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# Discussion on Theoretical Structure between Quantum Physics and Newtonian Mechanics

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## Abstract

In 1905, Einstein established relativity under the condition of denying the absolute space-time valid in Newtonian mechanics. In those days, the quantum theory was also established by accepting De Broglie's hypothesis of matter wave, regardless of the causality of Newtonian mechanics. It has thus caused some fundamental problems unsolved in the quantum theory, as the causality of quantum mechanics for Newtonian mechanics is unknown. In that situation, it was recently revealed that the quantum theory is reasonably established in accordance with the causality for Newtonian mechanics, and also that the hypothesis of De Broglie is theoretically valid. The fundamental problems unsolved for a long time were also reasonably elucidated then. Further, we then found that a micro particle has an imaginary mass in a local space, or its behavior depends on an imaginary time. In the present work, we thus review fundamental problems in quantum mechanics from a viewpoint of the theoretical structure of physics in accordance with the causality for Newtonian mechanics.

## Keywords

Quantum Teleportation, Imaginary Time and Mass, Essence of Wave Nature, Derivation of Schrödinger's Equation, Einstein-Bose Condensation

## 1. Introduction

In the early stage of the 20th century, relativity and quantum theory were established. The correlation between the theoretical structure of quantum mechanics and that of Newtonian mechanics had not been clarified. In those days, it was experimentally revealed that the light speed shows a constant value regardless of the coordinate system [1]. Einstein established the relativity by accepting it as a principle, and then the absolute space-time in Newtonian mechanics was denied [2]. We can thus understand the causality between relativity and Newtonian mechanics. On the other hand, the quantum theory had been established by accepting De Broglie's hypothesis relevant to the wave-particle duality, regardless of the theoretical structure of Newtonian mechanics [3]. We have thus been unable to understand the causality between quantum mechanics and Newtonian mechanics. Each of them has developed highly as modern physics. In that situation, however, we have been unable to understand the causality of quantum theory for the theoretical structure of Newtonian mechanics.

As mentioned above, we have been unable to understand the causality between quantum mechanics and Newtonian mechanics since the early stage of the 20th century. The matter gives the cause that understanding the quantum theory is difficult for beginners. Nevertheless, quantum mechanics has been highly applied to projects not only in material science but also in electronic engineering and even space engineering, leaving the fundamental problems unsolved. In other words, the fundamental problems have been accepted without theoretical evidence for a long time in accordance with the given principles or laws.

We have thus been unable to understand such theoretical evidence that:

1) Newtonian mechanics is not applicable to the behavior of a micro particle;

2) A micro particle corresponds to a matter wave of wave length $\lambda$;

3) The existence of a micro particle is obtained only as a probability;

4) The interference effect is shown in the behavior of a micro particle itself;

5) The energy $E$ and momentum $p$ of a micro particle are empirically accepted as not original quantities in physics but imaginary operators $i\hbar\frac{\partial}{\partial t}$ and $-i\hbar\nabla$ in accordance with the correspondence principle, where $\hbar\left(=h/2\pi\right)$ is the Planck constant.

Further, the following fundamental problems were also not reasonably solved.

6) We have been unable to understand the maximum size of a micro particle having a wave nature.

7) It has not been theoretically confirmed whether the relation itself of $p=h/\lambda$ proposed by De Broglie is really valid or not.

In that situation, it was recently revealed that the quantum theory is reasonably established in accordance with the causality for Newtonian mechanics, and then the density theorem of time valid in Newtonian mechanics was denied.

To discriminate between two micro particles of the same kind at a small distance of $d$, we must fundamentally observe them using a reflection light of wave length $\lambda$ satisfying the relation of $\lambda \le d$ then. On the other hand, it is well-known that the light of a frequency $\upsilon$ has an energy $E_\upsilon = h\upsilon\left(=hc/\lambda\right)$ proposed by Planck as a quanta, where $c$ is the light speed [4]. When $d$ is a very small value, therefore, the two micro particles are scattered by a large energy $E_\upsilon$. As a result, we cannot fundamentally discriminate between two micro particles of the same kind for a small $d$ value. Recently, we thus accepted the matter mentioned here as "impossible principle of discrimination" for two micro particles of the same kind [5]-[7].

It was revealed as discussed later that accepting the impossible principle of discrimination corresponds to existing a minimum time $t_0$ in the physics relevant to micro particles. Here, note that the fundamental problems of (1)-(7) mentioned above were reasonably solved by having noticed the existence of minimum time $t_0$ in physics. In other words, if we accept the relation $\Delta t \to \Delta t' = -i\Delta t$ in physics for the relation $0 \le \Delta t < t_0$ valid in mathematics, the quantum theory is reasonably established, regardless of the hypothesis proposed by De Broglie. The matter mentioned here means such an extremely rare situation that the uniqueness of solution for a partial differential equation valid in mathematics is not valid in physics relevant to a micro particle because of denying the density theorem of time in mathematics.

In the present situation, there is no room for doubt that a material is composed of atoms and/or molecules. It is, therefore, the most important subject for researchers in the material science to investigate behavior of micro particles in a material. Here, note that the description used as a micro particle is not exactly defined. We have thus vaguely used the word of micro particle for a particle having a wave nature. In that situation, the word of micro particle will be concretely defined in the present text.

It is considered that the diffusion equation relevant to a collective motion of micro particles correlates with the quantum theory relevant to a micro particle [8]. In fact, Einstein, Bohm and others in the early stage of establishing the quantum theory investigated a possibility of transformation from the diffusion equation of Fick's law into the wave equation of Schrödinger [9] [10]. However, their projects did not succeed in those days.

Recently, Okino thought that their failures were caused by unconditionally accepting the diffusion equation as a law of Fick. Then, the diffusion equation itself was first theoretically derived from the Markov theory in mathematics [11] [12]. It was then for the first time revealed that the diffusivity correlates with an angular momentum of a micro particle in a local space in the diffusion field. In addition, we have never noticed till then that the diffusion equation is expressed as a fixed coordinate system or a moving one, and also that the discussion between these coordinate systems is indispensable for understanding the well-known Kirkendall effect [13].

Using the diffusivity expression obtained from the derivation process of diffusion equation, it was reasonably confirmed that the diffusion equation and the Schrödinger equation are transformable into each other [5] [14]. In that case, a Brown particle has a wave nature, because of the validity of transformation from the equation of micro particles into the wave equation [5] [6].

Generally in physics, we have accepted a relation as a law if its relation is always applicable to analyzing the concerned phenomena, when we cannot theoretically elucidate the validity of relation. Further, we have also accepted a relation necessary for the theoretical structure of physics as a hypothesis, when there is no experimental evidence for the relation.

In each stage of developments in physics for a long time, the proposed hypothesis should be rewritten as a law in accordance with the theoretical structure of physics, after the concerned phenomena were experimentally confirmed. In a similar meaning, if we can theoretically derive a relation having been accepted as a law, the relation should be afterward accepted as a basic equation important in physics, which corresponds to a theorem in mathematics.

For example, the hypothesis of De Broglie should be accepted as a law relevant to the matter wave in accordance with the theoretical structure of physics after we confirmed the wave nature of micro particles in physical experiments. Further, it should then be rewritten from the description of law to the basic equation of matter wave after we have theoretically derived its relation [6] [7].

We usually encounter the description of the hypothesis of De Broglie in textbooks, regardless of the theoretical structure of physics at that time. Such a description will be unsuitable for students learning elementary physics, because of their confusion caused by the inconsistency with the theoretical structure of physics. From a viewpoint of education, therefore, the author thinks that the description of a hypothesis, a law, and a basic equation in textbooks should always be consistent with the theoretical structure of physics at that time. For example, such a description as

{the law of matter wave (hypothesis proposed by De Broglie in 1923)}

or

{the basic equation of matter wave (hypothesis proposed by De Broglie in 1923)} should be written in textbooks in accordance with the given situation, judging from the theoretical structure of physics. In addition, it seems that there is no such problem discussed here in mathematics.

In addition to the above fundamental problems (1)-(7), for example, the problem of well-known quantum teleportation in relation to the quantum entanglement has not also been theoretically solved [15]-[17]. The impossible principle of discrimination revealed that there is a minimum value $t_0$ as a real time in physics relevant to a micro particle. In other words, the behavior of a micro particle in a local space depends on an imaginary time as an independent variable. We cannot understand an imaginary quantity in physics. We cannot thus directly understand behavior of a micro particle depending on an imaginary time.

When the impossible principle of discrimination is incorporated into behavior of a micro particle, we must take account of existing an imaginary time $t' = -it$ in a physical equation and also a minimum time $t_0$ as a real time must be considered in analyzing a partial differential equation relevant to a micro particle. In that situation, the matter of an imaginary time as an independent variable is replaced by one of imaginary differential operator, as can be seen from the Schrödinger equation.

It seems that the quantum entanglement is caused by replacing the matter of imaginary time with one of imaginary differential operator. In that situation, the theoretical evidence that quantum teleportation occurs as a matter of fact is tentatively discussed in the present text [18] [19].

## 2. Bohr's Model of Hydrogen Atom

Bohr in 1913 proposed such an atomic model of hydrogen that an electron makes a circular motion around the nuclei of a proton [20]. He then assumed that the electron having a momentum $p$ on an orbit of radius $r_n$ satisfies a relation of $r_n p = n\hbar$, where $n$ is a positive integer. Further, he thought that the electron jumps between the orbits of $r_m$ and $r_n$ by the absorption or emission of the energy $E_{mn} = (n-m)h\upsilon$ resulting from the energy quanta $h\upsilon$ proposed by Planck, where $\upsilon = c/\lambda$ is a frequency of light [4].

In addition, the conception of energy quanta as a photon was used for investigating the photo-electric effect by Einstein [21]. After that, Compton confirmed its validity by investigating a collision problem between an electron and photon [22]. It was thus accepted that a light having been considered as an electro-magnetic wave also has a particle image. It was also revealed in those days that the so-called frequency condition of $r_n p = n\hbar$ is consistent with the empirical equation called the Balmer series [23]. It was thus accepted that Bohr's frequency condition gives reliability in physics.

The magnitude of angular momentum $|L\rangle = |r_n \times p\rangle$ corresponds to the frequency condition proposed by Bohr. Using the relation of $\Delta r = r_n - r_{n-1}$ for $r_0 = 0$, we have the equation given by

$$|\Delta L\rangle = |r_n \times p\rangle - |(r_n - \Delta r) \times (p - \Delta p)\rangle = |\Delta r \times p\rangle .$$

The frequency condition is thus able to rewrite as

$$\Delta r p = \hbar \tag{1}$$

for the electron on an arbitrary orbit of hydrogen. After that, the experimental facts gave evidence that an electron has a wave nature [24]. In accordance with the experimental results, therefore, it is considered that the orbital electron has the relation of

$$\lambda = 2\pi\Delta r , \tag{2}$$

using a wave length $\lambda$ of the electron as a matter wave.

In accordance with the law of momentum conservation, the momentum $p$ of electron just at a moment after breaking away from the atom is equal to one just at a moment before breaking away. Equation (1) is thus valid for the free electron after breaking away from the atom. In other words, Equation (1) holds for an arbitrary moving electron because of the conservation of $p$ before and after breaking away from the atom. Judging from the matter mentioned here, Equation (1) reveals that an electron is never in the standstill state, and also that an electron moves as a wave packet having always an intrinsic space $\Delta r$ in the translational motion. At the same time, we can confirm that Equation (2) is also valid then.

## 3. Derivation of Hypothesis $p = h/\lambda$ Proposed by De Broglie

It is well known as principle of equipartition that the micro particle having degree of freedom $\alpha$ has the energy $\alpha k_B T/2$ in the collective motion at an absolute temperature $T$, where $k_B$ is the Boltzmann constant [25].

Using Equation (1) valid in an arbitrary state of electron for such a free electron as a state of electron gas, the correspondence between the kinetic energy and the principle of equipartition yields the minimum time $\Delta t$ given by

$$\Delta t = \hbar/\alpha\left(\varepsilon + k_B T\right) \quad \text{for} \quad \alpha = 3 \tag{3}$$

where $\varepsilon$ is a correction factor at the temperature $T = 0$.

