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Physical Vacuum Is Dense in Matter

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Abstract

Analysis of annihilation process of electron and positron reveals the existence of residue particle as an end product of the annihilation. Such particle is neutral in electrics and gravitation and momentumless in kinematic motion but polarizable under any field of any nonzero strength. In physical vacuum comprised of such particles, number density of the particles is estimated up to 5×10^{19} kilomoles per cubic meter. Propagation of electromagnetic wave in such vacuum is along transverse direction of electric fields of the electromagnetic wave and shall accrue energy loss in association with such propagation.

Keywords

Annihilation of Particles, Physical Vacuum, Hubble-Lemaître Correlation

1. Introduction

Physical vacuum is one of the simplest but mysterious subject/object. Literally, *vacuum* means empty/void region of space in absence of matter of any kind. While Newton's law of universal gravitation [1] was a leap forward in physics, it was inconceivable that a mass body could exert its gravitation force to other mass bodies in empty region of space in absence of matter of any kind. In comprehension of the phenomenon of action at distance, notion of ubiquitous aether was adopted to mediate gravitation force [2]. However, none of the aether theories was/is satisfactory in explaining the mechanisms for aether to mediate gravitation force. Maxwell electrodynamics [3] assumed implicitly that physical vacuum is a dielectric medium, which has physical properties such as vacuum electric permittivity, vacuum magnetic permeability, characteristic impedance of vacuum, etc., and can also accommodate displacement current. Such medium was known as luminiferous aether [4] [5].

It was mainly due to the null result of Michelson-Morley experiment [6] that notion of aether faded away from mainstream physics, with the understanding that alteration of velocity of measurement setup with respect to stationary aether should have caused observable variances in speed of light c but that contradicted to the finding of Michelson and Morley. In rescue of aether from the contradiction, Lorentz proposed spacial contraction along direction of motion [7]. However, it can be shown that any version of Michelson-Morley experiment shall yield null result even if c is not constant, due to time dilation caused by motion of test setup in finite and boundaryless space and/or immersion in centripetal force field [8]. Einstein commented that relativity theories do not deny aether but only mechanical characteristics of aether [9].

The concept of force field was invented during the eighteenth century, originally as a mathematical object for computational convenience (assuming fields are vector additive) [10]. Fields are entities associated with sources, e.g., mass, charge, current, etc., and extending everywhere in physical space according to relevant laws of physics. Thus, if there is a mass object then, in association with the object, there is a gravitation field, and it is the field at location of other mass object that causes that mass object to fell the gravitation attraction of the source object.

Nevertheless, Maxwell predicted from his equations the existence of electromagnetic waves traveling in space at c that are completely detached from source of any kind [3], confirmed later by experiments [11]. Therefore, electric and magnetic fields propagate in vacuum at velocity of c. Einstein presumed that gravitation field also propagates at velocity of c in empty space by itself [12].

Since light is but electromagnetic waves [11] and light possesses energy and momentum, therefore, field shall possess energy and, if in motion, momentum. Planck found that energy of light is discrete and proportional to frequency of the light [13]. Thus, energy of electromagnetic wave is packed in quanta. Such electromagnetic wave is known as photon, *i.e.*, invariant pattern of electromagnetic field traveling in empty/void region of space at velocity c. The concept of field is thus evolved from an original mathematical construct to become a real physical object [14]. In quantum electrodynamics, physical vacuum is regarded as an otherwise empty/void region of space filled with photons of zero point energies [15] (implying physical space is finite and boundaryless).

In the process of discovering positron, Dirac hypothesized physical vacuum as a sea of electrons, known as Dirac Sea, in which, all the electrons are in their respective ground state with negative energy [16]. As a physical vacuum, it is necessary that the electrons in such state are massless, momentumless, and charge neutral so that the vacuum appears to mass objects as free space with respect to kinematic motions of the objects. If sufficient energy is provided, an electron in the ground state can be excited to generate a normal electron with positive energy but leaves a vacancy of negative energy in the sea, which should be observable [17], known later as positron. Conversely, a normal electron can occupy a vacancy state by releasing excess energy to become a member of the sea, and such process was coined by Dirac as annihilation [17]. It is then evident that the sea of Dirac is but a particular type of aether [18].

In 1905, Einstein discovered the law of mass energy conversion [19]. Since most of the classical laws of physics were discovered before then therefore not automatically compatible with the law of Einstein, e.g., the law of Newton gravitation. It is therefore necessary to refine and/or reinterpret the classical laws of physics to assure their compliance with the law of Einstein [8] [20]. On the other hand, a pair of electron and positron is probably the simplest binary particle system having the simplest internal interactions, and annihilation of particles of such pair provides an illustrative example for the general conversion between *mass* and *energy*. Thus, utilizing the laws of physics complying with the law of Einstein, this essay is to reanalyze electron-positron pair as a system and annihilation of such pair as a process to further enhance the understanding of the system, the process, and the physical vacuum as well.

The essay is organized as the follows. **Section 2** derives the energy expression for electrostatic interaction between electron and positron under the law of Coulomb electrostatics complying with the law of mass-energy conservation. Refined law of Coulomb electrostatics and that of Newton gravitation enable convergent field energies of electron and positron in **Section 3**. **Section 4** analyzes typical electron-positron annihilation process, focusing on energy and matter balance aspects of the process, which reveals the existence of a particle of unknown type, named for now as annihilation residue particle (ARP), and shows that the particle is a necessary product of electron-positron annihilation in addition to photons. **Section 5** analyses basic properties of ARP that are unique to the particle. **Section 6** analyzes internal interactions of ARP. **Section 7** derives energy states of ARP via wave mechanics complying with the law of mass-energy conservation. **Section 8** asserts that physical vacuum is comprised of ARPs, in consideration of the perfect match of all known properties of physical vacuum with that of ARP filled space, and further estimated number density of ARPs in the vacuum by analyzing polarization states of physical vacuum under external fields. **Section 9** predicts that photon deflection shall also occur in vacuum polarized by electrostatic field. As an application of electrogravitation vacuum, **Section 10** derives a simple model based on ARP interaction to explain the origin of Hubble-Lemaître Correlation.

2. Electron-Positron Pair

Consider an electron-positron pair at rest in $\mathbf{S}^3$ (a three-dimensional finite space without boundary) on a geodesic containing internal origin of the space. Suppose particles of the pair are located at opposite sides of the origin with equal distance to the origin, and the direction pointing from the origin to the positron is assigned as positive direction. Such arrangement of electron-positron pair is referred to as symmetric particle configuration. By the law of Coulomb electrostatics [20] [21], under infinite space approximation,

$$\begin{matrix} \boldsymbol{F}_{+,x} = -f_{\pm}\left(\boldsymbol{\rho}_{+} - \boldsymbol{\rho}_{-}\right) \\ \boldsymbol{F}_{-,x} = -f_{\mp}\left(\boldsymbol{\rho}_{-} - \boldsymbol{\rho}_{+}\right) \end{matrix}, f_{\pm} \equiv \frac{E_{e,\boldsymbol{i}}}{r_e} \frac{\beta_{-,x}^{1/2}\beta_{+,x}^{1/2}}{\left\|\boldsymbol{\rho}_{+} - \boldsymbol{\rho}_{-}\right\|^3}, \begin{matrix} \boldsymbol{\rho}_{\pm} \equiv \dfrac{\boldsymbol{r}_{\pm}}{r_e}, \quad r_e \equiv \dfrac{\alpha_e \hbar c_{\boldsymbol{i}}}{E_{e,\boldsymbol{i}}} \\ \beta_x \equiv \dfrac{c_x}{c_{\boldsymbol{i}}}, \quad x = f, 0, e \end{matrix}. \tag{1}$$

$\mp$: Subscript indicating the associated entity refers to the electron or the positron. $\boldsymbol{F}_x$: Electrostatic force experienced by a particle of an electron-positron pair at rest in self field of the pair. $\boldsymbol{\rho}$: Location vector of particle of the pair, in reduced unit. $E_{e,i}$: Self energy of particle of the pair in Rest State [22]. r_e : Characteristic length of static electric field of particle of electron-positron pair in Rest State, also known as classical electron radius. $\beta_{e,x}$: Coulomb Factor at location of particle of electron-positron pair at rest in self field of the pair. $\boldsymbol{r}$: Location vector of particle of the pair. α_e : Fine structure constant. $\hbar$: Planck constant, an invariant in centripetal force field [8]. c_i : Speed of light in vacuum (SLV) defined/measured in Rest State. c_x : SLV as measured at location of particle of an electron-positron pair at rest in self field of the pair. x : State indicator.

Relocate particle of the electron-positron pair in self field of the pair in infinitesimal displacement and velocity. By the law of energy conservation,

$$\begin{matrix} dE_{+,x} = -\boldsymbol{F}_{+,x}\cdot d\boldsymbol{r}_+ = E_{e,i}\dfrac{\beta_e}{2s^2}ds \\ dE_{-,x} = -\boldsymbol{F}_{-,x}\cdot d\boldsymbol{r}_- = E_{e,i}\dfrac{\beta_e}{2s^2}ds \end{matrix}, \quad \begin{matrix} \beta_e \equiv \beta_{\pm,x} \\ \boldsymbol{s} \equiv \boldsymbol{\rho}_+ - \boldsymbol{\rho}_- \end{matrix} \rightarrow \frac{d\epsilon_{\pm,x}}{ds} = \frac{\beta_e}{2s^2},\ \epsilon_{\pm,x} \equiv \frac{E_{\pm,x}}{E_{e,i}}. \tag{2}$$

E_x : Self energy of particle of electron-positron pair at rest in self field of the pair. s : Distance between particles of the electron-positron pair, in reduced unit. ϵ_x : Self energy of particle of electron-positron pair at rest in self field of the pair, in reduced unit.

It has been shown previously [4] that, under the law of mass-energy conservation of Einstein [19],

$$\epsilon_x = \beta_x \rightarrow \frac{d\beta_e}{ds} = \frac{\beta_e}{2s^2} \rightarrow \beta_e = \exp\left[-\frac{1}{2s}\right] \rightarrow \rho_{\text{ESS}} \equiv \rho\big|_{\beta_e\to 0} = 0 \le s < \infty. \tag{3}$$

That is, no electric Schwarzschild Sphere shall exist in electrostatic interaction between particles of an electron-positron pair.

From Equation (1),

$$\boldsymbol{F}_{\pm,x} = \mp\frac{E_{e,i}}{r_e}\frac{\beta_e}{s^2}\hat{\boldsymbol{s}} \rightarrow \boldsymbol{\mathcal{F}}_\pm \equiv \frac{\boldsymbol{F}_{\pm,x}}{E_{e,i}/r_e} = \mp\frac{\beta_e}{s^2}\hat{\boldsymbol{s}} \rightarrow \begin{matrix} s_{\mathcal{F},\max} = \dfrac{1}{4} \\ \mathcal{F}_{\max} = \dfrac{16}{e^2} \end{matrix}. \tag{4}$$

That is, strength of electrostatic attraction between particles of an electron-positron pair is not a monotonic function of the distance between the particles as the classical law of Coulomb electrostatics would have indicated but instead shall have maxima at ¼ of the length unit and diminish towards complete merge of the particles. Therefore, electron and positron in symmetric electrostatic interaction shall appear to each other as if charge of an entity is mainly distributed in an envelope of ⅛ classical electron radius. Therefore, around and below such length scale, particle model for electron/position may no longer be appropriate in describing such entities. Further, towards complete merge of an electron and a position, Coulomb Factor of the entities shall become zero hence self mass of the entities shall approach infinity, which is, however, unphysical. Therefore, other types of interactions between the entities, e.g., gravitation, should not be omitted in consideration (**Appendix A**).

Figure 1 plots out self energy of a particle of an electron-positron pair in self field of the pair, which is the same as/identical to potential energy of the particle in same, and electrostatic force between particles of the pair versus distance between the particles, in comparison with that of Coulomb potential energy and Coulomb force. It can be seen from the plot that deviation of the potential energy and the electrostatic force from that of Coulomb is significant below a length scale of ~10^0 length units. Further, the potential energy and the electrostatic force are both approaching zero in length scale of 10^{-2} length unit and shorter while that of Coulomb are divergent towards $s \to 0$.

