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Analysis and Reinterpretation of the Minimal Higgs Sector

Richard B. Holmes

Innoven Energy, Inc., Solana Beach, USA
Email: rholmes001@aol.com

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Abstract

The time evolution of the Higgs, the Faddeev-Popov ghost fields, and the gauge phase functions of the standard model are analyzed using the ghost Lagrangian density from Chapter 11 of Taylor's "Gauge Theories of Weak Interactions". The results assume that the amplitudes of the vector bosons are negligible for homogeneous solutions. With a broken SU(2) × U(1) symmetry, 3 of the 4 Higgs fields have no physical significance independent of the 3 vector bosons W_1, W_2 and Z. However, with a literal interpretation of the ghost Lagrangian density, the Higgs ghost fields are found to have independent physical significance. It is found that the Higgs, ghosts, and gauge functions are both the occupants and the generators of gratings which can be interpreted as the "potential wells" of mass matrices of an anomaly-free quantum field theory. This implies that the charged and uncharged ghosts of the minimal Higgs sector can be interpreted as preons. One set of consequences of this standard-model-based analysis is that neutrinos are found to have mass and also mass oscillations.

Keywords

Higgs Fields, Faddeev-Popov Ghosts, Quantum Field Theory, Electroweak Theory

1. Introduction

The minimal Higgs sector has been a fixture of the standard model for roughly 50 years [1]-[3]. The Higgs model consists of one vacuum expectation value that arises from two complex Higgs states having four total degrees of freedom. One of the four massless Goldstone bosons becomes responsible for the mass field, and the other three become the spin-zero components of the three massive spin-one electroweak bosons. One electroweak boson remains massless. This conventional wisdom has served well for the computations of Higgs interactions in the context of accelerator-based observations. The minimal Higgs sector also includes four Faddeev-Popov ghost fields to go with the four bosons of the electroweak model, and the four local gauge functions that give rise to the longitudinal portions of the bosons in a non-Abelian local gauge theory. Though often considered fictitious artifacts, the ghosts are needed in the electroweak sector in order to avoid over-counting of states of the electroweak bosons or the bosons in any non-Abelian gauge theory ([3], Ch. 15; [4], Ch. 16). Altogether, there are 12 degrees of freedom in the minimal Higgs sector. These 12 degrees of freedom are required to have a renormalizable, self-consistent non-Abelian model for the electroweak force [1] [3]-[18].

The objective of this paper is to provide explanations of poorly-understood aspects of the Higgs sector of the standard model as well as a class of preon models. Within the standard model's Higgs sector, questions arise with the interpretation of scalar yet anticommuting Faddeev-Popov ghosts [1] [3] [4] [7], the values of the fermion masses [19], the extremely high value of the vacuum energy density implied by the vacuum expectation value of the Higgs [20] [21], and the final structure of the Higgs sector. For preon models, there are the questions of a self-consistent, anomaly-free quantum field theory for preons, the consistency of such a theory with measurements, and the question of the extremely small size required of preons as constituents of fundamental fermions. These specific issues are addressed in some measure in this paper.

The aforementioned 12 degrees of freedom of the minimal Higgs sector are required to interact in a certain way, and have various global and local symmetries, even in the absence of all other particles. This paper explores the consequences of these interactions using the ghost Lagrangian density given in ([1], Ch. 11). This paper will show the conventional vacuum-expectation solutions found in textbooks. However, other solutions are found that are purely oscillatory in nature. This analysis finds that these persistent, oscillatory solutions for the ghosts, as well as Higgs and gauge functions together can be interpreted as the familiar fermions such as the electrons and neutrinos. Evidence for this is provided by a fit of all the fermion masses using the solutions of this paper [22], including a requirement for 3 generations per fermion family. This result becomes evident in a particular anomaly-free quantum field theory that is a modest extension of the standard model [23]. In particular, the well-known property that the ghost fields only occur in loops is found to be related to the closed-loop properties of the mass matrices of the fermions in [23]. Further, the electroweak ghost fields are anticommuting scalar fields, and it will be seen that there are two which are charged and two which are uncharged. These features are expected for the particles that occupy the potential wells of the theory of ([23], Ch. 11), and are also a match for the basic rishon preon theory [24]-[26]. The diverse composite-particle preons models [24]-[30] have always seemed distinct from the standard model, so the proposed link is somewhat surprising.

One unavoidable issue of composite-particle models, which include preon models, is that the underlying preons are massless or nearly massless and so should have very large Compton wavelengths. This is despite the fact that observations require the preons to be very small, on the order of 10^{-18} m in size or less [31]. This key issue is addressed in Section 5. This issue is addressed using a Bohr-like condition, rather than an energy condition on which the Compton wavelength is based. In this approach, the wave functions are assumed to circulate in a ring. As is written by S. Weinberg, "These ghosts propagate around loops, with single vector bosons attached at each vertex along the loops" ([3], page 27). It will be seen below that the ghost Lagrangian also allows Higgs bosons to be attached at each vertex as well. Though Weinberg's language involves vertices, which is associated with the perturbative treatment, a similar physical interpretation applies here with a classical approach that is not perturbative in the usual diagrammatic sense. The vector bosons are set to zero here to explore the possibility of other solutions that are independent of such bosons. Such solutions are indeed found, as evidenced in Section 3.2 and 3.3 of this paper and in reference [22]. In this reference solutions are found that fit the masses of all fundamental fermions, and with 3 generations per fermion family, as is observed.

There are important symmetry and quantization requirements present in the Standard Model but not in most theories of nonlinear optics upon which this approach is based [32] [33]. Most of the symmetry requirements of the Higgs sector equations are encoded in the Higgs sector Lagrangian, which is properly captured here. The quantization requirements of a non-Abelian gauge theory are not fully addressed here but rely on references [23] and [3]. The treatment here is classical, but a similar Hilbert-space-based quantum-mechanical treatment can be found in the anomaly-free QFT of [23] in which the requirement for anticommuting scalar "ghosts" is implicit in the mass matrices. In [22] it is shown that qualitatively similar quantum solutions are obtained to that using the classical treatment given here. The approach for fermion field quantization of the ghosts is given in [3]. The quantum versions of the classical momentum states indicate that a similar result is possible using quantized fields.

Section 2 describes the approach, which starts with the well-known Lagrangian density for Higgs fields and the lesser-known Lagrangian density for the interaction of the Higgs fields, the ghosts, and the local gauge functions. The resulting dynamical equations are written and interpreted in Section 3. Persistent oscillatory solutions are also identified in this section. Section 4 identifies the properties of various stationary solutions. Section 5 addresses the paradox of the extremely small size of the Higgs sector wavefunctions and the corresponding preon states. Section 6 presents an example numerical result. Section 7 summarizes the results of this effort.

2. Calculational Approach

The well-known Lagrangian density L_H for the time evolution of the Higgs fields is given by [1]-[4]

$$L_H = 0.5\left|\left[\partial_\mu - i/(2\hbar c)\left(g\mathbf{t}\cdot\mathbf{W}_\mu + g'YB_\mu\right)\right]\begin{bmatrix} H_2 - iH_1 \\ H_4 - iH_3 \end{bmatrix}\right|^2 - V(H). \tag{1}$$

Here the indices for the Higgs degrees of freedom use the conventions of Taylor [1] for the four real-valued components of the Higgs fields, H_1 to H_4, with the exception of the sign for H_1. In this equation $\mathbf{t}$ is a vector of three weak-isospin generators and Y is the weak hypercharge, $\mathbf{W}_\mu$ are the three corresponding SU(2) gauge bosons, and B_μ is the familiar U(1) gauge field. The coupling constants g and g' follow the standard definitions, with units of (energy × length)$^{1/2}$. The coupling constant g is equal to $e/\sin(\theta_W)$ where e is the charge of the electron and g' is equal to $e/\cos(\theta_W)$, where θ_W is the Weinberg angle (also known as the weak mixing angle). The numerical value of $\sin^2(\theta_W)$ is 0.23122 ±0.00004 at the Z-mass energy scale in the $\overline{MS}$ framework [34]. As is customary, $\hbar$ is Planck's constant divided by 2π, and c is the speed of light.

This equation is manifestly invariant under SU(2) transformations before symmetry breaking, and this fact is often used to gauge away components of the Higgs fields [1]-[4] in presentations of the standard model. The potential term $V(H)$ has the form $-\mu/2\,H^H H + \lambda/4\left(H^H H\right)^2$ in the standard model, where μ and λ are positive real numbers, and "H" denotes Hermitian conjugation. The nominal mass M_H of the Higgs is set to 125.3 GeV/c^2 from measurements [35]. This mass is equal to $(2\mu)^{1/2}\hbar/c$ with the chosen conventions, after dividing the energy by c^2. The nominal vacuum expectation value is equal to $(\mu/\lambda)^{1/2}$ with the above notation.

In a self-consistent treatment of the Higgs fields and Faddeev-Popov ghosts, the above Lagrangian density L_H must be supplemented by a ghost Lagrangian L_{Gh} that includes the coupling of Higgs and ghost particles [1]:

$$\begin{aligned} L_{Gh} = &-(\boldsymbol{\eta}\,\Box\,\boldsymbol{\omega}) - (\eta_4\,\Box\,\omega_4) + 1/(\hbar c)^2\Big[-\xi M_W^2 c^4(\eta_1\omega_1 + \eta_2\omega_2) - \xi M_Z^2 c^4 \eta_Z\omega_Z \\ &+ g(\hbar c)\boldsymbol{\eta}\cdot(\partial_\mu\boldsymbol{\omega}\times\mathbf{W}_\mu) - 0.5\xi g M_W c^2 H_3(\eta_2\omega_1 - \eta_1\omega_2) \\ &- 0.5\xi g M_Z c^2 \eta_Z(H_1\omega_2 - H_2\omega_1) - 0.5\xi g M_W c^2 H_4(\eta_1\omega_1 + \eta_2\omega_2) \\ &+ 0.5\xi\left(g^2 + g'^2\right)^{1/2} M_Z c^2 H_4 \eta_Z\omega_Z - 0.5\xi g M_Z c^2 \omega_Z(\eta_1 H_2 - \eta_2 H_1)\Big] \end{aligned} \tag{2}$$

Here $\boldsymbol{\eta} \equiv (\eta_1, \eta_2, \eta_3)$ along with η_4 are the four real-valued ghost fields corresponding to the four gauge bosons (W_1, W_2, Z, A) of the electroweak sector, respectively. Similarly, the components of $\boldsymbol{\omega}$ along with ω_4 are the phase gauge functions corresponding to the four gauge bosons (W_1, W_2, Z, A) of the electroweak sector, respectively. These are ordered as in Taylor's text for the first three components, for weak isospin. The fourth component is the weak hypercharge component. In conformity with the conventions of Equation (2), $H_2 - iH_1$ is the first complex (charged) component of the Higgs 2-vector, and $H_4 - iH_3$ is the second complex (uncharged) component of the Higgs 2-vector. The units of all the scalar fields are (energy/length)$^{1/2}$, as they should be. The notation "$\Box$" denotes the D'Alembertian operator, as usual, for a scalar Lorentz-invariant wave-function. Note that with the conventions above, the Lagrangian density L_{Gh} has units of energy/length[3] as it should after accounting for a factor of $1/(\hbar c)^2$. Hence, the D'Alembertian operator must include a factor of $1/c^2$ for the double time derivative, and similarly a factor of $1/c$ for the single time derivative in the second line. The number ξ is the dimensionless gauge factor which can have any value from zero to infinity. In this paper, the interest is in non-zero values of ξ. In Equation (2) the η_Z and ω_Z are defined as follows, following Taylor:

$$\eta_Z = \cos(\theta_W)\eta_4 - \sin(\theta_W)\eta_3 \text{, and} \tag{3a}$$

$$\omega_Z = \cos(\theta_W)\omega_4 - \sin(\theta_W)\omega_3 \tag{3b}$$

At this point it is paramount to observe that the ghosts $\boldsymbol{\eta}$ are anticommuting scalars in the standard model [1] [3] [4], and the same for the gauge functions $\boldsymbol{\omega}$. In the following, these are treated in the usual way, as algebraic objects in which the anticommuting part of these fields ("Grassmann numbers") are separable from the numerical part. The equations below solve for the evolution of the scalar amplitudes, while keeping the anticommuting part in mind as needed. In Equation (2), M_W denotes the mass of the W boson, about 80.4 GeV/c^2, and M_Z denotes the mass of the Z-boson, about 91.2 GeV/c^2 [35]. $M_W/M_Z = \cos(\theta_W)$ also follows from the standard model. This completes the definitions of the terms in Equation (2).