Equation (3) is valid for an electron under an arbitrary condition, because of using Equation (1). Nevertheless, the right-hand side of Equation (3) has no information about the electron itself. Here, note that an electron itself is one of micro particles. In accordance with the causality in physics, this means that Equation (3) is also applicable to a micro particle having the same degree of freedom $\alpha$ used here. On the contrary, Equation (1) is valid then regardless of $\alpha$. As a result, it was thus revealed that relation of

$$\Delta r p = \hbar$$

is essentially applicable to a micro particle. In addition, the relation $\Delta r p = \hbar$ used here is one more limited than the uncertainty principle.

After the proposition of De Broglie's hypothesis, it has been experimentally revealed that many atoms and molecules have a wave nature [26] [27]. Further, as discussed later, it is also theoretically revealed that a micro particle has a wave nature, regardless of the above discussion [5]. Therefore, the relation of

$$\lambda = 2\pi\Delta r$$

is also valid for a micro particle.

Here, the relation of

$$p = h/\lambda \tag{4}$$

is reasonably derived from Equations (1) and (2) valid for a micro particle [6] [7]. As a result, the matter mentioned here gives evidence that Equation (4) is applicable to behavior of a micro particle. In other words, the hypothesis proposed by De Broglie was for the first time reasonably derived then.

## 4. Discrete Time in the Physics Relevant to a Micro Particle

In the following, we first discuss a problem of perfectly elastic collision between particles A and B of the same kind, where each particle of them has mass $m_A = m$ and $m_B = m$ on a smooth plane. In Newtonian mechanics, when the particle A moving with an initial velocity $\left|v_A\right\rangle = \left|v_0\right\rangle$ collides with the particle B in the standstill state of $\left|v_B\right\rangle = 0$, it is well-known that each velocity of particles A and B becomes $\left|v_A\right\rangle = 0$ and $\left|v_B\right\rangle = \left|v_0\right\rangle$ immediately after the perfectly elastic collision.

Using a direct force $\left| f_{\mathrm{BA}} \right\rangle$ acting on the particle A from the particle B and a direct force $\left| f_{\mathrm{AB}} \right\rangle$ acting on the particle B from the particle A, the relations between the impulse and the momentum variation are then expressed as

$$\left.\begin{array}{l} \left| f_{\mathrm{BA}} \right\rangle \Delta t_1 = -m_{\mathrm{A}} \left| \Delta v_{\mathrm{A}} \right\rangle \\ \left| f_{\mathrm{AB}} \right\rangle \Delta t_2 = m_{\mathrm{B}} \left| \Delta v_{\mathrm{B}} \right\rangle \end{array}\right\} \rightarrow \left| f_{\mathrm{BA}} \right\rangle \Delta t_1 + \left| f_{\mathrm{AB}} \right\rangle \Delta t_2 = m\left( -\left| v_0 \right\rangle + \left| v_0 \right\rangle \right) = 0,$$

where each time of $\Delta t_1$ and $\Delta t_2$ is a collision time of each particle.

Figure 1 shows the schematic diagram of the locus of motion $r(t)$ before and after the perfectly elastic collision between particles A and B, where $\Delta l_{\mathrm{A}}$ denotes a minimum displacement of the particle A for $\Delta t_1$, and $\Delta l_{\mathrm{B}}$ denotes that of the particle B for $\Delta t_2$. The collision time is usually accepted as $\Delta t_1 \rightarrow \Delta t,\ \Delta t_2 \rightarrow \Delta t$ in Newtonian mechanics. In that case, using the minimum quantity of second order $\Delta^2 r(t)$ of $r(t)$, $\Delta l_{\mathrm{A}}$ and $\Delta l_{\mathrm{B}}$ shown in Figure 1 are expressed as

$$\begin{cases} \Delta l_{\mathrm{A}} = \Delta^2 r_1(t) = -\Delta v_{\mathrm{A}} \Delta t = \dfrac{f_{\mathrm{BA}}}{m_{\mathrm{A}}} (\Delta t)^2 \rightarrow \dfrac{\Delta^2 r_1(t)}{(\Delta t)^2} \rightarrow \dfrac{\mathrm{d}^2 r_1(t)}{\mathrm{d}t^2} = \dfrac{f_{\mathrm{BA}}}{m_{\mathrm{A}}} \\ \Delta l_{\mathrm{B}} = \Delta^2 r_2(t) = \Delta v_{\mathrm{B}} \Delta t = \dfrac{f_{\mathrm{AB}}}{m_{\mathrm{B}}} (\Delta t)^2 \rightarrow \dfrac{\Delta^2 r_2(t)}{(\Delta t)^2} \rightarrow \dfrac{\mathrm{d}^2 r_2(t)}{\mathrm{d}t^2} = \dfrac{f_{\mathrm{AB}}}{m_{\mathrm{B}}} \end{cases},$$

where $r_1(t)$ or $r_2(t)$ corresponds to a part of $r(t)$ during the perfect elastic collision.

Generally, behavior of particles A and B between $\Delta t$ is not usually discussed in textbooks. It is physically considered that accelerations $a_1$ and $a_2$ become

$$\begin{cases} a_1 \rightarrow -\infty & \text{for } t_{\mathrm{C}} - \Delta t_1 \le t \le t_{\mathrm{C}} \\ a_2 \rightarrow \infty & \text{for } t_{\mathrm{C}} \le t \le t_{\mathrm{C}} + \Delta t_2 \end{cases}, \tag{5}$$

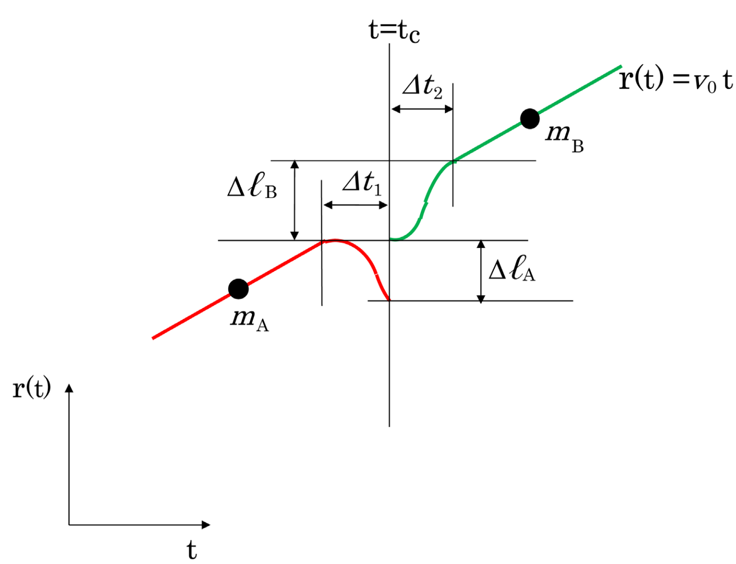


Figure 1. Perfectly elastic collision for particles of the same kind.

because of $f_{\mathrm{AB}} + f_{\mathrm{BA}} = 0$ for $\Delta t_1 = \Delta t_2 = \Delta t$, where $t = t_{\mathrm{C}}$ is just at a collision time between particles A and B.

Here, if we replace the problem of perfectly elastic collision mentioned above by a collision between two micro particles of the same kind, the impossible principle of discrimination discussed in the introduction should be considered for analyzing such a collision problem. In other words, we must then consider the possibility of $m_{\mathrm{B}} \rightarrow m_{\mathrm{A}}$ in Figure 1. If we consider behavior of micro particle $m_{\mathrm{A}}$ resulting from the impossible principle of discrimination then, we must thus physically accept the relation given by

$$\begin{cases} |v_{\mathrm{A}}\rangle = |v_0\rangle \rightarrow |v_{\mathrm{A}}\rangle = 0 \ \text{ for } t_{\mathrm{C}} - \Delta t_1 \le t \le t_{\mathrm{C}} \\ |v_{\mathrm{A}}\rangle = 0 \rightarrow |v_{\mathrm{A}}\rangle = |v_0\rangle \ \text{ for } t_{\mathrm{C}} \le t \le t_{\mathrm{C}} + \Delta t_2 \end{cases} \tag{6}$$

in the collision process.

Here, when we consider the mathematical behavior of locus of motion $r(t)$ in relation to Equation (6), it has a singular point at $t = t_{\mathrm{C}}$ and then $t = t_{\mathrm{C}}$ is an asymptotic line as shown in Figure 2. Judging from behavior of the asymptotic line $t = t_{\mathrm{C}}$ and the density theorem in mathematics, each functional behavior of $r_1(t)$ and $r_2(t)$ is mathematically convex upward as shown in Figure 2. In accordance with the definition of acceleration, the curvature of $r_2(t)$ then gives the acceleration of

$$a_2 = \frac{\mathrm{d}^2 r_2(t)}{\mathrm{d}t^2} = \lim_{\Delta t \to 0} \frac{\Delta^2 r_2(t)}{(\Delta t)^2} \quad \Rightarrow \quad a_2 < 0 . \tag{7}$$

Equation (7) obtained from the mathematical theory is inconsistent with the physical phenomena shown in Equation (6). Here, we introduce physically the imaginary time $\Delta t \rightarrow \pm i \Delta t$ into Equation (7) for the purpose of solving the inconsistency between physics and mathematics. In that case, the acceleration $a_2$ in Equation (7) becomes $a_2 > 0$ consistent with one in Equation (6) [5].

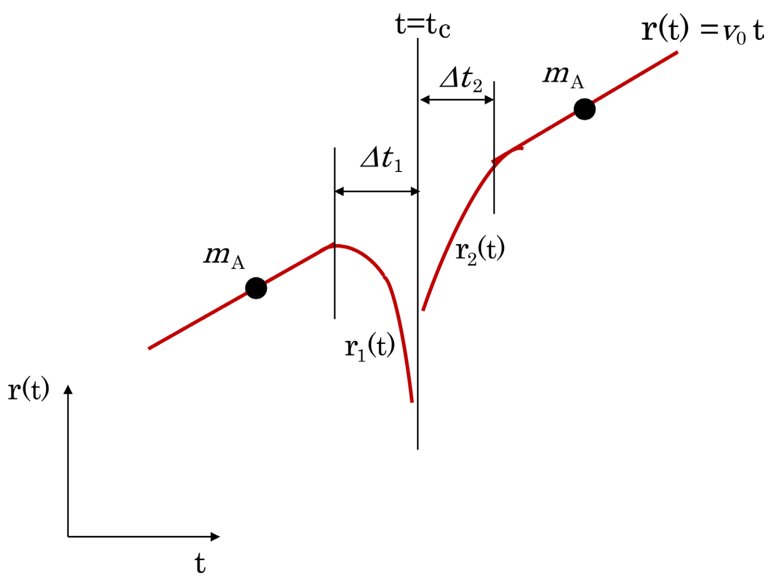


Figure 2. Mathematical Behavior of a Neighborhood of $r = r(t_{\mathrm{C}})$.

As a result, the impossible principle of discrimination indicates that there is a minimum time $t_0$ in physics. We thus accept the following relation of

$$\Delta t \to \Delta t' = \pm i \Delta t \tag{8}$$

in physics for the relation $0 \le \Delta t \le t_0$ satisfying the density theorem in mathematics. In that case, behavior of a micro particle depends on an imaginary time where we cannot understand from the usual conception in physics. The density theorem in mathematics is not thus applicable to a time in the physics relevant to micro particles because of existing a minimum time $t_0$.

Using $t_0$ for an arbitrary time $t_j$ in the physics relevant to a micro particle, we thus define the relation of

$$t_j = j t_0 \tag{9}$$

as a discrete time in physics, where $j$ is an arbitrary integer. Equation (9) is an important relation obtained from the impossible principle of discrimination.

In addition, the so-called Planck time $t_P \left(= 5.39 \times 10^{-44}\ \text{s}\right)$ is known as a minimum time in physics, where the value of $t_P$ is derived from the dimension analysis of the Planck constant $h$, the light speed $c$ and the gravitational constant $G$.

Here, note that the impossible principle of discrimination is not taken account of Equations (1), (2) and (4). It is thus considered that they are valid even in the case of $m_A \to m_B$ for $t > t_C$ in **Figure 2**. In other words, they are relations satisfying wave nature of a micro particle regardless of the impossible principle of discrimination.

## 5. Derivation of Schrödinger Equation

Schrödinger derived the wave equation applicable to behavior of a micro particle from accepting the hypothesis proposed by De Broglie [3] [9]. After that, the so-called Schrödinger equation of

$$i\hbar \frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi \tag{10}$$

has been highly applied to various phenomena in the physics relevant to a micro particle with a mass $m$. As a result, we found that the analyzed results having applied Equation (10) to behavior of a micro particle have no problems inconsistent with experimental results. The Schrödinger equation is thus widely accepted as a basic equation important in the quantum mechanics.

