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Warp Drive Time Ship

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Abstract

This paper proposes a novel approach to manipulate the spacetime metric $g_{\mu\nu}$ by employing gauge-transformed optical soliton beams to generate gravitational solitons, enabling a toroidal time machine to achieve apparent superluminal velocities ranging from $2c$ to $10^7 c$ and induce temporal regression (e.g., −32.8 years). Our findings demonstrate that the Lagrangian of two optical solitons can be transformed into that of a gravitational soliton via a generalized gauge transformation, involving the coupling of electromagnetic and gravitational fields, with dynamics governed by the invariant Einstein-Maxwell Lagrangian. Utilizing eight tangentially emitted optical soliton beams, combined with meticulous physical computations, we analyze the effects of polarization angle θ, energy density ρ_{EM}, and the number of gravitational soliton pairs (N) on apparent velocity and closed timelike curves (CTCs), while devising safety control strategies (e.g., precision in θ). This study explores the feasibility of this scheme from both theoretical and engineering perspectives, addressing the invariance of the gravitational soliton Lagrangian, curvature engine design, jump velocity mechanisms, and optimizations in structure and safety. The results reveal that this method can substantially enhance navigation speeds and enable time travel; however, challenges related to interstellar voyages and temporal paradoxes warrant further investigation. This work lays a theoretical foundation for future superluminal spacecraft and time machine technologies, heralding the vast potential of gravitational soliton research.

Keywords

Gravitational Solitons, Generalized Gauge Transformation, Warp Drive Ship, Time Travel Machine, Superluminal Speed

1. Introduction

Superluminal travel and time manipulation have long been the ultimate aspirations of humanity's quest to explore the cosmos and transcend temporal boundaries. However, Einstein's general theory of relativity establishes the speed of light (c) as the cosmic speed limit, rendering traditional methods insufficient to surpass this barrier [1]. In recent years, theoretical physicists have proposed innovative solutions, such as harnessing spacetime metric perturbations to achieve apparent superluminal effects or closed timelike curves (CTCs) to circumvent this constraint [2] [3]. Among these, solitons—stable nonlinear waves—exhibit unique localized properties in both electromagnetic and gravitational fields, offering a novel avenue for spacetime manipulation [4]. Optical solitons, as isolated electromagnetic waves, have been extensively studied experimentally [5]-[8], yet their transformation into gravitational solitons via gauge transformations remains underexplored.

Gravitational solitons are hypothesized strong-field excitations capable of profoundly altering local spacetime geometry, potentially generating high-speed effects akin to the Alcubierre Drive or inducing CTCs through circular trajectories [9] [10]. In 1994, Alcubierre introduced a theoretical model for superluminal travel by contracting spacetime ahead and expanding it behind, though its reliance on exotic (negative) energy was deemed impractical [9]. Furthermore, theoretical studies have demonstrated that circular beam configurations, such as optical soliton arrays, can induce temporal regression, laying a foundation for time machine designs [11]. In contrast to these, the gravitational soliton model based on optical solitons proposed in this article uses existing laser technology to generate gravitational perturbations through gauge transformations, potentially reducing the energy threshold significantly while offering a controllable space-time deformation solution [12].

Indeed, in this work, we reveal that the Lagrangian of two optical solitons can be converted into that of a gravitational soliton via generalized gauge transformations [13]-[16], entailing the coupling of electromagnetic and gravitational fields. Evidence supports the adoption of the Einstein-Maxwell Lagrangian, which encompasses the standard dynamical terms of both fields and their interactions. This Lagrangian remains invariant under gauge transformations and, notably in the weak-field approximation, describes the conversion of two polarized photons into a graviton [17] [18].

Building on this, we propose an innovative approach: utilizing eight tangentially emitted optical soliton beams, transformed via gauge methods into gravitational solitons, to manipulate the spacetime metric $g_{\mu\nu}$. This enables a toroidal spacecraft to achieve apparent velocities ranging from $2c$ to $10^7 c$ and realize temporal regression (e.g., −32.8 years). Through rigorous physical computations, we analyze the impacts of polarization angle θ, energy density ρ_{EM}, and the number of gravitational soliton pairs (N) on apparent velocity and CTCs, while designing safety control strategies (e.g., precision in θ). This paper aims to evaluate the feasibility of this scheme from both theoretical and engineering perspectives, providing a reference for future superluminal navigation and time travel technologies. In Section 2, we explore the invariance of the gravitational and optical soliton Lagrangians under gauge transformations; Section 3 examines key aspects of curvature engines and time machines, focusing on superluminal apparent velocities; Section 4 addresses the coordinate velocity of curvature bubbles and CTC calculations; Section 5 investigates the properties and motion of curvature bubbles; Section 6 proposes and designs a jump velocity mechanism; Section 7 details structural scheme designs; Section 8 focuses on safety design considerations; and Section 9 presents conclusions and future outlook. **Appendices A-C** offer detailed analyses and calculations for further reference.

2. Lagrangian of Gravitational Solitons and Optical Solitons

Considering the metric and gauge transformation constructed below, we propose to use the following Lagrangian to describe the process of two optical solitons transforming into a gravitational soliton, that is, the Lagrangian is:

$$L = \frac{1}{16\pi G}\sqrt{-g}R + \frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu} + \alpha\sqrt{-g}R_{\mu\nu}F^{\mu\sigma}F_{\sigma}^{\nu} \tag{1}$$

where R is the Riemann curvature scalar, describing the dynamics of the gravitational field; $F_{\mu\nu}$ is the electromagnetic tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2}$$

here A_μ is the electromagnetic potential; g is the determinant of the metric, ensuring that the Lagrangian is covariant in the framework of general relativity. While α is the coupling constant (dimension as [length]^2, controlling the coupling strength);If the solitons are close to merging, $F_{\mu\nu} \sim \operatorname{sech}(ku)$, then the perturbation term of $R_{\mu\nu} \sim \operatorname{sech}^2(ku)$ naturally appears, therefore, this coupling term provides a nonlinear feedback mechanism: electromagnetic self-action → gravitational disturbance → formation of localized gravitational solitons.

This Lagrangian combines the Einstein-Hilbert action (describing gravity) and the Maxwell action (describing electromagnetic fields), and naturally couples the two through the metric $g_{\mu\nu}$. In a strong field, it reflects the process of two optical solitons ω_U forming a gravitational soliton ω_V. In the weak field approximation, it can capture the process of two polarized photons transforming into a graviton through interaction. The reason is that it is invariant under the gauge transformation

$$g_{UV}(u) = \begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix} \tag{3}$$

and satisfies the gauge potential equation

$$\omega_V = g_{UV}^{-1}\omega_U g_{UV} + g_{UV}^{-1}\boldsymbol{d}g_{UV} \tag{4}$$

and the gauge transformation converts the gauge potential ω_U of two optical solitons into the gauge potential ω_V of one gravitational soliton, that is,

$$\omega_U = \operatorname{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \overset{g_{UV}(u)}{\Rightarrow} \omega_V = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{5}$$

We need to verify whether the above reasons are correct: First of all, from the theoretical framework, the Lagrangian combines the Einstein-Hilbert effect (describing gravity) and the Maxwell effect (describing electromagnetic fields), and is covariant in the framework of general relativity. The above gauge transformation (3) $g_{UV}(u)$ is a rotation matrix that belongs to the $SO(2)$ group and is isomorphic to $U(1)$, and can act on the polarization states ω_U (two optical solitons) and ω_V (gravitational solitons). We need to verify whether the Lagrangian can capture this transformation. In fact, from the perspective of metric and polarization state, we can analyze the gravitational soliton metric here [18]:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + H(u,x,y)\mathrm{d}u^2 + \mathrm{d}x^2 + \mathrm{d}y^2 \tag{6}$$

where

$$H(u,x,y) = \mathrm{sech}^2(ku)\left[A\left(x^2 - y^2\right) + 2Bxy\right] \tag{7}$$

We can find that this metric satisfies Einstein's vacuum equation $R_{\mu\nu} = 0$.

In fact, the gravitational soliton metric here is analyzed from the above metric and polarization state (6), so we can find that this metric satisfies the Einstein vacuum equation [18]. Here $H(u,x,y)$ describes a gravitational soliton, whose polarization state is:

$$\omega_V = \mathrm{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{8}$$

The corresponding polarization state of the optical soliton is:

$$\omega_U = \mathrm{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

And they are converted to each other through the gauge transformation g_{UV} of formula (3).

Under the weak field approximation, this transformation corresponds to the process of two polarized photons being transformed into one graviton. Therefore, we can analyze the transformation form of ω and calculate $Tr\left(\omega'^2\right)$:

Suppose original $\omega^2 = \omega\omega$, the trace is $Tr\left(\omega^2\right)$, after the transformation, $\omega' = g_{UV}\omega g_{UV}^{\mathrm{T}}$, then $\omega'^2 = \left(g_{UV}\omega g_{UV}^{\mathrm{T}}\right)\left(g_{UV}\omega g_{UV}^{\mathrm{T}}\right) = g_{UV}\omega\omega g_{UV}^{\mathrm{T}}$, considering $g_{UV}^{\mathrm{T}} g_{UV} = I$, hence $Tr\left(\omega'^2\right) = Tr\left(g_{UV}\omega\omega g_{UV}^{\mathrm{T}}\right) = Tr\left(\omega\omega g_{UV}^{\mathrm{T}} g_{UV}\right) = Tr(\omega\omega)$, and it allows $Tr\left(\omega'^2\right) = Tr\left(\omega^2\right)$, which proves the trace invariance.

Furthermore, the coupling term $R_{\mu\nu}F^{\mu\sigma}F_{\sigma}^{\nu}$ becomes under the transformation: $R_{\mu\nu}g_{\rho}^{\mu}F^{\rho\alpha}\cdot g_{\sigma}^{\nu}F_{\alpha}^{\sigma} = R_{\mu\nu}g_{\rho}^{\mu}g_{\sigma}^{\nu}F^{\rho\alpha}F_{\alpha}^{\sigma}$, but since $R_{\mu\nu}$ is a tensor and the rotation only acts on the polarization subspace (such as x, y), then as long as we limit the discussion to the non-zero $R_{\mu\nu}$ components in the polarization direction, such as R_{xx}, R_{yy}, this combination is a contraction of symmetric second-order tensors and remains unchanged under orthogonal transformations.

Hence g_{UV} is a rotation transformation that acts on the polarization states

ω_U and ω_V, ensuring that the Lagrangian remains invariant under the transformation, because it relies on scalar invariants (such as $Tr\left(\omega^2\right)$ or $Det(\omega)$ that remain invariant under $SO(2)$. Here the Lagrangian L is the scalar density, which remains invariant under coordinate transformations. The gauge transformation $g_{UV}(u)$ corresponds to a rotation in the x-y plane, *i.e.* the gravitational part is $\frac{1}{16\pi G}\sqrt{-g}R$, where R is the Riemann curvature scalar and is invariant under coordinate transformation; the electromagnetic part $\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$ is a scalar and remains unchanged; the interaction term is $\alpha\sqrt{-g}R_{\mu\nu}F^{\mu\sigma}F_{\sigma}^{\nu}$ which is also invariant under orthogonal transformations, therefore, the Lagrangian is unchanged under g_{UV}.

The gauge potential Equation (4) above, $\omega_V = g_{UV}^{-1}\omega_U g_{UV} + g_{UV}^{-1}\boldsymbol{d}g_{UV}$, is similar to the gauge potential transformation in non-Abelian gauge theory:

- $g_{UV}^{-1}\omega_U g_{UV}$ is the rotation part, keeping $Tr\left(\omega^2\right)$ unchanged;
- $g_{UV}^{-1}\boldsymbol{d}g_{UV}$ introduces additional terms, such as $g_{UV}^{-1}\boldsymbol{d}g_{UV} = \begin{pmatrix} 0 & \theta'(u) \\ -\theta'(u) & 0 \end{pmatrix}$,

  so the Lagrangian is based on the scalar invariant and remains unchanged.

But here it is a generalized gauge transformation, which means that it can transform optical solitons belonging to the electromagnetic field into gravitational solitons belonging to the gravitational field. For the concept and framework of this unified field theory, see references [13]-[18], that is, through the generalized gauge transformation g_{UV}, ω_U (two optical solitons) is converted into ω_V (a gravitational soliton), see details in **Appendix A**. In quantum field theory, this corresponds to two photons generating a graviton through gravitational interaction. In classical field theory, the energy-momentum tensor $T_{EM}^{\mu\nu}$ of the electromagnetic field is the source of the gravitational field:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}^{EM} \tag{10}$$

Therefore, two optical solitons (electromagnetic wave packets) may generate gravitational solitons (gravitational wave packets) through energy-momentum coupling. The unexpected detail is that this model implies the process in quantum field theory under the weak gravity field approximation, that is, two photons (the quantum counterpart of optical solitons) can generate one graviton (the quantum counterpart of gravitational solitons) through the generalized gauge transformation. This differs from the description of classical field theory and provides a potential connection from classical to quantum [19]-[21].

3. Key Points about Warp Drive and Time Machine

The above studies show that using the "Einstein-Maxwell type Lagrangian" and generalized gauge transformation to transform optical solitons into gravitational solitons may directly control the curvature of spacetime through the electromagnetic field and avoid the need for negative energy. This method assumes that the electromagnetic tensor directly affects the curvature (such as the Weyl tensor [22]-[24]) and does not completely rely on the traditional path of Einstein's field equations. For this purpose, we assume that:

1) The electromagnetic field is transformed into gravitational solitons through gauge transformation (such as formula (4)), which directly controls the curvature of space-time;

2) This mechanism does not rely on negative energy, but instead converts the electromagnetic tensor into curvature (such as the Weyl tensor), bypassing the energy-momentum coupling of the traditional Einstein field equations.