One may simplify the D'Alembertian using standard momentum states for each component of $\boldsymbol{\eta}$ or $\boldsymbol{\omega}$, e.g.,

$$\omega_j = \omega_{G,j}\omega_{j0}(t)\cos\left(\boldsymbol{p}_{\omega j}\cdot \boldsymbol{x}/\hbar + \vartheta\right). \tag{4}$$

Here $\omega_{j0}(t)$ is a potentially rapidly-varying function of time t alone and is a pure real scalar gauge function. $\boldsymbol{p}_{\omega j}$ is the gauge function's 3-momentum. The variable $\boldsymbol{x}$ denotes spatial position and ϑ is a phase set by initial conditions. The final solution may of course involve sums or integrals of terms of the form of Equation (4). This is the most general form for a wave solution in terms of momentum states. This solution applies for both retarded and advanced frames. The cosine may be replaced by a complex exponential for a complex scalar field. These oscillatory solutions should have negligible nonlinear gain or loss (and thus are stable) for momentum-matched solutions. This fact is widely known in nonlinear optics as "phase-matched" solutions which have no gain or loss for the scalar interactions such as those present here [33]. The nonlinear terms such as in the Higgs potential can mix momentum states, but stable solutions can nonetheless be found [22].

The dependence on Grassmann numbers $\omega_{G,j}$ is removed by integrating the ghost Lagrangian by a sum of all pairwise integrals over Grassmann numbers, $\sum_{i,j}\left(\iint \mathrm{d}\eta_{Gi}\mathrm{d}\omega_{Gj}L_{Gh}\right)$ and using Equation (9.64) of [4]. This gives the form of the equations below and also ensures that all the coherent interaction channels of the Lagrangian are properly included. There are no other assumptions except the standard model's Lagrangians for the Higgs sector, given by Equations (1) and (2), and the electroweak bosons are set to zero.

When the electroweak bosons are set to zero, it is well-known that the resulting Lagrangian density Equation (1) produces a D'Alembertian operator for the Higgs fields accounting for the implicit 4-vector dot product in Equation (1). The same form for the Higgs fields will be used as for the ghost and gauge fields, using Equation (4) above, but without the Grassmann number. Also, it will be helpful to use a dimensionless time. One may scale the Lagrangian by dividing it by $\left(M_Z c^2/\hbar\right)^2$ and absorb this factor into the double time derivatives, so that the dimensionless time is

$$t' = \left(M_Z c^2/\hbar\right)t. \tag{5}$$

This then provides equations with dimensionless coefficients. Note that $\Delta t' = 1$ then corresponds to a time interval Δt of 7.20 × 10^{-27} sec. Applying this dimensionless time, setting $\boldsymbol{W}_\mu = 0$, and differentiating Equation (2) with respect to η_j, one obtains equations for ω_j:

$$\begin{aligned}\partial_{t'}^2\omega_{10} &= -c^2\Big/\left[\left(M_Z c^2/\hbar\right)^2(\hbar c)^2\right]\left[\xi M_W^2 c^4\omega_1 - 0.5\xi g M_W c^2 H_3\omega_2\right.\\ &\quad\left. + 0.5\xi g M_W c^2 H_4\omega_1 + 0.5\xi g M_Z c^2\omega_Z H_2 - \left|\boldsymbol{p}_{\omega1}\right|^2 c^2\omega_1\right]\\ &= -\xi\big/\left(M_Z c\right)^2\left[M_W^2 c^2\omega_1 - 0.5 g M_W H_3\omega_2\right.\\ &\quad\left. + 0.5 g M_W H_4\omega_1 + 0.5 g M_Z\omega_Z H_2\right] - \left|\boldsymbol{p}_{\omega1}\right|^2\big/\left(M_Z c\right)^2\omega_1,\end{aligned} \tag{6a}$$

$$\begin{aligned}\partial_{t'}^2\omega_{20} &= -\xi\big/\left(M_Z c\right)^2\left[M_W^2 c^2\omega_2 + 0.5 g M_W H_3\omega_1\right.\\ &\quad\left. + 0.5 g M_W H_4\omega_2 - 0.5 g M_Z\omega_Z H_1\right] - \left|\boldsymbol{p}_{\omega2}\right|^2\big/\left(M_Z c\right)^2\omega_2,\end{aligned} \tag{6b}$$

$$\begin{aligned}\partial_{t'}^2\omega_{30} &= +\xi\sin\left(\theta_W\right)\big/\left(M_Z c\right)^2\left[M_Z^2 c^2\omega_Z + 0.5 g M_Z\left(H_1\omega_2 - H_2\omega_1\right)\right.\\ &\quad\left. - 0.5\left(g^2 + g'^2\right)^{1/2} M_Z H_4\omega_Z\right] - \left|\boldsymbol{p}_{\omega3}\right|^2\big/\left(M_Z c\right)^2\omega_3,\end{aligned} \quad \text{and} \tag{6c}$$

$$\begin{aligned}\partial_{t'}^2\omega_{40} &= -\xi\cos\left(\theta_W\right)\big/\left(M_Z c\right)^2\left[M_Z^2 c^2\omega_Z + 0.5 g M_Z\left(H_1\omega_2 - H_2\omega_1\right)\right.\\ &\quad\left. - 0.5\left(g^2 + g'^2\right)^{1/2} M_Z H_4\omega_Z\right] - \left|\boldsymbol{p}_{\omega4}\right|^2\big/\left(M_Z c\right)^2\omega_4.\end{aligned} \tag{6d}$$

The factors $\cos\left(\boldsymbol{p}_{\omega j}\cdot\boldsymbol{x}/\hbar + \vartheta\right)$ are omitted on the left-hand side to improve readability. They will be omitted going forward under the assumption that the equations are spatially frequency-matched (which can be achieved mathematically by integrating each equation over all space to obtain a delta function that enforces conservation of 3-momentum). The terms involving products of two fields then become convolutions over momentum. To further simplify these equations, one may set $\alpha_Z = 1\big/\left(M_Z c\right)^2$. Next, integrating the time components of the D'Alembertian terms of Equation (2) by parts twice inside the Lagrangian integral and then differentiating with respect to ω_j, one obtains the following for the ghost fields by an identical procedure:

$$\begin{aligned}\partial_{t'}^2\eta_{10} &= -\xi\alpha_Z\left[M_W^2 c^2\eta_1 + 0.5 M_W g H_3\eta_2 + 0.5 M_W g H_4\eta_1 - 0.5 M_Z g H_2\eta_Z\right]\\ &\quad - \alpha_Z\left|\boldsymbol{p}_{\eta1}\right|^2\eta_1,\end{aligned} \tag{7a}$$

$$\partial_{t'}^2\eta_{20} = -\xi\alpha_Z\left[M_W^2c^2\eta_2 - 0.5M_W gH_3\eta_1 + 0.5M_W gH_4\eta_2 + 0.5M_Z gH_1\eta_Z\right] - \alpha_Z\left|\boldsymbol{p}_{\eta 2}\right|^2\eta_2, \tag{7b}$$

$$\partial_{t'}^2\eta_{30} = +\xi\sin\left(\theta_W\right)\alpha_Z\left[M_Z^2c^2\eta_Z + 0.5M_Z g\left(H_2\eta_1 - H_1\eta_2\right) - 0.5M_Z\left(g^2+g'^2\right)^{\frac{1}{2}}H_4\eta_Z\right] - \alpha_Z\left|\boldsymbol{p}_{\eta 3}\right|^2\eta_3, \tag{7c}$$

$$\partial_{t'}^2\eta_{40} = -\xi\cos\left(\theta_W\right)\alpha_Z\left[M_Z^2c^2\eta_Z + 0.5M_Z g\left(H_2\eta_1 - H_1\eta_2\right) - 0.5M_Z\left(g^2+g'^2\right)^{\frac{1}{2}}H_4\eta_Z\right] - \alpha_Z\left|\boldsymbol{p}_{\eta 4}\right|^2\eta_4. \tag{7d}$$

Following the same procedure with Equation (1) for the Higgs fields and the ghost Lagrangian of Equation (2), and differentiating with respect to H_{j0}, one obtains the following (with the same choice of dimensionless time):

$$\partial_{t'}^2 H_{10} = \alpha_Z\left\{0.5\xi gM_Z\left(\omega_Z\eta_2 - \eta_Z\omega_2\right) + \hbar^2\left[\mu - \lambda\left(H_1^2+H_2^2\right)\right]H_1\right\}, - \alpha_Z\left|\boldsymbol{p}_{H1}\right|^2 H_1, \tag{8a}$$

$$\partial_{t'}^2 H_{20} = \alpha_Z\left\{-0.5\xi gM_Z\left(\omega_Z\eta_1 - \eta_Z\omega_1\right) + \hbar^2\left[\mu - \lambda\left(H_1^2+H_2^2\right)\right]H_2\right\}, - \alpha_Z\left|\boldsymbol{p}_{H2}\right|^2 H_2, \tag{8b}$$

$$\partial_{t'}^2 H_{30} = \alpha_Z\left\{-0.5\xi gM_W\left(\eta_2\omega_1 - \eta_1\omega_2\right) + \hbar^2\left[\mu - \lambda\left(H_3^2+H_4^2\right)\right]H_3\right\} - \alpha_Z\left|\boldsymbol{p}_{H3}\right|^2 H_3, \tag{8c}$$

$$\partial_{t'}^2 H_{40} = \alpha_Z\left\{-0.5\xi gM_W\left(\eta_1\omega_1 + \eta_2\omega_2\right) + 0.5\left(g^2+g'^2\right)^{1/2}M_Z\eta_Z\omega_Z + \hbar^2\left[\mu - \lambda\left(H_3^2+H_4^2\right)\right]H_4\right\} - \alpha_Z\left|\boldsymbol{p}_{H4}\right|^2 H_4. \tag{8d}$$

In Equations (6) and (7) it is worth noting that the leading terms on the right-hand side are of the form $\xi\alpha_Z M_W^2c^2$ or $\xi\alpha_Z M_Z^2c^2$, leading to temporal oscillations of the form $\exp\left(\pm i\sqrt{\xi}M_W c^2 t/\hbar\right)$, in the former case, for example. With this observation, one might be tempted to ascribe a mass to ω_1 or η_1 with this example. This interpretation seems inappropriate, however, since this "mass" depends on the gauge ξ. Moreover, if the Higgs attain values near the vacuum expectation value, so that $0.5g\left|H\right|$ approaches $M_W c^2$, one sees that the nominal oscillation frequency of the ghost and gauge fields will vary for fixed ξ. Note that when the Higgs state is at the nominal vacuum expectation value, one has $0.5g\left|H\right| = M_W c^2 \approx 80\ \text{GeV}$, which is comparable to the masses of both the W and Z bosons. This is properly accounted for in Sections 3.2 and 3.3.