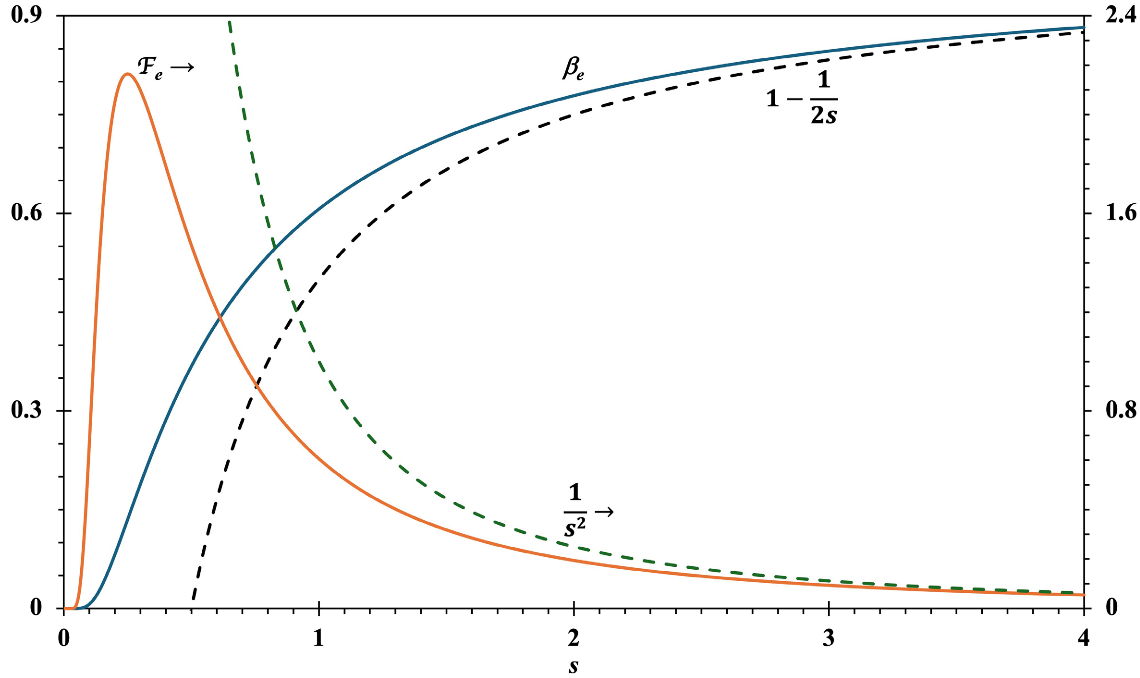

Figure 1. Comparison of potential energy and electrostatic force complying with the law of mass-energy conservation and that of Coulomb for particle of electron-positron pair in self field of the pair. Units of the figure are all expressed in reduced units, e.g., energy in unit of $E_{e,i}$, force in unit of $E_{e,i}/r_e$, and distance in unit of r_e.

3. Field Energy of Electron-Positron Pair

Consider an electron-positron pair in Rest State in origin-antipode configuration in $\mathbf{S}^3$. According to the law of Coulomb electrostatics, electrostatic force probed by a charge probe in field of the pair is [20]

$$\boldsymbol{F}_{p,x} = \frac{q_{e,x} q_{p,x} f_r}{4\pi \varepsilon_{0,e,x}^{1/2} \varepsilon_{0,p,x}^{1/2}} \boldsymbol{e}_\varphi, f_r \equiv \frac{1}{R^2 \sin^2 \varphi}, \varphi \equiv \frac{r}{R}, x = f, 0, e_{\text{EPP in OAC}}. \quad (5)$$

$\boldsymbol{F}_{p,x}$: Electrostatic force probed by charge probe at rest in field of an electron-positron pair in Rest State in origin-antipode configuration. q_e : Rest charge of electron. q_p : Rest charge of probe. ε_0 : Electric constant. r : Internal distance between probe and origin. R : External radius of $\mathbf{S}^3$. $\boldsymbol{e}_\varphi$: Unit vector of r at location of probe in $\mathbf{S}^3$.

By definition of electric field,

$$\boldsymbol{E} \equiv \frac{\boldsymbol{F}_p}{q_p} \rightarrow \boldsymbol{E}_{e,x} \equiv \frac{\boldsymbol{F}_{p,x}}{q_{p,x}} = \frac{q_{e,x} f_r}{4\pi\varepsilon_{0,e,x}^{1/2}\varepsilon_{0,p,x}^{1/2}} \boldsymbol{e}_\varphi . \tag{6}$$

$\boldsymbol{E}$: Static electric field. $\boldsymbol{F}_p$: Electrostatic force probed by probe at rest in the field. $\boldsymbol{E}_{e,x}$: Static electric field of an electron-positron pair in Rest State in origin-antipode configuration.

By definition, energy density of static electric field is [23]

$$\Sigma_{\boldsymbol{E}} = \frac{1}{2}\varepsilon_0 \boldsymbol{E}\cdot\boldsymbol{E} \rightarrow$$
$$\Sigma_{\boldsymbol{E},e,x} = \frac{\varepsilon_{0,e,x}^{1/2}\varepsilon_{0,p,x}^{1/2}}{2}\boldsymbol{E}_{e,x}\cdot\boldsymbol{E}_{e,x} = \frac{\alpha_e\hbar c_{\boldsymbol{i}}}{8\pi} n_e^2 \beta_{e,e,x}^{1/2}\beta_{e,p,x}^{1/2} \frac{\boldsymbol{e}_{e,x}}{\boldsymbol{e}_{p,x}} f_r^2,\ \beta_{e,j} \equiv \frac{c_j}{c_{\boldsymbol{i}}} . \tag{7}$$

$\Sigma_{\boldsymbol{E}}$: Energy density of static electric field. n_e : Charge of electron/positron, in reduced unit. $\beta_{e,j}$: Coulomb Factor of entity j. $\boldsymbol{e}_j$: Elementary charge of entity j. c_x : SLV as measured at rest at location of entity in static electric field.

According to Maxwell electrodynamics [3], without prejudice towards electrics or magnetics,

$$\varepsilon_0\mu_0 c^2 = 1 \rightarrow \varepsilon_{0,x}\mu_{0,x}c_x^2 = 1 \rightarrow \frac{\varepsilon_{0,\boldsymbol{i}}}{\varepsilon_{0,x}} = \frac{\mu_{0,\boldsymbol{i}}}{\mu_{0,x}} = \beta_x . \tag{8}$$

μ_0 : Magnetic constant.

By definition,

$$\frac{\varepsilon_{0,\boldsymbol{i}}}{\varepsilon_{0,x}} = \frac{\alpha_e\hbar c_x}{\alpha_e\hbar c_{\boldsymbol{i}}}\frac{\boldsymbol{e}_{\boldsymbol{i}}^2}{\boldsymbol{e}_x^2} = \beta_x \frac{\boldsymbol{e}_{\boldsymbol{i}}^2}{\boldsymbol{e}_x^2} \rightarrow \boldsymbol{e}_x = \boldsymbol{e}_{\boldsymbol{i}} \equiv \boldsymbol{e} \subset \text{Field Invariant} . \tag{9}$$

That is, charge of elementary charge is field invariant. Therefore,

$$\frac{\boldsymbol{e}_{e,x}}{\boldsymbol{e}_{p,x}} = 1 \rightarrow \Sigma_{\boldsymbol{E},e,x} = \frac{\alpha_e\hbar c_{\boldsymbol{i}}}{8\pi} n_e^2 \beta_{e,e,x}^{1/2}\beta_{e,p,x}^{1/2} f_r^2 . \tag{10}$$

By specification, the electron is in Rest State, therefore [20],

$$\beta_{e,e,x} = 1,\ \beta_{e,p,x} = \left(1 - \frac{\cot\varphi}{2R_e}\right)^2,\ n_e = -1 \rightarrow$$
$$\sigma_{\boldsymbol{E},e,x} \equiv \frac{\Sigma_{\boldsymbol{E},e,x}}{E_{e,\boldsymbol{i}}/r_e^3} = \frac{\beta_{e,p,x}^{1/2}}{8\pi R_e^4 \sin^4\varphi} . \tag{11}$$

$\sigma_{\boldsymbol{E}}$: Energy density of static electric field in reduced unit. $E_{e,\boldsymbol{i}}$: Self energy of electron in Rest State. R_e : External radius of $\mathbf{S}^3$ in reduced unit.

Total energy of static electric field of electron in Rest State in $\mathbf{S}^3$ is then

$$\epsilon_{\boldsymbol{E},e,x} \equiv \frac{E_{\boldsymbol{E},e,x}}{E_{e,\mathtt{i}}} = \iiint_{V_e} \frac{\sigma_{\boldsymbol{E},e,x}}{r_e^3} dV_e = \frac{1}{2R_e}\int_{\varphi_{\text{ESS}}}^{\pi/2}\left(1 - \frac{\cot\varphi}{2R_e}\right)\frac{d\varphi}{\sin^2\varphi} = \frac{1}{2},\ \varphi_{\text{ESS}} \equiv \varphi\big|_{\beta_{e,p,x}=0} . \tag{12}$$

$\epsilon_{\boldsymbol{E},e,x}$: Total energy of static electric field of electron in Rest State in $\mathbf{S}^3$, in reduced unit. $E_{\boldsymbol{E},e,x}$: Total energy of static electric field of electron in Rest State in $\mathbf{S}^3$. V_e : Volume of hemisphere of electron in $\mathbf{S}^3$.

That is, total energy of static electric field of electron in Rest State in $\mathbf{S}^3$ is ½ $E_{e,\boldsymbol{i}}$, *i.e.*, one half of the energy unit if self energy of electron in Rest State is appointed as unit of energy. By symmetry, total energy of static electric field of positron in

Rest State in $\mathbf{S}^3$ is also ½ of the energy unit.

It can be shown symbolically/numerically that total energy of static electric field of an electron-positron pair is independent of the separation distance between particles of the pair. Alternatively, external work done to separate particles of an electron-positron pair is gained by the particles as increment of self energy of the particles, as embedded in energy balance equation, e.g., Equation (2). Therefore, total energy of static electric field of an electron-positron pair must not depend on separation distance between particles of the pair or otherwise violation of the law of energy conservation is unavoidable.

Electron in Rest State at internal origin of $\mathbf{S}^3$ has nonzero rest mass hence is active in gravitation. By the flux conservation theorem of Gauss, counterpart of the electron, *i.e.*, positron, at antipode of the origin shall also have nonzero rest mass, but of opposite type with respect to that of the electron [24]. Static gravitation field associated with an electron-positron pair can be probed by mass probe. According to the law of Newton gravitation, static gravitation force probed by mass probe in static gravitation field of electron-positron pair in Rest State in origin-antipode configuration in $\mathbf{S}^3$ is [8], assuming probe mass is of same type as that of electron,

$$\boldsymbol{F}_{p,x} = -G_{e,x}^{1/2} G_{p,x}^{1/2} m_{e,x} m_{p,x} f_r \boldsymbol{e}_\varphi,\ f_r \equiv \frac{1}{R^2 \sin^2\varphi},\ x = f, 0, g_{\text{EPP in OAC}}. \tag{13}$$

$\boldsymbol{F}_{p,x}$: Static gravitation force probed by a mass probe in static gravitation field of an electron-positron pair in Rest State in origin-antipode configuration in $\mathbf{S}^3$. $G_{e,x}$: Gravitation constant as measured by a particle of the electron-positron pair at rest at location of the particle in static gravitation field of the pair. $G_{p,x}$: Gravitation constant as measured by mass probe at rest at location of the probe in static gravitation field of the pair. $m_{e,x}$: Self mass of a particle of the electron-positron pair at rest at location of the particle in static gravitation field of the pair. m_p : Self mass of the mass probe at rest at location of the probe in static gravitation field of the pair.

By definition of gravitation field,

$$\boldsymbol{G} \equiv \frac{\boldsymbol{F}_p}{m_p} \ \rightarrow\ \boldsymbol{G}_{e,x} \equiv \frac{\boldsymbol{F}_{p,x}}{m_{p,x}} = -G_{e,x}^{1/2} G_{p,x}^{1/2} m_{e,x} f_r \boldsymbol{e}_\varphi. \tag{14}$$

$\boldsymbol{G}$: Static gravitation field. $\boldsymbol{F}_p$: Static gravitation force probed by a mass probe at rest in the field. $\boldsymbol{G}_{e,x}$: Static gravitation field of an electron-positron pair in Rest State in origin-antipode configuration in $\mathbf{S}^3$.

By definition, energy density of static gravitation field is [23]

$$\Sigma_{\boldsymbol{G}} = \frac{\boldsymbol{G}\cdot\boldsymbol{G}}{4\pi G} \ \rightarrow\ \Sigma_{\boldsymbol{G},e,x} = \frac{\boldsymbol{G}_{e,x}\cdot\boldsymbol{G}_{e,x}}{4\pi G_{e,x}^{1/2} G_{p,x}^{1/2}} = \frac{G_{e,x}^{1/2} G_{p,x}^{1/2} m_{e,x}^2 f_r^2}{4\pi}. \tag{15}$$

$\Sigma_{\boldsymbol{G}}$: Energy density of gravitation field. G : Gravitation constant.