On the other hand, the diffusion equation of

$$\frac{\partial}{\partial t}C = D\nabla^2 C \tag{11}$$

has been accepted as a law proposed by Fick since 1855, where $C$ and $D$ are a concentration of diffusion particles and a diffusivity [10]. In addition, the expression of $C$ is defined here as a concentration expressed by particle numbers per a unit volume, and further it is normalized by a total number of diffusion particles in the diffusion region. In that situation, recently Equation (11) was reasonably derived from applying the Markov theory in mathematics to the elementary process of diffusion phenomena [5] [11]. The diffusivity $D$ in Equation (11) was then theoretically obtained as

$$D = \frac{\hbar}{2m} \tag{12}$$

using a mass $m$ of a diffusion particle [28]. Equation (12) has a characteristic constant $\hbar$ of physical quantity relevant to the angular momentum of a micro particle, where the dependence of diffusivity $D$ on $T$ and $C$ is neglected (See Appendix).

The impossible principle of discrimination revealed that we must accept the relation $\Delta t' = \pm i\Delta t$ for $0 \le \Delta t \le t_0$ valid in mathematics. In accordance with the discussion relevant to Equation (8), we can reasonably obtain the differential operators in the physics relevant to a micro particle, as shown in the following.

$$\frac{\partial}{\partial t} = \lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} \Rightarrow \frac{\partial}{\partial t'} = \lim_{\Delta t \to 0} \frac{\Delta}{\pm i\Delta t} \to \frac{\partial}{\partial t'} = \mp i \frac{\partial}{\partial t} \tag{13}$$

and

$$\frac{\partial t}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta}{\Delta x} t \Rightarrow \frac{\Delta}{\Delta x'} t' = \lim_{\Delta x' \to 0} \frac{\Delta}{\Delta x'} t' = \lim_{\Delta x \to 0} \pm i \frac{\Delta}{\Delta x} t \Rightarrow \frac{\partial}{\partial x'} \to \pm i \frac{\partial}{\partial x}. \tag{14}$$

We consider independently $\Delta t' \to 0$ using a real time $\omega$ for $t' = i\omega$ satisfying $\lim_{\omega \to 0} i\omega \to 0$, only for the purpose of mathematical analysis regardless of existing the minimum time $t_0$. Therefore, note that the relation of $\Delta t' \to 0$ does not mean $t \to 0$ then. In Equation (13), replacing the matter of imaginary time itself $\Delta t'$ by that of imaginary differential operator of real time $\Delta t$ means that the density theorem for a real time $t$ is outwardly valid again not physically but mathematically for the partial differential equation, because of the validity of $\Delta t \to 0$ via the relation $\Delta t' = \pm i\Delta t$ satisfying $\Delta t' \to 0$. In addition, Equation (14) shows that the density theorem in mathematics is essentially valid for the space $(x, y, z)$ in physics.

Here, by substituting Equations (12)-(14) into Equation (11), the equation of

$$\mp i \frac{\partial}{\partial t} C = -\frac{\hbar}{2m} \nabla^2 C$$

is reasonably obtained. Note that solution $C$ of above equation then becomes mathematically a complex value function as expressed by $C = C_{\text{re}} \pm iC_{\text{im}}$ using real functions of $C_{\text{re}}$ and $C_{\text{im}}$ [5]. Then, multiplying the both sides of above equation by $\hbar$ after replacing $C$ by $\Psi\left(= \Psi_{\text{re}} \pm i\Psi_{\text{im}}\right)$, the equation of

$$\mp i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi \tag{15}$$

is obtained. The correspondence between $C$ and $\Psi$ should be accepted $C = \gamma\Psi$ using $\gamma = 1\left[L^{-3}\right]$ with the length dimension $L$ for the dimensionless function $\Psi$. The relation of $|C| = \gamma|\Psi|$ then indicates that $|\Psi|^2$ means an existence probability of a micro particle judging from the given definition of $C$ and the correspondence between an energy of a micro particle and that of a wave.

In relation to the eigenvalue obtained from using Equation (15) for analyzing a problem, the double sign in Equation (15) was determined as +. Therefore, Equation (8) becomes $\Delta t' = -i\Delta t$ and Equations (13) and (14) are thus determined as

$$\frac{\partial}{\partial t} \to i\frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x} \to -i\frac{\partial}{\partial x}. \tag{16}$$

At this point, the Schrödinger equation of

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi$$

was reasonably derived from the above discussion, regardless of the hypothesis proposed by De Broglie [5] [6].

It was confirmed in the above discussion that the diffusion equation and the Schrödinger equation are reasonably transformable into each other. The matter gives the most important evidence that a diffusion particle of micro particle has a wave nature because of the validity of transformation from the equation of micro particles into the wave equation. In other words, such a micro particle as a diffusion particle jumping at random in a material has a wave nature [6].

Judging from the discussion about derivation of imaginary operators of Equation (13) and (14), note that the Schrödinger equation itself is a proper partial differential equation in mathematics, where the density theorem is valid in the real space-time $(t, x, y, z)$. In other words, the uniqueness of solution of partial differential equation is valid in the Schrödinger equation itself. However, the physical solution obtained from the mathematical analysis of the Schrödinger equation should be taken account of the existence of a minimum time $t_0$ because of the existence of Equation (9).

As mentioned above, we can obtain mathematical solutions of the Schrödinger Equation (10) in accordance with the uniqueness of a solution of partial differential equation. However, Equation (9) reveals that we cannot accept the mathematical solution $\Psi(t, x, y, z)$ as a physical solution $\Psi_{\mathrm{P}}(t, x, y, z)$ as it is. Using a probability factor $A_j$ for the mathematical solution $\Psi(t_j, x, y, z)$, we should thus accept it as

$$\Psi_{\mathrm{P}}(t, x, y, z) = \sum_{j=m}^{n} A_j \Psi(t_j, x, y, z), \tag{17}$$

if the time interval necessary for an observation of micro particle is expressed as

$$t_m \le t \le t_n,$$

judging from Equation (9).

Here, the derivation of Equation (17) reveals that some fundamental problems in the quantum theory are reasonably solved as follows [7].

(i) The uniqueness of solution for a partial differential equation for a micro particle is valid in mathematics, but it is not valid in physics because of Equation (9). At the same time, the probability interpretation is essentially required for a physical solution because of the superposition of mathematical solutions having the probability factor $A_j$ in Equation (17).

Einstein had never accepted the probability interpretation in the quantum mechanics in relation to the uniqueness of solution for a partial differential equation. He established the relativity by accepting the constant principle of light speed being inconsistent with Newtonian mechanics. If we accept the impossible principle of discrimination as well as the constant principle of light speed, Equation (17) is reasonably obtained. Even if the existence of discrete time resulting from the impossible principle of discrimination had been known in those days, wouldn't Einstein accept that the probability interpretation is essentially indispensable for the quantum mechanics?

(ii) Since the physical solution of $\Psi_{\mathrm{P}}(t,x,y,z)$ for behavior of a micro particle is expressed by a superposition of mathematical solutions, even behavior of a micro particle itself has essentially an interference effect resulting from a superposition of wave functions.

It is well-known that a photon of frequency $\upsilon$ has an energy value $E_{\upsilon}=h\upsilon$ as proposed by Planck [4]. The energy $E_{\lambda}$ relevant to a micro particle having a wave length $\lambda$ and a speed $v$ is also obtained as a similar expression using Equation (4). Concretely, a speed $v$ of micro particle moving as a wave packet is expressed as

$$v=\frac{\mathrm{d}\upsilon}{\mathrm{d}(1/\lambda)}=\frac{\mathrm{d}\upsilon}{\mathrm{d}(mv/h)}=\frac{h}{m}\frac{\mathrm{d}\upsilon}{\mathrm{d}v}$$

in accordance with the definition of group velocity, where $\upsilon(=v/\lambda)$ is also used as a frequency of micro particle having a wave nature. The integration of above equation gives

$$E_{\lambda}=h\upsilon\left(=\frac{1}{2}mv^{2}\right). \tag{18}$$

In addition, using Equation (2) for $\upsilon(=v/\lambda)$, the relations are obtained as

$$\upsilon=\frac{v}{\lambda}\rightarrow\upsilon=\frac{\Delta r}{\Delta t}\frac{1}{2\pi\Delta r}=\frac{1}{2\pi\Delta t}, \tag{19}$$

$$\Delta tE_{\lambda}=\hbar. \tag{20}$$

Equation (20) as well as Equation (1) is one more limited than the uncertainty principle.

When the momentum $p$ and energy $E_{\lambda}$ of a micro particle act on a physical quantity $\Psi$ as operators, the relations of

$$p_x\rightarrow-i\hbar\frac{\partial}{\partial x}\quad\text{and}\quad E_{\lambda}\rightarrow i\hbar\frac{\partial}{\partial t} \tag{21}$$

have been empirically accepted in accordance with the correspondence principle.

(iii) In the present discussion, however, simultaneously using Equation (16) for equations (1) and (2) or (19) and (20), the Equation (21) having been accepted without a theoretical evidence for a long time is reasonably derived as shown in the following [5] [6].

$$p_x\Psi = \hbar\frac{\Delta}{\Delta x}\Psi \to \hbar\frac{\partial}{\partial x}\Psi \quad \Rightarrow \quad p_x\Psi = \hbar\frac{\partial}{\partial x}\Psi \to -i\hbar\frac{\partial}{\partial x}\Psi$$

and

$$E_\lambda\Psi = \hbar\frac{\Delta}{\Delta t}\Psi \to \hbar\frac{\partial}{\partial t}\Psi \quad \Rightarrow \quad E_\lambda\Psi = \hbar\frac{\partial}{\partial t}\Psi \to i\hbar\frac{\partial}{\partial t}\Psi .$$

Since behavior itself of a micro particle in a local space depends essentially on an imaginary time, we cannot thus directly understand such a momentum or energy corresponding to a physical quantity having time-dimension. We can then indirectly understand the behavior from replacing them by imaginary operators in the physics. In addition, Equation (21) should be accepted as a result obtained from not the correspondence principle but the impossible principle of discrimination, judging from the above discussion.

In accordance with the theoretical structure of physics, Equations (1), (2) and also Equations (19) and (20) are relevant to the wave-particle duality, judging from the derivation process. They thus have a possibility of the case of $m_{\mathrm{A}} \to m_{\mathrm{B}}$ for $t > t_{\mathrm{C}}$ in **Figure 2**, because they have no connection with the impossible principle of discrimination. On the other hand, the fact that behavior of micro particle depends on an imaginary time resulting from the impossible principle of discrimination, which corresponds to the case shown in **Figure 2**, results in the relations $\Delta t \to \Delta t' = -i\Delta t$ and/or Equations (9), (21) regardless of the correspondence principle. In other words, Equation (21) means in mathematics that matter of an imaginary time as an independent variable is replaced by one of the imaginary differential operator for a real time. As a result, the Schrödinger Equation (10) is a partial differential equation of a real time as an independent variable under the condition of Equation (9) in physics.

## 6. Investigation of Size of Micro Particle and Minimum Time

As well known in Newtonian mechanics, when a body of mass $m$ having a position vector $|r_{\mathrm{G}}\rangle$ of the mass center is composed of $n$ particles of each mass $m_j$ having the position vector $|r_j\rangle$, the relation of

$$m|r_{\mathrm{G}}\rangle = \sum_{j=1}^{n} m_j |r_j\rangle \tag{22}$$

is defined as a given coordinate system. We can then investigate behavior of the motion of body with mass $m$ using the only position vector $|r_{\mathrm{G}}\rangle$, regardless of $m_j$ and $|r_j\rangle$.

In the quantum theory, we first experimentally confirmed that the electron with mass $m$ of elementary particle having no internal structure has a wave nature [29]. After that, experimental results reveal that an atom with mass $m$ composed of elementary particles has also a wave nature [26]. Further, it was experimentally confirmed that even a molecule with mass $m$ composed of atoms has a wave nature [27].

Even when we make $m$ and $m_j$ in Equation (22) correspond to a micro particle and an elementary particle or an atom, it is confirmed that the Schrödinger equation is then applicable to analyzing behavior of a micro particle with mass $m$, regardless of the internal structure. This means that the conception itself of mass center relevant to a motion of body in Newtonian mechanics is also applicable to analyzing behavior of a micro particle in the quantum mechanics.