Based on the comments of the prior paragraph, terms such as $\eta_2\omega_1$ in Equations (8c) are found to have a frequency of $\pm 2\sqrt{\xi}M_W c^2/\hbar$ or 0 when $g\left|H\right| \ll M_W c^2$. Similarly, terms such as $\eta_Z\omega_Z$ are found to have a frequency of order $\pm 2\sqrt{\xi}M_Z c^2/\hbar$ or 0 when $g\left|H\right| \ll M_W c^2$. For the same reason, H_1 and

H_2 in Equations (8a) and (8b) have forced frequencies of $\pm i\sqrt{\xi}\left(M_Z - M_W\right)c^2/\hbar$ from terms like $\eta_Z\omega_2$ since the product has such frequencies. Next, inspection of Equations (8) indicate that the leading behavior is a decaying or growing solution with complex frequencies $\pm ic\sqrt{\mu}$, when the first terms of Equations (8) are small compared to $\hbar^2\mu|H|$. This paragraph indicates that a near-zero frequency is appropriate for H_3 and H_4 when $g|H| \ll M_W c^2$.

The statements of the prior two paragraphs are verified by numerical integration of Equations (6) to (8). Taken together, these equations have significant similarities to the effective Lagrangians that have been used in nonlinear optics, in which two optical fields with definite frequency bands will interfere in a gas to excite a coherent material oscillation with a smaller frequency equal to the difference frequency of the two fields [32] [33]. It is well-known that such equations need not be chaotic but instead have solutions that may be oscillatory or exponentially growing or decaying.

3. Analysis in Terms of Charged States

The above approach can be applied to charged states. These states for the ghosts are of the form $\eta^\pm = \left(\eta_1 \pm i\eta_2\right)$ and for the gauge functions $\omega^\pm = \left(\omega_1 \pm i\omega_2\right)$. As discussed at the end of the last section, two different regimes are identified. The first regime is when $g|H| \ll 2M_W c^2$, for all four Higgs components. This case results in clearly-defined frequencies for the ghost fields and the gauge functions. The second regime is when $g|H|$ is comparable to $2M_W c^2$. In this case a different set of nominal frequencies appears. These cases are treated in Sections 3.1 and 3.2, respectively, below. A sub-case of the second regime, $g|H| \approx 2M_W c^2$ for only the uncharged components, is treated in Section 3.3.

3.1. Analysis for the Case $g|H| \ll M_W c^2$ for all Higgs Components

This is the case corresponding to values of the Higgs states near a local extremum in the Higgs potential, $|H| = 0$ and corresponds to Equations (6) and (7) above. Utilizing the results of the prior paragraphs, the nominal complex frequencies of the various slowly-varying envelopes in this case are summarized in **Table 1**. The photon column was not discussed but is well known and can be seen directly from Equation (A3) in the **Appendix**. A numerical analysis of the equations in this case indicates instability.

Table 1. Nominal complex frequencies in units of rad/sec, for the variables in Equations (6) to (8), for $g|H| \ll 2M_W c^2$. $|p_\phi|$ denotes the photon momentum.

Variable/Subscript	1	2	Z	photon
ω	$\pm\sqrt{\xi}M_W c^2/\hbar$	$\pm\sqrt{\xi}M_W c^2/\hbar$	$\pm\sqrt{\xi}M_Z c^2/\hbar$	$\pm\lvert\boldsymbol{p}_\phi\rvert c/\hbar$
η	$\pm\sqrt{\xi}M_W c^2/\hbar$	$\pm\sqrt{\xi}M_W c^2/\hbar$	$\pm\sqrt{\xi}M_Z c^2/\hbar$	$\pm\lvert\boldsymbol{p}_\phi\rvert c/\hbar$
H	$\pm ic\sqrt{\mu}$, $\pm\sqrt{\xi}\left(M_Z - M_W\right)c^2/\hbar$	$\pm ic\sqrt{\mu}$, $\pm\sqrt{\xi}\left(M_Z - M_W\right)c^2/\hbar$	$\pm ic\sqrt{\mu}$, 0	0

Equations (8) may be rewritten more succinctly in terms of the Higgs' "charge" states $H_{12,0}^{\pm} = (H_2 \mp iH_1)$ and $H_{34,0}^{\pm} = (H_4 \mp iH_3)$. The result is

$$\partial_{t'}^2 H_{12,0}^{+} = \alpha_Z \left\{ 0.5\xi g M_Z \left(\eta_Z \omega^+ - \omega_Z \eta^+ \right) + \hbar^2 \left[\mu - \lambda \left| H_{12}^{+} \right|^2 \right] H_{12}^{+} \right\} - \alpha_Z \left| \boldsymbol{p}_{H12} \right|^2 H_{12}^{+}, \tag{9a}$$

$$\begin{aligned} \partial_{t'}^2 H_{34,0}^{+} &= \alpha_Z \left\{ -0.5\xi g M_W \eta^- \omega^+ + 0.5\xi \left(g^2 + g'^2 \right)^{1/2} M_Z \eta_Z \omega_Z \right. \\ &\quad \left. + \hbar^2 \left[\mu - \lambda \left| H_{34}^{+} \right|^2 \right] H_{34}^{+} \right\} - \alpha_Z \left| \boldsymbol{p}_{H34} \right|^2 H_{34}^{+}, \\ &= \alpha_Z \left\{ 0.5\xi g M_Z \left[-\cos(\theta_W) \eta^- \omega^+ + \eta_Z \omega_Z / \cos(\theta_W) \right] \right. \\ &\quad \left. + \hbar^2 \left[\mu - \lambda \left| H_{34}^{+} \right|^2 \right] H_{34}^{+} - \left| \boldsymbol{p}_{H34} \right|^2 H_{34}^{+} \right\}. \end{aligned} \tag{9b}$$

Similar equations for the anti-charge Higgs states can be written, with all superscript signs reversed in the above equations. The charged states ω^+ and η^- are defined in the preceding paragraphs. These equations can also be derived from the **Appendix** as well.

One may observe that the evolution equations for ω and η are simplified with this notation. One then obtains the following versions involving charged states:

$$\begin{aligned} \partial_{t'}^2 \omega_0^+ &= -0.5\xi M_W \alpha_Z \left[2M_W c^2 \omega^+ + gH_3 i\omega^+ + gH_4 \omega^+ + gM_Z / M_W H_{12}^{+} \omega_Z \right] \\ &\quad - \alpha_Z \left| \boldsymbol{p}_{\omega 12} \right|^2 \omega^+ \\ &= -0.5\xi M_W \alpha_Z \left[2M_W c^2 \omega^+ + gH_{34}^{-} \omega^+ + gM_Z / M_W H_{12}^{+} \omega_Z \right] - \alpha_Z \left| \boldsymbol{p}_{\omega 12} \right|^2 \omega^+ \end{aligned} \tag{10a}$$

and

$$\begin{aligned} \partial_{t'}^2 \omega_{Z0} &= -0.5\xi M_Z \alpha_Z \left[2M_Z c^2 \omega_Z + g \left(H_1 \omega_2 - H_2 \omega_1 \right) - \left(g^2 + g'^2 \right)^{\frac{1}{2}} H_4 \omega_Z \right] \\ &\quad - \alpha_Z \left| \boldsymbol{p}_{\omega Z} \right|^2 \omega_Z \\ &= -0.5\xi M_Z \alpha_Z \left[2M_Z c^2 \omega_Z - gRe\left(H_{12}^{-} \omega^+ \right) - gM_Z / M_W H_4 \omega_Z \right] \\ &\quad - \alpha_Z \left| \boldsymbol{p}_{\omega Z} \right|^2 \omega_Z . \end{aligned} \tag{10b}$$

Similarly, for η one also finds a simpler set of equations:

$$\partial_{t'}^2 \eta_0^+ = -0.5\xi M_W \alpha_Z \left[2M_W c^2 \eta^+ + gH_{34}^{+} \eta^+ - gM_Z / M_W H_{12}^{+} \eta_Z \right] - \alpha_Z \left| \boldsymbol{p}_{\eta 12} \right|^2 \eta^+, \tag{11a}$$

$$\partial_{t'}^2 \eta_{Z0} = -0.5\xi M_Z \alpha_Z \left[2M_Z c^2 \eta_Z + gRe\left(H_{12}^{-} \eta^+ \right) - gM_Z / M_W H_4 \eta_Z \right] - \alpha_Z \left| \boldsymbol{p}_{\eta Z} \right|^2 \eta_Z . \tag{11b}$$

Equations (9) offer interesting interpretations that are not to be found in the literature. In the literature, one often finds that one of the "chargeless" states H_3 or H_4 can be gauged away in favor of the other. Surprisingly, Equation (2) and the following equations are not consistent with this view. The ghost Lagrangian density is not invariant under a complex phase change of $H_4 - iH_3$, as seen in both Equation (2) and the **Appendix**, but it is invariant in Equation (1) as is well-known. Further, the overall magnitude of H_{34}^{+} can be altered by the presence of chargeless combinations of ghost fields and gauge functions such as $\eta^{-}\omega^{+}$, as seen from Equation (9b). It is also worth observing that the three relevant terms of the Lagrangian of Equation (2) are invariant under the cyclic permutation $\eta \to H_{1,2} \to \omega \to \eta$. Perhaps even more interesting, Equation (9) to (11) indicates *that* $\omega^{\pm}$ *and* $\eta^{\pm}$ *are charged*, by conservation of charge, since H_{12}^{+} is a well-known charged state. Finally, numerical investigation of these equations indicate that the solutions are unstable near $|H|=0$, as might be expected in the vicinity of a local maximum of the Higgs potential.