3) The "ring time machine" uses rotation and electromagnetic fields to generate closed timelike curves (CTCs), which can realize time travel.

In the specific construction, first according to the form of $\omega_V = \text{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} = \begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix}$, it can be found that its mapping to the lateral perturbation is:

$$h_{xx} = A\text{sech}^2(ku);\ \ h_{xy} = h_{yx} = B\text{sech}^2(ku);\ \ h_{yy} = -A\text{sech}^2(ku) \tag{11}$$

The corresponding perturbation is expanded to the metric as:

$$\begin{aligned} \text{d}s^2 = &-2c^2\text{d}u\text{d}v + [1+h_{xx}]\text{d}x^2 + 2h_{xy}\text{d}x\text{d}y + [1 - A\text{sech}^2(ku)]\text{d}y^2 \\ &+ c^2\text{sech}^2(ku)\left[A(x^2-y^2)+2Bxy\right]\text{d}u^2 \end{aligned} \tag{12}$$

where $H = \text{sech}^2(ku)\left[A(x^2-y^2)+2Bxy\right]$ is dimensionless, but $c^2H\text{d}u^2$ has the dimension of m².

Our goal now is to calculate the exact apparent velocity (*i.e.* coordinate velocity) v_{eff} (including non-weak field conditions) under this metric perturbation (11):

(1) Used Parameters:

$$h_{xx} = A\text{sech}^2(ku)$$

$$h_{xy} = h_{yx} = B\text{sech}^2(ku)$$

$$h_{yy} = -A\text{sech}^2(ku)$$

- $H = \text{sech}^2(ku)\left[A(x^2-y^2)+2Bxy\right]$ (dimensionless)
- $c^2H\text{d}u^2$: unit m²

(2) Calculation Target:

- Along the x direction, $v_{eff} = 2c$ (c is the intrinsic speed of the photon). Consider how h_{xy}, h_{xy}, h_{yy} affect the apparent velocity (coordinate velocity) v_{eff}.

(3) Calculation method:

- Apparent velocity definition:
  - $v_{eff} = \frac{\Delta x}{\Delta t}$, where Δt is the propagation time of the photon in the perturbed spacetime;
  - Derived from the metric component g_{xx} and the photon path.

- Steps:
  - o Determine the photon propagation along the *x*-direction ($\mathrm{d}y = 0$);
  - o Calculate the compression effect of g_{xx} on distance;
  - o Derive v_{eff} and adjust the parameter to $2c$.

According to the above settings, firstly we calculate v_{eff} along the x direction, since $\mathrm{d}y = 0$ we simplify the metric to:

$$\mathrm{d}s^2 = -2c^2\mathrm{d}u\mathrm{d}v + \left[1 + h_{xx}\right]\mathrm{d}x^2 + c^2 H\mathrm{d}u^2 \quad (13)$$

where h_{xy} cross term contribution is zero (due to $\mathrm{d}x\mathrm{d}y = 0$). Then according to formula (13), the metric component is:

$$g_{uu} = c^2 H = c^2 \operatorname{sech}^2(ku)\left[A\left(x^2 - y^2\right) + 2Bxy\right]$$

$$g_{uv} = g_{vu} = -c^2$$

$$g_{xx} = 1 + h_{xx} = 1 + A\operatorname{sech}^2(ku)$$

- o $x = y = 0$ (disturbance center):

$$H = 0$$

$$h_{xx} = A$$

$$g_{xx} = 1 + A$$

$$\mathrm{d}s^2 = -2c^2\mathrm{d}u\mathrm{d}v + (1 + A)\mathrm{d}x^2$$

Then the apparent velocity can be derived as follows: From flat spacetime: $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 = 0$, we can get the g_{xx} of perturbed spacetime to change the spatial distance:

$$\Delta x_{phys} = \sqrt{g_{xx}}\Delta x \quad (14)$$

where, Δx is the coordinate distance, defined in the coordinate system as $x_2 - x_1$, in units of length (e.g. meters), but does not reflect the actual geometry, Δx_{phys} is the proper distance, the actual measured physical distance, scaled by the metric g_{xx} , and the apparent velocity is defined as shown above:

$$v_{eff,x} = \frac{\Delta x}{\Delta t} \quad (15)$$

where Δt is the coordinate time of the photon's propagation, and Δx is the coordinate distance. Here the coordinate time (Δt) means that the propagation time of the photon recorded by the observer (such as $t_2 - t_1$), corresponding to the propagation of Δx . The proper time of the photon ($\Delta\tau$) is related to $\mathrm{d}s^2 = -c^2\mathrm{d}\tau^2 = 0$. So the current calculation gives

$$\Delta t = \frac{\sqrt{g_{xx}}\Delta x}{c} \quad (16)$$

It is the coordinate time of photon propagation, that is, Δt is the time recorded by an external observer (such as $u = t$ coordinate system), which can be compared with the intrinsic time $\Delta\tau = 0$ (light-like), and is different from

$\Delta t \neq 0$. Be careful not to confuse: Δt contains $\sqrt{g_{xx}}\Delta x$ (proper distance), which is not similar to the construction of proper time:

$$\Delta \tau = \int \sqrt{-g_{tt}}\,\mathrm{d}t \quad \text{(time-like)} \tag{17}$$

So Δt is the external time of photon propagation, not the proper time of the path itself. If in the superluminal scenario, assuming $v_{eff,x} = \frac{\Delta x}{\Delta t} = 2c$, then it should be $g_{xx} = 1/4$, $\Delta t = \frac{\sqrt{1/4}\Delta x}{c} = \frac{\Delta x}{2c}$. That is, the coordinate time Δt of photon propagation is shortened, and the apparent speed increases due to space compression. The apparent speed is the "coordinate speed" (such as the $u = t$ coordinate system), so Δt is the coordinate time, reflecting the photon propagation time seen by the observer ($u = t$). Thus, if the photon propagates along x, considering the relationship between $\mathrm{d}u$ and $\mathrm{d}v$, we can use coordinate time (based on light cone coordinates) to parameterize $u = t$, $v = v(t)$, $x = x(t)$, $\frac{\mathrm{d}u}{\mathrm{d}t} = 1$, $\frac{\mathrm{d}x}{\mathrm{d}t} = v_{eff,x}$, and

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 = -2c^2\frac{\mathrm{d}v}{\mathrm{d}t} + c^2 H + \left(1 + h_{xx}\right)v_{eff}^2 \tag{18}$$

Then the geometric effect can be found:

$$g_{xx} = 1 + h_{xx} = 1 + A \tag{19}$$

The photon propagation time is affected by g_{xx} as shown in (16), and the apparent velocity is:

$$v_{eff,x} = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\frac{\sqrt{1+A}\Delta x}{c}} = \frac{c}{\sqrt{1+A}} \tag{20}$$

If we assume $v_{eff,x} = 2c$, we have

$$\frac{c}{\sqrt{1+A}} = 2c \Rightarrow A = -\frac{3}{4} \tag{21}$$

Therefore, as long as the gravitational soliton $\omega_V = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} = \begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix}$ takes the maximum value (that is, in the region of $ku \ll 1$), we can get

$$g_{xx} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$v_{eff,x} = \frac{c}{\sqrt{\frac{1}{4}}} = 2c$$

As for the influence of the h_{xy} cross term, since $\mathrm{d}y = 0$ along the x direction, h_{xy} has no contribution, so we can calculate in the y direction by taking

$A = -\frac{3}{4}$, maximum disturbance $ku \ll 1$ and $\operatorname{sech}^2(ku) \approx 1$, then we can obtain

$$h_{yy} \approx \frac{3}{4}\operatorname{sech}^2(0) = \frac{3}{4} \tag{22}$$

$$g_{yy} = 1 + h_{yy} = 1 + 3/4 = 7/4 \tag{23}$$

Since along the y direction, $\mathrm{d}x = 0$, the metric (12) becomes:

$$\mathrm{d}s^2 = -2c^2\mathrm{d}u\mathrm{d}v + \left(1 + h_{yy}\right)\mathrm{d}y^2 + c^2H\mathrm{d}u^2 \tag{24}$$

and because of $x = y = 0$, so $H = 0$, we get

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 = -2c^2\mathrm{d}u\mathrm{d}v + \left(1 + h_{yy}\right)v_{eff,y}^2 \tag{25}$$

Then the apparent velocity in the y direction is

$$v_{eff,y} = \frac{c}{\sqrt{g_{yy}}} \tag{26}$$

Again, because of $g_{yy} = 1 + h_{yy} = 1 - A = \frac{7}{4}$, so we get:

$$v_{eff,y} = \frac{2c}{\sqrt{7}} \approx 0.756c \tag{27}$$

Now we have $v_{eff,x} = v_x = 2c$ (along x); $v_{eff,y} = v_y \approx 0.756c$ (along y); Assuming x and y are perpendicular, we have

$$v_{eff} = v_x^2 + v_y^2 \tag{28}$$

$$v_{eff} = (2c)^2 + \left(\frac{2c}{\sqrt{7}}\right)^2 \approx 2.14c \tag{29}$$

Here the direction angle $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ is

$$\theta = \tan^{-1}\left(\frac{0.756c}{2c}\right) = \tan^{-1}(0.378) \approx 20.7^\circ \tag{30}$$

Therefore, we obtain $v_{eff} \approx 2.14c > c$, and the superluminal goal is achieved.

Furthermore, we can use a more rigorous method and consider the influence of cross terms to solve the above apparent velocity again, and the detail process is shown following 4 steps:

(1) Metric simplification and the photon equation of motion

In the central region ($x \approx y \approx 0$) ignoring the higher-order terms ($H \approx 0$), the metric (12) simplifies to:

$$\begin{aligned}\mathrm{d}s^2 = &-2c^2\mathrm{d}u\mathrm{d}v + \left(1 - \frac{3}{4}\operatorname{sech}^2(ku)\right)\mathrm{d}x^2 + 2\cdot\frac{\sqrt{7}}{4}\operatorname{sech}^2(ku)\mathrm{d}x\mathrm{d}y \\ &+ \left(1 + \frac{3}{4}\operatorname{sech}^2(ku)\right)\mathrm{d}y^2\end{aligned} \tag{31}$$

The photon trajectory (note: the intrinsic speed of the photon is constant at *c*, and superluminal speed is only the coordinate speed) satisfies $\mathrm{d}s^2=0$, so by substituting it into (31) we get:

$$0=-2c^2\mathrm{d}u\mathrm{d}v+\left(1-\frac{3}{4}\operatorname{sech}^2(ku)\right)\mathrm{d}x^2+2\cdot\frac{\sqrt{7}}{4}\operatorname{sech}^2(ku)\mathrm{d}x\mathrm{d}y+\left(1+\frac{3}{4}\operatorname{sech}^2(ku)\right)\mathrm{d}y^2 \tag{32}$$

(2) Parametric path and speed definition

Let $\mathrm{d}x=v_x\mathrm{d}u$ and $\mathrm{d}y=v_y\mathrm{d}u$ (Note: v_x and v_y defined in this way are related to the coordinate time $u=t$, the coordinate quantities $\mathrm{d}x$ and $\mathrm{d}y$, hence they are coordinate velocities, $v_x=\frac{\mathrm{d}x}{\mathrm{d}u}$ and $v_y=\frac{\mathrm{d}y}{\mathrm{d}u}$. They are the velocities measured in the global coordinate system, which may exceed the speed of light *c*, but do not violate the theory of relativity. Substituting them into Equation (32) yields:

$$0=-2c^2\mathrm{d}v+\left[\left(1-\frac{3}{4}\operatorname{sech}^2(ku)\right)v_x^2+\frac{\sqrt{7}}{2}\operatorname{sech}^2(ku)v_xv_y+\left(1+\frac{3}{4}\operatorname{sech}^2(ku)\right)v_y^2\right]\mathrm{d}u \tag{33}$$

After rearranging, we get the differential equation:

$$\frac{\mathrm{d}v}{\mathrm{d}u}=\frac{1}{2c^2}\left[\left(1-\frac{3}{4}\operatorname{sech}^2(ku)\right)v_x^2+\frac{\sqrt{7}}{2}\operatorname{sech}^2(ku)v_xv_y+\left(1+\frac{3}{4}\operatorname{sech}^2(ku)\right)v_y^2\right] \tag{34}$$

(3) Nonlinear coupling of velocity components

Considering the influence of cross terms, since the h_{xy} term in the metric leads to nonlinear coupling of v_x and v_y, a joint solution is required, therefore an auxiliary variable $k=\frac{v_y}{v_x}$ is introduced and Equation (34) becomes:

$$\frac{\mathrm{d}v}{\mathrm{d}u}=\frac{v_x^2}{2c^2}\left[\left(1-\frac{3}{4}\operatorname{sech}^2(ku)\right)+\frac{\sqrt{7}}{2}\operatorname{sech}^2(ku)k+\left(1+\frac{3}{4}\operatorname{sech}^2(ku)\right)k^2\right] \tag{35}$$

Here notice if $ku\ll 1$, $\operatorname{sech}^2(ku)\approx 1$, the above Equation (35) can be simplified to:

$$\frac{\mathrm{d}v}{\mathrm{d}u}=\frac{v_x^2}{2c^2}\left[\frac{1}{4}+\frac{\sqrt{7}}{2}k+\frac{7}{4}k^2\right] \tag{36}$$

(4) Synthesis and extreme value analysis of equivalent velocity

We apply an extremum condition, which corresponds to the direction in which the photon path has the least energy consumption or the most stable propagation under perturbations. We choose the minimum rather than the maximum because the physical system tends to be in a low energy state, so we can let

$$\frac{\mathrm{d}}{\mathrm{d}k}\left[\frac{1}{4}+\frac{\sqrt{7}}{2}k+\frac{7}{4}k^2\right]=0 \tag{37}$$

The extreme value condition helps determine the ratio of the velocity components $k=-\frac{\sqrt{7}}{7}$, so that the photon is least affected by disturbances when propagating in this direction, that is,

$$k=-\frac{\sqrt{7}}{7}\Rightarrow v_y=-\frac{\sqrt{7}}{7}v_x \tag{38}$$

then the modulus of the composite velocity is:

$$v_{eff}=v_x^2+v_y^2=v_x\sqrt{1+\left(\frac{\sqrt{7}}{7}\right)^2}\approx 1.06v_x\approx 2.12c \tag{39}$$

So it can be concluded that in the central region where x, y are small, the cross term causes the velocity direction to deflect, but the effect on the total velocity norm is small, and the total apparent velocity v_{eff} is still superluminal. Considering futher that the solution of gravitational solitons (see **Appendix A**) requires $A^2+B^2=1$, we take $B=\frac{\sqrt{7}}{4}$ to be symmetric with $|A|$, enhancing the coupling between x and y, but h_{xy} does not change v_{eff}.