Here, note that we must then consider a relation of $n \le n_0$ in Equation (22), where $n_0$ is a value of a maximum number of particles composing a micro particle with a wave nature. In that case, judging from experimental results obtained until recently, it is no room for doubt that an arbitrary atom has a wave nature, because of the existence of molecule $C_{60}$ having a wave nature [27].

The maximum size of micro particle having a wave nature has been unknown for a long time. As it were, the description of micro particle has been thus vaguely accepted without the theoretical evidence in the quantum mechanics. In the above discussion, therefore, we determine the largest size of micro particle having a wave nature by corresponding $m_j$ in Equation (22) to an atom composing a micro particle.

The collective motion of molecules in the gas state is known as diffusion phenomena. It was revealed in the present text that a diffusion particle has a wave nature. In the following, therefore, we tentatively determine the maximum value $n_0$ by investigating a motion of molecule in the ideal gas state [7].

The Avogadro's law shows that molecules of $N_{\mathrm{A}}\left(=6.02\times10^{23}\right)$ numbers coexist in the volume of $V_{\mathrm{A}}\left(=2.24\times10^{-2}\ \mathrm{m}^3\right)$ at the room temperature $T_n\left(=290\ \mathrm{K}\right)$. The size $l_n$ of a local space occupied by a molecule is thus estimated as

$$l_n = \left(V_{\mathrm{A}}/N_{\mathrm{A}}\right)^{1/3}\left(=3.34\times10^{-9}\ \mathrm{m}\right).$$

If a velocity of molecule moving through a local space in the state of ideal gas is $v_n$, the necessary time $t_n$ is given by the relation of $t_n = l_n/v_n$. In that case, we tentatively assume that $t_n$ is equal to $\Delta t$ of Equation (3) and also that $t_n \to t_0$ is valid. The following relation is then obtained as

$$t_0 = \frac{\hbar}{\alpha\left(\varepsilon + k_{\mathrm{B}}T\right)}. \tag{23}$$

Judging from the definition of degree of freedom for a micro particle, therefore, the relation of $v_n\left(=l_n/t_0\right) < c$ yields

$$3 \le \alpha < \frac{c\hbar}{l_n\left(\varepsilon + k_{\mathrm{B}}T\right)}. \tag{24}$$

Substituting the above values of $l_n$, $T$, $c$ and $\varepsilon = 0$ into Equation (24), the relation of

$$3 \le \alpha \le 780 \tag{25}$$

is thus obtained. Equation (25) reveals that such a micro particle as a giant molecule and/or a fine particle of metal composed of atoms less than 260 numbers has a wave nature because of $\alpha = 3n_0$. When a micro particle having a wave nature is composed of atoms, the maximum number $n_0$ of atoms is thus determined as $n_0 = 260$.

Judging from the relation discussed above, the relation of

$$v_n = l_n/t_0 \to v_n < c \to t_0 > l_n/c$$

is obtained. We then estimate a value of $t_0$ given by

$$t_0 > 1.11\times10^{-17}\,[\mathrm{s}].$$

On the other hand, Equations (23) and (25) yields

$$t_0 \le \frac{\hbar}{3(\varepsilon + k_B T)} \to t_0 \le 8.66\times10^{-15}\,[\mathrm{s}].$$

The minimum time $t_0$ dependent on $\alpha$ at the temperature $T = 290\ \mathrm{K}$ is thus expressed as

$$1.11\times10^{-17}\,[\mathrm{s}] < t_0 \le 8.66\times10^{-15}\,[\mathrm{s}]. \tag{26}$$

In the quantum theory, there is thus a minimum time $t_0$ dependent on the degree of freedom of a micro particle itself and on an absolute temperature.

Here, Equation (23) shows that a value of $t_0$ becomes extremely large one at an extremely low temperature. In that situation, the time interval $\Delta t = t_{j+1} - t_j \left(= t_0\right)$ of a real time becomes a large value in the conception of Newtonian mechanics, but there is no such a conception of time interval in the behavior itself of a micro particle, because of the situation in the world of imaginary time in accordance with the impossible principle of discrimination. On the other hand, we know phenomena of the superfluidity and superconductivity, where each of them is a collective motion of micro particles at an extremely low temperature and is a macro behavior resulting from the quantum effect in accordance with the Bose-Einstein condensation [30]-[33]. It seems that the impossible principle of discrimination has no direct relation to a collective motion of micro particles. Equation (20) is thus valid in the collective motion of micro particles because of no connection with the impossible principle of discrimination in the derivation process. On the other hand, it is considered that the minimum time $t_0$ correlates with the Bose-Einstein condensation, judging from an extremely low temperature in Equations (3) or (23). As a result, Equation (20) indicates that each energy of micro particles $E_\lambda$ becomes a normal energy level because of a large value $\Delta t$, so as to become the Einstein-Bose condensation.

## 7. Discussion and Conclusions

In the present text, we found that the quantum theory is reasonably established by accepting the impossible principle of discrimination between two micro particles of the same kind. Concretely, we must then accept that a minimum time $t_0$ is existent in the physics relevant to a micro particle. As a result, the fundamental problems (1)-(7) mentioned in the beginning were reasonably solved.

The well-known problems of Schrodinger's cat and a double-slit are discussed in the following.

**1) Collapse of a Wave Function and Probability Interpretation**

Equation (1) reveals that an arbitrary micro particle having a wave nature moves always in the situation having an intrinsic space $\Delta r$, regardless of whether it makes a circular motion or has a translational motion. In other words, we can never find a micro particle in the standstill state. When a micro particle is captured by a material, therefore, it is considered that the micro particle is not in standstill state but makes circular motion within an intrinsic space $\Delta r$ in the material. In fact, the atomic vibration is thus well known in a crystal material.

In relation to the density theorem of time in mathematics, the theoretical structure of quantum theory is different from that of Newtonian mechanics. As mentioned in the present text, therefore, we must accept Equation (17) resulting from Equation (9) as a physical solution. Further, we cannot also determine a strict position of a micro particle within the intrinsic space $\Delta r$ because of existing the minimum time $t_0$. The matter mentioned here indicates that we should not have such an image as a particle in the standstill state for behavior of a micro particle having a wave nature in a local space.

Even for a micro particle of particle image, Equation (17) reveals that the physical solution $\Psi_{\mathrm{P}}(t,x,y,z)$ in the intrinsic space $\Delta r$ is composed of some mathematical solutions, for example, $l$ solutions, because of $l \geq 2$ resulting from the resulting existence of a minimum time $t_0$, except for a special case of $\Psi_{\mathrm{P}}(t,x,y,z) = \Psi(t_j,x,y,z)$ conceived to be almost impossible. In that case, the physical solution is expressed as

$$\Psi_{\mathrm{P}}(t,x,y,z) = \sum_{j=1}^{l} A_j \Psi(t_j,x,y,z), \tag{27}$$

using a solution $\Psi(t_j,x,y,z)$ obtained from the mathematical analysis [7].

When a moving micro particle having an intrinsic space $\Delta r$ encounters such a force as an impact in a collision with others, it is considered that the influence of force on the intrinsic space $\Delta r$ generates a collapse of wave function $\Psi_{\mathrm{P}}$, and then $|\Psi_{\mathrm{P}}|^2$ gives theoretically an existence probability of a micro particle in the space. As far as a collapse of wave function does not occur, behavior of a micro particle should be just accepted as a wave image in the intrinsic space $\Delta r$.

In the quantum theory, a micro particle is not in the standstill state but always in the moving state even after a collapse, judging from Equation (1). Regardless of whether a value of $|\Psi_{\mathrm{P}}|^2$ in a collapse state is experimentally measured or not, some mathematical wave functions having composed $\Psi_{\mathrm{P}}$ expressed by Equation (27) propagates again as a material wave from a new wave source or the micro particle makes a circular motion in a material captured then and continuously moves at random in the material as a diffusion particle after the collapse.

**2) Quantum Teleportation and Imaginary Time and Mass**

In the following, we consider such a physical system composed of two micro particles A and B having an interaction between them. When the particles A and B start to move for an opposite direction with each other at a time $t = t_i$, we discuss a state $\Psi_{AB}$ of the physical system at a time $t = t_f$ after the time $t = t_i$ in relation to the quantum entanglement [15]-[17].

At a time $t = t_f$ for $t_f \gg t_i$, the particle A exists at a position separated sufficiently from the particle B. In that situation, when we observe a state $\Psi_A$ of the particle A, it is known that a state $\Psi_B$ of the particle B becomes a collapse state just at the same time resulting from the interaction with the particle A. If we think the case of $t_f \gg t_i$, it is considered that a finite time is necessary for transmitting an influence of the interaction caused by the observation of a state $\Psi_A$ to a state of $\Psi_B$. Nevertheless, it is experimentally revealed that a physical quantity in the state of $\Psi_B$ is determined by the influence just at a moment having observed a state $\Psi_A$. The matter mentioned here is inconsistent with the usual conception in the exiting physics. Although we cannot understand the theoretical evidence of its behavior of a micro particle, the fact is accepted as a problem of the quantum teleportation [18] [19].

In the following, however, the theoretical evidence that the quantum teleportation occurs in behavior of a micro particle is proposed as an opinion, based on the discussion of the theoretical structure of physics.

In accordance with the impossible principle of discrimination, it was revealed that the wave Equation (10) of Schrödinger and the diffusion Equation (11) are transformable into each other [7]. We then used the relation of $t' = -it$ between an imaginary time $t'$ and a real time $t$ resulting from the impossibility of discrimination between micro particles of the same kind. Further, it was also revealed that there is a minimum time $t_0$ in the real time and also that its value depends on a degree of freedom $\alpha$ of a micro particle and an absolute temperature $T$ as shown in Equation (23). In addition, it may be considered that the minimum time $t_0$ depends also relatively on a mass $m$ of micro particle resulting from the correlation between $\alpha$ and $m$. For behavior of a micro particle in the derivation process of Equation (14), it was accepted that the density theorem in mathematics is valid for each of components of a coordinate system in the real space $(x, y, z)$.

Judging from the theoretical structure of physics mentioned above, when we considered behavior of a micro particle in an inside of local space $(x, y, z)$, the impossible principle of discrimination gives evidences that we should accept an imaginary time $t'$ as a relation of $t' = -it$ for a real time $t$. In that situation, a micro particle moves in the space-time $(t', x, y, z)$ satisfying the limit relation $t' \to 0$ not in physics but in mathematics then. However, the limit relation $t \to 0$ is not acceptable in physics because of existing a minimum time $t_0$ in spite of accepting the relation of $t = it'$ for $t' \to 0$ in mathematics. We can thus formally accept the limit relation $t \to 0$ only in mathematical analysis for an equation related to behavior of a micro particle in the space-time $(t', x, y, z)$. Here, note that the density theorem is not still valid in the conception of time in physics.

As mentioned in the present text, it was revealed that Equation (1) indicates a wave nature of micro particle. In the following, a momentum $p'$ of a micro particle in the space-time $(t', x, y, z)$ is investigated using the relation of

$$p = m\frac{\Delta r}{\Delta t} \to p' = m'\frac{\Delta r'}{\Delta t'} \text{ for } \Delta r' = \Delta r,\ \Delta t' = -i\Delta t,$$

in accordance with the impossible principle of discrimination. In that case, the relations of $m' = -im$ in the space-time $(t', x, y, z)$ and $m(= im')$ in the space-time $(t, x, y, z)$ are obtained as shown in the relation of

$$m'\frac{\Delta r'}{\Delta t'} \to m'\frac{\Delta r}{-i\Delta t} = im'\frac{\Delta r}{\Delta t} \to m\frac{\Delta r}{\Delta t}.$$

In a theoretical equation of a micro particle in conception of Newtonian mechanics, as a result, the replacement of $m \to m'$ under the condition of $m' = -im$ leads to an equation in the quantum mechanics, regardless of the space-time.

Discussion about an imaginary mass of a micro particle correlate with a subject of whether a tachyon exists or not [34]. At present, the existence of tachyon is widely denied in physics. In that situation, it seems that a micro particle moves in a local space $\Delta r$ having an imaginary mass $m'(= -im)$. As a result, we cannot thus directly recognize behavior itself of a micro particle as a particle image in Newtonian mechanics.

As a matter of fact, what the imaginary number $i$ emerges in the quantum mechanics essentially means that the impossible principle of discrimination is really valid in behavior of a micro particle. Since we cannot fundamentally understand an imaginary quantity in physics, the determinism is not thus essentially applicable to the quantum theory.