3.2. Analysis near the Vacuum Expectation Value, All Components

In this subsection the case of relatively large Higgs amplitude is treated, and in particular a stability analysis is performed near the second local extremum of the Higgs potential at the vacuum expectation value. In this case, it is well known that the Higgs states will have zero or near-zero frequencies in the standard model. The magnitudes of the Higgs are comparable to the vacuum expectation value $(\mu/\lambda)^{1/2}$, which is equal to $2M_W c^2/g$, so that $g|H| \approx 2M_W c^2$. As in the previous sections, keeping the second time derivatives of the D'Alembertian, allows a more accurate analysis. In accord with the assumption of this subsection, the notation $H_{34}^{\pm} = (\mu/\lambda)^{\frac{1}{2}} - \delta H_{34}^{\pm}$ is used, and similarly for $\delta H_{12}^{\pm}$, and δH_4. Then for the gauge functions in dimensionless time as in Equation (10), one finds

$$\begin{aligned}\partial_{t'}^2 \omega_0^{\pm} = -\xi\alpha_Z \Big[& \left(2M_W^2 c^2 - 0.5 M_W g \delta H_{34}^{\mp} + \left|\boldsymbol{p}_{\omega 12}\right|^2/\xi\right)\omega^{\pm} \\ & + \left(M_W M_Z c^2 - 0.5 M_Z g \delta H_{12}^{\pm}\right)\omega_Z \Big],\end{aligned} \tag{12a}$$

and

$$\begin{aligned}\partial_{t'}^2 \omega_{Z0} = +\xi\alpha_Z \Big\{ & \left(M_W M_Z c^2 - 0.5 M_Z g \delta H_{12}^{-}\right)\omega^{+}/2 \\ & + \left(M_W M_Z c^2 - 0.5 M_Z g \delta H_{12}^{+}\right)\omega^{-}/2 \\ & + \left[-\left|\boldsymbol{p}_{\omega Z}\right|^2/\xi - 0.5 M_Z g \delta H_4/\cos(\theta_W)\right]\omega_Z \Big\} \\ = +\xi\alpha_Z \Big\{ & Re\left[\left(M_W M_Z c^2 - 0.5 M_Z g \delta H_{12}^{-}\right)\omega^{+}\right] \\ & - \left[\left|\boldsymbol{p}_{\omega Z}\right|^2/\xi + 0.5 M_Z g \delta H_4/\cos(\theta_W)\right]\omega_Z \Big\}.\end{aligned} \tag{12b}$$

Note that that the $2M_W^2 c^2$ in Equation (12a) arises from the sum of two terms of the Lagrangian density of Equation (2): $M_W^2 c^4 (\eta_1\omega_1 + \eta_2\omega_2)$ and $+0.5 g M_W c^2 H_4 (\eta_1\omega_1 + \eta_2\omega_2)$, while using $gH_4 \approx 2M_W c^2$ in the latter. The omission of $M_Z^2 c^4$ occurs in the third term on the right hand side of Equation (12b) because contributions cancel from the terms $-M_Z^2 c^4 \eta_Z \omega_Z$ and $+0.5\xi\left(g^2 + g'^2\right)^{1/2} M_Z c^2 H_4 \eta_Z \omega_Z$ of the Lagrangian.

Similar to Equations (12), for η one also finds a relatively simple set of equations for the ghost fields:

$$\begin{aligned}\partial_{t'}^2 \eta_0^{\pm} = -\xi\alpha_Z \Big[& \left(2M_W^2 c^2 - 0.5 M_W g \delta H_{34}^{\pm} + \left|\boldsymbol{p}_{\eta 12}\right|^2/\xi\right)\eta^{\pm} \\ & - \left(M_W M_Z c^2 - 0.5 M_Z g \delta H_{12}^{\pm}\right)\eta_Z \Big]\end{aligned} \tag{13a}$$

and

$$\begin{aligned}\partial_{t'}^2\eta_{Z0} &= -\xi\alpha_Z\Big\{\left(M_W M_Z c^2 - 0.5M_Z g\delta H_{12}^-\right)\eta^+/2 \\ &\quad + \left(M_W M_Z c^2 - 0.5M_Z g\delta H_{12}^+\right)\eta^-/2 \\ &\quad - \left[-\left|\boldsymbol{p}_{\eta Z}\right|^2/\xi - 0.5M_Z g\delta H_4/\cos(\theta_W)\right]\eta_Z\Big\} \\ &= -\xi\alpha_Z\Big\{Re\left[\left(M_W M_Z c^2 - 0.5M_Z g\delta H_{12}^-\right)\eta^+\right] \\ &\quad + \left[\left|\boldsymbol{p}_{\eta Z}\right|^2/\xi + 0.5M_Z g\delta H_4/\cos(\theta_W)\right]\eta_Z\Big\}.\end{aligned} \quad (13b)$$

With the assumptions of this subsection, one may differentiate Equations (A3) and (A5) of the **Appendix** with respect to H_{12}^- to obtain

$$\begin{aligned}\partial_{t'}^2 H_{12,0}^+ &= \alpha_Z\left[-\left|\boldsymbol{p}_{H12}\right|^2 + \hbar^2\mu\left(1-\left|1-\delta H_{12}^+/(\mu/\lambda)^{1/2}\right|^2\right)\right]\left[(\mu/\lambda)^{1/2} - \delta H_{12}^+\right] \\ &\quad + 0.5\alpha_Z\xi g M_Z\left(\eta_Z\omega^+ - \omega_Z\eta^+\right).\end{aligned} \quad (14a)$$

Similarly, one obtains an equation for H_{34}^+:

$$\begin{aligned}\partial_{t'}^2 H_{34,0}^+ &= \alpha_Z\left[-\left|\boldsymbol{p}_{H34}\right|^2 + \hbar^2\mu\left(1-\left|1-\delta H_{34}^+/(\mu/\lambda)^{1/2}\right|^2\right)\right]\left[(\mu/\lambda)^{1/2} - \delta H_{34}^+\right] \\ &\quad + 0.5\alpha_Z\xi g M_Z\left[-\cos(\theta_W)\eta^-\omega^+ + \eta_Z\omega_Z/\cos(\theta_W)\right].\end{aligned} \quad (14b)$$

Terms of order $|\delta H|^2$ and higher are maintained in Equations (14) to support a more exact solution. The equations for H_{12}^- and H_{34}^- are very similar to those above; the only difference is that superscript pluses and minuses are exchanged.

The Equations (12) to (14) can be analyzed in the limit when $g|\delta H|$, $g|\eta|$, and $g|\omega|$ are all much less than $M_W c^2$. One observes in this case that the nominal frequency approaches $\pm|\boldsymbol{p}_{H12}|c/\hbar$ for H_{12}^+. A similar result applies for H_{34}^+. One may also note that when all the $g|\delta H|$ are much less than $M_W c^2$, it follows from Equations (12) and (13) that

$$\partial_{t'}^2\left(\eta_0^+ - \eta_0^-\right) = 0, \text{ and } \partial_{t'}^2\left(\omega_0^+ - \omega_0^-\right) = 0 \quad (15)$$

i.e., these modes have zero or nearly zero frequency. When this behavior is included in Equations (12b) and (13b), one then finds solutions for η_Z and ω_Z that have the same frequency content as $\eta^\pm$ and $\omega^\pm$, respectively, when these variables have a non-zero real part.

In the limit when $g|\delta H|$ are all much less than $M_W c^2$, Equations (12) and (13) become linear equations which can easily be solved. One closed-form analytic solution can be obtained with Equation (13a) and the second form of Equation (13c). First set all the δH's and momentums to zero Then take the double second time derivative of Equation (13a) and use the second form of Equation (13c) for $\partial_{t'}^2\eta_Z$ in the resulting equation. One finds the following equation:

$$\partial_{t'}^4\eta^+ = -\xi\alpha_Z 2M_W^2 c^2\partial_{t'}^2\eta^+ - \left(\xi\alpha_Z M_W M_Z c^2\right)^2 Re\left(\eta^+\right). \quad (16)$$

This can be written as a pair of differential equations for the real and imaginary parts of η^+, which can be solved directly. The resulting frequencies Ω in scaled time for the real and imaginary parts, η_1 and η_2 respectively, are given by

$$\Omega = \pm\sqrt{\xi}\left(M_W/M_Z\right)\left\{1\pm\left[1-\left(M_Z/M_W\right)^2\right]^{1/2}\right\} \text{ for } \eta_1 \text{, and} \tag{17a}$$

$$\Omega = \pm 0,\ \ \Omega = \pm\left(2\xi\right)^{1/2}\left(M_W/M_Z\right) \text{ for } \eta_2 . \tag{17b}$$

Similar solutions also apply to η^- and $\omega^\pm$ as well. The solution $\Omega = 0$ is evidently spurious, as may be seen by substituting a solution $\eta^+ = iC$, C a constant, into Equation (13a), for the imaginary part of η^+. Since η_Z should be pure real in this analysis, there is no opportunity for cancellation between the two terms on the right-hand side with an imaginary first term. On the other hand, the left hand side must be zero in the case of η^+ = constant, so the substitution of an imaginary constant is contradictory.

The oscillatory solutions of Equation (17b) in particular imply that there are persistent solutions for charged states of the ghost fields and gauge functions. In dimensional time, the oscillatory solution Equation (17b) is $\Omega = \pm 1.414\sqrt{\xi}\left(M_W c^2/\hbar\right) = \pm 113.7\sqrt{\xi}$ GeV/$\hbar$ rad/sec. When ξ is set to 1, the resulting energy is 113.7 GeV. There has been evidence of a broad resonance near this energy [36]. There are also decaying and growing solutions, as seen in Equation (17a). The normalized complex frequencies for these latter states are numerically given by $\pm\sqrt{\xi}\left(1.0330\pm 0.2592i\right)\left(M_W c^2/\hbar\right)$. The decaying and growing solutions might be expected as the system decays or grows to an equilibrium value such as the Higgs vacuum expectation value. Also, the frequencies of the η_Z and ω_Z states are nominally the same as that of $\omega^\pm$ and $\eta^\pm$ states, when they are present, due to the parametric generation of the former states from the first terms of Equations (12b) or (13b). When the $\omega^\pm$ and $\eta^\pm$ states are not present and the δH 's are negligible, the frequencies of the η_Z and ω_Z states are just given by their momentum, $\pm\left|\boldsymbol{p}_{\eta Z}\right|c/\hbar$, or $\pm\left|\boldsymbol{p}_{\omega Z}\right|c/\hbar$, respectively. The results for this subsection for the nominal frequencies (including the imaginary parts) are summarized in Table 2. It should be noted that for the charged states $\eta^\pm$ and $\omega^\pm$, there are always a pair of persistent purely oscillatory solutions. Also noteworthy from the table is that such solutions could multiply to give a near-zero frequency contribution to the Higgs, based on Equations (14).

Table 2. Nominal complex frequencies in units of rad/sec of the variables in Equations (12) to (14), for $g\left|H\right| \approx 2M_W c^2$ (broken Higgs) and when $g\left|\delta H\right|$, $g\left|\eta\right|$, $g\left|\omega\right|$ and the momenta are all much less than $M_W c^2$. An imaginary part indicates a decaying or growing solution.

