

# The Nine Charged Standard Model Fermions, Treated as Spheres Have Masses Specified by Five Fundamental Constants: $\hbar$ , $G$ , $\alpha$ , $\Lambda$ , and $\Omega_\Lambda$

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## Abstract

The Standard Model of particle physics, assuming that fundamental fermions are point particles, does not explain why there are only nine charged fundamental fermions nor does it easily explain the masses of those nine charged fermions. This holographic analysis (based on quantum mechanics, general relativity, thermodynamics, and Shannon information theory) treats the nine charged fundamental fermions (three leptons and six quarks) as spheres. The analysis specifies charged fundamental fermion masses by five fundamental constants: Planck's constant  $\hbar$ , gravitational constant  $G$ , fine structure constant  $\alpha$ , cosmological constant  $\Lambda$ , and vacuum energy fraction  $\Omega_\Lambda$ .

## Keywords

Standard Model Fermions, Holographic Analysis, Fundamental Particles as Spheres

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## 1. Introduction

The Standard Model of particle physics describes fundamental fermions as point particles in three charge states  $qe = -e, 2e/3$ , or  $-e/3$ , for a total of nine charged fundamental fermions. The Standard Model has been very successful, but has some shortcomings. First, point particles defined to have zero volume, and thus infinite matter density, are problematic. Second, points cannot rotate, so ascribing angular momentum to point particles is problematic. Third, the Standard Model does not explain why there are only nine charged fundamental fermions, nor does it easily determine the masses of those nine charged fermions.

Holographic analysis [1] based on four of our best mathematical theories

(quantum mechanics, general relativity, thermodynamics, and Shannon information theory) was developed in recent years. Holographic analysis shows that all the changing bits of information that can ever describe the observable part of our universe are available on an observer's event horizon, at the furthest distance the observer can ever see out into the universe. Holographic analysis also indicates that all bits of information necessary to describe a system with definite mass are available on a spherical surface surrounding the system at the holographic radius proportional to the square root of system's mass.

Masses of charged fermions (treated as spheres rotating around an axis through their center with holographic radii proportional to the square root of their mass) have volume components, surface components, and axial components. The resulting cubic equation for the radius of charged fundamental fermions in each charge state has three (and only three) solutions corresponding to holographic radii of the three fundamental fermions in that charge state.

This paper: 1) discusses some aspects of holographic analysis, 2) presents a brief holographic analysis specifying electron mass in terms of five fundamental constants of the universe, and 3) specifies the rest of the nine experimentally determined charged fundamental fermion masses in terms of the same five fundamental constants.

## 2. Holographic Analysis

The radius of the event horizon of any observer in our vacuum dominated universe, with cosmological constant  $\Lambda = 1.088 \times 10^{-56} \text{ cm}^{-2}$ , is

$$R_H = \sqrt{\frac{3}{\Lambda}} = 1.661 \times 10^{28} \text{ cm} . \text{ Using Particle Data Group PDG 2024 data [2] with}$$

Hubble constant  $H_0 = 67.4 \text{ km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$ , critical energy density

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \text{ g/cm}^3 = 8.533 \times 10^{-30} \text{ g/cm}^3 , \text{ gravitational constant}$$

$G = 6.67430 \times 10^{-8} \text{ cm}^3 \cdot \text{g}^{-1} \cdot \text{sec}^{-2}$ , and vacuum energy fraction  $\Omega_\Lambda = 0.685$ , the mass of the observable universe within the event horizon is

$$M_H = \frac{4}{3} \pi (1 - \Omega_\Lambda) \rho_{crit} R_H^3 = 5.155 \times 10^{55} \text{ g} = \left( \frac{0.187 \text{ g}}{\text{cm}^2} \right) R_H^2 . \text{ With Planck length}$$

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.61625 \times 10^{-33} \text{ cm} , \text{ holographic analysis finds only}$$

$$N = \left( \frac{\pi}{\ln(2)} \right) \left( \frac{R_H}{l_p} \right)^2 = 4.741 \times 10^{122} \text{ bits of information on the event horizon will}$$

ever be available to describe the observable universe within the event horizon, so the mass associated with a bit of information on the event horizon is

$$m_{bit} = \frac{M_H}{N} = 1.078 \times 10^{-67} \text{ g} . \text{ This holographic analysis assumes the bits of information describing a system with definite mass } m \text{ within the universe are avail-}$$

able on a spherical surface surrounding the system with radius  $r = \sqrt{\frac{m}{M_H}} R_H$ , so

holographic radii of particles with definite mass  $m$  are  $r = \sqrt{\frac{m}{0.187 \text{ g/cm}^2}}$ .