We have understood behavior of micro particle from replacing the problem of imaginary time as an independent variable by one of the imaginary differential operators as shown in the Schrödinger equation using a usual conception of mass $m$ in the outside of a local space. In other words, the Schrödinger equation itself is thus essentially one expressed by the coordinate system $(t', x, y, z)$. On the other hand, since the diffusion equation for a collective motion of micro particles does not directly correlate to the discrimination between micro particles, the effect of impossible principle of discrimination is neglected. In addition, the diffusivity expression of Equation (12) has not been noticed until recently in the concerned field.

Judging from the matter discussed here, we consider introducing the impossible principle of discrimination into the basic diffusion equation of

$$\frac{\partial}{\partial t}C = \frac{\hbar}{2m}\nabla^2 C.$$

In that case, the wave equation of Schrödinger is then reasonably derived only from rewriting the mass having no time-dimension as $m \to m'(= -im)$ and $C \to \Psi/\hbar$ in Equation (11).

On the contrary, Equation (10) is essentially one in the coordinate system $(t', x, y, z)$, where it should be accepted that the relation of $t' = -it$ is already incorporated into Equation (10) as partial differential operators. Therefore, after substituting $m = im'$ into Equation (10), if we rewrite the mass as $m' \to m$, it is easily confirmed that Equation (10) is consistent with the diffusion Equation (11) then.

In addition, the distinction between $m$ and $m'$ has been neglected in the exiting physics concerned. Based on the theoretical structure of physics, it was thus revealed that the difference between the diffusion equation and the Schrödinger equation is fundamentally an only problem of whether we accept $m$ or $m'$ in the diffusion Equation (11). Further, judging from the discussion about a mass of micro particle, the particle image in Newtonian mechanics is not valid for a micro particle in a local space, because of having an imaginary mass $m'$.

As can be seem from the discussion mentioned above, we could reveal that a meaningful knowledge in the fundamental physics is obtained from the diffusion equation, where such theoretical discussion results from not the diffusion equation having been unconditionally accepted as a law but one derived reasonably from the general theory in physics and/or in mathematics (See Appendix).

As a result, it was revealed that the relation of $t' = -it$ or $m' = -im$ is acceptable in a local space $(t', x, y, z)$ in relation to behavior of a micro particle. Here, note that the conception of large or small value for an imaginary quantity is not essentially defined in mathematics. The conception of time lag between collapses of $\Psi_A$ and $\Psi_B$ is essentially accepted in Newtonian mechanics. However, there is not such a conception in the world of a micro particle having an imaginary time unable to understand itself in Newtonian mechanics. We cannot understand an imaginary quantity in physics. The matter itself gives evidence that we cannot understand the quantum teleportation having really occurred in the world of a micro particle [18] [19]. We should thus accept the quantum teleportation as a physical reality shown in an essential nature of a micro particle, because we can never observe an imaginary quantity in physics.

**3) Problem of the Schrodinger's cat**

In relation to the well-known problem of the Schrodinger's cat, the decay of a radioactive isotope corresponds to a collapse of wave function $\Psi_P$ in the quantum theory. This means that the collapse occurs probabilistically in accordance with the decay of a radioactive isotope. However, the probability interpretation of $\Psi_P$ has entirely no connection with whether a collapse of the wave function is probabilistic or not. As mentioned above, the probability interpretation of $\Psi_P$ is caused by the existence of the minimum time $t_0$. Opening the door of room occupied by Schrodinger's cat corresponds to confirming whether the collapse of wave function has occurred or not. The obtained results may depend on a time interval before and after just at a moment of the collapse of wave function in the quantum theory because of the existence of $t_0$. Therefore, the opening time in the problem of Schrodinger's cat should be just at a moment of the decay of a radioactive isotope.

Incorporating the conception of the discrete time $t_0$ resulting from the impossible principle of discrimination into the problem of Schrodinger's cat having the conception of Newtonian mechanics is not suitable to begin with, even if it would be a project to investigate a probability interpretation.

Here, we assume that Schrodinger's cat dies at a time $t = t_d$ $(t_m < t_d < t_n)$ for $k \le d \le k+1$ in the world of Newtonian mechanics, where the time interval $t_n - t_m$ is necessary time for confirming life or death of the cat. In that case, as far as we do not open the door of room occupied by the cat, the situation of cat is essentially in the state of superposition between physical solution $\Psi_{\mathrm{P1}}$ of life and physical solution $\Psi_{\mathrm{P2}}$ of death in the world of quantum mechanics. The situation of cat is thus expressed as

$$\Psi_{\mathrm{P}}(t,x,y,z) = \Psi_{\mathrm{P1}}(t,x,y,z) + \Psi_{\mathrm{P2}}(t,x,y,z),$$

where

$$\Psi_{\mathrm{P1}}(t,x,y,z) = \sum_{j=m}^{k} A_j \Psi(t_j,x,y,z), \quad \Psi_{\mathrm{P2}}(t,x,y,z) = \sum_{j=k+1}^{n} A_j \Psi(t_j,x,y,z).$$

For the life or death of the cat, the collapse $|\Psi_{\mathrm{P}}|^2$ of wave function, which corresponds to opening door, is expressed as

$$|\Psi_{\mathrm{P1}}|^2 = 1, |\Psi_{\mathrm{P2}}|^2 = 0 \quad \text{or} \quad |\Psi_{\mathrm{P1}}|^2 = 0, |\Psi_{\mathrm{P2}}|^2 = 1$$

in the world of quantum mechanics then. In relation to the probability interpretation of wave function, it is not suitable to determine probabilistically $t_d$ from using a radioactive isotope. Further, Equation (9) reveals that we cannot essentially determine an exact time $t = t_d$ in the physics relevant to micro particles, to begin with.

**4) Double Slit Problems of a Micro Particle**

If we perform repeatedly such an experiment that only electron moves through a double-slit, it is experimentally confirmed that an interference fringe yields on the screen behind the double-slit [24]. Even if the electron has a wave nature in the conception of Newtonian mechanics, we have not been able to understand why the interference fringe is formed by an only electron going through the double-slit. Judging from the matter mentioned above, however, we must consider that the electron reaches the double-slit as a wave packet having an intrinsic space $\Delta r$. Here, note that we cannot then essentially confirm the exact position of electron in the intrinsic space $\Delta r$ because of the existence of $t_0$.

When the wave packet of electron just collides with the double-slit, it becomes once in the collapse state expressed by $|\Psi_{\mathrm{P}}(t,x,y,z)|^2$ shown in Equation (27). If we do not then estimate an experimental value of $|\Psi_{\mathrm{P}}(t,x,y,z)|^2$, continuously a new wave function $\Psi_{\mathrm{P}}^{\mathrm{A}}$ composed of some mathematical wave functions having composed $\Psi_{\mathrm{P}}$ of Equation (27) in the intrinsic space $\Delta r$, which has a wave source at the slit A, propagates from the slit A to the screen. In the same meaning, a new wave function $\Psi_{\mathrm{P}}^{\mathrm{B}}$ propagates from the slit B to the screen.

The trace of electron having collided with the screen is then determined by an interference effect of the superposition between wave functions $\Psi_{\mathrm{P}}^{\mathrm{A}}$ and $\Psi_{\mathrm{P}}^{\mathrm{B}}$

just at a moment having reached the screen. When we perform repeatedly the above experiments, the interference fringe is statistically formed from accumulating each trace of electrons generated by the interference effect mentioned here.

The impossible principle of discrimination reveals that we cannot find whether the electron goes through the slit A or the slit B. As a matter of fact, regardless of whether the electron is observed at a detector set near the slit A or not, the collapse of wave functions results in

$$\left|\Psi_{\mathrm{P}}^{\mathrm{A}}\right|^2=1 \text{ and } \left|\Psi_{\mathrm{P}}^{\mathrm{B}}\right|^2=0 \text{ or } \left|\Psi_{\mathrm{P}}^{\mathrm{A}}\right|^2=0 \text{ and } \left|\Psi_{\mathrm{P}}^{\mathrm{B}}\right|^2=1.$$

Even if the concerned electron propagates again to the screen after the observation, there is no room for doubt that the interference effect afterward disappears because of no appearance of superposition between wave functions $\Psi_{\mathrm{P}}^{\mathrm{A}}$ and $\Psi_{\mathrm{P}}^{\mathrm{B}}$ for the sake of $\left|\Psi_{\mathrm{P}}^{\mathrm{A}}\right|\left|\Psi_{\mathrm{P}}^{\mathrm{B}}\right|=0$.

In accordance with the matter discussed here, it is thus meaningless to think whether an electron goes through the slits A or slit B regardless of a collapse of wave function, even if an electron has a particle image in the conception of Newtonian mechanics.

**5) Collective Motion of Micro Particles with a Wave Nature**

We understand the superfluidity and superconductivity, where each of them is a collective motion of micro particles at an extremely low temperature and is a macro behavior resulting from the quantum effect in accordance with the Bose-Einstein condensation [30]-[33]. In that case, Equation (20) shows that each of micro particles has a normal energy level resulting from a large value of $\Delta t\left(=t_0\right)$ in Equation (23) because of an extremely low temperature. On the other hand, the collective motion of micro particles at a high temperature is expressed by the diffusion equation. In that situation, Equation (20) shows that each of micro particles has a large energy resulting from a small value of $\Delta t\left(=t_0\right)$.

In the present text, we revealed that the diffusion equation and wave equation of Schrödinger are transformable into each other. This means that the diffusion particle of micro particle in a material has a wave nature. In other words, the so-called Brown motion having been known since 1827 is a macro behavior resulting from the quantum effect because of the diffusion particles having a wave nature.

The essential behavior of a micro particle can be seen from Equations (1), (2), (19) and (20) in relation to a wave nature and/or from the relation $t'=-it$ or $m'=-im$ and Equation (9) in accordance with the impossible principle of discrimination. That is, a micro particle having a wave nature is never in the standstill state even in a material. A micro particle then makes a circular motion in the intrinsic space $\Delta r$ in a material resulting from a central force generated by the other micro particles surrounding the micro particle itself, because of Equation (1). In that situation, the micro particle jumps from the intrinsic space $\Delta r$ into such a neighborhood space as a vacancy generated by the thermal fluctuation in the diffusion field. The diffusion phenomena are thus a collective motion of micro particles dependent on a temperature in the diffusion field and a diffusivity gradient in a local space (See Appendix).

**6) Double Slit Problems of a Photon**

From a viewpoint of the theoretical structure of physics, Maxwell unified the electronic and magnetic fields into the field of an electro-magnetic wave [35]. He then found that a light is also an electro-magnetic wave given by the equation of

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u , \tag{28}$$

where $u$ is a wave function.

Here, an electron is a substance having the mass $m$ and the energy $E = mc^2$ obtained from the relativity. We can thus accept it as a particle image, but it moves always as a state expressed by wave function having an intrinsic space $\Delta r$. As a result, we cannot determine an exact position of electron because of the existence of $t_0$. On the other hand, a light of frequency $\upsilon$ is a physical quantity of state having no mass and the energy $E = h\upsilon$ distributed to a space of the wave length $\lambda$. However, a light of an electro-magnetic wave propagates even in a space having no medium, which is different from behavior of other waves. If a light has a particle image as a photon, we can then understand the behavior.

When we perform repeatedly such an experiment that an only photon moves through a double-slit, it is experimentally confirmed that an interference fringe yields on the screen behind the double-slit [36]. Even if the photon has a wave nature in the conception of Newtonian mechanics, we have not been able to understand why the interference fringe is formed from an only photon going through the double-slit. However, we must consider that the photon reaches the double-slit as a wave having an intrinsic space $\lambda$. Here, note that we cannot then essentially confirm the exact position of photon having a particle image, because the photon itself is essentially not a substance but a physical quantity of state having an energy distributed to the intrinsic space $\lambda$.

When a photon just collides with the double-slit, it becomes once in a collapse state, and the photon is divided into a photon A and a photon B just at a moment having reached the double-slit. In fact, the Compton effect reveals that a light of energy $E = h\upsilon$ becomes a different one of $E = h\upsilon'$ after the collision with electron. Therefore, continuously a new photon A having energy $E_{\mathrm{A}} = h\upsilon_{\mathrm{A}}$ propagates as a wave function $u^{\mathrm{A}}$ from the slit A corresponding to a new wave source to the screen. In a similar situation, a new photon B having energy $E_{\mathrm{B}} = h\upsilon_{\mathrm{B}}$ propagates from the slit B to the screen as a wave function $u^{\mathrm{B}}$.