By specification, the electron is in Rest State, therefore [8],

$$\begin{matrix} m_{e,x} = m_{e,\boldsymbol{i}} \\ G_{e,x} = G_{\boldsymbol{i}} \end{matrix},\ G_{p,x} = \frac{G_{\boldsymbol{i}}}{\beta_{g,p,x}^4},\ \beta_{g,p,x} \equiv \frac{c_{p,x}}{c_{\boldsymbol{i}}} = \left(1 - \frac{4\cot\varphi}{R_g}\right)^{1/4},\ R_g \equiv \frac{R}{r_g},\ r_g \equiv \frac{G_{\boldsymbol{i}} m_{e,\boldsymbol{i}}}{c_{\boldsymbol{i}}^2}. \tag{16}$$

$m_{e,\boldsymbol{i}}$: Self mass of electron in Rest State. $G_{\boldsymbol{i}}$: Gravitation constant as measured in Rest State free of any field. $\beta_{g,p,x}$: Schwarzschild Factor associated with mass probe at rest in static gravitation field of an electron-positron pair in Rest State in origin-antipode configuration in $\mathbf{S}^3$. $c_{p,x}$: SLV as measured at rest at location of mass probe in static gravitation field of an electron-positron pair. r_g : Characteristic length of static gravitation field of an electron in Rest State. R_g : External radius of $\mathbf{S}^3$ in unit of r_g.

Total energy of static gravitation field of electron in Rest State in $\mathbf{S}^3$ is then

$$\epsilon_{\boldsymbol{G},e,x} = \iiint\limits_{V_e} \frac{\Sigma_{\boldsymbol{G},e,x}}{E_{e,\boldsymbol{i}}} dV_e = \frac{1}{R_g} \int_{\varphi_{g,\mathrm{SS}}}^{\pi/2} \frac{d\varphi}{\beta_{g,p,x}^2 \sin^2\varphi} = \frac{1}{2},\ \varphi_{g,\mathrm{SS}} \equiv \varphi\big|_{\beta_{g,p,x}=0}. \quad (17)$$

$\epsilon_{\boldsymbol{G},e,x}$: Total energy of static gravitation field of electron in Rest State in $\mathbf{S}^3$ in reduced unit.

That is, total energy of static gravitation field of electron in Rest State in $\mathbf{S}^3$ is ½ of the energy unit. By symmetry, total energy of static gravitation field of positron in Rest State in $\mathbf{S}^3$ is also ½ of the unit. It can be shown symbolically/numerically that total energy of static gravitation field of an electron-positron pair is independent of separation distance between particles of the pair, and must be so by the law of energy conservation.

4. Electron-Positron Annihilation Process

Low energy electron-positron annihilation process is commonly expressed as [25]

$$\text{Electron}_{\boldsymbol{i}} + \text{Positron}_{\boldsymbol{i}} \ \rightarrow \sum_j \gamma_j,\ \sum_j h\nu_j = 2E_{e,\boldsymbol{i}}. \quad (18)$$

$\boldsymbol{i}$: Subscript indicating associated entity is in Rest State. γ : Photon released during low energy electron-positron annihilation process. ν : Frequency of a photon released during the low energy electron-positron annihilation process. $E_{e,\boldsymbol{i}}$: Self energy of electron/positron in Rest State, assigned as unit of energy.

That is, an electron-positron pair initially in Rest State in origin-antipode configuration shall transform to plurality of photons having energy totaling two energy units at the end of a low energy electron-positron annihilation process, and nothing else is left there at the end of the process hence *annihilation* of particles of the electron-positron pair.

From previous section, if field energies of particles of an electron-positron pair were all of the same sign, e.g., all being positive, then field energy of the pair should be two units in total. Therefore, as a whole system, total energy of an electron-positron pair in Rest State in origin-antipode configuration should be four units (two units in form of field energy and two units in form of self/potential energy). However, total energy of an electron-positron pair at the end of a low energy electron-positron annihilation process is only two units (in form of photon energy) according to Expression (18). Therefore, per the expression, there would be violation of the law of energy conservation in low energy electron-positron annihilation process. However, by definition of field energy, Expression (7) and (15), field energy is and must be a positive attribute since such is in positive proportion to inner product of field vector with itself and field vector is real in mathematical sense.

Energy deficit of Expression (18) reflects a general issue in all annihilation processes involving charge particles. That is, there exists electric field of the system hence nonzero field energy of the system before annihilation and no field hence zero field energy after annihilation but the missing field energy is not accounted for. Similarly, in all annihilation processes involving mass particles, there exists gravitation field of the system hence nonzero field energy of the system before annihilation and no field hence zero field energy after annihilation but the missing field energy is not accounted for. In retrospect, field energy of charge/mass particle is a divergent entity under classical laws of physics hence ignored from relevant considerations. Now, with those laws refined/reinterpreted in compliance with the law of Einstein, field energy is no longer divergent hence becomes an undeniable/unignorable entity.

Therefore, for a low energy electron-positron annihilation process to comply with the law of energy conservation, field energy of a particle of an electron-positron pair has to be regarded as of opposite sign of that of its counterpart. That is, if energy of static electric field of electron is regarded as positive then energy of static electric field of positron must be regarded as negative; if energy of static gravitation field of electron is assigned as negative then energy of static gravitation field of positron must be viewed as positive; and vice versa. However, static field of electron and that of positron are one and the same field hence assignment of signs in such manner is unnatural/artificial, which also causes discontinuity in field energy density at interface of the fields. Alternatively, energy of static gravitation field has to be regarded as of opposite sign as that of static electric field. In any case, *energy* as a signed attribute is unavoidable. Since photon energy is regarded as positive, therefore electromagnetic field energy hence electric field energy must be viewed as positive. Accordingly, gravitation field energy has to be regarded as negative energy, so as to avoid violating the law of energy conservation in annihilation process.

Expression (18) is also imbalanced in material/substance/matter. An electron-positron pair possesses rest charges at beginning of a low energy electron-positron annihilation process but there were only photons at the end of the annihilation process and nothing else is left there according to the expression. Since photon is neutral in charge nor does it carry charge, annihilation deficit of rest charges of the particles cannot be accounted for by the photons.

Classical law of Coulomb electrostatics does not allow oppositely charged particles to merge with each other, for such shall cause divergences hence is unphysical. Or, it is said the law shall break down at merge of the particles hence is inapplicable to the situation hence should be ignored. Since, according to Expression (18), nothing is left there after annihilation, the charges must have been destructed completely, *i.e.*, annihilated in literal sense, even though elementary charges are understood as indestructible entities.

However, if a proton were to capture an electron to form a hydrogen atom, rest charge of the proton and that of the electron are not regarded as being annihilated but still existing there, *i.e.*, in the hydrogen atom. Likewise, if an electron jumps into a proton to form a neutron, rest charge of the electron and that of the proton are not regarded as being annihilated but still existing there, *i.e.*, in the neutron. Therefore, charges in an annihilation process can still be understood as annihilated but only in the sense as the word originally meant by Dirac, *i.e.*, becomes neutral, invisible, undetectable, etc., with respect to the corresponding probes instead of the nothingness as commonly understood.

Under the refined/reinterpreted laws of physics complying with the law of Einstein, the combined interactions of electrostatics and gravitation allows complete merge of electron and positron without divergence of any kind (**Appendix A**). Therefore, at the end of electron-positron annihilation process, there must leave an entity there, which combines the charges hence is neutral in charge.

An electron-positron pair also possesses rest masses at beginning of a low energy electron-positron annihilation process but there were only photons at the end of the annihilation process and nothing else is left there according to expression (18). Since rest energy of particles of an electron-positron pair in initial state is converted to energy of the photons via low energy electron-positron annihilation process, then rest mass of the particles in final state of the process must be nonzero [8] [20] and such mass cannot be accounted for by the photons emitted from the annihilation process since rest mass of photon is none by definition of photon. Such imbalance in matter further indicates the incompleteness of the description of Expression (18) on low energy electron-positron annihilation process.

Classical law of Newton gravitation does not allow mass particles to merge with each other, for such shall cause divergences whether or not the particles are of same or opposite mass type. However, for pair of electron and positron, such merge is allowable under electrogravitation interaction (**Appendix A**). Since electron and positron are repulsive to each other gravitationally [24], the law of Gauss gravitation, *i.e.*, nonlocal simultaneous integration of vector field of gravitation over any enclosure of simple connectivity is proportional to total rest mass enclosed by the enclosure, dictates that rest mass of an electron-positron pair is and must be zero whether or not particles of the pair merge with each other. Similar to the situation of rest charges, therefore, at the end of electron-positron annihilation process, there must exist an entity there, which is of zero rest mass. In other words, the entity is massless hence neutral, invisible, undetectable, etc., with respect to mass probes instead of nothing.

Accordingly, Expression (18) needs to be revised to

$$\text{Electron}_{\boldsymbol{i}} + \text{Positron}_{\boldsymbol{i}} \;\rightarrow\; \text{ARP} + \sum\nolimits_j \gamma_j,\; \sum\nolimits_j h\nu_j = 2(1-\kappa)E_{e,\boldsymbol{i}}. \tag{19}$$

ARP: Annihilation residue particle. κ: A dimensionless constant, $\kappa \equiv \eta^{1/7}$, $\eta \equiv r_g/r_e$.

That is, in addition to photons released from a low energy electron-positron annihilation process, there must exist an ARP at the end of the process. For two-photon low energy electron-positron annihilation process, from known data [26] [27],

$$\eta \approx 2.400\ 60\times10^{-43},\ E_{e,i} = 510{,}998.950\ 00(15)\ \text{eV}\ \rightarrow$$
$$h\nu_{\text{LEPA}} \approx 510{,}998.533\ 23(15)\ \text{eV},\ \Delta E_{\text{LEPA}} \equiv E_{e,i} - h\nu_{\text{LEPA}} \approx 0.417\ \text{eV} \quad (20)$$

Therefore, precision measurement of energies of the photons released from a two-photon electron-positron annihilation process at the low energy limit, *i.e.*, initial separation of the particles is sufficiently large and initial velocities of the particles are sufficiently low, shall observe an energy difference of ~0.4 electron volts if the parameter n_g in refined law of Newton gravitation [8] is indeed $n_g = -2$. Such measurement shall also provide an alternative route for determining the parameter n_g for the gravitation constant *G*.

Figure 2 plots out comparison of Planck Factor and Coulomb Factor for particles of an electron-positron pair in self fields of the pair. It can be seen from the plot that the two factors are essentially identical in most regions of inter-particle distance except for $s < 0.04$ length units. On the other hand, self energy of constituent particles of an ARP is nonzero towards $s \rightarrow 0$, therefore, self mass of same shall not become divergent at formation of the ARP.

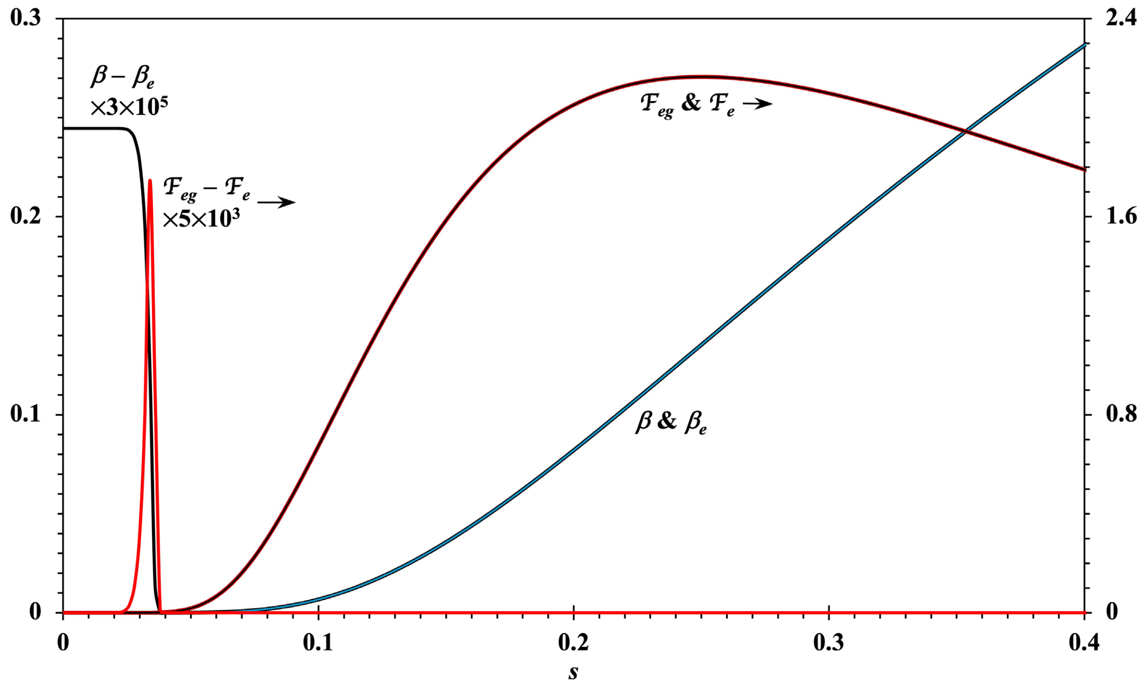

Figure 2. Comparison of Planck Factor and Coulomb Factor of particles of an electron-positron pair in self fields of the pair. Units of the figure are all expressed in reduced units, e.g., energy in unit of $E_{e,i}$, force in unit of $E_{e,i}/r_e$, and distance in unit of r_e.