Variable/Subscript	1	2	Z	photon
ω	$\pm\sqrt{\xi}\left(1.03\pm 0.26i\right)M_W c^2/\hbar$	$\pm\left(2\xi\right)^{1/2} M_W c^2/\hbar$	$\sim\pm\left|\boldsymbol{p}_{\omega z}\right|c/\hbar$, also any frequency in columns 1 or 2	$\pm\left|\boldsymbol{p}_\phi\right|c/\hbar$
η	$\pm\sqrt{\xi}\left(1.03\pm 0.26i\right)M_W c^2/\hbar$	$\pm\left(2\xi\right)^{1/2} M_W c^2/\hbar$	$\sim\pm\left|\boldsymbol{p}_{\eta z}\right|c/\hbar$, also any frequency in columns 1 or 2	$\pm\left|\boldsymbol{p}_\phi\right|c/\hbar$
H	$\sim\pm\left|\boldsymbol{p}_H\right|c/\hbar$	$\sim\pm\left|\boldsymbol{p}_H\right|c/\hbar$	$\sim\pm\left|\boldsymbol{p}_H\right|c/\hbar$	0

The results in Table 2 match with the expectation that the broken Higgs is nearly a constant in this section, *i.e.*, a near zero-frequency excitation. This is analogous to the broken symmetry associated with phonons in ordinary matter [37]. This also is in further accord with the nonlinear optics analogy, in which relatively low-frequency sound waves interact with very-high frequency electromagnetic waves, as mentioned above. For the ghost fields and the gauge functions, there are both oscillatory and decaying and growing solutions. The decaying and growing solutions are tentatively matched with interacting cases. The growing solutions will undergo saturation, and in the quantum world, probably correspond to the creation of new particles. The purely oscillatory solutions represent persistent solutions.

It is also worth noting that there are three-wave terms. For example, for ω_Z in Equation (12b), there is a $\delta H_4 \omega_Z$ term which contributes a three-wave term via Equation (14b). This term can have as many as six distinct frequencies. These frequencies are $\pm\left|\boldsymbol{p}_{\eta Z}\right| c/\hbar$ and $\pm 2\left|\boldsymbol{p}_{\omega Z}\right| c/\hbar \pm \left|\boldsymbol{p}_{\eta Z}\right| c/\hbar$. For example, if $\left|\boldsymbol{p}_{\omega Z}\right| = 3\Delta$ and $\left|\boldsymbol{p}_{\eta Z}\right| = 2\Delta$, this leads to $\pm 2\Delta c/\hbar$, $\pm 4\Delta c/\hbar$, and $\pm 8\Delta c/\hbar$ as the distinct frequencies in the equation for ω_Z, assuming frequency matching. In this example, the distinct non-zero frequencies for η_Z are $\pm\Delta c/\hbar$, $\pm 3\Delta c/\hbar$ and $\pm 7\Delta c/\hbar$.

There is an alternative limit in which the $g\left|\delta H\right|$, $g\left|\eta\right|$, or $g\left|\omega\right|$ are *not* all much less than $M_W c^2$. These solutions become highly nonlinear and also violate the assumptions of this section, that $g\left|H\right| \approx 2M_W c^2$. These cases will not be explored further in this paper and might best be analyzed numerically or via Feynman diagrams.

A key point of this section is that there are small-amplitude solutions that are purely oscillatory, *i.e.*, which represent persistent solutions for the ghost and gauge functions. This highlights that these fields could be the constituents of the long-lived fundamental fermions. But how can such solutions create the diverse spectrum of fermion solutions? For the uncharged states associated with neutrinos, this is addressed explicitly in Section 3.3. For the charged states there are purely oscillatory solutions for the imaginary parts of the charged fields, *i.e.*, η_2 and ω_2, that can feed into the neutral Higgs via the $H_4 \eta_2 \omega_2$ term of Equation (2). With non-zero and non-equal momentum for η_2 and ω_2, this in turn creates gratings in the Higgs. Note that $H_4 \eta_2 \omega_2$ is equal to $H_4 Im\left(\eta^+\right) Im\left(\omega^+\right)$ which is in turn equal to $H_4 Im\left(\eta^-\right) Im\left(\omega^-\right)$. Hence, depending on initial conditions, this term can support grating formation for either positively or negatively charged ghosts and gauge functions. These gratings can persist and lead to self-consistent solutions that exhibit a number of the measured properties of fermions, as shown explicitly in the next subsection.

3.3. Analysis near the Vacuum Expectation Value, Uncharged Components Only

Under the assumptions of the previous section, it was seen that $Re\left(\eta^+\right)$ and η_Z are tightly coupled, e.g., see Equations (13b) or (16). This seems contrary to the concept that the charged and uncharged solutions should decouple. It was seen that $Im(\eta^+)$ and η_Z are indeed decoupled and give persistent oscillatory solutions for the former. This subsection identifies another solution in which the charged and uncharged solutions do indeed decouple and η_Z is purely oscillatory. These solutions are then applied to the neutrino family.

Consider the case in which $H_{12}^{\pm}$, $\omega^{\pm}$ and $\eta^{\pm}$ are all identically zero. This condition might be expected when charge is quantized and conserved. In this case, Equations (9) to (11) simplify significantly:

$$\partial_{t'}^2 \eta^{\pm} = \partial_{t'}^2 \omega^{\pm} = \partial_{t'}^2 H_{12}^{\pm} = 0\,, \tag{18}$$

$$\partial_{t'}^2 \omega_{Z0} = -\alpha_Z \left[\left| \boldsymbol{p}_{\omega Z} \right|^2 + 0.5\xi M_Z g \delta H_4 / \cos(\theta_W) \right] \omega_Z\,, \tag{19}$$

$$\partial_{t'}^2 \eta_{Z0} = -\alpha_Z \left[\left| \boldsymbol{p}_{\eta Z} \right|^2 + 0.5\xi M_Z g \delta H_4 / \cos(\theta_W) \right] \eta_Z \quad \text{and} \tag{20}$$

$$\begin{aligned} \partial_{t'}^2 H_{34,0}^{+} = {} & \alpha_Z \left[-\left| \boldsymbol{p}_{H34} \right|^2 + \hbar^2 \mu \left(1 - \left| h_{34}^{+} - \delta H_{34}^{+} / (\mu/\lambda)^{1/2} \right|^2 \right) \right] \left[(\mu/\lambda)^{1/2} h_{34}^{+} - \delta H_{34}^{+} \right] \\ & + 0.5\alpha_Z \xi g M_Z / \cos(\theta_W) \eta_Z \omega_Z\,. \end{aligned} \tag{21}$$

In Equation (21), H_{34}^{+} is set to $(\mu/\lambda)^{1/2} h_{34}^{+} - \delta H_{34}^{+}$ to account for the fact that this equation will lead to a zeroth-order solution h_{34}^{+} which is not equal to 1.

Note that in accord with the interpretation of the Higgs states as mass fields, the Higgs terms on the right-hand sides of Equations (19) and (20) can be viewed as mass-like terms. In the especially simple limit in which δH_4 is so small that the second terms on the right-hand sides are negligible compared to the first (momentum) terms, a solution for the temporal evolution of ω_Z and η_Z is then

$$\omega_Z = \omega_{Z1} \cos\left(\left| \boldsymbol{p}_{\omega Z} \right| ct/\hbar + \vartheta_{\omega Z} \right), \text{ and } \eta_Z = \eta_{Z1} \cos\left(\left| \boldsymbol{p}_{\eta Z} \right| ct/\hbar + \vartheta_{\eta Z} \right), \tag{22}$$

where $\vartheta_{\omega Z}$ and $\vartheta_{\eta Z}$ are constant phases determined by initial conditions. Here, ω_{Z1} and η_{Z1} are constant or slowly varying in time. Substituting these purely oscillatory solutions into Equation (21), and initially setting the Higgs to its vacuum expectation value and the Higgs momentum to zero, and integrating twice with respect to time gives

$$\begin{aligned} H_{34,0}^{+} = {} & 0.5\xi g M_Z c^2 / \left[\hbar^2 \cos(\theta_W) \right] \eta_{Z1} \omega_{Z1} \\ & \times \iint \mathrm{d}t_2 \mathrm{d}t_1 \cos\left(\left| \boldsymbol{p}_{\omega Z} \right| ct_1/\hbar + \vartheta_{\omega Z} \right) \cos\left(\left| \boldsymbol{p}_{\eta Z} \right| ct_1/\hbar + \vartheta_{\eta Z} \right). \end{aligned} \tag{23}$$

When $\left| \boldsymbol{p}_{\omega Z} \right| = \left| \boldsymbol{p}_{\eta Z} \right|$, there is a secular, long-term increase in δH_{34}^{+}. When $\left| \boldsymbol{p}_{\omega Z} \right|$ and $\left| \boldsymbol{p}_{\eta Z} \right|$ are sufficiently different, there is negligible secular increase due to the periodic solutions. In this case, the ghost and gauge function form sinusoidal gratings, as discussed earlier in the paper. These then form the equivalent of potential wells. Also in this case, there arises the need for a periodic boundary condition when the fields are propagating in a ring, as discussed in Section 5. Using Equations (19) to (21), one can estimate the masses of the neutrinos that are consistent with measured mass differences. To show this, assume a steady-state oscillatory solution for the Higgs field δH_{34}^{+}, with frequency $\Omega_{\delta H}$. Then $\partial_{t'}^2 \delta H_{34}^{+} = -\Omega_{\delta H}^2 \delta H_{34}^{+}$.

Substituting this into Equation (21) and simplifying this equation one finds a zeroth-order solution and first-order solution in δH_{34}^{+}. A zeroth-order solution h_{34}^{+} is $\exp\left(i\left|\boldsymbol{p}_H\right|c/\hbar\right)$. The first-order, dimensionless solution is then equal to

$$\begin{aligned}&\left[M_H^2c^4/2+\left|p_H\right|^2c^2-\hbar^2\Omega_{\delta H}^2\right]\delta h_{34}^{+}+\left(M_H^2c^4/2\right)\delta h_{34}^{+*}h_{34}^{+2}\\&=-\xi M_Z^2c^4\eta_Z'\omega_Z'+O\left(\left|\delta h_{34}^{+}\right|^2\right).\end{aligned}\tag{24}$$

Here δh_{34}^{+} is equal to δH_{34}^{+} normalized by the vacuum expectation value of the Higgs field, $(\mu/\lambda)^{1/2}$, and similarly, $\eta_Z'=\eta_{Z1}/(\mu/\lambda)^{1/2}$ and $\omega_Z'=\omega_{Z1}/(\mu/\lambda)^{1/2}$. Note that $Re\left(\delta h_{34}^{+}\right)$ is equal to $\delta H_4/(\mu/\lambda)^{1/2}$. The frequency $\Omega_{\delta H}$ may be set to the frequencies of $\eta_Z'\omega_Z'$ in Equation (24) to get the frequency response of $Re\left(\delta h_{34}^{+}\right)$. The temporal frequencies of η_Z' and ω_Z' might be considerably larger than the contribution from the momentum terms $|\boldsymbol{p}|c$ alone due to the Higgs mass terms in Equations (19) and (20). The mass of the Higgs M_H has been defined earlier, and the "$O(\,)$" notation is the "big-Oh" notation to indicate that the residual error is comparable to or of smaller order than the quantity in parentheses. It is worth noting that the inclusion of these second- and third-order terms lead to a cubic equation in δh_{34}^{+}, which can lead to solutions for three masses in a family [22].

