(1+\alpha F)\left[1+\alpha F + \frac{m^2c^2(1+\alpha F)}{K+Ft}\right]^2 \\ &+(4+3\alpha F)\left[1+\alpha F + \frac{m^2c^2(1+\alpha F)}{K+Ft}\right] < 0. \end{aligned} \tag{59}$$

However, this is not possible because all of the quantities are no negatives and the time varies as $0 < t < \infty$. This means that even with the modified proper time it is not possible for a body to travel faster that the speed of light.

Of course, taking a force or mass depending on time, position or velocity, the analysis becomes much more complicated, but one can guess that, in any case, it is not possible for a body to travel faster than the speed of light. In addition, note from (52) that all the terms on the right hand side after the constant 1 are nonpositive since β, α, and F are non-negative. Therefore, one expects that a body under external force never travels even close to the speed of light.

7. Results and Comments

Using the usual relativistic equation of motion of a body moving in 1-D under external force (for example, solar wing acting on a space ship) or without external force but loosing its mass somehow (for example, gas or ions expulsion with some energy from the ship), there is no way that this body could be travel faster than the speed of light. These usual relativistic equations of motion define the special relativity and appear from the concept of **proper time** of the body τ, which is the time the body measures on its own inertial reference system S', and the pseudo metric used is invariant under Poincaré-Lorentz transformation. However, as soon as there is an external force acting on the body, this system S' is no longer an inertial reference system. Thus, the usual relativistic equations of motion of the special relativity (5) or (8) or (9) are not correct, and it is necessary to make the correction of the pseudometric that must be invariant between two non-inertial systems of reference. The proposed pseudometric in the space-time is used when a constant force is acting on a body, and it looks reasonable, defining a new proper time τ' (47) which brings about a new relativistic equation (50) which explicitly considers the effect of the force, and has the right limit when the force is zero. Using this new relativistic equation of motion one arrives at the same conclusion: **under an external force, a body can not travel faster than the speed of light**. In fact, due to this observation it suggests that the usual relativistic equations of motion of Classical and Quantum Mechanics are approximations when they are used with external force.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

Ferrari-Cardano method for fourth order equation.

Let the fourth order equation be of the form

$$ax^4 bx^3 + cx^2 + dx + e = 0, \tag{60}$$

where the coefficients are real numbers, $a, b, c, d, e \in \Re$. Defining new coefficients and a new variable as

$$p = \frac{8ac - 3b^2}{8a^2}, \tag{61}$$

$$q = \frac{8a^2 d - 4abc + b^3}{8a^3}, \tag{62}$$

$$r = \frac{256a^3 e - 64a^2 bd + 16ab^2 c - 3b^4}{256a^4}, \tag{63}$$

and

$$z = x + \frac{b}{4a}, \tag{64}$$

the equation (60) is transformed as (so called reduced form)

$$z^4 + pz^2 + qz + r = 0 \tag{65}$$

The associated cubic equation of (65) is given by

$$y^3 + 2py^2 + \left(p^2 - 4r\right) y - q^2 = 0. \tag{66}$$

Now, let y_1 the real root of this equation such that $y_1 > 0$. Then, the four roots of (60) are given by

$$x_1 = -\frac{b}{4a} + \frac{1}{2}\left[\sqrt{y_1} + \sqrt{-y_1 - 2p - \frac{2q}{\sqrt{y_1}}}\right], \tag{67}$$

$$x_2 = -\frac{b}{4a} + \frac{1}{2}\left[\sqrt{y_1} - \sqrt{-y_1 - 2p - \frac{2q}{\sqrt{y_1}}}\right], \tag{68}$$

$$x_3 = -\frac{b}{4a} + \frac{1}{2}\left[-\sqrt{y_1} + \sqrt{-y_1 - 2p + \frac{2q}{\sqrt{y_1}}}\right], \tag{69}$$

and

$$x_4 = -\frac{b}{4a} + \frac{1}{2}\left[-\sqrt{y_1} - \sqrt{-y_1 - 2p - \frac{2q}{\sqrt{y_1}}}\right]. \tag{70}$$