The trace of photon having collided with the screen is then determined from an interference effect of the superposition between wave functions $u^{\mathrm{A}}$ and $u^{\mathrm{B}}$ just at a moment having reached the screen. When we perform repeatedly the above experiments, the interference fringe is statistically formed from accumulating each trace of photons generated by the interference effect mentioned here. Generally in physics, the interference effect has been understood as a superposition between waves expressed as a shape variation of a collective motion of continuous medium, as shown in a sound wave, water wave and so on. However, the formation mechanism of interference fringe generated by a photon as well as an electron going through the double-slit is entirely different from that of a sound wave and water wave.

We have used not Schrödinger equation but Equation (28) for investigating a light behavior in spite of the light itself having a wave-particle duality. In the present text, using Equation (16) for the diffusion Equation (11), Schrödinger equation was reasonably derived. On the other hand, even if we substitute Equation (16) into Equation (28), Equation (28) is unchangeable as it is. The so-called determinism for Equation (28) is thus valid in behavior of light in spite of having a wave-particle duality, which is different from a micro particle.

**7) Discrete Time**

The impossible principle of discrimination reveals that the existence of minimum time $t_0$ is indispensable for investigating behavior of a micro particle. In the present text, the relation of

$$t_0 = \frac{\hbar}{\alpha\left(\varepsilon + k_{\mathrm{B}} T\right)}.$$

was tentatively used for understanding the quantum theory. In the theoretical structure of physics discussed in the present text, a subject of the quantum theory transfers into one of Newtonian mechanics under the condition of $t_0 \to 0$. On the other hand, we find in usual textbooks that the quantum theory transfers into Newtonian mechanics under the condition of $h \to 0$ in spite of the constant value of physical quantity. As a matter of fact, however, we should accept that the quantum theory transfers into Newtonian mechanics under the condition of $n > n_0$ in Equation (22).

There is a large difference between the minimum time $t_0$ shown in Equation (26) and the Planck time $t_{\mathrm{P}}$, even if the value $t_0$ of Equation (26) is not exactly but tentatively investigated. Judging from the theoretical structure of physics, however, the determination of value $t_{\mathrm{P}}$ has an unsuitable problem as follows.

(a) The physical constant $\hbar$ is shown in the quantum mechanics, where the quantum mechanics is established by denying the density theorem of time valid in Newtonian mechanics.

(b) The physical constant $c$ is shown in the relativity, where the relativity is established by denying the absolute space-time valid in Newtonian mechanics.

Based on the matter described in the above (a) and (b), the gravitational constant $G$ shown in Newtonian mechanics is a different situation from the physical constants $\hbar$ and $c$ judging from the theoretical structure of physics. Therefore, we seem to be unsuitable for using simultaneously $G$ with $\hbar$ and $c$ from a viewpoint of the theoretical structure of physics, even if those are used for the dimension analysis. The further investigation will be thus necessary for the determination of minimum time [37] [38].

**8) Theoretical Structure of Modern Physics**

The quantum theory and the relativity established in the early stage of the 20th century, which are not understandable in the theoretical structure of Newtonian mechanics, have highly developed as modern physics. The relativity was established by Einstein under the only condition of accepting the constant principle of light speed unacceptable in Newtonian mechanics. The absolute space-time in Newtonian mechanics was denied then. The theoretical structure of relativity is thus compact and distinct and the causality between the relativity and Newtonian mechanics is also understandable.

On the other hand, the quantum theory has been established by many giant physicists for a long time under the conditions of accepting principles, laws and hypothesizes unacceptable in Newtonian mechanics. We have been thus unable to understand the causality between the quantum theory and Newtonian mechanics. The theoretical structure of quantum theory is thus complicated and indistinct. In other words, the unclearness of theoretical structure in quantum theory results from the unknown of causality between the quantum mechanics and Newtonian mechanics. The matter has caused the fundamental problems (1)-(7) as discussed in the beginning.

Generally in physics, as far as we unconditionally accept a relation without the theoretical evidence as a principle, a law or a hypothesis for physical phenomena, we cannot find the essence of physics itself. For example, as far as we accept the diffusion equation as a law of Fick, we cannot understand the essential meaning of diffusivity. In that situation, however, since we derived the diffusion equation from general theories in physics and/or mathematics, it was first revealed in the derivation process that the diffusivity is a physical quantity relevant to an angular momentum of a diffusion particle in a local space in the diffusion field. At the same time, Equation (A-12) shown in Appendix was also then obtained as an important relation expressing the essence of diffusion phenomena, which has not been known since the proposition of Fick's law (See Appendix). As an important finding relevant to the fundamental theory of the quantum mechanics, furthermore, it was also theoretically revealed that the diffusion equation and the Schrödinger equation are transformable into each other resulting from the derivation of diffusion equation.

Figuratively speaking, it seems that the existing quantum theory and the relativity correspond to a statue of parquetry and one of a simple wood carving, respectively. Here, the author does not understand whether one of their statues is more worthy than another as an artwork. From a viewpoint of theoretical structure of physics, however, we think that the physical theory should be formed as simple structure as possible.

In that situation, it was recently revealed that the quantum theory is reasonably and clearly established in accordance with the causality for Newtonian mechanics corresponding to the relativity established by Einstein. If we accepted the only impossible principle of discrimination between two micro particles of the same kind, we found that the quantum mechanics is reasonably established. As a result, we must then accept such a minimum time $t_0$ that the relation of $\Delta t \to i\Delta t'$ is valid in the quantum theory for $0 \le \Delta t < t_0$ in Newtonian mechanics. In other words, the matter mentioned here just corresponds to the causality between the quantum mechanics and Newtonian mechanics.

Figure 3 shows that the theoretical structure of quantum theory is essentially compact and distinct in similar to relativity. The fundamental problems (1)-(7) discussed in the beginning have been accepted without theoretical evidence for a long time. In the present text, we might merely reconfirm the theoretical validity of those matters. From a viewpoint of theoretical structure of physics, however, we believe that the schematic diagram shown in Figure 3 is meaningful in its own way like Maxwell unified the existing electronic and magnetic fields into the field of electro-magnetic wave, including a light wave.

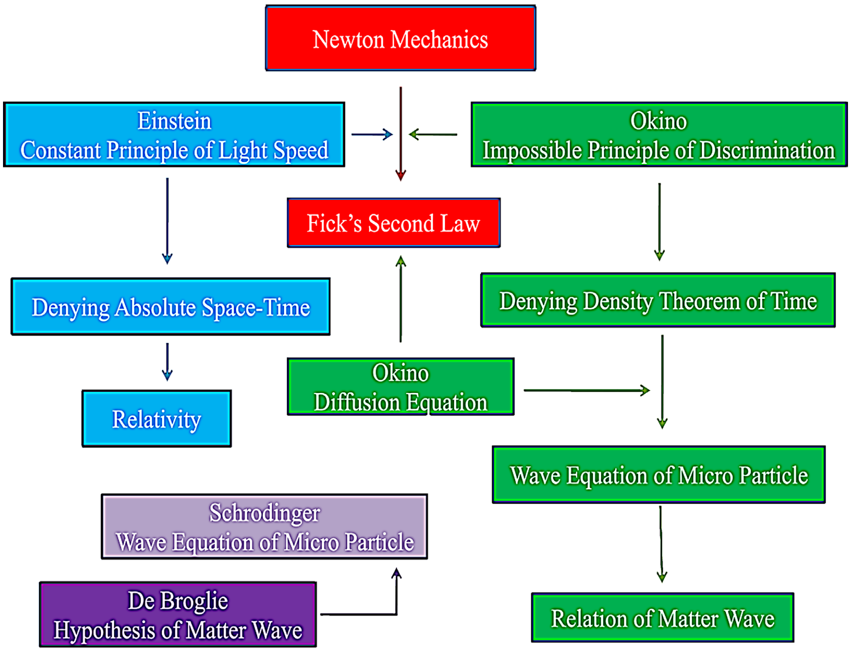


**Figure 3.** Theoretical structure of modern physics.

From the consideration of fundamental problems in accordance with the theoretical structure, the new findings were obtained in the present text as follows. The impossible principle of discrimination revealed that physical quantities having a time-dimension for a micro particle in a local space depend on an imaginary time $t'(=-it)$ different from a real time $t$ in Newtonian mechanics and the relativity. As a result, it was also revealed that a micro particle has an imaginary mass $m'(=-im)$ in a local space, which is not understandable in the conception of Newtonian mechanics. The matter gives evidence that Newtonian mechanics is not applicable to behavior of a micro particle.

The Schrödinger Equation (10) can be rewritten as

$$\frac{\partial \Psi}{\partial t'} = -\frac{\hbar}{2m}\nabla^2\Psi \quad \text{for} \quad t' = -it\,. \tag{29}$$

Equation (29) is a partial differential equation for a micro particle with a mass $m$ in a space-time $(t', x, y, z)$. In addition, it can also be rewritten as

$$\frac{\partial \Psi}{\partial t} = \frac{\hbar}{2m'}\nabla^2\Psi \quad \text{for} \quad m' = -im\,. \tag{30}$$

Equation (30) is a partial differential equation for a micro particle with a mass $m'$ in a space-time $(t, x, y, x)$. Equation (30) then is consistent with the diffusion Equation (11) if we rewrite as $m' \to m$ and $\Psi \to C$, because of a collective motion of diffusion particles having no connection with the impossible principle of discrimination. The matter discussed here gives evidence that the conception of a mass in Newtonian mechanics is not applicable to the behavior of a micro particle in a local space.

Hereinbefore, we have investigated fundamental problems unsolved and left in quantum mechanics. In that case, the reasonable derivation of the diffusion equation, which has been unconditionally accepted as a law of Fick since 1855, played a dominant role in understanding the theoretical structure of physics in the establishment of quantum theory. Further, the new findings relevant to the diffusion theory itself were also revealed, as shown in Appendix. In addition, it is reported that we have some serious problems having been wrongly accepted in the concerned field for a long time (See Appendix).

The diffusion equation has been used for investigating the behavior of a collective motion of micro particles. In the present discussion, however, the elementary process of a diffusion particle in a local space was investigated. As a result, we could then understand the essential meaning of diffusivity. Further, it was also revealed that a diffusion particle corresponds to a micro particle having a wave nature. In addition, the size of diffusion particles moving at random in a material has not been discussed in the concerned field. In that situation, we found in the present text that a collective motion of micro particles composed of atoms less than 260 shows diffusion phenomena.

Finally, we hope that the theoretical evolution discussed in the present text for fundamental theory in quantum mechanics is highly useful not only for the education of beginners but also for further development of fundamental research from a viewpoint of the theoretical structure in quantum theory.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References


[1] Michelson, A.A. and Morley, E.W. (1887) On the Relative Motion of the Earth and the Luminiferous Ether. *American Journal of Science*, **3**, 333-345. https://doi.org/10.2475/ajs.s3-34.203.333

[2] Einstein, A. (1905) Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, **322**, 891-921. https://doi.org/10.1002/andp.19053221004

[3] De Broglie, L. (1923) Waves and Quanta. *Nature*, **112**, 540-540. https://doi.org/10.1038/112540a0

[4] Planck, M. (1900) Ueber das Gesetz der Energieverteilung im Normalspectrum. *Annalen der Physik*, **309**, 553-563. https://doi.org/10.1002/andp.19013090310

[5] Okino, T. (2013) Correlation between Diffusion Equation and Schrödinger Equation. *Journal of Modern Physics*, **4**, 612-615. https://doi.org/10.4236/jmp.2013.45088

[6] Okino, T. (2019) Relation of Matter Wave Verified in Diffusion Theory. *International Journal of Fundamental Physical Sciences*, **9**, 48-54. https://doi.org/10.14331/ijfps.2019.330131

[7] Okino, T. (2023) Reality of Schrodinger's Cat. *International Journal of Fundamental Physical Sciences*, **13**, 10-16. https://doi.org/10.14331/ijfps.2023.330158

[8] Brown, R. (1828) A Brief Account of Microscopical Observations Made in the Months of June, July and August 1827, on the Particles Contained in the Pollen of Plants and on the General Existence of Active Molecules in Organic and Inorganic Bodies. *The Philosophical Magazine*, **4**, 161-173. https://doi.org/10.1080/14786442808674769

[9] Schrödinger, E. (1926) Quantisierung als Eigenwertproblem. *Annalen der Physik*, **384**, 361-376. https://doi.org/10.1002/andp.19263840404

[10] Fick, A. (1855) On Liquid Diffusion. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **10**, 31-39.