Next, use the circular periodic boundary condition of Equation (38) below, so that $\left|\boldsymbol{p}_{\eta,n}\right|$ and $\left|\boldsymbol{p}_{\omega,n}\right|$ are equal to $n\hbar/(2r)$, where n must be a positive integer and where r is the radius of the neutrino "loop". The values of n should not be equal to avoid secular growth of the Higgs as noted above. A straightforward choice is then $n = 1$ for $\left|\boldsymbol{p}_{\eta,n}\right|$ and $n = 2$ for $\left|\boldsymbol{p}_{\omega,n}\right|$ (It is worth mentioning that another reasonable choice is $n = 2$ for $\left|\boldsymbol{p}_{\eta,n}\right|$ and $n = 3$ for $\left|\boldsymbol{p}_{\omega,n}\right|$). This then implies that the frequencies of $\eta_Z'\omega_Z'$ are $\pm 1c/(2r)\pm 2c/(2r)$. The ± factors must occur because η_Z' and ω_Z' are pure real, implying that both positive and negative frequencies must be present. This implies that at a minimum the frequencies $c/(2r_n)$ and $3c/(2r_n)$ and their negatives must be present. Higher frequencies may be introduced by parametric generation from the $\delta H_4\omega_Z$ and $\delta H_4\eta_Z$ terms in the equations for ω_Z and η_Z, respectively, as discussed in the previous section. However, higher frequencies will be suppressed by the double time integrals seen in Equation (23), which will lead to factors of 1/frequency^2 which obviously suppresses higher frequencies. With this in mind, one may write the Higgs perturbation field as a Fourier series in multiples of the frequency $\Omega_0=c/(2r)=\left|\boldsymbol{p}_H\right|c/\hbar$:

$$\delta h_{34}^{+}=\sum\nolimits_n A_n\exp\left(in\Omega_0 t\right).\tag{25}$$

where the A_n are the coefficients of the Fourier series. Note that t might represent the retarded or advanced time in this formula, thus implying a sum over momentum states as well. Inserting this into Equation (24) leads to the following relations for $n = 1$ and $n = 3$, by matching frequencies:

$$\left[M_H^2c^4/2+\left|\boldsymbol{p}_H\right|^2c^2-\hbar^2\Omega_0^2\right]A_1+\left(M_H^2c^4/2\right)A_1^{*}=-\xi M_Z^2c^4\eta_Z'\omega_Z'\left(\Omega_0\right),\tag{26a}$$

$$\left[M_H^2c^4/2+\left|\boldsymbol{p}_H\right|^2c^2-9\hbar^2\Omega_0^2\right]A_3+\left(M_H^2c^4/2\right)A_{-1}^*=-\xi M_Z^2c^4\eta_Z'\omega_Z'\left(3\Omega_0\right). \quad (26b)$$

Here $\eta_Z'\omega_Z'\left(n\Omega_0\right)\equiv B\left(n\Omega_0\right)$ denotes the n^{th} frequency component of the product $\eta_Z'\omega_Z'$. Note that there are no frequency components other than $n = 1$ and $n = 3$ to lowest order, based on the discussion of the preceding paragraph. That said, one might expect other frequencies, including $n = 2$ to arise from parametric frequency generation, though of lesser magnitude. This then generates at least 3 frequencies of energies 1, 2, and 3 times $\hbar\Omega_0$. These frequencies will be associated with the masses of the three generations of the neutrino family for this uncharged case. To solve Equations (26), one may use the fact that the right hand sides should be pure real with proper choice of initial time, so that A_i should be pure real or nearly so. One then obtains:

$$A_1=-\xi\left(M_Z^2/M_H^2\right)B\left(\Omega_0\right), \quad (27a)$$

$$A_3=\xi\left(M_Z^2/M_H^2\right)\left[-2B\left(3\Omega_0\right)+B\left(\Omega_0\right)\right]. \quad (27b)$$

Here, terms of order $\left(\left|\boldsymbol{p}_H\right|^2c^2-9\hbar^2\Omega_0^2\right)\big/\left(M_H^2c^4\right)$ are neglected, which seems reasonable for the neutrino family in particular, because of their known small masses. These coefficients then may be related to the masses of the particles by inserting the results of Equations (26) and (27) into Equations (19) and (20):

$$\hbar^2\partial_t^2\omega_{Z0}=-\left\{\left|\boldsymbol{p}_{\omega Z}\right|^2c^2+\xi\left(M_Zc^2\right)^2 Re\left[\sum\nolimits_n A_n\exp\left(in\Omega_0t\right)\right]\right\}\omega_Z, \quad (28a)$$

$$\hbar^2\partial_t^2\eta_{Z0}=-\left\{\left|\boldsymbol{p}_{\eta Z}\right|^2c^2+\xi\left(M_Zc^2\right)^2 Re\left[\sum\nolimits_n A_n\exp\left(in\Omega_0t\right)\right]\right\}\eta_Z. \quad (28b)$$

From Equations (28) it is evident that that effective masses for η_Z and ω_Z are $\sqrt{\xi}M_Zc^2A_n$ and that there are oscillations between these mass terms.

To check these various assertions in a specific example, consider neutrino masses of $m_1=0.005$ eV/c^2, $m_2=0.01$ eV/c^2, and $m_3=0.0505$ eV/c^2 for the three respective generations of neutrinos. These masses satisfy the recent best measurements of $\Delta m_{32}^2=m_3^2-m_2^2$ and $\Delta m_{21}^2=m_2^2-m_1^2$ for neutrinos, which are 245 × 10^{-5} and 7.53 × 10^{-5} (eV/c^2)^2, respectively, with the normal hierarchy [38]. The normal hierarchy seems appropriate given the above integer progression of the mass energies, which leads to a quadratic variation in n for the squares of masses. It is obvious that $m_2/m_1=2$, which matches expectations for an integer progression of mass energies. However, this integer progression implies that $m_3=0.015$ eV, which does not match the above example masses. One solution to this is to allow the particle radius of Equation (38) to vary with neutrino generation. Another approach is to use the higher-order terms shown in Equation (24), and to solve the cubic equation in δh_{34}^+. This can exactly fit all three 3 generations of neutrino masses as given in [22], given the sum of the masses. For the non-neutrino fermion families, the sums of the masses can be determined from the electroweak boson masses, as discussed in ([23], Ch. 11). It should also be noted that for the charged ghosts the leading frequencies are $\left(2\xi M_W^2c^4+\left|\boldsymbol{p}_\eta\right|^2c^2\right)^{1/2}\big/\hbar$ rather than $\left|\boldsymbol{p}_\eta\right|c/\hbar$. This implies that the difference frequencies feeding into the

Higgs dynamics are of order $\left|\boldsymbol{p}_\eta\right|^2 / \left(4\hbar\sqrt{\xi}M_W\right)$ which is much less than $\sqrt{2\xi}M_W c^2/\hbar$, when $\xi > 1$. Hence for the charged ghosts the Higgs frequencies are much less than the ghosts' nominal frequencies.

The results of this subsection can be summarized as follows. First, the above shows that chargeless, purely oscillatory states are possible within the context of this theory. The two chargeless states are η_Z and ω_Z in this case. This chargeless case should match up with the neutrino family in the context of the theory of [23] when chargeless preons are assumed. Second, the above indicates that the neutrinos should have mass because of the coupling to the Higgs sector, as seen in Equations (19) to (21). Third, it seems that the masses of the neutrinos can be quite small: plausible neutrino masses can be matched with the above approach. Fourth, with proper choice of the momenta, the 3-wave mixing of this approach can generate 3 frequencies that look like mass oscillations for the neutrinos. Fifth, it seems that the normal hierarchy of mass ordering is appropriate with this approach. Sixth, one does not find explicit antiparticles for the neutrino family, but the spin ±1/2 solutions of Section 5 should be topologically the same as for the other families if the above arguments are correct.

Overall, this section shows that charged and chargeless ghost and gauge functions can exist in persistent oscillatory states, and that the interference of these states can create "gratings" that form potential wells in which stable states of non-zero energy are possible. This is in accord with the nonlinear optics analogy. This implies that the form of such potential wells is sinusoidal or a Fourier series of such sinusoids. Fits to example neutrino family mass parameters are found, with partial explanations. There are also unstable solutions associated with the extremum at $|H| = 0$. The results of this subsection have been checked numerically. Sample numerical results are shown in Section 6.

4. Stationary, Constant-Amplitude Oscillatory Solutions

A subset of the solutions in the momentum representation are in fact persistent oscillatory solutions, shown both analytically and numerically in Section 6, when the variations of the Higgs about its minima is small. That the time-averaged impact on the Higgs equations is zero is further demonstrated in this section. This is done by setting all the time derivatives to zero and all the momenta to zero. One expected stationary solution is when H_4 is equal to the vacuum expectation value and all the other variables are zero. It was seen from Sections 3.2 and 3.3 that such a solution is indeed a stationary solution for the neutral Higgs, but there are also purely oscillatory solutions about this constant. Stationary or purely oscillatory solutions are explored analytically in this section, with implications for the mutual exclusion of various charged and neutral states in stationary solutions. Note that purely-oscillatory solutions can be viewed as "stationary" solutions in a time-averaged sense, and these are a focus of this section.

The equations of Sections 2 and 3 can separately yield conditions for stationary solutions in four separate cases. These are (i) charged Higgs components, (ii) uncharged Higgs components, (iii) charged ghost and gauge function components, and (iv) uncharged ghost and gauge function components. These are obtained by setting the right-hand sides of Equations (12) to (14) to zero. These distinct cases are investigated briefly in this section.

4.1. Stationary Solutions for the Higgs Components

A basic requirement for a stationary solution for the Higgs components can be seen from Equations (9a) and (9b) to be at a local extremum of the Higgs potential:

$$\left|H_{12}^{+}\right|^{2}=\mu/\lambda \quad \text{or} \quad \left|H_{12}^{+}\right|=0\text{, and } \left|H_{34}^{+}\right|^{2}=\mu/\lambda \quad \text{or} \quad \left|H_{34}^{+}\right|=0\,. \tag{29}$$

From Equation (9a) it can be seen that a second condition for stationary solutions for the charged Higgs is that

$$\eta_Z\omega^{\pm}-\omega_Z\eta^{\pm}=0\,. \tag{30}$$

From Equation (9b) it can be seen that an additional condition for stationary solutions for the neutral Higgs is that

$$\cos(\theta_W)\eta^{\mp}\omega^{\pm}-\eta_Z\omega_Z/\cos(\theta_W)=0\,. \tag{31}$$

The "±" superscripts cover cases for both $H_{12}^{\pm}$ in Equation (30) and both $H_{34}^{\pm}$ in Equation (31). As noted above, one obvious solution to these equations for stationary solutions for the Higgs states is to set

$$\omega^{+}=\omega^{-}=\eta^{+}=\eta^{-}=\eta_Z=\omega_Z=0\,. \tag{32}$$

This corresponds to the standard case of a broken SU(2) × U(1) symmetry. In this case, the ghost Lagrangian is identically zero and only Equation (1) applies, in the absence of the Z and W bosons.