4. Coordinate Velocities of Curvature Bubbles and CTCs

The two most important factors of this curvature engine time ship are that the apparent speed of the curvature bubble is faster than the speed of light and the CTCs produce time reversal in the bubble. These induced several concepts need to be explained here:

1) The nature of apparent velocity:
- $v_{eff}\approx 2.14c$ is the geometric effect caused by the metric perturbations (h_{xx}, h_{xy}, h_{yy}), not the intrinsic speed of the curvature bubble.
- Similar to the Alcubierre drive [9] [25] as the space compresses in front and expands in the back, the objects are stationary inside the bubble and the observers in an external coordinate system (such as the Earth's coordinate system) to see the objects "moving" faster than the speed of light.
- The annular beam generates a dynamic $h_{\mu\nu}$ in the x-y plane, and the curvature bubble envelops the central region of the annular ring as a local spacetime deformation zone, without requiring the annular device to be physically moved.

2) Reasons why the inherent velocity is zero:
- The passenger cabin is stationary in the center of the curvature bubble. Its proper time $\Delta\tau$ is different from the external coordinate time Δt, but the cabin itself has no spatial displacement in the curvature bubble.
- Time travel relies on closed timelike curves (CTCs), generated by the global effect of a ring beam rotating around the circumference of the ring, rather than the motion of the capsule.

3) Is the curvature bubble moving?
- In the current design, the curvature bubble wraps around the center of the

ring, the ring-shaped light beam rotates around the circumference of the ring, the manned cabin is fixed at the center of the ring ($x = y \approx 0$), and the gravitational solitons generated by polarized light solitons compress or stretch space-time to produce superluminal motion, but all objects in the curvature bubble have no relative motion and are all stationary.

- o Later, we hope that the curvature bubble will wrap the annular spacecraft and move along the Earth coordinate system at an apparent speed (such as along the x direction), while keeping the CTCs in the curvature bubble in "virtual" movement, that is, we can design 8 polarized light beams to rotate around the circumference of the ring to produce this "virtual".

4) Time travel mechanism:

- o CTCs do not rely on the bulk motion of the bubble, but rather on the twisting of spacetime induced by the rotation of the ring-shaped beam (similar to a rotating universe).
- o The passenger cabin is inside the bubble, and the external time is "advanced" because $v_{eff} > c$, while the time inside the cabin flows backward (*i.e.*, $\Delta\tau < 0$).
- o Further analysis shows that $v_{eff,y} < c$ means that light propagates slower in the (y) direction, which does not directly lead to time advancement, but is the result of space-time stretching. To achieve "time to the future", if we adjust $g_{00} = 0.5$, $\mathrm{d}s^2 = -0.5c^2\mathrm{d}t^2 + \frac{7}{4}\mathrm{d}y^2$, $\mathrm{d}y = 0$ (time component compression), we can make the proper time $\mathrm{d}\tau = \sqrt{\frac{-\mathrm{d}s^2}{c^2}} = -0.5\mathrm{d}t$, that is, $\mathrm{d}\tau < \mathrm{d}t$, the apparent time is greater than the proper time, and the time in the cabin is advanced. However, in order to simplify the problem, this article only involves the problems of time reversal and superluminal speed in the subsequent analysis and design. Here we just mention that the time ring machine we designed can advance time and rewind time in both directions.

So here $v_{eff} \approx 2.14c$ is the geometric effect, not the physical speed of the curvature bubble or ring. The ring device should be fixed in the cabin, the curvature bubble is generated in the center, the passenger cabin is stationary inside, and the time effect is driven by CTCs. Below we prove that the CTCs of this model do exist:

First, the perturbation metric (12) is expressed in polar coordinates

$$\mathrm{d}s^2 = -2c^2\mathrm{d}u\mathrm{d}v + c^2H\mathrm{d}u^2 + g_{rr}\mathrm{d}r^2 + 2g_{r\varphi}\mathrm{d}r(\omega\mathrm{d}u) + g_{\varphi\varphi}(\omega\mathrm{d}u)^2 \tag{40}$$

here, through the polar coordinate transformation $x = r\cos\varphi$, $y = r\sin\varphi$, the components of the metric are expressed as:

$$g_{uu} = c^2H = c^2\operatorname{sech}^2(ku)\left[-\frac{3}{4}r^2\cos 2\varphi + \frac{\sqrt{7}}{2}r^2\sin 2\varphi\right] \tag{41}$$

$$g_{rr} = 1 + \operatorname{sech}^2(ku)\left[-\frac{3}{4}r^2\cos 2\varphi + \frac{\sqrt{7}}{2}r^2\sin 2\varphi\right] \tag{42}$$

$$g_{\varphi\varphi} = r^2\left[1 + \operatorname{sech}^2(ku)\left(\frac{3}{4}r^2\cos 2\varphi - \frac{\sqrt{7}}{2}r^2\sin 2\varphi\right)\right] \tag{43}$$

Then we calculate the closed integral $\oint \mathrm{d}s^2$, where the circular path is: $r = R = 5\ \mathrm{m}$, $\varphi = \omega t$, $u \approx t$, and the integration steps are as follows:

1) Metric simplification: By ($\mathrm{d}r = 0$), and $\mathrm{d}u = \frac{\mathrm{d}\varphi}{\omega}$, the above metric (40) is simplified to:

$$\mathrm{d}s^2 = \left(-2c^2 + c^2H + g_{\varphi\varphi}\omega^2\right)\frac{\mathrm{d}\varphi^2}{\omega^2} \tag{44}$$

2) Substitute the H component into

$$H = \operatorname{sech}^2(ku)\left[-\frac{3}{4}R^2\cos 2\varphi + \frac{\sqrt{7}}{2}R^2\sin 2\varphi\right] \tag{45}$$

$$\oint \mathrm{d}s^2 = \frac{1}{\omega^2}\int_0^{2\pi}\left(-2c^2 + c^2H + g_{\varphi\varphi}\omega^2\right)\mathrm{d}\varphi = \frac{-4\pi c^2}{\omega^2} + 0 + 2\pi R^2 \tag{46}$$

where $\frac{1}{\omega^2}\int_0^{2\pi}\left(c^2H\right)\mathrm{d}\varphi = 0$, since $\int_0^{2\pi}\operatorname{sech}^2\left(\frac{k\varphi}{\omega}\right)\cos 2\varphi \mathrm{d}\varphi = 0$, $\int_0^{2\pi}\operatorname{sech}^2\left(\frac{k\varphi}{\omega}\right)\sin 2\varphi \mathrm{d}\varphi = 0$, therefore we obtain:

$$\oint \mathrm{d}s^2 = \frac{-4\pi c^2}{\omega^2} + 2\pi R^2 \tag{47}$$

Note that here defining the integral of the quadratic form along the parameterized path as $\oint \mathrm{d}s^2 := \oint\left(\frac{\mathrm{d}s}{\mathrm{d}\varphi}\right)^2\mathrm{d}\varphi$. Substituting the numerical values for calculation ($R = 5\ \mathrm{m}$, $\omega = \frac{c}{5} = 6\times10^7\ \mathrm{s}^{-1}$, $c = 3\times10^8\ \mathrm{m/s}$), the formula (46) becomes

$$\oint \mathrm{d}s^2 = \frac{-4\pi c^2}{\omega^2} + 2\pi R^2 = -2\pi R^2 < 0 \tag{48}$$

If $R = 10\ \mathrm{m}$, then

$$\oint \mathrm{d}s^2 = -\pi R = -314.16\left(\mathrm{m}^2\right) < 0 \tag{49}$$

That is, the more negative it is, the larger R is, and the higher the energy required.

From the above calculations, we can see that: the metric component of the central region $g_{xx} = \frac{1}{4} \Rightarrow 2c$; and the circular closed integral $\oint \mathrm{d}s^2 = \frac{4\pi c^2}{\omega^2} + 2\pi R^2$; time reversal is driven by CTCs and exists independently of superluminal speed. Therefore, this model can mathematically satisfy both superluminal speed and CTCs at the same time, and the both are realized through different metric components without direct conflict. Specifically, the metric perturbation of the gravitational soliton is separable: $h_{xx}(u)$, dominates the space compression (2 times the superluminal speed in the x direction); $h_{xy}(u)$ and $H(u)$, dominate the circular closed integral (CTCs). Their parameters are compatible: $A = -\frac{3}{4}$, $B = \frac{\sqrt{7}}{4}$, satisfying $A^2 + B^2 = 1$, without additional constraint conflicts. Therefore, in the structure of this model, it is necessary to consider designing 8 polarized beams evenly distributed on the circumference of the ring, with a difference of 45 degrees between each. In this way, energy demand and disturbance stability are guaranteed, and energy focusing and central area compression coexist with annular disturbances; the total power required is estimated to be about $P \sim 10^{14}\ \mathrm{W}$, which is mapped to metric perturbations through gauge transformation. So the numerical simulations show that metric perturbations are stable in the short term ($\operatorname{sech}^2(ku)$ slowly changing). In this way, a curvature engine time spacecraft can be constructed in which time reversal and superluminal physics coexist. The time travel mechanism here are that CTCs allow closed timelike paths, and passengers can cycle through the past and the future; superluminal speed shortens the external observation time, and the internal time flow is controlled by the CTC. No doubt, if the CTC is confined to a local bubble, global causality may be maintained and further topological analysis is required. However, the calculation of time reversal can be performed, namely if let the return time be $\Delta\tau$, then we can get

$$\Delta\tau = \frac{\sqrt{-\oint \mathrm{d}s^2}}{c} \tag{50}$$

By taking $R = 5\ \mathrm{m}$, $\omega = 6\times10^7\ \mathrm{s}^{-1}$, from the above, we know that $\oint \mathrm{d}s^2 = -2\pi R^2 = -157.08\left(\mathrm{m}^2\right)$, hence we have

$$\Delta\tau = \frac{\sqrt{157.08}}{3\times10^8} \approx 4.18\times10^{-8}\ \mathrm{s} \tag{51}$$

That is, the period

$$T = 2\pi/\omega = 2\pi/6\times10^7 \approx 1.047\times10^{-7}\ \mathrm{s} \tag{52}$$

and the backflow time of each cycle is 4.18×10^{-8} s.

5. Questions about Movement of the Curvature Bubble

Question 1: Does the curvature bubble envelop the spacecraft and make it travel at superluminal apparent speed?

Although the apparent velocity $v_{eff} \approx 2.14c$ is a geometric effect caused by the metric perturbation, but there is no the intrinsic motion speed of the curvature bubble or the spacecraft. Similar to the Alcubierre drive, the space is compressed in the front and expanded in the back, the spacecraft is stationary in the curvature bubble, and the external observer (such as the earth coordinate system) should see the spacecraft moving at v_{eff} [16]. However, due to the fixed ring device in the current design, the 8 soliton beams rotate around the ring in the x-y plane, generating a curvature bubble at the center ($x = y = 0$); the passenger cabin is stationary in the bubble (intrinsic velocity = 0), and the time reversal is driven by CTCs; therefore, $v_{eff} \approx 2.14c$ is a local geometric effect, and the curvature bubble itself is not designed to move as a whole. However, we do not want this "virtual geometric speed", but hope that the curvature bubble wraps the entire spacecraft and moves along the earth coordinate system at a superluminal apparent speed (such as $v_{eff} \approx 2.14c$ or higher) while retaining the time reversal effect of CTCs. To this goal, the following adjustments from theoretical and engineering aspects are made to the structure.

Firstly, we clarify the current limitations:

1) Static properties of curvature bubble:

- In the current model, the curvature bubble is generated by the interference of 8 light beams, and the perturbation peak is fixed at $x \approx y \approx 0$, which is the center of the ring.
- The spatial distribution of $h_{\mu\nu} = \operatorname{sech}^2(ku)$ is static (or only changes periodically with time) and is not designed to move along the Earth's coordinate system (such as in the x direction).
- $v_{eff} \approx 2.14c$ is the apparent speed of the test signal (such as a photon) in the distorted spacetime inside the bubble, not the speed at which the curvature bubble itself moves.

2) Sources of CTCs:

- CTCs are generated by the spacetime twist induced by the beam's rotation around the ring (angular velocity ω), relying on the global effect of $g_{t\varphi}$ or $g_{\varphi\varphi}\omega^2$.
- This twist does not require the curvature bubble to move as a whole, but is a geometric property of the circular path.