5. Annihilation Residue Particle

From Expression (A3), self energy of an ARP is the lowest among all possible configurations of the electron-positron pair. Therefore, ARP is a stable form of existence of such pair. Further, strength of static electric field of an ARP is zero everywhere in space, due to exact cancellation of the fields of constituent particles of the ARP. Therefore, ARP is neutral in electrics hence its existence is invisible/undetectable via charge probe. Likewise, strength of static gravitation field of an ARP is zero everywhere in space, due to exact cancellation of the fields of constituent particles of the ARP. Therefore, ARP is neutral in gravitation hence its existence is invisible/undetectable via mass probe.

From Expression (A3), rest mass of a constituent particle of an ARP is large, ~673 atomic mass units, due to the law of mass-energy conservation and minute value of η. If such rest mass is identical/equivalent/same to momentum rest mass (also known as inertia rest mass), *i.e.*, the proportion parameter in definition of *momentum* of mass object at the limit of zero velocity, then ARP would have long been discovered since it would be rather difficult to have missed observing such massive particles in electron-positron annihilation experiments, regardless of how small a region of space an ARP might occupy. Therefore, the null results of electron-positron annihilation experiments, *i.e.*, failure to have observed existence of massive particles (of ~1,345 atomic mass units) at the end of any electron-positron annihilation process, indicate that ARP must be a momentumless particle no matter how massive its constituent particles may be. In other words, rest mass of ARP must be zero hence momentum mass of ARP is none, since the latter is proportional to the former by definition of the entities. Consequently, ARP cannot exchange momentum with other entities hence is invisible/undetectable to/by momentum probe.

However, there is nothing special about rest mass of a constituent particle of an ARP, which is of exactly the same property and shall have exactly the same effect as that of any ordinary mass entity. The only aspect that might be unique of ARP is that strength of the static gravitation field associated with an ARP is zero everywhere in space due to exact cancellation of the field of gravitation matter and that of inverse matter constituting ARP. Therefore, momentumlessness of ARP, derived from null result of electron-positron annihilation experiments, reveals that inertia rest mass of an object must be in direct association with self field of the object. That is, if an object has self field then the object shall have inertia rest mass; if the object has no self field, or equivalently, strength of its self field is zero everywhere in space, then the object shall have no or zero inertia rest mass. Field strength of ARP is zero everywhere in space, therefore, momentum rest mass of ARP is zero hence momentumlessness of ARP. On the other hand, by the law of Newton gravitation, gravitation rest mass of an object is in direct association with gravitation field of same. Since field strength of ARP is zero everywhere in space, therefore, gravitation rest mass of ARP must be zero no matter how massive its constituent particles may be. Therefore, momentum rest mass of an object is and must be identical/equivalent/same to gravitation rest mass of same. That is, momentum rest mass and gravitation rest mass are and must be one and the same attribute of any mass object.

Further, since rest mass of ARP is zero while that of its constituent particles is nonzero, therefore, rest mass of a constituent particle of ARP must have opposite sign with respect to that of its counterpart. In other words, rest mass of an object, hence mass of same, must be a signed attribute of same. Accordingly, refinement of the law of mass-energy conservation is in demand, since *mass* referred to by the law is commonly understood as an unsigned entity while *energy* in the law is also an unsigned entity.

6. Electrogravitation Force between Constituent Particles of ARP

From Equation (A1), electrogravitation force between particles of an electron-positron pair is, under infinite space approximation,

$$\boldsymbol{F}_{\pm} = \mp \frac{E_{e,i}}{r_e}\left(\beta - \frac{\eta}{\beta^6}\right)\frac{\hat{\boldsymbol{s}}}{s^2} \rightarrow \boldsymbol{\mathcal{F}}_{\pm} \equiv \frac{\boldsymbol{F}_{\pm}}{\mathbb{U}_{\boldsymbol{F}}} = \mp\left(\beta - \frac{\eta}{\beta^6}\right)\frac{\hat{\boldsymbol{s}}}{s^2},\ \mathbb{U}_{\boldsymbol{F}} \equiv \frac{E_{e,i}}{r_e}. \quad (21)$$

$\boldsymbol{F}_{\mp}$: Electrogravitation force experienced by particle of electron-positron pair due to interaction with counterpart of same. $\boldsymbol{\mathcal{F}}_{\mp}$: Electrogravitation force expressed in reduced unit.

Therefore, if particles of the pair are sufficiently apart from each other then, with Equation (A2),

$$s \gg 1 \rightarrow \mathcal{F}_{eg} \equiv \left\|\boldsymbol{\mathcal{F}}_{\pm}\right\| = \frac{1}{s^2}\frac{1-\eta}{\beta^6}e^{-\frac{7}{2s}} \simeq \frac{1}{s^2}. \quad (22)$$

That is, far field interaction between constituent particles of ARP shall resemble Coulomb attraction. For shorter separation distance between the particles, the attraction force shall be increasing but rate of the increase is slower than that of the Coulomb interaction and the force shall reach maxima at

$$s_{\mathcal{F}_{eg},\max} \approx \frac{1}{4} + 1.471\ \eta^{6/7},\ \mathcal{F}_{eg,\max} \approx \frac{16}{e^2},\ \beta_{\mathcal{F}_{eg},\max} \approx \frac{1}{e^2}. \quad (23)$$

That is, at and below ¼ of the length unit, electrogravitation attraction between particles of an electron-positron pair shall attenuate with reducing distance between the particles and the force shall diminish at particle amalgamation configuration. Further,

$$\lim_{s\to 0}\frac{d^n\mathcal{F}_{eg}}{ds^n} = 0,\ 0 \le n,\ n \in \text{Integers}. \quad (24)$$

That is, at particle amalgamation configuration, electrogravitation force and any order of derivatives thereof between particles of an electron-positron pair is none. Therefore, any external field with any nonzero strength shall polarize an ARP. It can be seen from Figure 2 that electrogravitation force and electrostatic force between particles of electron-positron pair are essentially identical except in range $0.02 < s < 0.04$.

7. Energy State of Annihilation Residue Particle

From Equation (A4), total energy of a constituent particle of an ARP in Rest State is

$$\begin{gathered}\epsilon = \frac{\beta}{\beta_u} \rightarrow u^2 + \frac{\beta^2}{\epsilon^2} = 1 \rightarrow -p^2 + \frac{\epsilon^2}{\beta^2} = 1 \rightarrow \\ \left(\frac{\Delta_\rho}{\alpha_e^2} + \frac{\epsilon^2}{\beta^2} - 1\right)\Psi = 0,\ \kappa \le \epsilon \le 1\end{gathered}. \quad (25)$$

ϵ : Total energy of constituent particle of an ARP in Rest State, in reduced unit. β : Planck Factor of constituent particle of an ARP in self field of same in Rest State. β_u : Lorentz Factor of constituent particle of an ARP in motion in self field of same in Rest State. u : Reduced velocity of constituent particle of an ARP in motion in self field of same in Rest State. p : Field momentum of constituent particle of an ARP in motion in self field of same in Rest State, in reduced unit. ρ : Distance between constituent particle of an ARP and symmetry center of same, in reduced unit. Δ_ρ : Laplace operator, in reduced unit. α_e : Fine structure constant. Ψ: Wave function of constituent object of an ARP in Rest State.

This is the wave mechanic equation in compliance with the law of mass-energy conservation for energy state of constituent object of an ARP in Rest State, ignoring spin of the electron and that of positron, interaction between self fields of the object and internal motion of same, field delay and retardation, etc. The last condition of Equation (25) constrains the ARP to regular bound states. Such entity is commonly known as positronium [28].

For Equation (25), define a complete and irreducible set of base functions as

$$\Phi_{n,l,m} \equiv R_n^l Y_l^m,\ R_n^l \equiv \sqrt{\frac{\nu_n (n-l-1)!}{n(n+l)!}} \frac{e^{-\nu_n \rho}}{\rho} (2\nu_n \rho)^{l+1} L_{n-l-1}^{2l+1}[2\nu_n \rho] . \qquad (26)$$
$$|m| \le l,\ 0 \le l < n,\ m,l,n \in \text{Integers}$$

Φ : Base function for Equation (25). Y_l^m : Normalized harmonic function. L_a^b : Generalized Laguerre polynomial.

Wherein,

$$\nu_n = \alpha_e \sqrt{1-\epsilon_n^2},\ \epsilon_n^2 = \frac{\sqrt{1+4x}-1}{2x},\ x \equiv \left(\frac{(1-\eta)\alpha_e}{4n}\right)^2 \rightarrow$$
$$\epsilon_n = 1 - \frac{1}{2}x + \frac{7}{8}x^2 - \frac{33}{16}x^3 + \cdots \qquad (27)$$

With the Laplace operator expressed in spherical polar coordinates,

$$\frac{\Delta_\rho R_n^l}{R_n^l} = \frac{l(l+1)}{\rho^2} + \alpha_e^2 (1 - \epsilon_n^2 f_\rho),\ f_\rho \equiv 1 + \frac{1-\eta}{2\rho} . \qquad (28)$$

Therefore, let

$$\Psi \equiv \sum_{n,l,m} c_{n,l,m} \Phi_{n,l,m},\ \sum_{n,l,m} c_{n,l,m}^2 \ne 0 \rightarrow$$
$$\sum_{n,l,m} c_{n,l,m} \left(\Delta_\rho + \frac{\alpha_e^2 \epsilon^2}{\beta^2} - \alpha_e^2 \right) \Phi_{n,l,m} = 0 . \qquad (29)$$

Ψ : Wave function of Equation (25). $c_{n,l,m}$: Coefficients for linear combination of the base functions.

Substitute Expression (28) into the equation,

$$\sum_{n,l,m} c_{n,l,m} \left(\frac{\epsilon^2}{\beta^2} - f_\rho \epsilon_n^2 \right) \Phi_{n,l,m} = 0,\ \beta = \left(\eta + (1-\eta) \exp\left[-\frac{7}{4\rho} \right] \right)^{1/7} . \qquad (30)$$

Multiply the equation with a base function of Expression (26) and integrate over entire domain, using Dirac notation,

$$\sum_{n,l,m} c_{n,l,m} \left\langle \Phi_{n',l',m'} \left| \frac{\epsilon^2}{\beta^2} - f_\rho \epsilon_n^2 \right| \Phi_{n,l,m} \right\rangle = 0 \rightarrow \\ \sum_n c_{n,l} \left\langle R_{n'}^l \left| \frac{\epsilon^2}{\beta^2} - f_\rho \epsilon_n^2 \right| R_n^l \right\rangle = 0,\ n, n' > l,\ l = 0, 1, \cdots \quad (31)$$

Each set of the linear equations for a given l can be expressed in matrix form,

$$\left(\epsilon^2 \boldsymbol{\mu} - \boldsymbol{\nu\varepsilon}\right)\boldsymbol{c} = \boldsymbol{0}, \quad \begin{array}{ll} \mu_{j,k} \equiv \left\langle R_{j+l}^l \left| \beta^{-2} \right| R_{k+l}^l \right\rangle, & \varepsilon_{j,k} \equiv \delta_{j,k} \epsilon_{k+l}^2 \\ \nu_{j,k} \equiv \left\langle R_{j+l}^l \left| f_\rho \right| R_{k+l}^l \right\rangle, & j, k = 1, 2, \cdots \end{array} \quad (32)$$

$\delta_{j,k}$: Kronecker delta function.