### 3. Electron Mass from $G, \hbar, \alpha, \Lambda$ , and $\Omega_\Lambda$

Electrostatic potential energy of electron charge  $e$  and positron charge  $-e$  separated by distance  $2r_e$  is  $V = -\frac{e^2}{2r_e} = -\frac{\alpha\hbar c}{2r_e}$  with fine structure constant  $\alpha = e^2/(\hbar c)$ . Two adjacent spheres with holographic radius  $r_e$ , a precursor for electron-positron pair production, have total energy  $E = 2m_e c^2 - \frac{\alpha\hbar c}{2r_e} = 0$  when

$$r_e = \frac{\alpha\hbar c}{4m_e c^2}. \text{ Two equations for } r_e \text{ result in } \frac{\alpha\hbar c}{4m_e c^2} = \sqrt{\frac{m_e}{M_H}} R_H \text{ and the equation}$$

$$\text{for electron mass } m_e = \left[ \left( \frac{(\alpha\hbar)^2}{32} \right) \left( \frac{1 - \Omega_\Lambda}{G\Omega_\Lambda} \sqrt{\frac{\Lambda}{3}} \right) \right]^{1/3}.$$

Particle Data Group values indicate electron mass 0.5% higher than actual electron mass to three significant figures [3]. Setting  $\Lambda = 1.08800 \times 10^{-56} \text{ cm}^{-2}$  and increasing  $\Omega_\Lambda$  by 0.5% to 0.6883855, within Particle Data Group error bars, specifies electron mass to six significant figures. Gravitational constant  $G$  is only known to six significant figures, so the calculation cannot be extended to greater precision until  $G$  is measured more precisely.

### 4. Charged Fundamental Fermion Masses from $G, \hbar, \alpha, \Lambda$ , and $\Omega_\Lambda$

Fundamental fermions described as spheres with holographic radius  $r$ , mass  $m = (0.187 \text{ g/cm}^2)r^2$ , and matter density  $\rho = m/(4/3\pi r^3)$  have volume mass components, surface mass components, and axial mass components along their rotation axis. Charge on the rotation axis of fundamental fermions avoids energy loss from accelerated charge.

A cubic equation for fermion holographic radius in charge state  $q$  is  $4/3\pi\rho r^3 = 4/3\pi\rho_V r^3 + 4\pi\rho_S r^2 + 2\rho_A r$ , or  $\rho r^3 = \rho_V r^3 + 3\rho_S r^2 + \frac{3}{2\pi}\rho_A r$ , where  $\rho_S$  has dimensions  $\text{g/cm}^2$  and  $\rho_A$  has dimensions  $\text{g/cm}$ . The equation rewritten as  $ar^3 + br^2 + cr = 0$  with  $a = \rho - \rho_V$ ,  $b = -3\rho_S$ , and  $c = -3/2\pi\rho_A$ , has positive discriminant  $b^2c^2 - 4ac^3$  and the three real roots of the equation correspond to holographic radii of the three fundamental fermions in charge state  $q$ .

With electron holographic radius  $r_e$ , this holographic analysis specifies holographic radii of the three lowest mass fundamental fermions as  $r_e$ ,  $r_u = 2r_e$  and  $r_d = 3r_e$ , predicting up quark mass  $m_u = 4m_e$  and down quark mass  $m_d = 9m_e$ . These up and down quark masses are within quark mass estimate ranges, based on intricate lattice quantum chromodynamic calculations, shown in “ideograms” on pages 2 and 4 of PDG 2024 “Particle Listings, Light Quarks”.