3) Challenges of wrapping a spaceship:

- The current curvature bubble range is about $1/k \approx 1\,\text{cm}$ ($k = 10^2\,\text{m}^{-1}$), which is not enough to wrap the entire 10 m diameter spacecraft.
- To achieve superluminal movement of the entire spacecraft, the curvature bubble needs to expand to cover the spacecraft and dynamically adjust to propel it along the Earth's coordinate system.

So from above 1) - 3) analysis, in order to wrap the toroidal spacecraft in a curvature bubble and fly along the Earth coordinate system at v_{eff} while preserving CTCs, we need:

(a) Extended curvature bubble size:

- Adjust (k): From the current $k = 10^2\,\text{m}^{-1}$ and the perturbation range $1/k \approx 0.01\,\text{m}$ to reduce $k = 0.1\,\text{m}^{-1}$ and to extend the range to $\frac{1}{k} \approx 10\,\text{m}^{-1}$, which allow the range enough larger to wrap around the ship so that $h_{xx} = A\operatorname{sech}^2(ku)$ is still possible. This reducing (k) does not change (A) or v_{eff}, but requires increasing the beam energy to maintain the perturbation strength.
- Energy requirement: Since $\rho_{EM} \propto h_{xx} \propto P/V$, the volume ($V$) increases

by $(10/0.01)^3 = 10^6$ times, so the power needs to be increased from 10^{14} W to about 10^{19} W.

(b) Dynamically adjust the curvature bubble position:

- o Virtual movement becomes real movement:
  - ▪ The current "virtual move" is that the beam focus is adjusted so that the $h_{\mu\nu}$ peak moves in the *x*-*y* plane (e.g., in a circle).
  - ▪ Instead, move along a straight line (e.g., x direction): adjust the phase and emission direction of the 8 beams so that the center of the curvature bubble moves from (0, 0) to $(x_c(t), 0)$, e.g., $x_c(t) = v_{eff} t$.
  - ▪ Phase modulation: $\varphi_i(t) = k(x_i - x_c(t))$, the beam interference peak shifts along the *x*-axis with time.
- o Metric changes:

Since $h_{xx} = A\text{sech}^2(k(x - v_{eff} t))$, $h_{xy} = B\text{sech}^2(k(x - v_{eff} t))$, $h_{yy} = -A\text{sech}^2(k(x - v_{eff} t))$, this means that the curvature bubble moves at v_{eff}, similar to the dynamic bubble driven by Alcubierre. The consistent movement of h_{xy} does not affect v_{eff}; the curvature bubble moves at v_{eff} precisely because the central area (disturbance peak) moves at v_{eff}, which is achieved by the dynamic control of beam interference. The original solution is $h_{xx} = A\text{sech}^2(ku)$, $u = t$ or $u = t - x/c$, and the dynamic solution now is $h_{xx} = A\text{sech}^2(k(x - v_{eff} t))$, if $u = x - v_{eff} t$ is defined. So $ku = k(x - v_{eff} t)$ is formally equivalent. Key difference is that the original $u = t - x/c$ indicates that the soliton propagates at the speed of light, while the new solution moves at v_{eff} (superluminal). The mathematical form of the solution remains unchanged (still sech^2), but the propagation speed changes from *c* to v_{eff}. Physical consistency is that the gravitational soliton generated by the gauge transformation depends on the soliton properties of $\text{sech}^2(ku)$, not on the specific speed. $v_{eff} = 2.14c$ is a geometric effect, determined by $A = -3/4$, and does not become invalid due to the change in the definition of *u*.

(c) Preserve CTCs:

- o The beam continues to rotate around the ring (angular velocity $\omega = 6 \times 10^7\ \text{s}^{-1}$), maintaining $g_{t\varphi}$ and $\oint \text{d}s^2 < 0$ inside the bubble.
- o When the spacecraft moves as a whole, the annular structure is fixed inside the bubble, and the CTCs effect moves with the bubble.

(d) Implementation:

- o Structure: The spacecraft is a 10 m sphere with 8 lasers distributed on the sphere and a circular path retained inside.
- o Propulsion: The front beam is enhanced $h_{xx} < 0$ (compressed space), and weakened at the back (expanded space), driving the bubble to move along the *x*-axis at $v_{eff} = 2.14c$ (in a direction of 20.7 degrees).
- o Energy: 10^{19} W (fusion or antimatter level) and material stress 10^{13} Pa (graphene composite) are required.

From these we get a desired result: The curvature bubble wraps the spacecraft, flying along the Earth coordinate system at $v_{eff} = 2.14c$, and the passenger cabin inside the bubble is stationary, and CTCs keep time flowing backwards.

Question 2: The generation of curvature bubble, the relationship between velocity and apparent velocity

1) How is a curvature bubble created?

- Physical mechanism:
  - The curvature bubble is generated by the interference of 8 polarized soliton beams, through the gauge transformation (4) converting electromagnetic disturbances into gravitational disturbances $h_{\mu\nu}$.
  - The beam is emitted along the ring tangentially, with polarization rotated (rotation of polarization $\omega = 10^6\ \mathrm{s}^{-1}$ and rotation around the ring circumference with an angular velocity of 6 × 10^7 s^{-1}, forming an energy density peak in the $x \approx y \approx 0$ region, so that $\rho_{EM} = \frac{1}{2} 2\epsilon_0 E^2$, $E \propto \sqrt{P}$, 8 beams superimposed, and $\rho_{EM} \sim 10^{19}\ \mathrm{J/m^3}$, see detail calculation of energy in **Appendix B**.
- Gravitational effect: The gravitational field is generated through the energy-momentum tensor $T_{\mu\nu}$ of the optical soliton. This relationship is supported by the Einstein field equations.
- Under strong fields, the nonlinear effect of $h_{\mu\nu}$ is significant, and the form of ω_V provides a localized space-time curvature, which is consistent with the physical image of gravitational solitons.
  - $h_{xx} = -3/4\,\mathrm{sech}^2(ku)$ and other disturbances form a curvature bubble, and local spacetime is compressed or stretched.

2) What is the speed of the curvature bubble?

- Intrinsic velocity: In the current optimized design, the curvature bubble wraps around the entire ring-shaped time machine, with intrinsic velocity = 0 (relative to the spacecraft or ring). But the position of its central region changes with time to make CTCs motion. The beam rotates around the ring (velocity c), but does not drive the bubble to move.
- Apparent velocity is defined as the speed at which a bubble or signal moves as seen by an external observer (e.g., in the Earth coordinate system). The apparent velocity is the "moving speed" of the bubble, and $v_{eff} = c/\sqrt{g_{xx}}$ is controlled by h_{xx}, and related to the beam polarization angle. The optimized curvature bubble moves at $v_{eff} = 2.14c$ (or higher) along the Earth coordinate system.

6. Sudden Jump in Apparent Velocity

From the above calculations and analysis, we can know that the curvature engine time spacecraft we designed has good performance at about 2.14 times the speed of light. If the radius of the spacecraft is 5 meters, the reflow time is

$\Delta\tau = \frac{\sqrt{157.08}}{3\times10^8} \approx 4.18\times10^{-8}\ \mathrm{s} = 4.18$ nanoseconds. However, considering the requirements of real long-distance interstellar flight, the apparent speed and reflow time of this curvature bubble are not enough. **For this reason, we envision a jump speed solution.** That is, considering the apparent speed is

$$v_{eff} = \frac{c}{\sqrt{g_{xx}}} = \frac{c}{\sqrt{1+H_{xx}}} = \frac{c}{\sqrt{1+A}}$$

If H_{xx} is very close to −1, then v_{eff} may be very large. For this reason, we can consider that it tends to −1 in 1 nanosecond and then returns to its original value. Then the v_{eff} may tend to an extremely amazing value in 1 nanosecond, for example, the apparent velocity reaches 1000c or more. The following analysis is made:

(1) $A \to 0.9999$ sudden jump risk and feasibility

The relevant background parameters are given by the above analysis and calculation, here the Metric perturbation form as formula (12), namely

$$\mathrm{d}s^2 = -2c^2\mathrm{d}u\mathrm{d}v + \left[1+h_{xx}\right]\mathrm{d}x^2 + 2h_{xy}\mathrm{d}x\mathrm{d}y + \left[1+h_{yy}\right]\mathrm{d}y^2 + c^2H\mathrm{d}u^2$$

where

$$h_{xx} = A\,\mathrm{sech}^2(ku)$$

$$h_{xy} = B\,\mathrm{sech}^2(ku)$$

$$h_{yy} = -A\,\mathrm{sech}^2(ku)$$

$$H = \mathrm{sech}^2(ku)\left[A\left(x^2-y^2\right)+2Bxy\right]$$

- $A^2 + B^2 = 1$ (e.g. $A = -3/4$, $B = \sqrt{7}/4$)

The apparent velocity is

$$v_{eff} = \frac{c}{\sqrt{g_{xx}}}, g_{xx} = 1 + h_{xx} = 1 + A$$

when $A \to -0.9999$, we have

$$g_{xx} = 1 + (-0.9999) = 0.0001$$

$$v_{eff} = \frac{c}{\sqrt{0.0001}} = 100c$$

The sudden jump concept is that the ultra-high v_{eff} and significant time reversal are produced, by briefly jumping A from a small value (such as -0.9) to -0.9999 and then quickly returning.

(2) Potential dangers

Even assuming unlimited energy and material strength, the following risks may exist for sudden jumps:

1) Singularity Risk

- Problem: When $A \to -1$ or exceeds −1 (for example, due to control error becoming −1.0001), $g_{xx} = 1 + h_{xx}$, which causes the metric sign to flip (from (−, +, +, +) to an unphysical state), possibly forming a naked singularity.
- Consequences: The fabric of spacetime could collapse, creating a black

hole-like effect that could destroy the ship or the surrounding area.

- Mitigation: Short jumps (e.g. $\Delta t = 1\,\text{ns}$) may not allow the singularity to form completely in time, but require ultra-precise (A) control (accuracy < 10^{-4}), such as the control technology of elementary particle flow in elementary particle experiments.

2) Causal destruction

- Problem: Extreme space-time distortion with $v_{eff} = 100c$ may induce closed time-like curves (CTCs), especially in the ring design where $\oint \text{ds}^2 < 0$.
- If the abrupt jump amplifies the CTCs effect, it may cause excessive time reversal and trigger causal paradoxes (such as the "grandfather paradox").
- Consequences: Timeline confusion, the ship may go back in time and change its own history.
- Mitigation: temporarily limit the scope of CTCs (such as backflow within the cabin only), or accept the many-worlds interpretation (return to the parallel universe).

3) Gravitational gradient and tidal forces

- Problem: g_{xx} suddenly drops from 1 to 0.0001, the space is compressed violently, and the gravitational gradient (tidal force) may reach an extreme value: curvature $\propto \partial^2 h_{xx} / \partial x^2 \propto k^2 A \text{sech}^2(ku) \tanh(ku)$. If $k = 0.1\,\text{m}^{-1}$, $A = -0.9999$, the curvature surges.
- Consequences: Even if the material strength is sufficient, objects or occupants in the cabin may be torn apart by tidal forces.
- Mitigation method: The jump time is extremely short (<10^{-9} s), or a uniform disturbance field is designed to reduce the gradient.

4) Quantum gravitational effects

- Question: When $g_{xx} \to 0^+$, the curvature of space-time tends to infinity and may enter the Planck scale ($l_p \approx 10^{-35}\,\text{m}$), where classical general relativity becomes invalid.
- Consequences: Unknown quantum gravity effects could collapse the perturbation, or trigger an unpredictable collapse.
- Mitigation: Short jumps may allow a "jump start" through the danger zone, but the instantaneous response of quantum effects remains unknown.

(3) Physical impossibility

Even if energy and materials were unlimited, there are still potential obstacles:

1) Control accuracy limit

- $A = -0.9999$ needs to be accurate to 10^{-4} level, any small deviation (such as −1.0001) will make $g_{xx} < 0$.
- Current technologies (such as laser phase control) may be less accurate than this, and in the future, quantum computing plus AI-level control may be required.

2) Soliton stability

- The soliton form of $h_{xx} = A \text{sech}^2(ku)$ may be unstable, decompose or

dissipate when $A \to -1$.
- o It is necessary to verify whether the conversion of optical solitons to gravitational solitons can sustain such extreme perturbations.

3) Causal Protection Hypothesis
- o Hawking proposed [26]-[28] that quantum fluctuations may prevent CTCs or extreme superluminal effects, automatically destroying the state of $g_{xx} \to 0$.
- o Short jumps may be circumvented, but this still needs to be verified experimentally.

(4) Feasibility of short jumps
- Advantages: If the jump is completed within $\Delta t = 1\,\text{ns}$, singularity and causality problems may not have time to develop.
- Travel distance: $\Delta x = v_{eff}\Delta t = 100c \times 10^{-9} = 300\,\text{m}$.
- Time reversal: The $\Delta\tau$ of a single jump needs to be calculated by $\oint ds^2$, which may be small, but multiple jumps can be accumulated. The key is to design an instantaneous return mechanism (such as laser pulse switching), for example, to ensure that the time window < 10^{-10} s.

Moreover, repeated numerical calculations have found that increasing the number of rings or beam superposition effect is a feasible optimization direction, which can not only increase the time return flow $\Delta\tau$ to the target value (for example, the return time to the Andromeda Galaxy is −410 years), but also try to maintain the main calculation results of the original model (such as v_{eff} and beam surround mechanism), while considering energy saving. The following is a detailed optimization design and calculation process:

1) Optimization objectives and constraints

Target
- $v_{eff} = 10^7 c$, to the Andromeda Galaxy (2.5 × 10^6 light-years):
- Sailing time: 91.44 days
- Reflow time: -410 years $\approx -1.293 \times 10^{10}$ s

Constraint
- $\omega = c/R$ (the speed of the light beam around the ring is the speed of light c), which is not easy to adjust.
- $h_{\mu\nu} = A\,\text{sech}^2(ku)$ (A determines v_{eff}), maintaining the gravitational soliton form.
- $\oint ds^2 = -2\pi R^2$ is the single-ring base value.
- Energy saving: avoid large increases in beam power (e.g. 10^{19} W to higher).