Another set of nonzero solutions that give a stationary Higgs are when any one of $\omega^{+},\omega^{-},\eta^{+},\eta^{-},\eta_Z$ or ω_Z is not zero and all of the other values are zero. This particular set of stationary solutions is consistent with the notion of charged and uncharged preons that may be persistent solutions. Yet another possibility for a stationary solution is that any one of $\eta^{\pm}$ or η_Z is non-zero while some subset of $\omega^{\pm}$ and ω_Z are zero, or vice versa. This alternative seems more consistent with the results of Section 3. More specifically, the time-averaged non-zero stationary solutions are when only the oscillatory pair (η^{+}, ω^{-}) is non-zero, or when the oscillatory pair (η_Z, ω_Z) is non-zero. From this section and Section 3.3, it also seems that there can be persistent oscillatory solutions for the ghosts and gauge fields for which the Higgs fields remain stationary, at least in a time-averaged sense. For example, a persistent oscillatory solution with both η_Z and ω_Z non-zero was found in Section 3.3, but the time average of the product is zero.

There are other possibilities for non-zero stationary solutions. For example, for the purely oscillatory solutions found in Sections 3.2 and 3.3, one finds that Equations (30) and (31) are zero in a time-averaged sense. As a second example, Equation (30) leads to a condition

$$\left|\eta_Z\right|\left|\omega^{\pm}\right|=\left|\omega_Z\right|\left|\eta^{\pm}\right|, \tag{33}$$

as a necessary condition for a stationary charged Higgs state. This is consistent with Section 3.2 when $Re(\omega^{\pm}) = Re(\eta^{\pm}) = \eta_Z = \omega_Z = 0$ and $Im(\omega^{\pm})$ or $Im(\eta^{\pm})$ are non-zero, for example.

4.2. Stationary Solutions in the Charged Ghost and Gauge Function Portions of the Higgs Sector

In this case it can be seen from Equations (10a) and (11a) that the sufficient conditions for stationary solutions (at least in a time-average sense) are

$$\left(2M_W c^2 + gH_{34}^-\right)\omega^+ + gH_{12}^+\omega_Z / \cos(\theta_w) = 0 \tag{34a}$$

and

$$\left(2M_W c^2 + gH_{34}^-\right)\eta^+ - gH_{12}^+\eta_Z / \cos(\theta_w) = 0 . \tag{34b}$$

This leads to the following relations:

$$\omega^+ = -gH_{12}^+\omega_Z \Big/ \left[\cos(\theta_w)\left(2M_W c^2 + gH_{34}^-\right)\right] \tag{35a}$$

and

$$\eta^+ = +gH_{12}^+\eta_Z \Big/ \left[\cos(\theta_w)\left(2M_W c^2 + gH_{34}^+\right)\right]. \tag{35b}$$

Since the denominators in both Equations (35a) and (35b) have equal magnitudes, this leads to a stationary solution with $|\eta_Z||\omega^{\pm}| = |\omega_Z||\eta^{\pm}|$ as in Equation (33) above, even when H_{12}^+ is zero. Since the signs are reversed between the two product terms in Equations (30) and (35), it is evident that both product terms must be zero, at least on average, for stationary or purely-oscillatory solutions for both the Higgs and charged ghosts and gauge functions in such a special overlapping case when both equations apply.

4.3. Stationary Solutions in the Un-Charged Ghost Portion of the Higgs Sector

In this case it can be seen from Equations (10b) and (11b) that the sufficient conditions for stationary or purely oscillatory solutions are

$$-gRe\left(H_{12}^-\omega^+\right) + \left[2M_Z c^2 - gH_4/\cos(\theta_w)\right]\omega_Z = 0 \tag{36a}$$

and

$$gRe\left(H_{12}^-\eta^+\right) + \left[2M_Z c^2 - gH_4/\cos(\theta_w)\right]\eta_Z = 0 . \tag{36b}$$

Note that the terms in square brackets can be zero when H_4 is equal to the vacuum expectation value. Hence in this case, one obtains $Re\left(H_{12}^-\omega^+\right) = 0$ or $Re\left(H_{12}^-\eta^+\right) = 0$, at least in a time-averaged sense, as might be expected in this uncharged case. This is consistent with Section 3.3 above. When the term in square brackets is non-zero, and ω_Z or η_Z is non-zero, this leads to the following relation for a stationary solution:

$$Re\left(H_{12}^-\eta^+\right)\omega_Z = -Re\left(H_{12}^-\omega^+\right)\eta_Z . \tag{37}$$

This result is similar to but not identical to that derived in the two preceding sections. Overall, the results of this section identify stationary solutions which are consistent with specific charged states or uncharged states for the ghosts and gauge functions being non-zero, with all other states having zero amplitude, at least in time average.

5. Circular Periodic Boundary Conditions

The fermionic ghosts of the Higgs sector should return to the negative of their original value after circulating 2π radians around a loop. This condition is similar to the Bohr condition but not identical, because this condition implies a winding number of two to return to the initial phase modulo 2π. With this condition, one has that

$$mv\gamma r' = n\hbar/2\,, \tag{38}$$

where m is the particle mass, v is the speed of the particle and γ is the relativistic gamma, equal to $\left[1-(v/c)^2\right]^{-1/2}$. The variable r' denotes the "radius" of the excitation and n must be a positive integer, in order for the field to return the negative of the original value. From the discussions of Section 3, the most relevant values of n are 1, 2, and 3. When $v \approx c$ it can be seen that r' is evidently related to the Compton wavelength $r = 2\pi\hbar/mc$ by the relation

$$r' = nr/(4\pi\gamma)\,. \tag{39}$$

Since Equation (38) should be applied to the hypothesized composite particles found in Section 3, it should be applied to the collective oscillation as a whole. Accordingly, the relevant masses are the mass parameters of all 4 families (up, down, electron, and neutrino), based on the approach of ([23], Ch. 2). The mass parameters are the average of the masses in a family. These family masses, generically denoted m_f, are shown in **Table 3** using the 2016 PDG values of the masses for the up and down families, which uses the minimal subtraction scheme at an energy scale of 2 GeV. For the neutrinos, particle masses of 0.0054, 0.0102, and 0.0515 eV/c^2 are assumed for the three generations.

From ([23], Ch. 11), one has an estimate for r' of about 10^{-21} meters, based on electrostatic repulsion. Using this and a mass parameter, one can then compute $v\gamma$ using Equation (38). One finds that $v \approx c$ for all families, and this simplifies the computation. With γ determined, one can then compute r from Equation (39). **Table 3** shows these results. For the neutrino family in particular, there is no charge, so the estimate of r based on electrostatic repulsion need not apply. However, arguments based on values of the PMNS matrix imply that the neutrino size should be similar to that for the electron [39]. That said, the resulting value of the relativistic γ seems extreme for neutrinos. To address this, a third measure of particle size is introduced. This measure is $1/\sqrt{\sigma_f}$, where σ_f is the family-dependent correction to the Higgs potential from [22], which has units of 1/length^2. It is used to obtain a precise fit to the masses in the respective fermion families. This effective radius is more consistent with the PMNS and CKM matrix parameters using the approach of [39].

Table 3. Estimates of the relativistic γ from Equation (38) and an assumed value of r' equal to 10^{-21} meters. The Compton wavelength is computed using Equation (39). The family mass parameters are discussed above. The effective radii are from [22].

Family or Particle	Particle/Family Mass Parameter $m_f\,(\mathrm{GeV}/c^2)$	γ (unitless) for $r' = 10^{-21}$ m	Compton wavelength r (fm)	Effective radius $1/\sqrt{\sigma_f}$ (fm)
electron particle	5.11×10^{-4}	1.928×10^{8}	2426	1.84×10^{-3}
up family	58.127	1695	0.0213	1.65×10^{-3}
down family	1.4269	6.903×10^{4}	0.8675	1.79×10^{-3}
electron family	0.6277	1.569×10^{5}	1.9720	1.83×10^{-3}
neutrino family	2.273×10^{-11}	4.339×10^{15}	5.452×10^{10}	3.81×10^{-3}

Equations (38) and (39) give values of r that are consistent with the Compton wavelength of the family mass parameter when $v \approx c$, by construction. These results are also approximately consistent with Heisenberg's uncertainty principle, which requires that $(\Delta E \Delta r)/c \geq h/4\pi$. Here ΔE is the standard deviation of the particle's energy distribution, and Δr is the standard deviation of the particle's spatial distribution. This result applies for a relativistic particle for which $E/c \approx p$ where p is the magnitude of the particle's spatial momentum and uses the usual recipe $\Delta E \sim E$ and $\Delta r \sim r'$.

All the Compton wavelengths of Table 3 are much larger than the 10^{-21} m estimated from electrostatic repulsion of preon-like particles in neighboring potential wells. The simple-minded electrostatic repulsion calculations assume that the particles are at rest relative to each other. However, this is not the case when the particles are moving in potential wells distributed at angles $2\pi/3$ around a circle. This implies that any two particles in different potential wells are not in the same rest frame, so a Lorentz contraction occurs. This Lorentz contraction is consistent with Equation (39), without the factor of 4π. The extra factor of 4π occurs because of the arrangement of potential wells around a double ring, accounting for the distinction between particle radius and well separation.

To summarize this section, the radii for fermions are consistent with the Compton wavelength and also approximately consistent with Heisenberg's uncertainty relation. The approach uses a circular periodic boundary condition for fermions and accounts for special relativity including Lorentz contraction, assuming the scalar fields circulate in a double ring in a classical picture. The particular choice of mass of these scalar fields is not particularly crucial with this approach, though it should be a positive number based on Equation (38). This is consistent with Section 3.

6. Example Numerical Result

Equations (19) to (21) of Section 3.3 for a chargeless state may be solved numerically. The second derivatives are approximated by forward second-order finite differences. Equations (19) and (20) pose no particular challenge when

$0.5\xi M_Z g\delta H_4/\cos(\theta_W)$ is comparable to $|\boldsymbol{p}_{\eta Z}|^2$ and $|\boldsymbol{p}_{\omega Z}|^2$ or less, which is the case here for neutrinos. Equation (21) however, has frequencies $M_H c^2/(\sqrt{2}\hbar)$, which is vastly larger than the frequency of the solutions being sought (a factor of 10^{13} for the neutrino family). Thus equation (21) is a very "stiff" equation, and specialized techniques such as Runge-Kutta-Nystrom approaches may be advantageous for a numerical solution [40]. Because the equation is so stiff, the approach for Equation (21) is to apply the exact forced solution in each integration step, an approach also mentioned in [40]. This is equivalent to Equation (24) but all the higher powers in δh_{34}^+ are here included in the numerical solution.

The result of a numerical integration using Euler's method or the trapezoidal method is shown in **Figure 1** versus time in picoseconds. In the example shown, purely oscillatory solutions are evident. It should be noted that there are cases where the numerical solutions are apparently unstable, but it is unclear whether this is a numerical instability or a genuine instability of the equations. This issue will be left to future work. Note also that Equations (21) or (24) can be written in terms of the small parameter $(|\boldsymbol{p}_H|/M_H c)^2$. Neglecting summands proportional to this *very* small parameter (including the derivatives) leaves a cubic equation in δh_{34}^+ and a forcing term involving the product $\eta_Z \omega_Z$. This leads to the algebraic approach of [22] which yields exact fits to the measured three masses in each fermion family (for the neutrino family, fits are to example masses).