Since the base function set is complete and irreducible, inverse of matrix $\boldsymbol{\nu}$ exists. Therefore,

$$\left(\boldsymbol{\lambda} - \frac{1}{\epsilon^2}\mathbf{I}\right)\boldsymbol{c} = \boldsymbol{0},\ \boldsymbol{\lambda} \equiv \boldsymbol{\varepsilon}^{-1}\boldsymbol{\nu}^{-1}\boldsymbol{\mu} \rightarrow \left|\boldsymbol{\lambda} - \frac{1}{\epsilon^2}\mathbf{I}\right| \left|\boldsymbol{c}\right| = 0. \quad (33)$$

$\mathbf{I}$: Identity matrix.

From Condition (29),

$$\left|\boldsymbol{c}\right| \neq 0 \rightarrow \left|\boldsymbol{\lambda} - \frac{1}{\epsilon^2}\mathbf{I}\right| = 0 \rightarrow \epsilon_{n,l} = \frac{1}{\sqrt{\lambda_{n,l}}}. \quad (34)$$

$\lambda_{n,l}$: The nth eigenvalue of matrix $\boldsymbol{\lambda}$.

Therefore, total energy of constituent particle of positronium in bound state hence that of positronium is discrete by wave mechanics. Table 1 lists energy level of some near ground states. Comparing results from Equation (34) and that from Expression (27), the correction term is on the order of α_e^4 for $l = 0$ and smaller for $l \neq 0$. Note also that spectral pattern of positronium is identical to that of hydrogen atom except photon energies from positronium are ~1/16 of that of corresponding ones from hydrogen atom.

With Expression (26),

Table 1. Energy level of near ground states of positronium, in reduced unit, scaled by α_e^3, and ϵ_∞ is set as zero.

n \ l	ϵ_n	0	1	2	3	4	5	6
9	−0.052 142	−0.052 920	−0.052 869	−0.052 869	−0.052 869	−0.052 869	−0.052 869	−0.052 869
8	−0.067 422	−0.066 986	−0.066 912	−0.066 912	−0.066 912	−0.066 912	−0.066 912	−0.066 912
7	−0.087 467	−0.087 505	−0.087 395	−0.087 395	−0.087 395	−0.087 395	−0.087 395	−0.087 395
6	−0.118 585	−0.119 130	−0.118 955	−0.118 955	−0.118 955	−0.118 955	−0.118 955	
5	−0.171 593	−0.171 597	−0.171 295	−0.171 295	−0.171 295	−0.171 295		
4	−0.267 951	−0.268 240	−0.267 649	−0.267 649	−0.267 649			
3	−0.475 905	−0.475 815	−0.475 820	−0.475 820				
2	−1.070 581	−1.075 366	−1.070 595					
1	−4.282 347	−4.320 930						

$$\bar{\rho}_{n,l} \equiv \langle R_n^l | \rho | R_n^l \rangle = \frac{2}{\alpha_e^2} \frac{3n^2 - l(l+1)}{(1-\eta)\epsilon_n^2}. \tag{35}$$

$\bar{\rho}_{n,l}$: Mean radius of a base function of Equation (25), in reduced unit.

Figure 3 plots out some base functions for near ground states of positronium. It can be seen from the plot that the base functions extend out about four times further than that of an electron in hydrogen atom, consistent with the higher energy levels with respect to that of same.

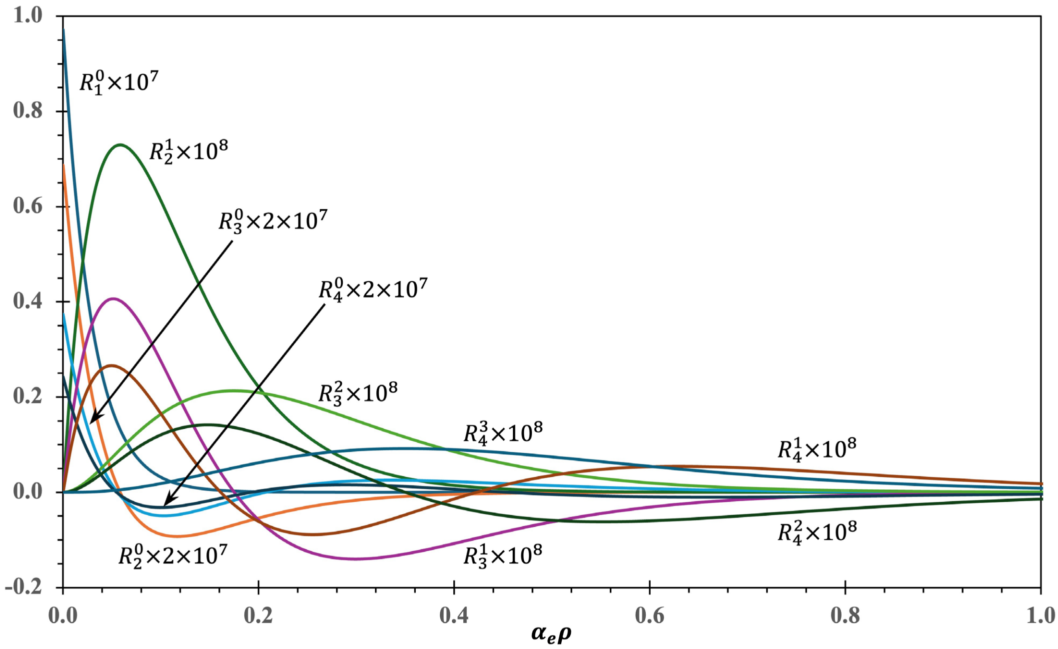

Figure 3. Base functions for near ground states of positronium.

The wave function Ψ_n^0 is a linear combination of the base function R_k^0, $\Psi_n^0 = \sum_k c_{n,k} R_k^0$, corresponding to the states of $l = 0$, which are the same as/identical to that for an ARP under linear polarization. From Expression (27) or Table 1, the lowest energy corresponds to $n = 1$, which is close to unity. Therefore, Ψ_1^0 does not represent the true ground state of positronium. On the other hand, from Equation (25), there is a solution, corresponding to *the* ground state of positronium,

$$\left(\frac{\Delta_\rho}{\alpha_e^2} + \frac{\epsilon^2}{\beta^2} - 1\right)\delta_{D,\rho} = 0 \rightarrow \Psi_0^0 = \delta_{D,\rho},\ \epsilon_{0,0} = \lim_{\rho\to 0}\beta = \kappa. \tag{36}$$

δ_D : Dirac delta function [29].

That is, with Dirac delta function as the wave function, an electron-positron pair in the particle amalgamation configuration is permissible by wave mechanics, and total energy of *the* ground state of constituent object of ARP is κ. Therefore, with respect to *the* ground state, all other wave functions are in association with excited states of ARP, *i.e.*, positronium. Since, for Ψ_0^0, $l = 0$, transition from excited states is allowable but only if their quantum number l is also zero, per rules of wave mechanics. Therefore, annihilation of an electron-positron pair can happen but only if the electron is in head to head collision course with respect to the positron. Accordingly, the photons released from the annihilation must be along transverse direction of the collision course.

8. Electrogravitation Vacuum

Empty space or region therein filled with plurality of ARPs is referred to hereinafter as electrogravitation vacuum. It is plausible that other types of vacuum may also exist, as an otherwise empty space or region therein filled with other sorts of residue particles formed via annihilation of other types of particle pairs. A common feature of all such vacuums, if exist, is their neutrality in electrics and gravitation and momentumlessness of the residue particles filled therein. Therefore, electrogravitation vacuum is a momentumless object neutral to both electrics and gravitation. Consequently, kinematic motion of any object in such vacuum shall not be affected by presence of the ARPs existing therein, since no exchange of momentum shall occur among the entities thereat. Therefore, electrogravitation vacuum is identical, equivalent, and indifferent to free space with respect to kinematic motion of object therein.

If ARPs in electrogravitation vacuum would all be in amalgamation configuration then there should not exist an up limit on how many of such ARPs may be packed into a finite but nonzero volume. However, ARPs are polarizable by any field of any nonzero strength, and such fields do and always exist in physical space. Therefore, in reality, ARPs in electrogravitation vacuum can and only exist in form of oscillating charge/mass dipoles (**Appendix B**) hence should have "volume" of some sort, *i.e.*, there must exist an up limit on number density of ARPs in electrogravitation vacuum, even though any field of none zero strength shall attract hence compact polarized ARPs. Further, if strength of an external field is sufficiently strong then ARPs in electrogravitation vacuum shall be dissociated under such field (**Appendix C**). In case of static electric field, such phenomenon is known as vacuum breakdown, which may be comprehended as due to regrouping of constituent particles of ARPs with counterpart of neighboring ARPs under same field.

As a simple model for polarization of electrogravitation vacuum under external field, consider an array of ARP dipoles evenly distributed along a straight line parallel to the direction of an external field. Under infinite space approximation,

$$\begin{gathered}\mathcal{F}_{+,A} = -\frac{1}{l^2}\left(\Gamma_D^1[x] - \Gamma_D^1[1-x]\right)\left(\beta_A - \frac{\eta}{\beta_A^6}\right), x \equiv \frac{s}{l}, 0 \le x \le \frac{1}{2} \rightarrow \\ \frac{d\beta_A}{dx} = \frac{1}{2l}\left(\Gamma_D^1[x] - \Gamma_D^1[1-x]\right)\left(\beta_A - \frac{\eta}{\beta_A^6}\right)\end{gathered} \quad . \tag{37}$$

$\mathcal{F}_{+,A}$: Electrogravitation force experienced by positron of an ARP under self field of the dipole array, in reduced unit. β_A: Planck Factor for constituent particle of an ARP under self field of the dipole array. s: Length of an ARP dipole, in reduced unit. l: Distance between symmetry centers of adjacent dipoles of the ARP array, in reduced unit. Γ_D^n : n^{th} derivative of digamma function.

Therefore,

$$\beta_A = \left(\eta + (1-\eta)\exp\left[\frac{7}{2l}\left(\Gamma_D^0\left[|x|\right] + \Gamma_D^0\left[1-|x|\right] + 2\gamma_E\right)\right]\right)^{1/7}, \ |x| \le \frac{1}{2}. \quad (38)$$

γ_E : Euler's constant γ.

Under infinite space approximation, uniform static field does not alter Planck Factor of a particle therein. Therefore, condition for breakdown of electrogravitation vacuum under external static electric field is

$$\beta_A^{1/2}\mathcal{E}_0 + \mathcal{F}_{+,A} = 0, \ \frac{d}{dx}\left(\beta_A^{1/2}\mathcal{E}_0 + \mathcal{F}_{+,A}\right) = 0. \quad (39)$$

$\mathcal{E}_0$: Strength of external static electric field, in reduced unit.

If the strength of static electric field measured at ~3 × 10^8 volts per meter [30] was indeed due to breakdown of physical vacuum then

$$\mathcal{E}_{0,\text{VB}} \approx 1.65\times10^{-12}\frac{E_{e,i}}{\boldsymbol{e}r_e} \rightarrow \begin{matrix} s_{\text{VB}} \approx 0.037\,5 \\ l_{\text{VB}} \approx 0.079\,1 \end{matrix} \rightarrow n_A \equiv \frac{1}{l_{\text{VB}}} \approx 12.6\ r_e^{-1}. \quad (40)$$

$\mathcal{E}_{0,\text{VB}}$: Strength of static electric field applied to physical vacuum, at which, electrical breakdown of the vacuum gap between the electrodes was observed [30]. s_{VB} : Dipole length of an ARP in ARP dipole array at breakdown of electrogravitation vacuum, in reduced unit. l_{VB} : Distance between symmetry centers of adjacent ARPs in the dipole array at breakdown of the vacuum, in reduced unit. n_A : Number density of ARP dipole array along field line of applied static electric field.

Static gravitation field shall also polarize ARPs therein. Condition for the breakdown of electrogravitation vacuum under external static gravitation field is

$$\frac{\mu}{\eta\rho_G^2\beta_A^3} + \mathcal{F}_{+,A} = 0, \ \frac{d}{dx}\left(\frac{\mu}{\eta\rho_G^2\beta_A^3} + \mathcal{F}_{+,A}\right) = 0, \ \mu \equiv \frac{m_{e,i}}{M_i}, \ \rho_G \equiv \frac{r}{r_G}, \ r_G \equiv \frac{G_i M_i}{c_i^2}. \quad (41)$$

$m_{e,i}$: Rest mass of electron in Res State. M_i : Rest mass causing the static gravitation field, in Rest State. r : Distance between symmetry center of an ARP in the ARP dipole array and center of the gravitation field. r_G : Characteristic length of the gravitation field.