2) Increase the number of rings or beam superposition effect

Principle
- Single loop reflux:
  - o When $R = 5\,\text{m}$, $\omega = 6 \times 10^7\,\text{s}^{-1}$, $\oint ds^2 = -157.08\,\text{m}^2$ (Single loop path integration), the time reflux for single loop is $\Delta\tau = -4.18 \times 10^{-8}\,\text{s}$;
  - o When $R = 10\,\text{m}$, $\omega = 6 \times 10^7\,\text{s}^{-1}$, $\oint ds^2 = -628.32\,\text{m}^2$, $\Delta\tau = -8.36 \times 10^{-8}\,\text{s}$.

- Multiple rings stacking:
    - (N) rings run in parallel, each ring contributes $\oint ds^2 = -2\pi R^2$, total time return is $\Delta\tau_{total}$, where $\oint ds_{total}^2 = N \times (-2\pi R^2)$ is defined as N loop path integration, hence $\Delta\tau_{total}$ is $\Delta\tau_{total} = -\frac{\sqrt{N \times 2\pi R^2}}{c}$
    - Advantages: It does not change the single-ring $h_{\mu\nu}$ or ω, and only amplifies the CTCs effect through superposition.

Calculate the number of rings required (N)

- Target: Andromeda return −410 years;
- For $R = 5\ \text{m}$, we have:

Total number of cycles: $91.44 \times 8.25 \times 10^{11} \approx 7.55 \times 10^{13}$, Period (T) of a single cycle (the time it takes for the beam to go around the ring): $T = \frac{2\pi}{\omega} = \frac{2\pi}{6 \times 10^7} \approx \frac{6.2832}{6 \times 10^7} \approx 1.0472 \times 10^{-7}\ \text{s}$; Number of cycles per second (frequency (f)): $f = \frac{1}{T} = \frac{1}{1.0472 \times 10^{-7}} \approx 9.549 \times 10^6\ \text{Hz}$ (or cycles/second); There are 24 × 60 × 60 = 86,400 s in a day, n_{day} (Number of cycles per day) $= f \times 86400 = 9.549 \times 10^6 \times 86400 \approx 8.25 \times 10^{11}$; flight time: $91.44\ \text{days} \times 86400\ \text{s/day} = 7.9 \times 10^6\ \text{s}$; total number of cycles: $n = f \times \text{flight time} = 9.549 \times 10^6 \times 7.9 \times 10^6 \approx 7.55 \times 10^{13}$. Hence

- Single reflow time: total reflow time/total number of cycles $= \frac{-1.293 \times 10^{10}}{7.55 \times 10^{13}} \approx -1.71 \times 10^{-4}\ \text{s}$,
- N loop integration paths $\oint ds_{total}^2 = -(3 \times 10^8 \times 1.71 \times 10^{-4})^2 \approx -2.63 \times 10^9\ \text{m}^2$,
- The number of N loops is $N = \frac{2.63 \times 10^9}{157.08} \approx 1.67 \times 10^7$.

- For $R = 10\ \text{m}$:
    - Number of cycles: $91.44 \times 4.13 \times 10^{11} \approx 3.78 \times 10^{13}$,
    - Single reflux: $\frac{-1.293 \times 10^{10}}{3.78 \times 10^{13}} \approx -3.42 \times 10^{-4}\ \text{s}$,
    - $\oint ds_{total}^2 = -(3 \times 10^8 \times 3.42 \times 10^{-4})^2 \approx -1.05 \times 10^{10}\ \text{m}^2$,
    - $N = \frac{1.05 \times 10^{10}}{628.32} \approx 1.67 \times 10^7$.

However, we later found from the energy calculation that $N = 1.67 \times 10^7$ is too large and the energy requirement may be too high, so it is more realistic to adjust the number of loops to $N \sim 10^5$, then we have:

- For $R = 5\ \text{m}$:

$$\oint ds^2 = -10^5 \times 157.08 = -1.57 \times 10^7\ \text{m}^2$$

$$\Delta\tau = -\frac{\sqrt{1.57 \times 10^7}}{3 \times 10^8} \approx -1.32 \times 10^{-6}\ \text{s}$$

- Daily Reflux: $-1.32 \times 10^6 \times 8.25 \times 10^{11} \approx -1.09 \times 10^6\ \text{s} \approx -12.6\ \text{days}$,
- Total return flow to the Andromeda Galaxy: −99.8 years.

- For $R = 10\ \text{m}$:

$$\oint ds^2 = -10^5 \times 628.32 = -6.28 \times 10^7\ \text{m}^2$$

$$\Delta\tau = -\frac{\sqrt{6.28 \times 10^7}}{3 \times 10^8} \approx -2.64 \times 10^{-6}\ \text{s}$$

  - Daily reflux: $-1.27 \times 10^6 \approx -14.7\ \text{days}$,
  - Andromeda Total Reflux: −199.6 years,
  - Furthermore, when $N = 2.7 \times 10^5$, the total return time to Andromeda is: −413 years (reach the target). Therefore $N \sim 10^5$ does indeed superimpose and amplify CTCs well.

7. Design of Further Structural Solutions

According to the above calculations and analysis, we set the target as: $v_{eff} = 10^7 c$: the navigation speed is achieved by $A = -0.99999999999999$. Single loop: $\oint ds^2 = -2\pi R^2 = -628.32\ \text{m}^2$, single return flow: $\Delta\tau = -2.64 \times 10^{-6}\ \text{s}$; total return flow required −413 years: $\oint ds^2 = -6.28 \times 10^7\ \text{m}^2$ (selection: $R = 10\ \text{m}$, $N = 2.7 \times 10^5$).

I) Structural design

- **Concept:**
  - There are 8 main beams, each containing $N = 2 \times 10^5/8 = 25000$ optical solitons, and every 2 generate 1 gravitational soliton.
- **Specific design:**
  - Beam construction: Each beam is a high-density pulse sequence with a pulse frequency of 3 × 10^9 Hz (corresponding to $R = 0.1\ \text{m}$).
  - (25,000) optical solitons propagate in parallel within a radius 10 m stacking ring.
  - Gravitational Solitons: Each pair of optical solitons generates $h_{xx} = A\text{sech}^2(ku)$ through the gauge transformation $\omega_V = g_{UV}^{-1}\omega_U g_{UV} + g_{UV}^{-1} dg_{UV}$; total (12,500) gravitational solitons in each beam and are together 10^5 in 8 beams;
  - The path square of every singlering is $\oint ds^2 = -628.32\ \text{m}^2 \times 10^5 = -6.28 \times 10^7\ \text{m}^2$.
- **Energy:**
  - Each bundle power is $10^{14}\ \text{W} \times 25000/8 \approx 3.125 \times 10^{17}\ \text{W}$,
  - Total power is 2.5 × 10^{18} W.

II) Recommended solutions and construction details

1) Light source:

- 8 femtosecond lasers (wavelength 1550 nm, pulse width 100 fs), with power 10^{14}W.
- Each output (25,000) optical soliton sequences, with frequency 3 × 10^9 Hz.

2) Beam path:

- Main ring $R = 10\ \text{m}$, circumference 62.8 m.
- The optical soliton propagates along the ring, with a polarization angle of

$\theta \approx 70^\circ$ (Gauge transformation solution).

3) Gravitational soliton generation:

- (25,000) solitons in each beam, grouped into (12,500) pairs.
- Each pair is generated by interference and gauge transformation as $h_{xx} = -0.99999999999999 \operatorname{sech}^2(ku)$.

4) CTCs system:

- There are a total of 10^5 gravitational solitons superimposed, $\Delta\tau = -2.64 \times 10^{-6}$ s.
- Switch control: pulse pause, stop reflux.

5) Energy optimization:

- Total power 2.5×10^{18} W,
- Pulse operation (10% time): 2.5×10^{17} W.

III) Verification and Results

- Time reflux: Andromeda: −413 years, matches target.
- Velocity performance: $v_{eff} = 10^7 c$ unchanged, 91.44 days of navigation.
- Controllable devices: CTCs can be paused to ensure safety of energy in certain period of navigation.

IV) The following is a detailed calculation:

Question 1: Cumulative time of jump speed and return flow calculation

Concept clarification

- **Jump speed:**
  - In actual navigation, $v_{eff} = 10^7 c$ may be a "jump form", that is, the spacecraft reaches a very high apparent speed (close to the metric singularity) in a short period of time, and then pauses or slows down. This method may be because $h_{xx} \to -1$ will cause the metric to be unstable, and continuous operation is unrealistic.
  - Assuming that the duration of each jump is very short (e.g., microsecond or nanosecond level), multiple jumps are accumulated to reach Andromeda.
- **Target:**
  - Andromeda distance: 2.5×10^6 light-years $= 2.37 \times 10^{22}$ m,
  - Smooth sailing: $t = 91.44$ days $= 7.9 \times 10^6$ s.

Calculate cumulative time

- **Single jump:**
  - Assume $h_{xx} = -0.99999999999999$ ($v_{eff} = 10^7 c$), jump time $t_{jump} = 10^{-6}$ s (1 microsecond, close to the controllable limit of the singularity).
  - Single displacement:

$$\Delta x = v_{eff} \times \Delta t_{jump} = 3 \times 10^{15} \times 10^{-6} = 3 \times 10^9 \text{ m}$$

- **Number of jumps:**
  - Total distance: 2.37×10^{22} m,
  - Required times:

$$N_{jumps} = \frac{2.37 \times 10^{22}}{3 \times 10^9} = 7.9 \times 10^{12}$$

- **Cumulative time:**
  - Assumed kick interval $\Delta t_{pause} = 10^{-3}$ s (1 milliseconds, cooldown/reset time):

$$t_{total} = N_{jumps} \times \left(\Delta t_{jump} + \Delta t_{pause}\right) = 7.9\times10^{12} \times \left(10^{-6} + 10^{-3}\right)$$
$$\approx 7.9\times10^{9}\ \text{s} \approx 250\ \text{years}$$

  - If the interval is shortened to 10^{-6} s:

$$t_{total} = 7.9\times10^{12} \times 2\times10^{-6} = 1.58\times10^{7}\ \text{s} \approx 183\ \text{days}$$

Reflow calculation

- Single-**shot backflow**:
  - Single ring: $\Delta\tau = -8.36\times10^{-8}$ s ($R = 10$ m),
  - $N = 10^{5}$: $\Delta\tau = -2.64\times10^{-6}$ s ,
  - Number of loops within the jump: $\omega = 3\times10^{7}\ \text{s}^{-1}$, $\Delta t_{jump} = 10^{-6}$ s ,

$$n = 3\times10^{7} \times 10^{-6} = 30$$

$$\Delta\tau_{jump} = -2.64\times10^{-6} \times 30 = -7.92\times10^{-5}\ \text{s}$$

- Total **reflux**:
  - $N_{jumps} = 7.9\times10^{12}$,
  - $\Delta\tau_{total} = -7.92\times10^{-5} \times 7.9\times10^{12} \approx -6.26\times10^{8}\ \text{s} \approx -19.8$ years .

Adjustment

- Original target −413 years, need to increase the jump frequency or (N):
  - If $N = 2.7\times10^{5}$:

$$\Delta\tau = -4.35\times10^{-6}\ \text{s}, \Delta\tau_{jump} = -1.305\times10^{-4}\ \text{s},$$

$$\Delta\tau_{total} = -1.305\times10^{-4} \times 7.9\times10^{12} \approx -1.03\times10^{9}\ \text{s} \approx -32.6\ \text{years}$$

- If reflux (−413 years) is required, the jump time or frequency needs to be extended.

So from the above analysis and calculation we can draw a conclusion: 183 days is the cumulative time of the jump, and the return flow is −32.6 years. If we want to optimize it to −413 years, we may need $\Delta t_{jump} = 10^{-5}$ s , $N = 5\times10^{6}$. These will greatly increase the energy requirements, so we change the goal to a total return flow of time as 32.6 years instead of −413 years. The following calculation is for this change (please also refer to Appendices C for detail calculations of time rewind).

V) Further precise calculations:

1) Defining Metrics

- Δt_{jump} (Time for a jump): The duration of a single $v_{eff} = 10^{7}c$ or the duration for a spacecraft to reach v_{eff} once. Formula: Manual setting (e.g. 10^{-6} s).
- Δt_{pause} (Bump Interval): The pause/cooldown time between two jumps. Formula: Manual setting (e.g. 10^{-6} s).
- Δt_{total} (Jump cumulative time): Total time to reach Andromeda, $N_{jumps} \times \left(\Delta t_{jump} + \Delta t_{pause}\right)$, $\Delta t_{total} = N_{jumps} \times \left(\Delta t_{jump} + \Delta t_{pause}\right)$

- N_{jumps} (Number of jumps): Total jump times,
  $N_{jumps} = \frac{\text{Total distance}}{\Delta x} = \frac{d}{v_{eff} \times \Delta t_{jump}}$.
- $\Delta\tau_{jump}$ (Single jump backflow time): CTCs reflux within a single jump,
  $\Delta\tau_{jump} = \Delta\tau_{single} \times n_{jump}$, where $n_{jump} = \frac{\omega}{2\pi} \times \Delta t_{jump}$.
- $\Delta\tau_{single}$ (Single cycle reflow time): The reflow time of the beam around the ring once, $\Delta\tau_{single} = -\frac{\oint \mathrm{d}s^2_{single}}{c}$, where $\oint \mathrm{d}s^2_{single} = -2\pi R^2$.
- $\Delta\tau_{total}$ (Total backflow time): Cumulative backflow of the entire Andromeda voyage, $\Delta\tau_{total} = \Delta\tau_{jump} \times N_{jumps}$.
- E_{total} (Total energy consumption): Total energy requirement,
  $E_{total} = P_{single} \times N \times N_{jumps} \times \Delta t_{jump}$, where P_{single} is the power of a single ring, N is the total number of rings.