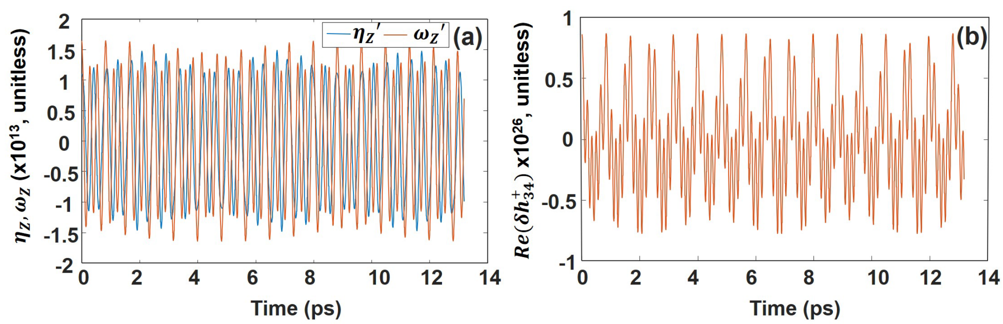

Figure 1. Example numerical solutions for the dimensionless variables $Re(\delta h_{34}^+)$, η_Z', and ω_Z' versus time (ps) for the neutrino family. Input parameters are $|\boldsymbol{p}_H|c = 0.005\,\text{eV}$, $|\boldsymbol{p}_{\eta Z}| = 2|\boldsymbol{p}_H|$, and $|\boldsymbol{p}_{\omega Z}| = 3|\boldsymbol{p}_H|$, and $\xi = 1$. Initial conditions are $\delta h_{34}^+(0) = (|\boldsymbol{p}_H|/M_H c)^2$, $\eta_Z'(0) = |\boldsymbol{p}_{\eta Z}|/M_Z c$, and $\omega_Z'(0) = |\boldsymbol{p}_{\omega Z}|/M_Z c$. All initial time derivatives are equal to zero. (a) time history of η_Z', and ω_Z'; (b) time history of just $Re(\delta h_{34}^+)$ with an expanded vertical scale.

7. Conclusions and Reinterpretation

Persistent, purely oscillatory solutions are found in the Higgs sector, both analytically and numerically. The assumptions are standard in treatments of nonlinear wave equations. The numerical solutions, which make no assumptions except that the fields can be expanded in states of definite momenta, show purely oscillatory solutions for all fields in a non-zero neighborhood of the minimum of the Higgs potential. The results of this paper show that the local gauge functions $\omega^\pm$ and ghost fields $\eta^{\pm}$ are charged states with persistent oscillatory solutions around the minima of the Higgs potential. It is not so unusual that the local gauge functions have charge, since this is a tenet of local gauge invariance for non-Abelian fields, e.g., the charged W boson and its corresponding gauge function. The experimental evidence indicates that their absolute value of quantized charge is $e/3$, since that is the minimum absolute value of observed charge of fundamental fermions (for the down family). The charges of all other fundamental fermions are a multiple of this charge. With three potential wells as in Section 3, one can have up to three anticommuting particles circulating in a ring, based on the Pauli exclusion principle. Hence, the maximum charge allowed is $\pm e$ for a fundamental fermion in this paradigm.

The issue of the small size of bound scalars compared to the Compton wavelength is addressed here using a circular periodic boundary condition, rather than an energy condition (upon which the Compton wavelength is based). For this condition to be applicable to spin-1/2 particles, the wavefunctions are found to circulate in a double ring. This provides an explanation for the small size that involves a non-zero mass and special relativity in a classical picture.

This effort provides a rationale for why there should be three potential wells. From Section 3, the three-wave mixing of the Higgs, ghosts, and gauge functions in a ring implies that there should be three primary spatial and temporal frequencies. This finding is reinforced in [22] which fits all fermion masses.

The conventional picture of the Higgs sector might be reinterpreted as follows, based on the above analyses. The Higgs itself might be viewed as an elastic material, which takes the form of a ring. The vacuum expectation value of the Higgs represents a stable state of tension of this "rubber band" which forms the "bond" for the ghosts and gauge functions, which circulate and oscillate in this ring. The characterization of the Higgs as a bond is consistent with a bosonic scalar field with units of $[\text{Force}]^{1/2}$. This ring is localized, so this vacuum expectation value does not imply a pervasive vacuum energy density that is extremely high. There are persistent oscillatory states that are stationary, in at least a time-averaged sense, which are readily interpreted as preons. These solutions have any one of $\eta^{+}, \eta^{-}, \eta_{Z}, \eta_{\phi}$ (and also the corresponding ω's) not equal to zero and all of the other values equal to zero. In the specific case where the only non-zero persistent oscillatory solutions are the pairs (η^{+}, ω^{+}), (η^{-}, ω^{-}) or (η_{Z}, ω_{Z}), these anticommuting states oscillate in a ring with potential wells generated through interaction with the Higgs field. For the uncharged states (η_{Z}, ω_{Z}), this is evident from Section 3.3. For the charged states, the discussions of Section 3.2 and Section 4 support this assertion.

A next step in this course of inquiry might be to compute the fermion masses from the electroweak parameters. This is done in [22] assuming the mass parameters (average of the masses) are given and assuming family-dependent modifications to the Higgs potential. The mass parameters can be computed from the electroweak parameters, by reversing the calculations of the electroweak boson masses and the Higgs masses from the mass parameters that already exists in ([23], Ch. 11). However, the neutrino family mass parameter is not estimated precisely using this approach due to its small value compared to the other family mass parameters. Nonetheless, the discussion of Section 3.3 and Section 5 separately provides example neutrino family mass properties, fitting measured neutrino mass-squared differences. The standard-model result of this paper also predicts masses and mass oscillations for neutrinos. This paper *does not* challenge the existing theories of the Higgs sector. It just finds new solutions within the existing Higgs model and shows how these solutions predict properties of the fundamental fermions. Regarding preon models, it provides a classical path to quantitative predictions of fermion properties. To summarize this paragraph, there is already a path from the Higgs sector to the calculation of a large subset of the fermion properties including specific mass properties in the context of the theory of [22] and [23]. The verification of this approach therefore already exists in the known properties of the fermions and extends the standard model by showing how neutrinos can have mass and mass oscillations.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix. Reformulation of the Higgs Sector Lagrangian in Terms of Charged States

An alternative formulation of Equations (1) and (2) in terms of charged states is given in this **Appendix** in order to verify the primary equations of this paper and to check conservation of charge. The ghost Lagrangian density may be rewritten in terms of charged states and Z boson and photon states. Use $\omega^{\pm} = (\omega_1 \pm i\omega_2)/\sqrt{2}$ and $\eta^{\pm} = (\eta_1 \pm i\eta_2)/\sqrt{2}$ for the charged ghost states and similarly for the charged Higgs states. Use Equation (3) for the Z-boson ghost and gauge functions. For the photon ghost and gauge functions, use

$$\eta_\phi = \sin(\theta_W)\eta_4 + \cos(\theta_W)\eta_3 \text{ and} \tag{A1a}$$

$$\omega_\phi = \sin(\theta_W)\omega_4 + \cos(\theta_W)\omega_3 . \tag{A1b}$$

Note that the factor of 1/2 is not used for $H_{34}^{\pm}$ since these are not charged states. One then finds the following Lagrangian density:

$$\begin{aligned} L_{Gh} = & -\eta^+ \Box \omega^- - \eta^- \Box \omega^+ - \eta_Z \Box \omega_Z - \eta_\phi \Box \omega_\phi \\ & + 1/(\hbar c)^2 \Big[-\xi M_W^2 c^4 (\eta^+\omega^- + \eta^-\omega^+) - \xi M_Z^2 c^4 \eta_Z \omega_Z \\ & + g(\hbar c)\boldsymbol{\eta}\cdot(\partial_\mu \boldsymbol{\omega} \times \mathbf{W}_\mu) - 0.5\xi g M_W c^2 (H_{34}^+ - H_{34}^-)(\eta^+\omega^- - \eta^-\omega^+)/2 \\ & + 0.5\xi g M_Z c^2 \eta_Z (H_{12}^+\omega^- + H_{12}^-\omega^+) \\ & - 0.5\xi g M_W c^2 (H_{34}^+ + H_{34}^-)(\eta^+\omega^- + \eta^-\omega^+)/2 \\ & + 0.5\xi (g^2 + g'^2)^{1/2} M_Z c^2 (H_{34}^+ + H_{34}^-)\eta_Z\omega_Z/2 \\ & - 0.5\xi g M_Z c^2 \omega_Z (H_{12}^+\eta^- + H_{12}^-\eta^+) \Big]. \end{aligned} \tag{A2}$$

This then leads to Equations (10) and (11). For example, differentiating Equation (A2) by η^- leads to an equation for ω^+. After integrating the leading term of Equation (A2) by parts twice inside the integral of the Lagrangian density, one can obtain an equation for η^+ by differentiating by ω^-. Similarly for all the other ghost fields and gauge functions. To obtain Equation (9a) for the Higgs fields, the leading factor of 0.5 in Equation (1) is absorbed into the complex scalar equation for the redefined $H_{12}^{\pm}$ that includes a factor of $1/\sqrt{2}$, as noted above. Equation (1) for $H_{12}^{\pm}$ then becomes

$$L_H = \left\| \left[\partial_\mu - i/(2\hbar c)(g\mathbf{t}\cdot\mathbf{W}_\mu + g'YB_\mu) H_{12}^+ \right] \right\|^2 + \mu H_{12}^- H_{12}^+ - \lambda (H_{12}^- H_{12}^+)^2 . \tag{A3}$$

Note that leading numerical factors have also been absorbed into the last two terms with this redefinition. With the neglect of the electroweak boson fields, and integrating the kinetic energy term by parts, one has a homogeneous Lagrangian density that has a D'Alembertian form similar to that of Equation (A2) for the charged ghost fields and gauge functions:

$$L_H = -H_{12}^- \Box H_{12}^+ + \mu H_{12}^- H_{12}^+ - \lambda (H_{12}^- H_{12}^+)^2 . \tag{A4}$$

A similar equation applies for H_{34}^+. From this, Equation (9a) is obtained after differentiation with respect to H_{12}^-. To recover Equation (9b) using this alternate approach, one can start with equations similar to Equations (A3) and (A4) for $H_{34}^{\pm}$ and carefully track factors of $\sqrt{2}$ but otherwise follow a similar approach. For either Equation (9a) or (9b), one needs to account for the $1/\sqrt{2}$ factors that differ between the definitions of the ghost and gauge function variables there and in this **Appendix** to obtain agreement.

In summary, this **Appendix** finds that the ghost Lagrangian can be written in terms of charged complex states, which is in conformity with the well-known result that complex states are appropriate to properly describe scalar charged particles ([4], Ch. 2). This **Appendix** also confirms that Equations (9) to (11) follow from Equations (1) and (2). Also, because the Lagrangian density's terms all have zero net charge, one expects conservation of charge for the system in the absence of the charged W bosons. A detailed calculation confirms this expectation, but it is not shown in this paper due to its length.