Therefore, if number density of ARPs in electrogravitation vacuum were indeed ~12.6 per classical electron radius then, within a distance of about 7.17×10^{10} m from center of the Sun, strength of Sun's gravitation field would be able to dissociate ARPs in the vacuum surrounding the Sun. Consequently, there should be a charge sphere within ~103 radius of the Sun. However, such sphere of charges has not been observed. Therefore, the density estimation of Expression (40) is overestimated. In other words, the breakdown voltage observed in the literature was not due to breakdown of the vacuum but instead other materials involved in the measurement device/setup. In general, existence of the charge sphere surrounding a black hole has never been observed. It is therefore reasonable to assume that even the strongest gravitation field, *i.e.*, the field at Schwarzschild Sphere of massive body, cannot dissociate ARPs in physical vacuum. Accordingly, let

$$\rho_G = 4, \ M_i = 3M_S \rightarrow l \approx 0.113\,892 \rightarrow n_A \approx 8.78\ r_e^{-1}. \quad (42)$$

M_S : Rest mass of Sun in Rest State.

That is, number density of ARPs in electrogravitation vacuum should not exceed 677 ARPs per reduced unit volume, or ~5 × 10^{19} kilomoles per cubic meter, which is still much denser than any ordinary matter known today. Accordingly, density of rest energy of electrogravitation vacuum is ~4 × 10^{27} J·m^{-3}. In contrast, density of rest mass of electrogravitation vacuum is zero in any case.

From Equation (38), self energy of constituent particles of an ARP array is the lowest when the ARPs are in particle amalgamation configuration, and the highest if length of an ARP dipole is one half of the distance between symmetry centers of adjacent ARPs, referred to herein as midpoint. Self energy of constituent particle of the ARPs at midpoint configuration is a function of number density of the ARP array. The denser the array is, the lower the energy at midpoint will be, hence the shallower the potential well of the particle will be, which also leads to flatter bottom of the potential well, as illustrated in **Figure 4(a)**. Further, potential well of constituent particle of an ARP array is a periodic function along the array, denoted as *z*-axis, as illustrated in **Figure 4(a)**. Accordingly, β_A^{-2} resembles a rectangular spacial wave along the array, as shown in **Figure 4(b)**, which is similar to a case of one-dimensional periodic potential well in classical wave mechanics, wherein, height of the energy barriers is finite and bottom of the potential wells nonzero. However, the most stable location for a constituent particle of ARP array is at middle of the "energy barrier" instead of bottom of the "potential well."

Since Planck Factor of constituent particle of ARP array is an even function with periodicity l along the array, cosine function can be used as base functions to express the wave functions,

$$\begin{gathered}\frac{d^2\Psi}{dx^2}+\frac{\alpha_e^2 l^2}{4}\left(\frac{\epsilon^2}{\beta_A^2}-1\right)\Psi=0,\ \Psi\equiv\sum_{m=0}^{\infty}c_m\cos[2m\pi x],\ \sum_{m=0}^{\infty}c_m^2\neq 0\ \rightarrow\\ \left(\frac{\epsilon}{\kappa}\right)^2\sum_{m=0}^{\infty}c_m\frac{\cos[2m\pi x]}{(\beta_A/\kappa)^2}-\sum_{m=0}^{\infty}c_m\gamma_m\cos[2m\pi x]=0,\ \gamma_m\equiv 1+\left(\frac{4m\pi}{\alpha_e l}\right)^2\end{gathered}. \tag{43}$$

Ψ : Wave function associated with constituent particle of an ARP array.

Therefore,

$$\left(\frac{\epsilon}{\kappa}\right)^2\sum_{m=0}^{\infty}\frac{b_{n+m}+b_{|n-m|}}{2}c_m=\sum_{m=0}^{\infty}\frac{\delta_{n+m,0}+\delta_{n,m}}{4}\gamma_m c_m,\ b_k\equiv\int_0^{1/2}\left(\frac{\kappa}{\beta_A}\right)^2\cos[2k\pi x]dx. \tag{44}$$

In matrix form,

$$\left(\boldsymbol{\mu}-\left(\frac{\kappa}{\epsilon}\right)^2\boldsymbol{\varepsilon}\right)\boldsymbol{c}=\boldsymbol{0},\ \begin{matrix}\mu_{0,0}=b_0, & \mu_{n,m}=\frac{1}{2}\left(b_{n+m}+b_{|n-m|}\right)\\ \varepsilon_{0,0}=\frac{1}{2}, & \varepsilon_{n,m}=\frac{1}{4}\delta_{n,m}\gamma_m\end{matrix},\ n,m=1,2,\cdots. \tag{45}$$

$\boldsymbol{c}$: Column vector of linear combination coefficients of cosine base functions, real numeral and constant, satisfying Condition (43). $\boldsymbol{0}$: Column vector of zeros.

Therefore,

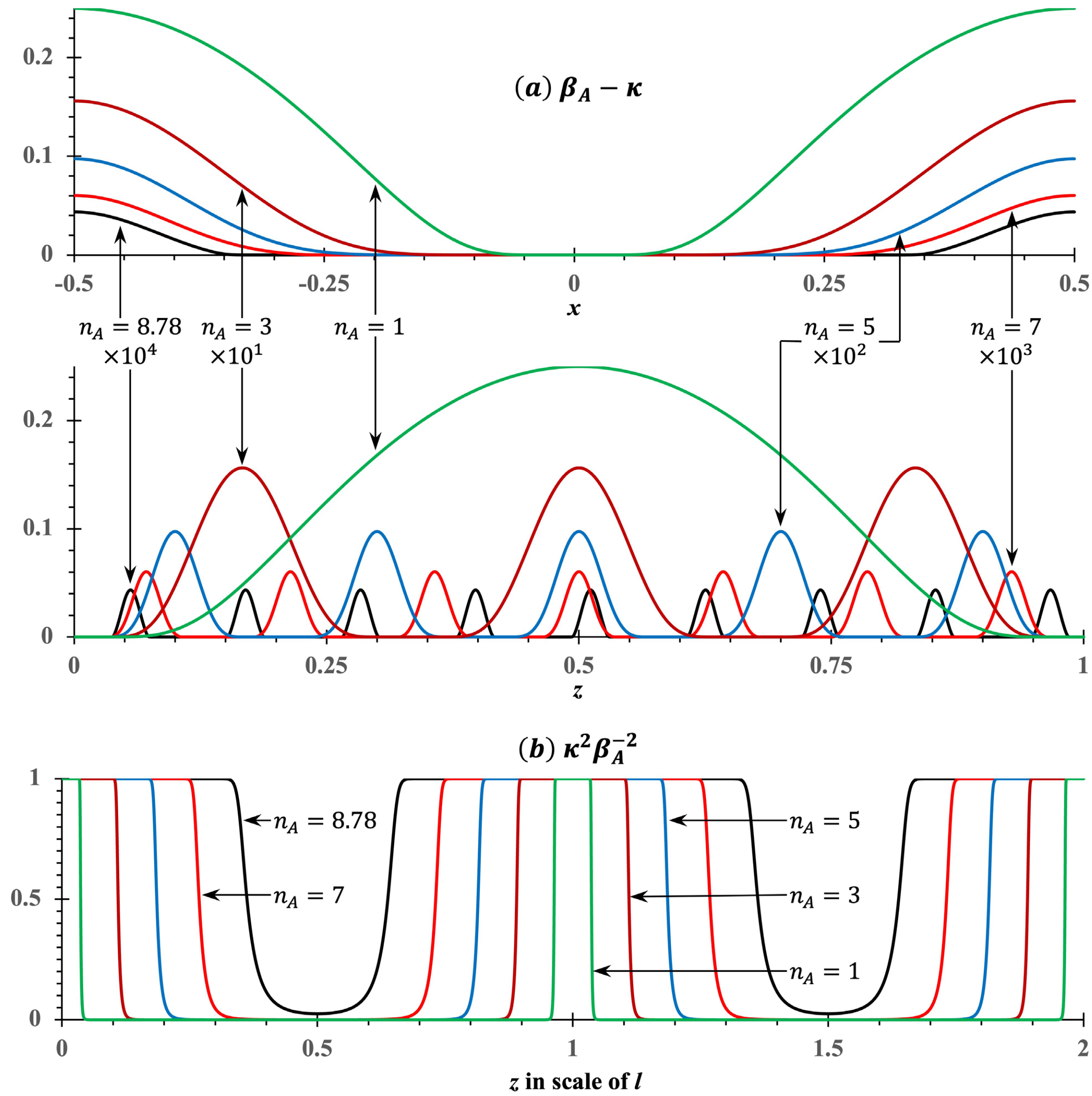

Figure 4. Potential well of constituent particle of ARP array. Units of the figure are all expressed in reduced units, e.g., energy in unit of $E_{e,i}$ and distance in unit of r_e unless specified otherwise.

$$\left(\lambda - \left(\frac{\kappa}{\epsilon}\right)^2 \mathbf{I}\right)c = \mathbf{0},\ \lambda \equiv \varepsilon^{-1}\mu \ \rightarrow\ \frac{\epsilon_n}{\kappa} = \frac{1}{\sqrt{\lambda_n}}. \tag{46}$$

$\mathbf{I}$: Identity matrix. λ_n: The nth eigenvalue of matrix λ.

Total energy of a constituent particle of ARP array is thus discrete by wave mechanics. Table 2 lists some energy levels of constituent particles in such array with ARP number densities of interest. It can be seen from the table that there is always a ground state, which corresponds to $n = 0$, in association with ARP of the array of various densities, for which, excess energy of the state, also known as zero-point energy, is nonzero. Therefore, in particle picture, oscillation of ARP array is natural, even in ground state under no influence of external field. On the other hand, such oscillation does not cause emission of photon as Maxwell electrodynamics would have demanded otherwise. Note also that, for $n_A \geq 3$, the first excited states are electric conduction states, since ϵ_1 exceeds the midpoint energy of the corresponding potential well. Therefore, particles in such states are not bounded locally but instead delocalized.

Table 2. Energy level of near ground states of constituent particle of ARP array, scaled with κ.

n \ n_A	8.78	7	5	3	1
$\epsilon_{x=1/2}$	6.342 82	74.835 2	1,197.36	19,157.8	306,525
⋮	⋮	⋮	⋮	⋮	⋮
4	80,221.2	88,272.6	91,546.8	92,787.7	93,096.7
3	60,644.7	66,337.0	68,750.2	69,670.5	69,900.0
2	40,636.7	44,286.2	45,875.8	46,484.5	46,636.5
1	20,375.9	22,161.3	22,950.5	23,253.4	23,329.1
0	1.161 39	1.356 08	1.632 78	2.121 75	3.680 95
x_ϵ	0.353 277	0.267 349	0.187 755	0.113 457	0.039 326
T_0	0.162 078	0.116 120	0.097 870	0.088 867	0.085 505

Figure 5 plots out some wave functions of constituent particle of an ARP array. As shown in Figure 5(a), Ψ_0 is essentially unity across the entire domain, including regions where total energy of the state is lower than that of the corresponding potential well. Such is commonly interpreted as tunneling of the particle associated with the wave function. However, as can be seen in Figure 5(b) and Figure 5(c), the wave functions have spacial ripples mainly in flat region of bottom of the potential well but essentially unity everywhere outside such region, and the latter is much wider than the former if line density of an ARP array is not too high. Thus, if $|\Psi|^2$ is interpreted as probability of particle presence at location of space then constituent particles of ARP array would have to be explained as most likely being present at outside of the corresponding potential well instead of the inside. Therefore, probability interpretation of physical meaning of wave function of wave mechanics is dubious in such case.