## 5. Relation between Fundamental Fermion Masses in the Three Charge States

Charged fundamental fermion masses and holographic radii shown in **Table 1** include electron, up quark, and down quark masses specified by this holographic analysis. Remaining lepton masses in **Table 1** are PDG 2024 values, with remaining quark masses within PDG 2024 90% confidence limits.

**Table 1.** Charged fundamental fermion masses and holographic radii.

Fermion	Charge	Mass (MeV/c <sup>2</sup> )	Holographic radius (fm)
electron	$-e$	<b>0.510999</b>	<b>0.698</b>
muon	$-e$	105.858	10.0
tau	$-e$	1776.93	41.1
up quark	$2e/3$	<b>2.04340</b>	<b>1.40</b>
charm quark	$2e/3$	1257	34.6
top quark	$2e/3$	172570	406
down quark	$-e/3$	<b>4.59899</b>	<b>2.09</b>
strange quark	$-e/3$	94.3	9.48
bottom quark	$-e/3$	4180	63.1

In each charge state  $qe$ , roots  $r_1(q), r_2(q)$  and  $r_3(q)$  of the cubic equation for holographic radii of the three fundamental fermions are projections on the  $r$  axis of vertices of an equilateral triangle [4] centered at

$r_N(q) = \frac{1}{3}(r_1(q) + r_2(q) + r_3(q))$ . The line from the center of the triangle to the vertex corresponding to the largest holographic radius  $r_3(q)$  makes angle

$$\theta(q) = \tan^{-1} \left( \frac{1}{\sqrt{3}} \left[ \frac{r_2(q) - r_1(q)}{r_3(q) - r_N(q)} \right] \right)$$

with the  $r$  axis.  $\theta(q)$  determines orientation of the equilateral triangle with respect to the  $r$  axis and is characteristic of charge state  $q$ . Holographic radii of fundamental fermions in **Table 1** then specify  $\theta(-1) = 2/9$ ,  $\theta(2/3) = 2/27$ , and  $\theta(-1/3) = 1/9$ . Electron holographic radius is  $r_e$  and lowest holographic radii in each charge state are  $r_1(1) = r_e$ ,  $r_1(2/3) = r_u = 2r_e$ , and  $r_1(-1/3) = r_d = 3r_e$ . When holographic radii of electrons, up quarks, and down quarks are specified, the respective values of  $\theta(q)$  then determine holographic radii and masses of the remaining fundamental fermions. Using holographic radii in **Table 1** as multiples of electron holographic radius, holographic analysis specifies all nine charged fundamental fermion masses in terms of five fundamental constants of the universe: Planck's constant  $\hbar$ , gravitational constant  $G$ , fine structure constant  $\alpha$ , cosmological constant  $\Lambda$ , and vacuum energy fraction  $\Omega_\Lambda$ .

## 6. Nucleon Internal Structure

Proton mass is 938.3 MeV/c<sup>2</sup> and neutron mass is 939.6 MeV/c<sup>2</sup>, so nucleon

holographic radius is 29.9 fm. With an attractive inverse square force  $-\frac{F_Q e^2}{r^2}$  between quarks bound within a nucleon holographic radius, a nucleon can be described as an  $s$ -wave bound state of a diquark (an  $s$ -wave bound state of an up quark and a down quark with mass  $13m_e$  and charge  $e/3$ ) and an up quark or a down quark. Bohr radii of  $s$ -wave bound states of quarks within nucleons are determined by Schrodinger equations analogous to the hydrogen atom Schrodinger equation. Bohr radius of a quark-diquark bound state is  $\frac{m_e \hbar^2}{F_Q \mu e^2 m_e}$ , where reduced mass  $\mu$  is  $3.06m_e$  for the quark-diquark bound state comprising a proton and  $2.25m_e$  for the quark-diquark bound state comprising a neutron. Bohr radii of quark-diquark bound states comprising neutrons are then  $\frac{3.05}{2.25} = 1.36$  Bohr radii of quark-diquark bound states comprising protons. Hydrogen atom van der Waals radius is 3.16 times the Bohr radius [5]. Setting nucleon holographic radius equal to 3.16 times neutron quark-diquark Bohr radius provides a lower bound estimate of the attractive inverse square force between quarks bound within a nucleon holographic radius of  $-\frac{2480e^2}{r^2}$ .

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### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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