2) Further optimization of calculations ($\Delta t_{total} \approx 183$ days **,** $\Delta\tau_{total} \approx -32.8$ years **)**

- Andromeda distance: $d = 2.37 \times 10^{22}$ m,
- $v_{eff} = 10^7 c = 3 \times 10^{15}$ m/s,
- $R = 10$ m, $\omega = 3 \times 10^7\ \mathrm{s}^{-1}$,
- Single ring: $\oint \mathrm{d}s^2_{single} = -628.32\ \mathrm{m}^2$.

Step 1 setup: $\Delta t_{total} = 183$ days

- $\Delta t_{total} = 183 \text{ days} = 1.58 \times 10^7$ s,
- $\Delta t_{jump} = 10^{-6}$ s, $\Delta t_{pause} = 10^{-6}$ s,
- $N_{jumps} = \frac{\Delta t_{total}}{\Delta t_{jump} + \Delta t_{pause}} = \frac{1.58 \times 10^7}{2 \times 10^{-6}} = 7.9 \times 10^{12}$
- Single displacement: $\Delta x = v_{eff} \times \Delta t_{jump} = 3 \times 10^{15} \times 10^{-6} = 3 \times 10^9$ m
- Verification distance: $d = N_{jumps} \times \Delta x = 7.9 \times 10^{12} \times 3 \times 10^9 = 2.37 \times 10^{22}$ m.

Step 2 calculation: $\Delta\tau_{total} = -32.8$ years

- $\Delta\tau_{total} = -32.8 \text{ years} = -1.035 \times 10^9$ s,
- $\Delta\tau_{jump} = \frac{\Delta\tau_{total}}{N_{jumps}} = \frac{-1.035 \times 10^9}{7.9 \times 10^{12}} \approx -1.31 \times 10^{-4}$ s,
- $n_{jump} = \frac{\omega}{2\pi} \times \Delta t_{jump} = \frac{3 \times 10^7}{6.2832 \times 10^{-6}} \approx 4.77 \times 10^6 \times 10^{-6} = 4.77$,
- $\Delta\tau_{single} = \frac{\Delta\tau_{jump}}{n_{jump}} = \frac{-1.31 \times 10^{-4}}{4.77} \approx -2.75 \times 10^{-5}$ s,
- (N): Solving the formula $\Delta\tau_{single} = -\frac{\sqrt{N \times 628.32}}{3 \times 10^8}$ for N is

$$N = \frac{6.80625 \times 10^7}{628.32} \approx 1.083 \times 10^5$$

- Verify that goals are achieved:

$$\Delta\tau_{single} = \frac{\sqrt{1.083 \times 10^5 \times 628.32}}{3 \times 10^8} \approx -2.75 \times 10^{-5}\ \mathrm{s},$$

$$\Delta\tau_{jump} = -2.75\times10^{-5}\times4.77 \approx -1.31\times10^{-4}\ \text{s},$$

$$\Delta\tau_{total} = -1.31\times10^{-4}\times7.9\times10^{12} \approx -1.035\times10^{9}\ \text{s} \approx -32.8\ \text{years}$$

Step 3 hazard control: ($h_{xx} \to -1$)

- $A = -0.99999999999999$ ($v_{eff} = 10^7 c$),
- Threshold: $A_{max} = -0.9999999999$ ($v_{eff} = 10^6 c$),
- $\Delta t_{jump} = 10^{-6}$ s short enough that feedback control (A) avoids singularities.

Step 4 energy control:

- Single ring: $P_{single} = 10^{14}$ W,
- $N = 1.083\times10^5$,
- Total power: $P_{total} = P_{single}\times N = 10^{14}\times1.083\times10^5 = 1.083\times10^{19}$ W,
- Pulse operation: $P_{total,eff} = 1.083\times10^{19}\times0.01 = 1.083\times10^{17}$ W,

$$E_{total} = P_{total,eff}\times N_{jumps}\times\Delta t_{jump} = 1.083\times10^{17}\times\frac{1.58}{2}\times10^7 = 8.56\times10^{22}\ \text{J}$$

- Pulse optimization (10% time): $E_{total} = 8.56\times10^{22}$ J.

The above calculation optimization results are listed as following Table 1:

Table 1. Optimize parameter calculation list.

Index	Definition	Formula	Value
Δt_{jump}	Time for a jump	Settings	10^{-6} s
Δt_{pause}	Time for a pause	Settings	10^{-6} s
N_{jumps}	Number of jumps	$\frac{d}{v_{eff}\times\Delta t_{jump}}$	7.9×10^{12}
Δt_{total}	Jumps cumulative time	$N_{jumps}\times\left(\Delta t_{jump}+\Delta t_{pause}\right)$	183 days (1.58×10^7 s)
$\Delta\tau_{single}$	Single cycle reflow time	$\frac{\sqrt{-\oint ds^2_{single}}}{c}$	-2.75×10^{-5} s ($N = 1.083\times10^5$)
$\Delta\tau_{jump}$	Single jump reflow time	$\Delta\tau_{single}\times\frac{\omega}{2\pi}\times\Delta t_{jump}$	-1.31×10^{-4} s
$\Delta\tau_{total}$	Total reflow time	$\Delta\tau_{jump}\times N_{jumps}$	−32.8 years (-1.035×10^9 s)
E_{total}	Total energy consumption	$P_{single}\times N\times N_{jumps}\times\Delta t_{jump}$	8.56×10^{22} J (Pulse Optimization)

3) Goals achieved

- $\Delta t_{total} = 183$ days, $\Delta\tau_{total} = -32.8$ years, *i.e.* half a year to reach Andromeda from the earth, making one 32.8 years younger, and a round trip takes one year, so the time flow back is −65.6 years, which is very suitable for human travel.
- Danger control: $v_{eff} = 10^7 c$, $\Delta t_{jump} = 10^{-6}$ s, feedback regulation (A) safety.
- Total energy for the entire voyage is required 8.56×10^{22} J; the specific calcu-

lation for the total energy is as follows:

- Volume calculation: beam cross section $1/k = 0.01\,\text{m}$ (soliton width), area $a = 0.01^2 = 10^{-4}\,\text{m}^2$; ring circumference $L = 2\pi R = 62.8\,\text{m}$, ($R = 10\,\text{m}$), volume $V = a \times L = 10^{-4} \times 62.8 = 6.28 \times 10^{-3}\,\text{m}^3$.
- Energy calculation for the single cycle time: $E_{single} = \text{Laser} \times \text{single cycle time} = 10^{14} \times 2.094 \times 10^{-7} = 2.094 \times 10^{7}\,\text{J}$.

Notice here 10^{14} is the laser power (actual high-power lasers can reach this level), and single cycle time is $T = \frac{2\pi}{\omega} = \frac{2\pi}{3 \times 10^7} \approx 2.094 \times 10^{-7}\,\text{s}$, therefore we have

$$\rho_{EMsingle} = \frac{E_{single}}{V} = \frac{2.094 \times 10^7}{6.28 \times 10^{-3}} \approx 3.33 \times 10^9\,\text{J/m}^3$$

If pulses are used, $\rho_{EMsingle}$ can be further reduced by 1%, that is, $\rho_{EMsingle} = 3.33 \times 10^7\,\text{J/m}^3$, then we can obtain:

- Total power: $P_{total} = P_{single} \times N = 10^{14} \times 1.083 \times 10^5 = 1.083 \times 10^{19}\,\text{W}$,
- Pulse operation: $P_{total,eff} = 1.083 \times 10^{19} \times 0.01 = 1.083 \times 10^{17}\,\text{W}$,
- $E_{total} = P_{total,eff} \times N_{jumps} \times \Delta t_{jump} = 1.083 \times 10^{17} \times \frac{1.58}{2} \times 10^7 = 8.56 \times 10^{22}\,\text{J}$.

Where assuming the laser power: $P_{single} = 10^{14}\,\text{W}$ (the reasonable range of actual high-power lasers), bypassing the overly complicated derivation of Einstein's equations, and obtaining reasonable values of $\rho_{EMsingle}, P_{single}, E_{total}$ and so on, the detailed calculation can be found in **Appendix C**.

8. Design for Safety

Safety is one of the most critical challenges in this design! When the apparent velocity $v_{eff} = 10^7 c$, the A of the gravitational soliton (*i.e.*, the amplitude of h_{xx}) is very close to −1, and the accuracy is required to reach the 14th decimal place (for example, $A = -0.99999999999999$), which not only places extremely high demands on the control system, but also needs to consider the influence of microscopic perturbations such as quantum fluctuations and the uncertainty principle. We propose to set a reasonable threshold by the polarization angle θ, and combine it with the analysis of the role of (N) to try to hit the key points. The following is a detailed analysis of how to control (A) not to reach −1, explore the relationship between (N) and v_{eff}, and draw on the exquisite methods in elementary particle physics or laser technology to propose a feasible solution.

(1) The core of the security issue: the risk of $A \to -1$

Apparent velocity and (A)

- $v_{eff} = \frac{c}{\sqrt{g_{xx}}}$, $g_{xx} = 1 + h_{xx} = 1 + A\,\text{sech}^2(ku)$, if $v_{eff} = 10^7 c$, then we have

$$\sqrt{g_{xx}} = 10^{-7}, g_{xx} = 10^{-14}, 1 + A = 10^{-14},$$
$$A = -1 + 10^{-14} = -0.99999999999999$$

- Existing risks:
  - If $A = -1$, $g_{xx} = 0$, $v_{eff} \to \infty$, the occurrence of metric singularities and spatiotemporal collapse may lead to spacecraft disintegration or uncontrol-

lable effects.

- Control accuracy requirement: The fluctuation of (A) should not exceed 10^{-14}, which is highly susceptible to quantum fluctuations or measurement errors.

- Quantum fluctuations:
  - The electromagnetic fields (E) and (B) of optical solitons are affected by vacuum fluctuations, $\Delta E \Delta t \geq \hbar/2$.
  - If $\rho_{EM} = 3.33\times10^7\ \mathrm{J/m^3}$ (pulse optimization value), fluctuations may introduce small disturbances, but amplification effects are significant under strong fields.
- Uncertainty principle:
  - The measurement accuracy of polarization angle θ is limited by $\Delta\theta\Delta p \geq \hbar/2$, which may lead to uncertainty in $A = \cos 2\theta$.

(2) Analysis of the relationship between (N) and v_{eff}

- We found that N is related to time reflux and power, but not to apparent velocity, the role of verification and clarification (N) is as follows:
  - $\Delta\tau_{single} = -\dfrac{\sqrt{N\times 2\pi R^2}}{c}$,
  - $N = 1.083\times10^5$: $\Delta\tau_{single} = -2.75\times10^{-5}\ \mathrm{s}$, $\Delta\tau_{total} = -32.8$ years
  - (N) amplifies the CTC effect, which is proportional to the reflux time.
- The Source of v_{eff}:
  - v_{eff} is determined by (A) and has no direct relationship with (N):

    $v_{eff} = \dfrac{c}{\sqrt{1+A}}$, $A = \cos 2\theta$, (Obtained by solving the gauge transformation equation).
  - $N = 10^4$ or 10^3 does not directly affect v_{eff}, but reduces $\Delta\tau_{total}$.
- Power relationship:
  - $P_{total} = P_{single}\times N$, the larger the value of (N), the higher the energy demand, but the v_{eff} is still controlled by θ (*i.e.* A).

(3) Strategy for controlling (A)

Key: Control the polarization angle θ

- $A = \cos 2\theta$ (assuming $\mathrm{sech}^2(ku)\approx 1$ at peak):
  - $v_{eff} = 10^7 c$: $1+\cos 2\theta = 10^{-14}$, $\cos 2\theta = -0.99999999999999$, $2\theta \approx 180^\circ - 2.56\times10^{-6}$ deg, $\theta \approx 90^\circ - 1.28\times10^{-6}$ deg
  - Ensuring that $\theta < 90^\circ$ is the key to avoiding $A = -1$.

Set threshold

- Safety threshold:
  - If setting $v_{eff} = 10^6 c$: $1+A = 10^{-12}$, $A = -0.999999999999$, $\cos 2\theta = -0.999999999999$, $\theta \approx 90^\circ - 1.28\times10^{-5}$ deg;
  - The precision requirement is reduced from 10^{-14} to 10^{-12}, which is more controllable than setting $v_{eff} = 10^7 c$.

- Control methods:

- Laser polarization precise modulation:

- Technical reference: Using high precision laser interferometers (such as LIGO) with polarization angle control accuracy of 10^{-9} rad (Approximately 5.7×10^{-8} deg).
- Method: Using a polarizing beam splitter and an electro-optic modulator (EOM) to adjust θ in real-time and maintain $\theta < 90^\circ$.

- Feedback system:
  - Monitor h_{xx} (through interference intensity or soliton power), if $|A| > 0.999999999999$ reduce θ or decrease laser power.
  - Response time: 10^{-9} s (picosecond laser technology) is required responding time period.
- Quantum fluctuation suppression:
  - Compressed state light: Compressed light technology in quantum optics can reduce ΔE and control the uncertainty of θ.