9. Photon Deflection in Static Electric Field

As analyzed, an ARP in ground state is neutral in both electrics and gravitation and momentumless in kinematic motion. On the other hand, ARP is polarizable under any field of any nonzero strength. Therefore, electrogravitation vacuum is neutral in electrics and gravitation and momentumless in kinematic motion but polarizable under any field of any nonzero strength. It is known that physical vacuum under static gravitation field shall deflect photon [8] [31]. If physical vacuum is indeed electrogravitation vacuum then such field shall polarize ARPs therein. Therefore, deflection of photon by gravitation field in physical vacuum

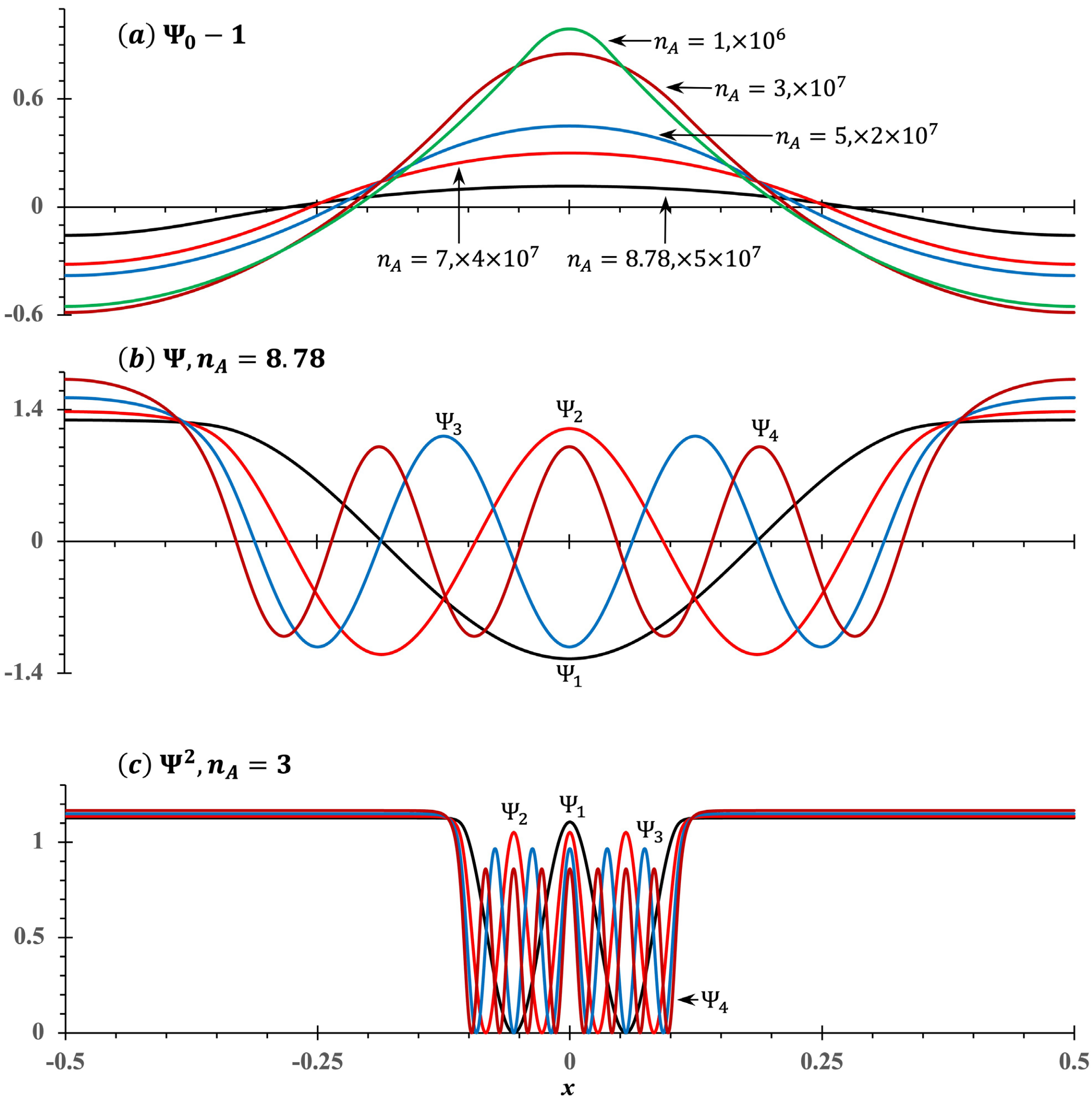

Figure 5. Wave functions of constituent particle of ARP array.

may be understood as caused by polarization of the vacuum under the field. Accordingly, electrogravitation vacuum in static electric field should also deflect photon if physical vacuum is indeed electrogravitation vacuum.

From Equation (39), polarization of ARP dipole array under static electric field is

$$\begin{gathered}\beta_A^{1/2}\mathcal{E}_0-\frac{1}{l^2}\left(\Gamma_D^1[x]-\Gamma_D^1[1-x]\right)\left(\beta_A-\frac{\eta}{\beta_A^6}\right)=0\ \rightarrow\\ \frac{1}{l^2}\left(\Gamma_D^1\left[x_{\mathcal{E}_0}\right]-\Gamma_D^1\left[1-x_{\mathcal{E}_0}\right]\right)\left(\beta_{A,\mathcal{E}_0}-\frac{\eta}{\beta_{A,\mathcal{E}_0}^6}\right)=\mathcal{E}_0\beta_{A,\mathcal{E}_0}^{1/2}\end{gathered}. \tag{47}$$

$\mathcal{E}_0$: Strength of external static electric field, in reduced unit. $\beta_{A,\mathcal{E}_0}$: Planck Factor of constituent particle of ARP array under external static electric field of strength $\mathcal{E}_0$. $x_{\mathcal{E}_0}$: Polarization fraction of ARP dipole under external static electric field having strength $\mathcal{E}_0$.

To achieve the same degree of polarization, from Equation (41),

$$\frac{\mu}{\eta\rho_G^2}\frac{1}{\beta_{A,\mathcal{E}_0}^3}-\frac{1}{l^2}\left(\Gamma_D^1\left[x_{\mathcal{E}_0}\right]-\Gamma_D^1\left[1-x_{\mathcal{E}_0}\right]\right)\left(\beta_{A,\mathcal{E}_0}-\frac{\eta}{\beta_{A,\mathcal{E}_0}^6}\right)=0 \rightarrow$$
$$\rho_{G,\mathcal{E}_0}^2=\frac{\mu}{\eta\beta_{A,\mathcal{E}_0}^3}\frac{l^2}{\left(\Gamma_D^1\left[x_{\mathcal{E}_0}\right]-\Gamma_D^1\left[1-x_{\mathcal{E}_0}\right]\right)\left(\beta_{A,\mathcal{E}_0}-\frac{\eta}{\beta_{A,\mathcal{E}_0}^6}\right)}. \tag{48}$$

$\rho_{G,\mathcal{E}_0}$: Distance from center of static gravitation field, in reduced unit, at which, ARPs polarized by gravitation field is of same degree as that by static electric field of strength $\mathcal{E}_0$.

Therefore,

$$\rho_{G,\mathcal{E}_0}=\sqrt{\frac{\mu}{\eta\mathcal{E}_0\beta_{A,\mathcal{E}_0}^{7/2}}}. \tag{49}$$

To first order approximation, angle of photon deflection in gravitation field is [8]

$$\theta_{D,G}\simeq\frac{4}{\rho_G} \rightarrow \theta_{D,\mathcal{E}_0}\simeq 4\sqrt{\frac{\eta\mathcal{E}_0\beta_{A,\mathcal{E}_0}^{7/2}}{\mu}}. \tag{50}$$

$\theta_{D,G}$: Angle of photon deflection in static gravitation field, in unit of radian. $\theta_{D,\mathcal{E}_0}$: Angle of photon deflection in static electric field, in unit of radian.

Under static electric field of practically realizable strength,

$$\mathcal{E}_0<1.65\times10^{-12} \rightarrow \beta_{A,\mathcal{E}_0}\simeq\kappa \rightarrow \theta_{D,\mathcal{E}_0}\simeq 4\sqrt{\frac{\mathcal{E}_0\eta^{3/2}}{\mu}}. \tag{51}$$

Thus, with the mass ratio from Expression (42) and field strength from Expression (40),

$$\theta_{D,\mathcal{E}_{0,\max}}\approx 0.03\ \text{arcsec}. \tag{52}$$

That is, in physical vacuum under static electric field of 3×10^8 V·m^{-1}, a photon passing through the vacuum shall be deflected by about 30 milliarcseconds by the electric field. While the deflection is minute, such should be measurable via, e.g., collimated laser beam in ring-down cavity immersed in the vacuum under the field. If spacing between reflection mirrors in such cavity is 5 meters long and light of the laser is bounced back and forth between the mirrors for a million times then drift of the light spot at the mirrors should be ~0.4 mm.

10. Hubble-Lemaître Law in Electrogravitation Vacuum

When external electric field is applied to an ARP, the ARP shall be polarized under such excitation. That is, constituent particles of the ARP shall move away from their ground state position towards along direction of the external field respectively. While transient, such motion is or is equivalent to an electrical current. According to electrodynamics of Maxwell, such electrical current shall cause the generation of a transient magnetic field surrounding the electrical current of the ARP. Consequently, ARPs in neighborhood of the ARP shall be polarized by the magnetic field. That is, constituent particles of these ARPs shall move away from their respective ground state positions towards along the same direction as that of the external electric field, respectively. Such motion of constituent particles of these ARPs shall cause further generation of a transient magnetic field, which shall cause further polarization of ARPs farther away, and so on and so forth. Therefore, electromagnetic wave shall propagate in electrogravitation vacuum, and propagation of such wave in such medium is along transverse direction of the excitation electric field. In contrast, polarization of ARP under gravitation field is merely variation of dipole length of the ARP along the field but not alteration of direction of polarization. Therefore, in electrogravitation vacuum, gravitation wave is a longitudinal wave.

From the analysis in **Appendix C**, oscillation frequency of an ARP dipole generally does not follow the frequency of the external electric field, and only the velocity profile, hence temporal profile of the electrical current, of constituent particles of ARPs under excitation that follows frequency of the excitation. Therefore, propagation of electromagnetic wave in electrogravitation vacuum is via coupling of ARPs in the vacuum through the magnetic fields induced by kinematic motions of constituent particles of the ARPs. Due to the differences in propagation mechanism, propagation velocity of gravitation wave in electrogravitation vacuum is not necessarily equal to c, unless pre-assumed so. Alteration of dipole length of ARP by gravitation wave may also cause generation and propagation of electromagnetic wave. However, the latter shall propagate along transverse direction of the gravitation wave.

Since propagation of electromagnetic wave in electrogravitation vacuum involves kinematic motions of massive particles, *i.e.*, constituent particles of ARPs involved in the polarization, propagation velocity of electromagnetic wave in electrogravitation vacuum must be finite. Further, speed of light c in field-free vacuum is constant. Therefore, if physical vacuum is electrogravitation vacuum then volumetric density of ARPs in the electrogravitation vacuum must be uniform on average at relevant length scale.

In addition to electromagnetic field, kinematic motion of constituent particles of ARP shall also cause motion of the gravitation fields associated with the particles, and motion of such field is energy dissipative due to retardation of self field to motion of mass object causing the field, and rate of such dissipation is proportional to at least first order of the velocity of the moving mass associated with the field [8]. Therefore, electrogravitation vacuum is energy dissipative to nonstatic excitation, including electromagnetic wave. Therefore, photon travel in electrogravitation vacuum shall accrue energy loss,

$$\begin{gathered}\frac{d\epsilon_p}{d\tau}=-\gamma\sum_k\left|u_k\right| \;\rightarrow\; d\epsilon_p=-\gamma\sum_k\left|u_k\right|d\tau \;\rightarrow\\ \epsilon_p\left[\tau+\frac{\delta\tau}{2}\right]-\epsilon_p\left[\tau-\frac{\delta\tau}{2}\right]=-\gamma\sum_k\int_{\tau-\delta\tau/2}^{\tau+\delta\tau/2}\left|u_k\right|d\tau\end{gathered}. \tag{53}$$

ϵ_p : Energy of a photon as measured at rest in Rest Frame [22], in reduced unit. τ : Rest Time in reduced unit. γ : Proportion parameter for gravitative energy dissipation of mass particle in motion driven by electromagnetic wave associated with the photon at same moment of Rest Time, in reduced units. u_k : Reduced velocity of constituent particles of ARPs in motion driven by the electromagnetic wave associated with the photon at same moment of Rest Time.

The integration term on the right hand side of the equation is but total distance traveled by a constituent particle of the ARPs during $\delta\tau$, which, as analyzed in **Appendix C**, is $2s_\varepsilon$ per n_ε excitation periods, and n_ε is inversely proportional to strength of the excitation electric field if the strength is not too high. Therefore, ignoring high order terms on the left hand side of Equation (53),

$$\delta\tau\frac{d\epsilon_p}{d\tau}\simeq-\gamma\sum_k\frac{2s_{\varepsilon,k}}{n_\varepsilon T_p}\delta\tau,\, n_\varepsilon\propto\frac{1}{\mathcal{E}_p}\;\rightarrow\;\frac{d\epsilon_p}{d\tau}\propto-\frac{\mathcal{E}_p}{T_p}\sum_k s_{\varepsilon,k}\,. \tag{54}$$

s_ε : Saturation length of an ARP dipole polarized by electric field of an electromagnetic wave, in reduced unit. T_p : Temporal period of the electromagnetic wave associated with a photon, in Rest Time in reduced unit. n_ε : Number of the temporal periods, during which, a constituent particle of the ARP completes one forced oscillation. $\mathcal{E}_p$: Maximum strength of the electric field of the electromagnetic wave associated with the photon, in reduced unit.