[11] Markov, A.A. (1960) Theory of Algorithms. *American Mathematical Society Translations*, **5**, 1-14.

[12] Okino, T. (2012) Theoretical Evidence for Revision of Fickian First Law and New Understanding of Diffusion Problems. *Journal of Modern Physics*, **3**, 1388-1393. https://doi.org/10.4236/jmp.2012.310175

[13] Smigelskas, A.D. and Kirkendall, E.O. (1947) Zinc Diffusion in Alpha Brass. *Transactions of the AIME*, **171**, 130-142.

[14] Okino, T. (2018) Quantum Effect on Elementary Process of Diffusion and Collective Motion of Brown Particles. *Journal of Modern Physics*, **9**, 1007-1028. https://doi.org/10.4236/jmp.2018.95063

[15] Bell, J.S. (1964) On the Einstein Podolsky Rosen Paradox. *Physics Physique Fizika*, **1**, 195-200. https://doi.org/10.1103/physicsphysiquefizika.1.195

[16] Pearle, P.M. (1970) Hidden-Variable Example Based Upon Data Rejection. *Physical Review D*, **2**, 1418-1425. https://doi.org/10.1103/physrevd.2.1418

[17] Fuchs, C.A., Mermin, N.D. and Schack, R. (2014) An Introduction to QBism with an Application to the Locality of Quantum Mechanics. *American Journal of Physics*, **82**, 749-754. https://doi.org/10.1119/1.4874855

[18] Furusawa, A., Sørensen, J.L., Braunstein, S.L., Fuchs, C.A., Kimble, H.J. and Polzik, E.S. (1998) Unconditional Quantum Teleportation. *Science*, **282**, 706-709. https://doi.org/10.1126/science.282.5389.706

[19] Yonezawa, H., Aoki, T. and Furusawa, A. (2004) Demonstration of a Quantum Teleportation Network for Continuous Variables. *Nature*, **431**, 430-433. https://doi.org/10.1038/nature02858

[20] Bohr, N. (1913) On the Constitution of Atoms and Molecules. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **26**, 1-25. https://doi.org/10.1080/14786441308634955

[21] Einstein, A. (1905) Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. *Annalen der Physik*, **322**, 132-148. https://doi.org/10.1002/andp.19053220607

[22] Compton, A.H. (1922) A Quantum Theory of the Scattering of X-Rays by Light Elements. *Physical Review*, **21**, 483-502. https://doi.org/10.1103/physrev.21.483

[23] Balmer, J.J. (1885) Notiz über die Spectrallinien des Wasserstoffs. *Annalen der Physik*, **261**, 80-87. https://doi.org/10.1002/andp.18852610506

[24] Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T. and Ezawa, H. (1989) Demonstration of Single-Electron Buildup of an Interference Pattern. *American Journal of Physics*, **57**, 117-120. https://doi.org/10.1119/1.16104

[25] Boltzmann, L. (1876) Überdie Naturder Gasmoleküle. *Wiener Berichte*, **74**, 533-560.

[26] Shimizu, F. (2000) Specular Reflection of Very Slow Metastable Neon Atoms from a Solid Surface. *Physical Review Letters*, **86**, 987-990. https://doi.org/10.1103/physrevlett.86.987

[27] Arndt, M., Nairz, O., Vos-Andreae, J., Keller, C., van der Zouw, G. and Zeilinger, A. (1999) Wave-Particle Duality of C60 Molecules. *Nature*, **401**, 680-682. https://doi.org/10.1038/44348

[28] Okino, T. (2015) Mathematical Physics in Diffusion Problems. *Journal of Modern Physics*, **6**, 2109-2144. https://doi.org/10.4236/jmp.2015.614217

[29] Thomson, G.P. and Reid, A. (1927) Diffraction of Cathode Rays by a Thin Film. *Nature*, **119**, 890-890. https://doi.org/10.1038/119890a0

[30] Khalatnikov, I. (1965) An Introduction to the Theory of Superfluidity. W. A. Benjamin Inc.

[31] Kamerlingh, O. (1911) The Superconductivity of Mercury. *Communications from the Physical Laboratory of the University of Leiden*, 122-124.

[32] Bose, N. (1924) Planck Gesetz und Lichtquantempothese. *Zectchrift fur Physik*, **26**, 178-181. https://doi.org/10.1007/bf01327326

[33] Einstein, A. (1925) Quantentheorie des einatomigen idealen Gases. Zweite Abhandlung. *Mathematical Physics*, **3**, 1914-1932.

[34] Feinberg, G. (1967) Possibility of Faster-Than-Light Particles. *Physical Review*, **159**, 1089-1105. https://doi.org/10.1103/physrev.159.1089

[35] Maxwell, J.C. (1865) A Dynamical Theory of the Electromagnetic Field. *Philosophical Transactions of the Royal Society*, **155**, 459-512.

[36] Tuchiya, Y., Inuzuka, E., Kurono, T. and Hosoda, M. (1986) Photo-Counting Imaging and Its Application. *Advance in Electronics and Electron Physics*, **64**, 21-31.

[37] Lévi, R. (1927) Théorie de l'action universelle et discontinue. *Journal de Physique et le Radium*, **8**, 182-198. https://doi.org/10.1051/jphysrad:0192700804018200

[38] Caldirola, P. (1980) The Introduction of the Chronon in the Electron Theory and a Charged-Lepton Mass Formula. *Lettere Al Nuovo Cimento Series* 2, **27**, 225-228. https://doi.org/10.1007/bf02750348

[39] Fourier, J.B.J. (1822) Theorie analytique de la chaleur. Didot Paris, 499-508.

[40] Einstein, A. (1905) Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen. *Annalen der Physik*, **322**, 549-560. https://doi.org/10.1002/andp.19053220806

[41] Dirac, P.A.M. (1958) Principles of Quantum Mechanics. Oxford Clarendon Press.

[42] Gauss, C.F. (1840) Allgemeine lehrsätze in beziehung auf die im verkehrten verhältnisse des quadrats der entfernung wirkenden anziehungs—Und abstossungs-kräfte. Springer.

[43] Skvortsov, A.A., Orlov, A.M. and Muradov, V.E. (2009) Studying Diffusion in Multilayer Thin-Film Structures on Silicon by the Contact Melting Technique. *Technical Physics Letters*, **35**, 606-609. https://doi.org/10.1134/s1063785009070062

[44] Ting, P.Y. and Iltis, R.A. (1994) Diffusion Network Architectures for Implementation of Gibbs Samplers with Applications to Assignment Problems. *IEEE Transactions on Neural Networks*, **5**, 622-638. https://doi.org/10.1109/72.298232

[45] You, Y.L., Xu, W.Y., Tannenbaum, A. and Kaveh, M. (1996) Behavioral Analysis of Anisotropic Diffusion in Image Processing. *IEEE Transactions on Communications*, **11**, 1539-1553.

[46] Das, I., Agrawal, N.R., Gupta, S.K., Gupta, S.K. and Rastogi, R.P. (2009) Fractal Growth Kinetics and Electric Potential Oscillations during Electropolymerization of Pyrrole. *The Journal of Physical Chemistry A*, **113**, 5296-5301. https://doi.org/10.1021/jp8064147

[47] Raj, A., Kuceyeski, A. and Weiner, M. (2012) A Network Diffusion Model of Disease Progression in Dementia. *Neuron*, **73**, 1204-1215. https://doi.org/10.1016/j.neuron.2011.12.040

[48] Zhou, Q. (2015) The Application of Fractional Brownian Motion in Option Pricing. *International Journal of Multimedia and Ubiquitous Engineering*, **10**, 173-182. https://doi.org/10.14257/ijmue.2015.10.1.16

[49] Boltzmann, L. (1872) Weitere studien uber das warmegleichwicht unter gasmolekuen. *Wiener Berichte*, **66**, 275-370.

[50] Correa da Silva, L.C. and Mehl, R.F. (1951) Interface and Marker Movements in Diffusion in Solid Solutions of Metals. *JOM*, **3**, 155-173. https://doi.org/10.1007/bf03397292

[51] Darken, L.S. (1948) Diffusion, Mobility and Their Interrelation through Free Energy in Binary Metallic System. *Transactions of the AIME*, **175**, 184-201.

[52] Okino, T. (2011) New Mathematical Solution for Analyzing Interdiffusion Problems. *Materials Transactions*, **52**, 2220-2227. https://doi.org/10.2320/matertrans.m2011137

[53] Cho, H., Yamada, K. and Okino, T. (2018) New Analytical Method of Interdiffusion Problems. *Journal of Modern Physics*, **9**, 130-144. https://doi.org/10.4236/jmp.2018.92009

[54] Okino, T. (2023) Establishment of New Diffusion Theory—Ending of Fick's Laws. Medical Oita Publishers.

## Appendix

The heat conduction equation proposed by Fourier in 1822 has been applied to investigating the temperature distribution in materials [39]. On the other hand, the so-called Brown motion was found in 1827, where the self-diffusion of water was visualized by behavior of pollen motions [8]. Nevertheless, the Brown motion had not been recognized as a diffusion problem until the Einstein theory of Brown motion in 1905, although it was a typical diffusion problem [40]. In 1855, Fick applied the heat conduction equation to diffusion phenomena as it had been [10]. It seems that he noticed concentration profiles as well as temperature profiles in a material satisfying the parabolic law then.

In that situation, it will be revealed that the so-called Fick's first law is not only unsuitable for accepting as a law but also wrong in the mathematical theory in the following. Further, the Fick's second law, which has been accepted as a law unable to reveal the validity, will be theoretically derived from the Markov theory in mathematics [12] [28].

In the history of diffusion field, we have never discussed the size of diffusion particle showing diffusion phenomena. In the present text, we found that a diffusion particle has a wave nature, because the diffusion equation and the wave equation of Schrödinger are transformable into each other. Based on the matter, we found that a diffusion particle showing the diffusion phenomena is composed of atoms less than 260 numbers.

In the following, an abbreviate notation of differential operator for an arbitrary independent variable $\xi$ and the well-known Dirac's bracket notation for an arbitrary vector given by

$$\partial_\xi = \frac{\partial}{\partial \xi},\ \left\langle \tilde{\nabla} \right| = \left(\partial_x, \partial_y, \partial_z\right)$$

are used in the present text [41]. If an operator $Q$ is Hermite one, $\left\langle Q \right| = \left\{ \left| Q \right\rangle \right\}^\dagger$ is valid using the notation of Hermite conjugate $\dagger$. Here, the notation $\left\langle \tilde{\nabla} \right|$ is thus defined as $\left\langle \tilde{\nabla} \right| = -\left\{ \left| \nabla \right\rangle \right\}^\dagger$ because of $\partial_\xi = -\left\{ \partial_\xi \right\}^\dagger$.

The Fick's first law of

$$\left| J \right\rangle = -D \left| \nabla \right\rangle C \tag{A-1}$$

and the second law of

$$\partial_t C = D \left\langle \tilde{\nabla} \middle| \nabla \right\rangle C \tag{A-2}$$

were proposed by Fick in 1855, where $\left| J \right\rangle, D$ and $C$ are a diffusion flux, a diffusivity and a concentration. After that, Equation (A-2) has been used for $D$ independent of $C$ and the generalized one of

$$\partial_t C = \left\langle \tilde{\nabla} \middle| D \nabla \right\rangle C \tag{A-3}$$

has been also accepted as Fick's second law.

In those days, the Gauss's divergence theorem of

$$\iint_S \langle A|n\rangle \mathrm{d}S = \iiint_V \langle\tilde{\nabla}|A\rangle \mathrm{d}V \tag{A-4}$$

had been already reported in 1840 [42], where $\langle A|$ and $|n\rangle$ are an arbitrary vector and a unit vector perpendicular to a surface element $\mathrm{d}S$. If we accepted Equation (A-1) as a law, therefore, Equation (A-3) is theoretically obtained from replacing $|A\rangle$ with $|J\rangle$ in Equation (A-4) and using the relation of

$$\iint_S \langle J|n\rangle \mathrm{d}S = -\iiint_V \partial_t C \mathrm{d}V. \tag{A-5}$$

In addition, Equation (A-5) was also widely known as continuous equation of Euler in those days. Judging from the theoretical structure of physics, therefore, the obtained Equation (A-3) is not a law but a basic equation corresponding to a theorem in mathematics, where the validity of theorem is theoretically proved but that of law is not. Nevertheless, Equation (A-3) has been accepted as Fick's second law since 1855. The author has not been unable to understand the unreasonable situation having been accepted as a law since his youth, judging from the theoretical structure of physics [28].