(4) Feasibility and Optimization

Threshold suggestion

- $v_{eff} = 10^6 c$:
  - $\Delta t_{total} = 1830$ days (It takes 5 years to get from Earth to the Andromeda Galaxy, but traveling within the Milky Way is sufficient), the total reflow time is $\Delta \tau_{total} = -328$ years (Scale by $N \sim 10^5$).
  - High safety, energy requirements may be relatively decreased.

Experimental reference

- Particle Physics: The magnetic field control accuracy of the LHC (10^{-6} T) can be used for electromagnetic field stabilization.
- Laser technology: Phase locking of femtosecond laser with an accuracy of 10^{-12} sensors θ stability.

Below, we will derive the value of the polarization angle θ and its control accuracy requirements for three cases of $v_{eff} = 10^3 c, 10^6 c, 10^7 c$, and analyze the corresponding (A) (*i.e.*, the magnitude of h_{xx}). Accuracy will be expressed in degrees or radians, while considering practical technical feasibility. The following is the detailed calculation process:

1) Calculation formulas and basic relationships

Firstly, the apparent speed can be calculated by

$$v_{eff} = \frac{c}{\sqrt{g_{xx}}}, g_{xx} = 1 + h_{xx} = 1 + A\operatorname{sech}^2(ku)$$

where we assume $\operatorname{sech}^2(ku) \approx 1$ (peak value) and $A = \cos 2\theta$. Then our targets are to calculate A, θ and to determine the control accuracy of θ (*i.e.* $\Delta\theta$) for ensuring that A does not reach −1.

2) Calculate θ and accuracy

Case 1: $v_{eff} = 10^3 c$

- g_{xx} and (A):

$$\sqrt{g_{xx}} = \frac{c}{v_{eff}} = 10^{-3}, g_{xx} = 10^{-6}, 1 + A = 10^{-6}, A = -1 + 10^{-6} = -0.999999$$

- Value θ and radians:

$$\cos 2\theta = -0.999999, 2\theta = \arccos(-0.999999) \approx 180^\circ - 0.02546^\circ,$$

$$\theta \approx 90^\circ - 0.01273^\circ; 2\theta \approx \pi - 4.445\times10^{-4}, \theta \approx \frac{\pi}{2} - 2.2225\times10^{-4}\ \text{rad}$$

- Control accuracy:
  - o If $A = -1$ (singularity): $\cos 2\theta = -1, 2\theta = 180^\circ, \theta = 90^\circ$;
  - o Maximum allowable deviation:

$$\Delta(2\theta) = 180^\circ - (180^\circ - 0.02546^\circ) = 0.02546^\circ,$$

$$\Delta\theta = 0.01273^\circ \approx 2.2225\times10^{-4}\ \text{rad}$$

  - o Accuracy requirement: θ should be controlled at the level of 10^{-4} rad.

Case 2: $v_{eff} = 10^6 c$

- o g_{xx} and (A):

$$\sqrt{g_{xx}} = 10^{-6}, g_{xx} = 10^{-12}, 1 + A = 10^{-12},$$

$$A = -1 + 10^{-12} = -0.999999999999$$

- o Value θ:

$$\cos 2\theta = -0.999999999999,$$

$$2\theta \approx 180^\circ - 0.00002546^\circ, \theta \approx 90^\circ - 0.00001273^\circ$$

- o Radians:

$$2\theta \approx \pi - 4.445\times10^{-7}, \theta \approx \frac{\pi}{2} - 2.2225\times10^{-7}\ \text{rad}$$

- o Control accuracy and maximum deviation:

$$\Delta(2\theta) = 0.00002546^\circ, \Delta\theta = 0.00001273^\circ \approx 2.2225\times10^{-7}\ \text{rad}$$

- o Accuracy requirement: Level 10^{-7} rad.

Case 3: $v_{eff} = 10^7 c$

- o g_{xx} and (A):

$$\sqrt{g_{xx}} = 10^{-7}, g_{xx} = 10^{-14}, 1 + A = 10^{-14},$$

$$A = -1 + 10^{-14} = -0.99999999999999$$

- o Value θ:

$$\cos 2\theta = -0.99999999999999,$$

$$2\theta \approx 180^\circ - 0.000002546^\circ, \theta \approx 90^\circ - 0.000001273^\circ$$

- o Radians:

$$2\theta \approx \pi - 4.445\times10^{-8}, \theta \approx \frac{\pi}{2} - 2.2225\times10^{-8}\ \text{rad}$$

- o Control accuracy $\Delta\theta$ and maximum deviation:

$$\Delta(2\theta) = 0.000002546^\circ, \Delta\theta = 0.000001273^\circ \approx 2.2225\times10^{-8}\ \text{rad}$$

- o Accuracy requirement: Level 10^{-8} rad.

Therefore the above results can be expressed in below **Table 2**:

Table 2. Control parameter calculation list.

v_{eff}	(A)	θ (degree)	θ (radian)	Control $\Delta\theta$ (degree)	Control $\Delta\theta$ (radian)
$10^3 c$	-0.999999	90˚ - 0.01273˚	$\frac{\pi}{2}-2.2225\times10^{-4}$	0.01273˚	2.2225×10^{-4}
$10^6 c$	-0.999999999999	90˚ - 0.00001273˚	$\frac{\pi}{2}-2.2225\times10^{-7}$	0.00001273˚	2.2225×10^{-7}
$10^7 c$	-0.99999999999999	90˚ - 0.000001273˚	$\frac{\pi}{2}-2.2225\times10^{-8}$	0.000001273˚	2.2225×10^{-8}

3) Technical feasibility analysis

Accuracy requirements

- $v_{eff} = 10^3 c$: $\Delta\tau \approx 10^{-4}$ rad (about 0.0057˚),
- $v_{eff} = 10^6 c$: $\Delta\theta \approx 10^{-7}$ rad (about × 10^{-6}˚),
- $v_{eff} = 10^7 c$: $\Delta\theta \approx 10^{8}$ rad (about 5.7 × 10^{-7}˚)。

Existing technology

- **Laser interferometer (such as LIGO)**:
  - Phase control accuracy: 10^{-9} rad,
  - Satisfying $v_{eff} = 10^3 c$ and $10^6 c$, but $10^7 c$ slightly exceeds the limit.
- **Femtosecond laser lock-in**:
  - Phase accuracy: 10^{-12} s (corresponding to an angle of 110^{-8} rad level) can cover all situations.
- **Quantum Optics**:
  - Compressed state light: reduce $\Delta\theta$ to 10^{-10} rad (theoretical limit).

9. Conclusion and Expectations

We have proposed a method for controlling the spacetime metric by generating gravitational solitons through gauge transformation of optical soliton beams, thereby enabling the apparent velocity of a circular time spacecraft to reach astonishing superluminal speeds of $2c$ to $10^7 c$, realizing the dream of exploring distant galaxies and participating in time travel to rejuvenate life. The data obtained in this article is astonishing, but there are also many paradoxes related to galaxy travel and time machines involved. This article only explores the design of physical calculations, structures, and security from the perspective of theoretical physics and technology, without delving into these paradoxes or philosophical aspects. But the author believes that with the ultimate discovery of gravitational solitons by humans, an era of using gravitational solitons or gravitons to design superluminal spacecraft or time machines will eventually arrive.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

Appendix A. Conversion of Optical Solitons into Gravitational Solitons

I) The calculation process of converting 2 optical solitons into 1 gravitational soliton

1) Initial optical soliton:

- Each laser emits an optical soliton, and the polarization states of the two optical solitons are $\omega_U = \mathrm{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,
- Spatial distribution $\mathrm{sech}^2(ku)$ represents the local envelope along the u (time) direction, where $k = 0.1\,\mathrm{m}^{-1}$.

2) Rotate polarization:

- The polarization rotator applies $g_{UV}(\omega(t))$:

$$g_{UV}(t) = \begin{pmatrix} \cos(\omega(t)) & -\sin(\omega(t)) \\ \sin(\omega(t)) & \cos(\omega(t)) \end{pmatrix} \tag{A1}$$

$$\frac{\mathrm{d}g_{UV}}{\mathrm{d}t} = \frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\begin{pmatrix} -\sin(\omega(t)) & -\cos(\omega(t)) \\ \cos(\omega(t)) & -\sin(\omega(t)) \end{pmatrix} \tag{A2}$$

Note that g_{UV} is in the form of zero in vector space, so the definition of the exterior differential is: $\mathrm{d}g_{UV}(Y) = Y(g_{UV}) = \mathrm{d}\frac{g_{UV}}{\mathrm{d}t}$, $Y = \mathrm{d}/\mathrm{d}t$ is the tangent vector in the time direction. So the derivative term of the gauge transformation here usually takes the form of a time derivative:

$$\omega_V(t) = g_{UV}^{-1}\omega_U g_{UV} + g_{UV}^{-1}(\mathrm{d}g_{UV}/\mathrm{d}t) \tag{A3}$$

3) Transformation calculation:

- Item 1:

$$\begin{aligned} g_{UV}^{-1}\omega_U g_{UV} &= \mathrm{sech}^2(kt)\begin{pmatrix} \cos(\omega(t)) & \sin(\omega(t)) \\ -\sin(\omega(t)) & \cos(\omega(t)) \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &\times \begin{pmatrix} \cos(\omega(t)) & -\sin(\omega(t)) \\ \sin(\omega(t)) & \cos(\omega(t)) \end{pmatrix} \\ &= \mathrm{sech}^2(kt)\begin{pmatrix} \cos(2\omega(t)) & -\sin(2\omega(t)) \\ -\sin(2\omega(t)) & -\cos(2\omega(t)) \end{pmatrix} \end{aligned} \tag{A4}$$

- Item 2:

$$g_{UV}^{-1}dg_{UV} = \frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{A5}$$

4) Mapping to $h_{\mu\nu}$, we get the equation:

$$\mathrm{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix}$$

$$= \operatorname{sech}^2(ku)\begin{pmatrix} \cos(2\omega(t)) & -\sin(2\omega(t)) \\ -\sin(2\omega(t)) & -\cos(2\omega(t)) \end{pmatrix} + \frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{A6}$$

- Matching of diagonal terms of equations:

$$\cos(2\omega t)\operatorname{sech}^2(ku) = A\operatorname{sech}^2(ku) \Rightarrow A = \cos(2\omega t) \tag{A7}$$

- Matching of off-diagonal terms of equations ($u = t$):

$$\text{Top right corner: } -\sin 2\omega(t)\operatorname{sech}^2(ku) - \frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = B\operatorname{sech}^2(ku) \tag{A8}$$

$$\text{Lower left corner: } -\sin 2\omega(t)\operatorname{sech}^2(ku) + \frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = B\operatorname{sech}^2(ku) \tag{A9}$$

After eliminating $\operatorname{sech}^2(ku)$ from the two equations, we can combine the above Equations (A8) and (A9):

$$-\sin 2\omega(t) + \frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\cdot\cosh^2(ku) = -\sin 2\omega(t) - \frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\cdot\cosh^2(ku) \tag{A10}$$

This allows

$$2\frac{\mathrm{d}\omega(t)}{\mathrm{d}t}\cdot\cosh^2(ku) = 0 \Rightarrow \frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = 0 \tag{A11}$$

Therefore, $\omega(t)$ must be a constant. Substituting $\frac{\mathrm{d}\omega(t)}{\mathrm{d}t}$ into Equation (A8) or (A9) yields:

$$B = -\sin 2\omega(t) \tag{A12}$$

So the solution to the differential Equation (A6) is a constant:

$$\omega(t) = \theta(u) = \frac{1}{2}\arccos(A) = -\frac{1}{2}\arcsin(B) \tag{A13}$$

where A and B satisfy $A^2 + B^2 = 1$.

Therefore, by solving Equation (A13), we can choose an appropriate gauge transformation matrix $g_{UV}(u)$ to convert the polarization state ω_U of the optical soliton into the form of the gravitational soliton ω_V, that is,

$$\omega_U = \operatorname{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \overset{g_{UV}(u)}{\Rightarrow} \omega_V = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} = \begin{pmatrix} H_{xx} & B_{xy} \\ B_{xy} & H_{yy} \end{pmatrix} \tag{A14}$$

Since $\cos(2\theta) = -\frac{3}{4} < 0$, and $\sin(2\theta) = \frac{\sqrt{7}}{4} > 0$, 2θ is located in the second quadrant. Solve for the principal value of 2θ, which is calculated by inverse trigonometric functions: $2\theta = \pi - \arccos\left(\frac{3}{4}\right)$, where $\arccos\left(\frac{3}{4}\right)$ is the solution of $\cos(x) = -\frac{3}{4}$ in the first quadrant. Considering periodicity, the general solution is $\theta = 2\pi - \frac{1}{2}\arccos\left(\frac{3}{4}\right) + k\pi\arccos\left(\frac{3}{4}\right)$.

$$\arccos\left(\frac{3}{4}\right) \approx 0.7227 \text{ radian}\left(\text{about } 41.41^\circ\right) \tag{A15}$$

Principal value solution is $\theta \approx 2\pi - 0.7227 \approx 1.2094 \text{ radian} \left(\approx 69.29^{\circ}\right)$.

All solutions that meet the conditions are: $B = \frac{\sqrt{7}}{4} \approx -\sin\left(2\theta\right)$,

$A = -\frac{3}{4} = \cos\left(2\theta\right)$, $\theta \approx 1.2094 \text{ radian} \left(\approx 69.29^{\circ}\right)$, $A^2 + B^2 = 1$;

$h_{xx} = -\frac{3}{4}\text{sech}^2\left(ku\right)$, $h_{xy} = \frac{\sqrt{7}}{4}\text{sech}^2\left(ku\right)$, $h_{yy} = \frac{3}{4}\text{sech}^2\left(ku\right)$.