A great many phenomena in various science fields are expressed by using the well-known evolution equations. The diffusion equation, the electro-magnetic wave equation and the Schrödinger equation emerge even in the present text. The diffusion equation corresponds mathematically to the Markov process in relation to the normal distribution rule [11]. In other words, the motion of diffusion particles corresponds to the well-known Brown movement satisfying the parabolic law. It is widely accepted that the Brown problem is a general category of investigating subjects in various science fields relevant to the Markov process, such as material science, information science, life science, social science and so on [43]-[48].

As far as a material composed of micro particles having a wave nature is conserved in the given region, the divergence theorem shows that the diffusion equation is applicable to diffusion phenomena for every material in an arbitrary thermodynamic state. The diffusion information about a material, such as crystal material or amorphous material, and/or solid, liquid or gas state, is incorporated into the diffusivity in the given diffusion equation. The diffusion equation with such an arbitrary diffusivity is thus investigated in the following.

The function $C(t,|r\rangle)$ is defined as a concentration expressed by particle numbers per a unit volume in the diffusion region. A diffusion particle moves at random in a material. We consider that a diffusion particle having a wave nature in the initial state at a space-time $(t_0,|r_0\rangle)$ exists in the state at a space-time $(t_j,|r_j\rangle)$ after $j$ times jumps. Judging from the principle of equipartition in the diffusion field under the condition of a constant temperature, it is thus considered that the jump displacement $\Delta r = \||r_j\rangle - |r_{j-1}\rangle| = \||\Delta r\rangle|$ and jump frequency $1/\Delta t$ $(\Delta t = \Delta t_j - \Delta t_{j-1})$ for the diffusion particle are equivalent in probability to their mean values of all diffusion particles in the collective system [25].

Further, it is also considered that the probability of diffusion-jump from the state of $(t_{j-1},|r_{j-1}\rangle)$ to $(t_j,|r_{j-2}\rangle)$ is equivalent to that of diffusion-jump from

$\left(t_{j-1},\left|r_{j-1}\right\rangle\right)$ to $\left(t_j,\left|r_j\right\rangle\right)$ in the isotropic space. The Markov process then shows that the relation of

$$C\left(t+\Delta t,\left|r\right\rangle\right)=\left\{C\left(t,\left|r\right\rangle-\left|\Delta r\right\rangle\right)+C\left(t,\left|r\right\rangle+\left|\Delta r\right\rangle\right)\right\}/2 \quad \text{(A-6)}$$

is valid. The Taylor expansion of both sides of Equation (A-6) yields

$$C\left(t+\Delta t,\left|r\right\rangle\right)=C\left(t,\left|r\right\rangle\right)+\Delta t\partial_t C\left(t,\left|r\right\rangle\right)+\cdots, \quad \text{(A-7)}$$

$$C\left(t,\left|r\right\rangle\pm\left|\Delta r\right\rangle\right)=C\left(t,\left|r\right\rangle\right)\pm\left\langle\Delta r\middle|\nabla C\left(t,\left|r\right\rangle\right)\right\rangle+\frac{\left(\Delta r\right)^2}{2}\left\langle\tilde{\nabla}\middle|\nabla C\left(t,\left|r\right\rangle\right)\right\rangle\pm\cdots. \quad \text{(A-8)}$$

The substitution of equations (A-7) and (A-8) into Equation (A-6) gives

$$\partial_t C=D\left\langle\tilde{\nabla}\middle|\nabla C\right\rangle, \quad \text{(A-9)}$$

where $D=\left(\Delta r\right)^2/2\Delta t$. Here, the diffusion equation having been unconditionally accepted as a law was theoretically obtained.

In case of neglecting the temperature dependence, equations (A-7) and (A-8) shows that the diffusivity $D$ in a local space is expressed as

$$D=\frac{\left(\Delta r\right)^2}{2\Delta t}=\frac{\Delta rp}{2m}=\frac{\hbar}{2m} \quad \text{for} \quad v=\frac{\Delta r}{\Delta t}, \quad \text{(A-10)}$$

where $m$, $v$ and $p\left(=mv\right)$ are a mass, a jumping velocity and a momentum of a diffusion particle, and $\Delta rp=\hbar$ of Equation (1) discussed in the Bohr model of hydrogen is used. Here, it was revealed that the diffusivity depends on an angular momentum of a diffusion particle in a local space $\Delta r$ in the diffusion field. In other words, a diffusion particle makes a circular motion in a local space in the diffusion field.

The existence probability of a diffusion particle in a local space corresponds to the well-known Boltzmann factor. Incorporating a potential energy $U$ of interaction force $F$ between a diffusion particle and the other particles surrounding the diffusion particle itself and an activation energy $Q$ in the diffusion field at an absolute temperature $T$ into the Boltzmann factor, the generalized diffusivity in a material is expressed as

$$D=\frac{\hbar}{2m}\exp\left[\frac{U-Q}{k_B T}\right], \quad \text{(A-11)}$$

where $k_B$ is the Boltzmann constant [49].

In addition, an effect of entropy $S\left(=k_B\ln W\right)$ is incorporated into $U$ as $U_S=ST$ using a state numbers $W$ in the diffusion field [28]. Equation (A-10) satisfying the well-known parabolic law of $\Delta r=\sqrt{2D\Delta t}$ indicates that a jumping velocity $v$ of diffusion particle is determined by the gradient of diffusivity $D$ in a local space $\Delta r$. Then, Equation (A-11) leads to the expression valid in a diffusion region as shown in the following.

$$v=\frac{\partial D}{\partial\Delta r}=\frac{\Delta r}{\Delta t} \quad \to \quad v=\left|\nabla\right\rangle D=-\frac{F}{k_B T}D \quad \text{for} \quad U=-\int F\mathrm{d}r+U_S. \quad \text{(A-12)}$$

We have not noticed Equation (A-12) obtained theoretically here in the history of diffusion field. In that situation, Equation (A-12) gives the essence of diffusion phenomena, where the diffusivity gradient corresponds to the jumping velocity of a diffusion particle in a local space.

The diffusion equation having been accepted as Fick's second law of

$$\partial_t C = \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C$$

is transformed into

$$\partial_t C = \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C = \left\{ \left\langle \tilde{\nabla} \middle| D \right\} \middle| \nabla \right\rangle C + D\left\langle \tilde{\nabla} \middle| \nabla \right\rangle C = \left\langle \tilde{v} \middle| \nabla \right\rangle C + D\left\langle \tilde{\nabla} \middle| \nabla \right\rangle C. \quad \text{(A-13)}$$

In Equation (A-13), the term $\left\langle \tilde{v} \middle| \nabla \right\rangle C = \left\langle \tilde{\nabla} \middle| v \right\rangle C$ means that the concentration profile itself moves in the diffusion region against the fixed coordinate system.

Using a moving coordinate system $\left(t', \left| r' \right\rangle\right)$ given by

$$r = r' + v_C t' \quad \text{for } t = t'$$

for the fixed coordinate system, the relations of differential operator of

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r'}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - v_C \frac{\partial}{\partial r} \quad \text{(A-14)}$$

are thus valid, where $v_C$ is a velocity of an origin of moving coordinate system for that of the fixed coordinate system. Substituting Equation (A-14) into Equation (A-13) yields

$$\partial_{t'} C' = \left\langle \tilde{v} + \tilde{v}_C \middle| \nabla' \right\rangle C' + D\left\langle \tilde{\nabla}' \middle| \nabla' \right\rangle C' \quad \text{(A-15)}$$

as a moving coordinate system. Here, note that the jumping velocity $v$ of diffusion particle correlates with the movement of diffusion region space. In other words, the relation $v + v_C = 0$ is physically valid as known from imaging a vacancy movement. In that case, Equation (A-15) of

$$\partial_{t'} C' = D\left\langle \tilde{\nabla}' \middle| \nabla' \right\rangle C'$$

is consistent with the diffusion Equation (A-9) derived from the Markov theory.

As a result, Equation (A-3) is one expressed by a fixed coordinate system and Equation (A-2) or (A-9) is acceptable as a moving coordinate system even if the diffusivity depends on a concentration. The matter discussed here reveals that they are transformable into each other. In addition, the experimental results known as Kirkendall effect indicate a migration of diffusion region space [13]. It is, therefore, indispensable for investigating diffusion phenomena to consider problems between coordinate systems [28].

Equations (A-3), (A-4) and (A-5) give the relation of

$$\iint_S \left\langle J \middle| n \right\rangle \mathrm{d}S = -\iiint_V \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C \mathrm{d}V \rightarrow \iint_S J_x \mathrm{d}y\mathrm{d}z = -\iint_S \left\{ \int \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C \mathrm{d}x \right\} \mathrm{d}y\mathrm{d}z.$$

Then, the mathematical theory gives

$$J = -D\nabla C + J(t) + J_{eq}, \quad \text{(A-16)}$$

because of

$$J_x = -\int \left\langle \tilde{\nabla} \middle| D\nabla \right\rangle C \mathrm{d}x = \int \partial_x \left\{ -D\partial_x C \right\} \mathrm{d}x \quad \rightarrow \quad J_x = -D\partial_x C + J_x(t) + J_{eq}.$$

Equation (A-1) having been accepted as a law since 1855 is different from Equation (A-16) derived from the mathematical theory, because of existence of $J(t)+J_{eq}$. Here, $J(t)$ means a migration of diffusion region space and $J_{eq}$ plays an important role in the self-diffusion problems because of $\partial_x C = 0$ [12] [28]. It is, therefore, revealed that the Fick's first law is not only unsuitable for a law judging from the theoretical structure but also wrong in the mathematical theory.

In relation to the diffusion phenomena, the obtained Equation (A-12) plays an important role [28]. Equation (A-12) is thus extremely useful for understanding behavior of diffusion particle in the well-known Brown motion. For example, using the Fick's first law, the vat't Hoff's law and the Stokes's theorem for the Brown problem, Einstein derive the well-known relation of

$$D = \frac{k_{\mathrm{B}}T}{k} \quad \text{for} \quad F = -kv \tag{A-17}$$

from the complicated analysis then, where $F$ is an external force acting on a pollen particle having visualized a movement of water molecules [40]. In the present theory, however, only substituting $F = -kv$ into Equation (A-12) gives directly Equation (A-17). This gives evidence that Equation (A-12) is a basic equation important in the diffusion phenomena.

On the other hand, the conception of intrinsic diffusivity has been widely accepted by researchers in the diffusion field in relation to the Kirkendall effect [13] [50]. The phenomena of Kirkendall effect are essentially just problems themselves between the fixed coordinate system and a moving one. The so-called Darken equation has been widely used for investigating the interdiffusion problems [51]. The results of intrinsic diffusivity obtained from numerical analysis of the Darken equation do not show profiles in accordance with the parabolic law. However, Equation (A-10) reveals that a diffusivity profile should become one resulting from the parabolic law. Judging from the matter mentioned here, a researcher should notice that the Darken equation is not applicable to analyzing the interdiffusion problems. In fact, it has been reported that the Darken equation is mathematically wrong in the derivation process [12] [52].

In the present text, the fundamental problems in quantum mechanics are discussed from a viewpoint of the theoretical structure. As a result, the validity of fundamental problems having been unconditionally accepted without theoretical evidence was reasonably reconfirmed by accepting the impossible principle of discrimination. However, the case of fundamental problems discussed in Appendix is quite different from those in quantum mechanics. As it were, the diffusion equation is now not a law but a basic equation in physics and further the diffusion flux having been also accepted as a law is not correct in the mathematical theory. Furthermore, it is extremely serious matters that the conception of intrinsic diffusion conceived in relation to the Kirkendall effect has been wrongly accepted by researchers in the field for a long time. The discussion relevant to coordinate systems of diffusion equation reveals that there is no such a conception of intrinsic diffusion, to begin with. In fact, the theoretical equation satisfying the parabolic law for the Kirkendall effect was reported [53].

From a viewpoint of education for students, not such a textbook discussed using the Fick's laws in the beginning but one discussed first deriving the diffusion equation in relation to the Markov theory is required as soon as possible [54].