II) The disturbance of gravitational solitons on space-time

1) Optical solitons through gauge transformation

In this model, gravitational solitons are converted from the electromagnetic disturbances of optical solitons through gauge transformation. The core equation of the gauge transformation above is (A1, A6), note that here $U \cap V \neq 0$, U represents the region of electromagnetic action of optical solitons, and V represents the region of gravitational action of gravitational solitons. These two regions are usually different, but now the intersection is not equal to 0.

- ω_U : The polarization state matrix of the initial optical soliton (represented by electromagnetic field),
- g_{UV} : Transformation matrix (rotation matrix),
- ω_V : The transformed matrix represents the gravitational perturbation:

$$\text{sech}^2\left(ku\right)\begin{pmatrix} \cos 2\theta\left(u\right) & -\sin 2\theta\left(u\right) \\ -\sin 2\theta\left(u\right) & -\cos 2\theta\left(u\right) \end{pmatrix} \sim \begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix} \quad \text{(A16)}$$

So ω_V is a matrix generated from the optical soliton ω_U through gauge transformation, representing the disturbance of the gravitational field. It directly reflects the local deformation of spacetime, corresponding to the perturbation component $h_{\mu\nu}$ of the metric. Gravitational solitons are not perturbations in weak fields, but excitons in strong gravitational fields, which can significantly change the geometry of spacetime (such as $h_{xx} \to -1$, $g_{xx} \to 0$ driving superluminal effects and time reflux.

2) Why does it correspond to h_{xx}, h_{xy}, h_{yy}?

- Role of ω_V: ω_V is the matrix obtained after the generalized gauge transformation, which represents the perturbation of the gravitational field and is directly related to the perturbation of the metric $h_{\mu\nu}$.
- Under strong fields, the nonlinear effect of $h_{\mu\nu}$ is significant, and the form of ω_V provides a localized space-time curvature, which is consistent with the physical image of gravitational solitons.
- The metric perturbation matrix is usually written as:

$$h_{\mu\nu} = \begin{pmatrix} h_{tt} & h_{tx} & h_{ty} & h_{tz} \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} \\ h_{yt} & h_{yx} & h_{yy} & h_{yz} \\ h_{zt} & h_{zx} & h_{zy} & h_{zz} \end{pmatrix} \quad \text{(A17)}$$

- In this model, the gravitational soliton mainly acts in the x-y plane (the plane of the ring beam), which is simplified to a 2 × 2 matrix, so ω_V is defined as:

$$\omega_V = h_{ij} = \begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix} \tag{A18}$$

where directly assignments are

- $h_{xx} = A\text{sech}^2(ku)$,
- $h_{xy} = B\text{sech}^2(ku)$,
- $h_{yy} = -A\text{sech}^2(ku)$.

3) Consistency

- Symmetry:
  - $h_{\mu\nu}$ is a symmetric tensor, $h_{xy} = h_{yx}$, which is consistent with the ω_V matrix form.
- Soliton characteristics:
  - $\text{sech}^2(ku)$ provides spatial locality, which is consistent with the physical picture of gravitational solitons.
- Metric perturbations:
  - $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = diag(-1,1,1,1)$, this is not limited to the weak field assumption.
  - $g_{xx} = 1 + h_{xx}$, $g_{xy} = h_{xy}$, $g_{yy} = 1 + h_{yy}$,
  - $v_{eff} = c/\sqrt{g_{xx}}$ depends on h_{xx}. In strong fields, when $h_{xx} \to -1$, $g_{xx} \to 0$, and the apparent velocity increases significantly.
  - ω_U represents the electromagnetic polarization of the optical soliton, and ω_V is converted to gravitational perturbations through g_{UV}. (A) and (B) are the transformed coefficients that determine the spatial distribution of the gravitational field.

Therefore, $\omega_V = \text{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix}$ is the result of the gauge transformation, which represents the gravitational perturbation matrix. It is directly mapped to the metric perturbation h_{ij}, so the physical meaning of the gravitational soliton is the local deformation of spacetime, $\omega_V \to h_{ij}$, and these components define the geometric effect of the curvature bubble.

Appendix B. Energy for $v_{eff} = 2.14c$

1) Objectives and Assumptions

- Target speed: $v_{eff} = 2.14c$
- Turn off the reflux system: that is, $N = 0$ or $N = 1$ (not relying on a large number of gravitational soliton pairs to produce CTCs), and only the superluminal effect remains.
- Destination: Fly to Proxima Centauri, which is approximately $d = 4.24\ \text{light-years} = 4.01 \times 10^{16}\ \text{m}$ away.
- Basic parameters:
  - $P_{single} = 10^{14}$ (single laser power, assumed above),
  - $R = 10\ \text{m}$, $\omega = 3 \times 10^7\ \text{s}^{-1}$,

- $\Delta t_{jump} = 10^{-6}$ s, $\Delta t_{pause} = 10^{-6}$ s.

2) Calculate the metric of $v_{eff} = 2.14c$ and (A)

- Metric: $v_{eff} = \frac{c}{\sqrt{g_{xx}}}$, $g_{xx} = 1 + h_{xx} = 1 + A \Rightarrow A = -\frac{3}{4} \approx -0.782$
- Polarization angle: $A = \cos 2\theta \Rightarrow 70.815^{\circ}$, which is much lower than $\theta \approx 90^{\circ}$ at $10^7 c$, and is safer.

3) Travel time calculation

- Coordinate time Δt_{total}:
  - Distance $d = 4.01 \times 10^{16}$ m,
  - $\Delta t_{total} = \frac{d}{v_{eff}} = \frac{4.01 \times 10^{16}}{6.42 \times 10^{8}} \approx 6.246 \times 10^{7}\ \text{s} = 1.98\ \text{years}$
  - Result: It only takes about 1.98 years (about 2 years) to fly to Proxima Centauri, which is much shorter than the time at sub-light speed.

4) Calculate E_{total}

- Number of jumps N_{jumps}:

$$N_{jumps} = \frac{d}{v_{eff} \times \Delta t_{jump}} = \frac{4.01 \times 10^{16}}{6.42 \times 10^{8} \times 10^{-6}} \approx 6.246 \times 10^{13}$$

- Total time verification:

$$\Delta t_{total} = N_{jumps} \times \left(\Delta t_{jump} + \Delta t_{pause} \right) = 6.246 \times 10^{13} \times 2 \times 10^{-6} = 1.249 \times 10^{8}\ \text{s}$$

- Close the backflow system:
  - In the previous article, $N = 1.083 \times 10^{5}$ was used for CTCs. Now, $N = 1$ (only single beam effect, no amplification of reflux);
  - Single energy: $E_{jump} = P_{single} \times \Delta t_{jump} = 10^{14} \times 10^{-6} = 10^{8}$ J
  - Total Energy:

$$E_{total} = E_{jump} \times N \times N_{jumps} = 10^{8} \times 1 \times 6.246 \times 10^{13} = 6.246 \times 10^{21}\ \text{J}$$

  - Pulse optimization (1% duty cycle, strategy as above):

$$E_{total} = 6.246 \times 10^{21} \times 0.01 = 6.246 \times 10^{19}\ \text{J}$$

Although this energy is about 1000 times that of a large nuclear explosion (e.g. 15 megatons of TNT, 6.3×10^{16} J), this is the energy required for the total distance traveled. The energy of each pulse is: 10^{8} J.Current laser technology (10^{14} W) and future energy sources (e.g. nuclear fusion) can gradually approach this goal.

Appendix C. Time Rewind

Assuming laser power: $P_{single} = 10^{14}$ W (the reasonable range of actual high-power lasers), we bypassed the overly complicated derivation of the Einstein equation and obtained reasonable values of $\rho_{EMsingle}, P_{single}, E_{total}$, etc. The following is a clear table containing parameter definitions, calculation formulas, and results to ensure that the logic is rigorous and easy to understand:

1) Confirm the acquisition of relevant parameters

- $V = 6.28 \times 10^{-3}\ \text{m}^3$: The volume of a single soliton's path.

- $V = a \times L = 10^{-4}\ \text{m}^2 \times 2\pi \times 10\ \text{m} = 6.28 \times 10^{-3}\ \text{m}^3$,
- $a = 0.01^2 = 10^{-4}\ \text{m}^2$ (beam cross-section), $L = 62.8\ \text{m}$ ($R = 10\ \text{m}$ ring circumference).

- $E_{single} = P_{single} \times T = 2.094 \times 10^7\ \text{J}$: Energy of a single loop.
  - $P_{single} = 10^{14}\ \text{W}$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3 \times 10^7} \approx 2.094 \times 10^{-7}\ \text{s}$,
  - $E_{single} = 10^{14} \times 2.094 \times 10^{-7} = 2.094 \times 10^7\ \text{J}$.
  - $\rho_{EMsingle} = \frac{E_{single}}{V} = 3.33 \times 10^9\ \text{J/m}^3$: Energy density of a single loop.
  - $\rho_{EMsingle} = \frac{2.094 \times 10^7}{6.28 \times 10^{-3}} \approx 3.33 \times 10^9\ \text{J/m}^3$.
- $\rho_{EMsingle} = 3.33 \times 10^7\ \text{J/m}^3$: After pulse optimization, the energy density is only 1% of the original.
  - Pulse duty cycle 1%: $3.33 \times 10^9 \times 0.01 = 3.33 \times 10^7\ \text{J/m}^3$.

Here:

- (V) is the volume of the soliton in the circular path, based on the cross-sectional area and circumference,
- E_{single} is the energy of a single loop, derived from the laser power and the cycle time.
- $\rho_{EMsingle} = 3.33 \times 10^9\ \text{J/m}^3$ is the energy density in continuous operation, and $3.33 \times 10^7\ \text{J/m}^3$ is the result after pulse optimization, reflecting the energy saving effect of 1% duty cycle.

2) Calculation details and optimization verification

- **Continuous operation:**
  - $P_{single} = 10^{14}\ \text{W}$,
  - $E_{single} = 2.094 \times 10^7\ \text{J}$,
  - $\rho_{EMsingle} = 3.33 \times 10^9\ \text{J/m}^3$
- Pulse optimization
  - Duty cycle 1%:
  - $P_{single,eff} = 10^{14} \times 0.01 = 10^{12}\ \text{W}$ (Effective power),
  - $\rho_{EMsingle} = 3.33 \times 10^9 \times 0.01 = 3.33 \times 10^7\ \text{J/m}^3$,
  - Total power: $P_{total} = P_{single} \times N = 10^{14} \times 1.083 \times 10^5 = 1.083 \times 10^{19}\ \text{W}$,
  - Pulse operation: $P_{total,eff} = 1.083 \times 10^{19} \times 0.01 = 1.083 \times 10^{17}\ \text{W}$,
  - $E_{total} = P_{total,eff} \times N_{jumps} \times \Delta t_{jump} = 1.083 \times 10^{17} \times \frac{1.58}{2} \times 10^7 = 8.56 \times 10^{22}\ \text{J}$.

3) Organize the parameter table

Below are the complete parameter definitions, calculation formulas and result table which cover all key indicators:

Parameter	Definition	Calculation formula	Results
(R)	Circular path radius	Settings	10 m
(ω)	Beam angular velocity around ring	$\frac{c}{R}$	$3 \times 10^7\ \text{s}^{-1}$

Continued

(T)	Single surround cycle	$\frac{2\pi}{\omega}$	2.094×10^{-7} s
(V)	Single orbit path volume	$a\times2\pi R$ ($a=0.01^2$)	6.28×10^{-3} m^3
P_{single}	Single laser beam power	Set value (assuming high-power laser)	10^{14} W
E_{single}	Single surround energy	$P_{single}\times T$	2.094×10^{7} J
$\rho_{EMsingle}$	Single round energy density (continuous)	$\frac{E_{single}}{V}$	3.33×10^{9} J/m^3
$\rho_{EMsingle}$	Single round energy density (pulse)	$\frac{E_{single}}{V}\times$ Duty cycle	3.33×10^{7} J/m^3 (1% Duty cycle)
(N)	Gravitational soliton pairs	$\frac{(\Delta\tau_{single}\cdot c)^2}{2\pi R^2}$	1.083×10^{5}
Δt_{jump}	Jumping time once	Setting	10^{-6} s
Δt_{pause}	Jump interval time	Setting	10^{-6} s
N_{jumps}	Jumping frequency	$\frac{\Delta t_{total}}{\Delta t_{jump}+\Delta t_{pause}}$	7.9×10^{12}
Δt_{total}	Cumulative time of jumps	$N_{jumps}\times(\Delta t_{jump}+\Delta t_{pause})$	183 days (1.58×10^{7} s)
n_{jump}	Number of single jump cycles	$\frac{\omega}{2\pi}\times\Delta t_{jump}$	4.77
$\Delta\tau_{single}$	Single cycle reflux time	$-\frac{\sqrt{N\times2\pi R^2}}{c}$	-2.75×10^{-5} s
$\Delta\tau_{jump}$	Single jump reflux time	$\Delta\tau_{single}\times n_{jump}$	-1.31×10^{-4} s
$\Delta\tau_{total}$	Total reflux time	$\Delta\tau_{jump}\times N_{jumps}$	−32.8 years (-1.035×10^{9} s)
E_{total}	Total energy consumption	$P_{single}\times N\times N_{jumps}\times\Delta t_{jump}\times$ Duty cycle	8.56×10^{22} J (1% Duty cycle)