At any moment of Rest Time, volume occupied by the electromagnetic wave associated with a photon is finite, therefore,

$$\overline{s}_{\varepsilon,p}\equiv\frac{\sum_k s_{\varepsilon,k}}{2V_p n_{\text{EGV}}},\, V_p=A_pN_p\lambda_p\;\rightarrow\;\frac{d\epsilon_p}{d\tau}\propto-\frac{\lambda_p\mathcal{E}_p}{T_p}\overline{s}_{\varepsilon,p}\,. \tag{55}$$

$\overline{s}_{\varepsilon,p}$: Average saturation length of ARP dipoles polarized by electric field of electromagnetic wave associated with a photon, in reduced unit. V_p : Volume of electrogravitation vacuum effected by the electromagnetic wave associated with the photon at same moment of Rest Time, in reduced unit. n_{EGV} : Volumetric number density of ARPs in electrogravitation vacuum, in reduced units. A_p : Cross sectional area of V_p along direction of the photon, in reduced unit. λ_p : Wavelength of the electromagnetic wave associated with the photon, in reduced unit. N_p : Number of spacial waves of the photon.

In reduced unit,

$$\epsilon_p=\frac{2\pi}{\alpha_e T_p}=\frac{2\pi}{\alpha_e\lambda_p}\;\rightarrow\;\frac{d}{d\tau}\frac{1}{\lambda_p}\propto-\mathcal{E}_p\overline{s}_{\varepsilon,p}\,. \tag{56}$$

α_e : Fine structure constant.

From Equation (A21),

$$s_\varepsilon\simeq-\frac{7}{4}W_{-1}\left[-\frac{7\eta^{3/7}\mathcal{E}_0^{1/2}}{4\sqrt{1-\eta}}\right]^{-1}\;\rightarrow\;\frac{d}{d\tau}\frac{1}{\lambda_p}\propto\mathcal{E}_pW_{-1}\left[-\frac{7\eta^{3/7}\mathcal{E}_p^{1/2}}{4\sqrt{1-\eta}}\right]^{-1}. \tag{57}$$

By definition,

$$\mathcal{E}_p^2\propto\frac{\epsilon_p}{V_p}=\frac{2\pi}{\alpha_eA_pN_p\lambda_p^2}\;\rightarrow\;\frac{d}{d\tau}\frac{1}{\lambda_p}=\frac{a}{\lambda_p}W_{-1}\left[-\frac{b}{\sqrt{\lambda_p}}\right]^{-1}. \tag{58}$$

a,b : Proportion constants to be determined.

For photon travel in vacuum, in reduced units,

$$\frac{ds}{d\tau}=1 \rightarrow \frac{d}{ds}\frac{1}{\lambda_p}=\frac{a}{\lambda_p}W_{-1}\left[-\frac{b}{\sqrt{\lambda_p}}\right]^{-1} \rightarrow$$
$$s=\frac{1}{a}\left(W_{-1}\left[-\frac{b}{\sqrt{\lambda_p}}\right]-W_{-1}\left[-\frac{b}{\sqrt{\lambda_0}}\right]\right)\left(2+W_{-1}\left[-\frac{b}{\sqrt{\lambda_p}}\right]+W_{-1}\left[-\frac{b}{\sqrt{\lambda_0}}\right]\right). \quad (59)$$

s : Path length of photon travel in vacuum, in reduced unit. λ_0 : Wavelength of the photon at $s=0$ in vacuum, in reduced unit.

By definition of redshift,

$$z\equiv\frac{\lambda_p-\lambda_0}{\lambda_0}=\frac{\lambda_p}{\lambda_0}-1,\ b_\lambda\equiv\frac{b}{\sqrt{\lambda_0}} \rightarrow$$
$$s=\frac{1}{a}\left(W_{-1}\left[-\frac{b_\lambda}{\sqrt{1+z_{\mathrm{HL}}}}\right]-W_{-1}\left[-b_\lambda\right]\right)\left(2+W_{-1}\left[-\frac{b_\lambda}{\sqrt{1+z_{\mathrm{HL}}}}\right]+W_{-1}\left[-b_\lambda\right]\right). \quad (60)$$

z : Redshift of photon. z_{HL} : Hubble-Lemaître redshift of photon, caused by photon travel in physical vacuum over path length s.

This is the expression for Hubble-Lemaître law [32] in electrogravitation vacuum. With the set of the characteristic peaks observed in the redshift distribution of quasars [32], parametric fitting of Equation (60) results in

$$b_\lambda\simeq\frac{1}{e} \rightarrow s_R=\frac{1}{a}\left(1+W_{-1}\left[-\frac{1}{e\sqrt{1+z_{\mathrm{HL}}}}\right]\right)^2,\ \left.\frac{ds}{dz_{\mathrm{HL}}}\right|_{z_{\mathrm{HL}}=0}=\frac{1}{a} \rightarrow. \quad (61)$$
$$a\approx 0.09595,\ z_G\approx 0.971,\ z_R=0.0831,\ z_D=0.1613$$

s_R : Path length of photon travel in physical vacuum, in unit of internal radius of physical space. z_G : Common gravitation redshift of quasars [32]. z_R : Hubble-Lemaître redshift of path length of internal radius of physical space. z_D : Hubble-Lemaître redshift of path length of internal diameter of physical space.

From Hubble-Lemaître Correlation [33] [34],

$$s=\frac{c_i}{H_0}z_{\mathrm{HL}} \rightarrow R_i\simeq\frac{ac_i}{H_0}\approx 1.34\ \text{Billion Lightyears}. \quad (62)$$

H_0 : Hubble constant, 70 km/Mpc [35]. R_i : Internal radius of physical space.

Therefore, internal radius of physical space is about 30% larger than that as estimated previously. **Figure 6** plots out relationship of path length of photon in electrogravitation vacuum and Hubble-Lemaître redshift of photon in same. Nonlinearility of the Hubble-Lemaître law in electrogravitation vacuum is evident.

11. Discussion

As analyzed in this essay, physical existence of ARP is inevitable under the law of energy conservation and the law of mass-energy conservation. It is due to their neutrality in electrics and gravitation and momentumless in kinematic motion that ARPs have never been observed in direct manner. On the other hand, the existence of electrogravitation vacuum has long been anticipated [3] [36], commonly known as aether, that can mediate forces over seemingly void/empty space

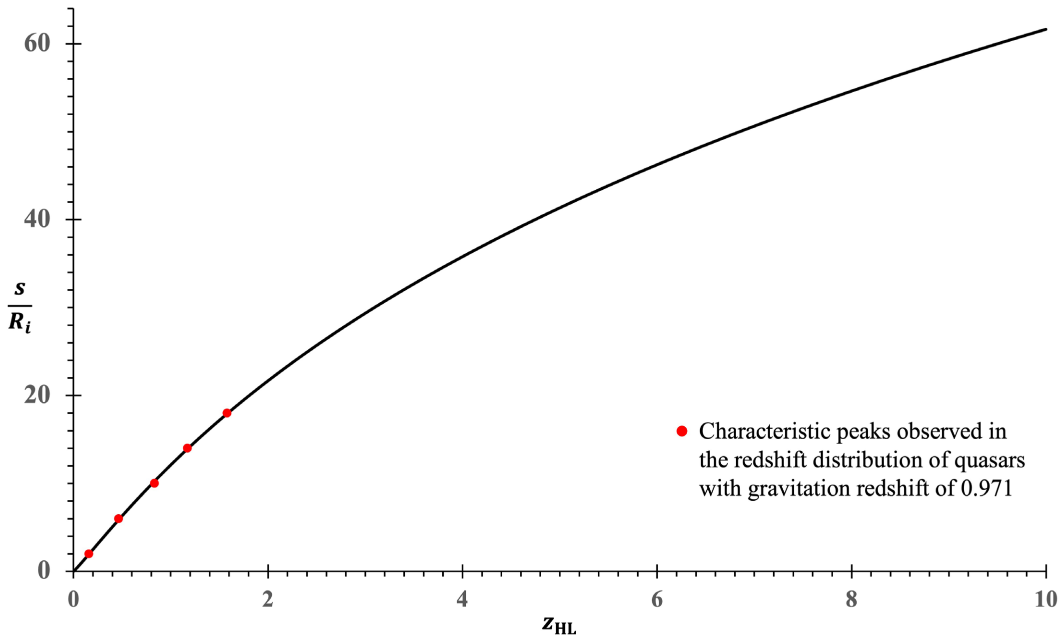

Figure 6. Hubble-Lemaître law in electrogravitation vacuum.

and propagate electromagnetic waves. However, all aether theories had to assume the existence of a medium of nonzero mass so that mechanical tools, such as Newtonian mechanics, can be applied thereto. Such assumption led inevitably to the notion of "vacuum dragging" or alike. A recent work even managed to create a unifying physical theory of everything based only on an aether of nonzero mass [37]. In contrast, physical vacuum is massless and so as the electrogravitation vacuum. Therefore, vacuum is momentumless to kinematic motion of mass objects therein, rendering mechanical tool of any type useless. Further, by the law of mass-energy conservation, object of nonzero rest mass can and only travel at speed less than that of light, and by logical consistency, light, *i.e.*, photon, a particle of zero rest mass, can but only travel at speed of light. In contrast, ARP is a massless particle and there is no law of physics nor regulation of any kind that constrains its motion in space. Therefore, all motion related concepts are inapplicable to ARP hence the vacuum made from ARPs. Since any mass body has a gravitation field in association, which shall polarize vacuum in surrounding, which shall alter properties of the vacuum in vicinity, hence cause photon deflection, which may appear as if vacuum in vicinity of mass body was dragged by the body. As a medium for propagation of electromagnetic wave, velocity of such propagation is and must be independent of velocity of light source, and constant if but only if environment or conditions of the vacuum are identical and invariant, as has been assumed by Einstein in his theory of relativity. On the other hand, while an ARP in ground state is massless, an ARP in polarization states has gravitation field in association and local strength of the field is nonzero, although rapidly alternating (**Appendix C**). Therefore, polarized ARP, hence polarized physical vacuum, shall have nonzero momentum mass hence is not momentumless, even though time averaged momentum mass is zero.

As shown previously [24], electron and positron are gravitationally repulsive to each other. Therefore, by the law of Gauss gravitation, total rest mass of an electron-positron pair must be zero whether or not the pair is in amalgamation configuration. Therefore, rest mass of electron and that of positron must be of opposite sign. However, the negative rest mass herein is fundamentally different from dark matter [38], for the former merely means direction of gravitation force caused by the mass is opposite to that of normal mass and gravitation inverse matter is as visible as that of gravitation normal matter while the latter is still gravitation normal matter but only invisible to current observer.

As analyzed in **Section 4**, energy of gravitation field has to be regarded as negative with respect to that of electrostatic field in order to maintain conservation of the energies during electron-positron annihilation process. This negative field energy is fundamentally different from dark energy [39], since the latter is still considered as positive energy and presumably has normal gravitation effect.

Existence of electrogravitation vacuum has also been anticipated by Dirac [17], known now as Dirac Sea. However, Dirac Sea is a hypothetic matter, in which, positron is regarded as a vacant state while electron an occupied state of the matter. In contrast, electrogravitation vacuum is real matter, in literal sense, which is comprised of ARPs that are real and physical particles. Nevertheless, all things considered, it can be recognized that electrogravitation vacuum is but concretization of Dirac Sea, and ARP is but Dirac's electron in ground state. Further, positron in electrogravitation vacuum is real particle, as real as electron. Therefore, kinetic energy of positron is positive, as positive as that of electron. From **Table 2**, density of excess energy, hence that of kinetic energy, of electrogravitation vacuum is ~6×10^{26} J·m^{-3}.

For simplicity and other reasons, this analysis ignored internal structure/property of elementary particles, e.g., spin, magnetic moment, etc. On the other hand, electron is known to have spin, so as positron. Further, spin energy of electron and that of positron are both positive and invariant regardless of state/configuration of an electron-positron pair. Therefore, even in ground state, an ARP hence electrogravitation vacuum shall have nonzero spin energy in addition to the excess energy. Given the number density of ARPs in electrogravitation vacuum, spin energy of physical vacuum may also be significant even though that of an individual particle may be minute.

12. Summary

In compliance with mass-energy conservation law, refined laws of Coulomb electrostatics and Newton gravitation eliminated divergence problem in field energy calculation. Finite and definitive field energies of electron and positron enable thorough checkup of energy balance of electron-positron annihilation process without leakage. Instead of electrostatic interaction alone, combined interactions of electrostatics and gravitation eliminated divergence problem of rest mass of electron and position upon annihilation hence enabled thorough checkup of matter balance of the annihilation process without leakage. Examination of electron-positron annihilation process reveals the existence of a particle of unknown type, named ARP, as a necessary product of the process in addition to photons. Under the law of mass-energy conservation, in conjunction with the null result of annihilation experiments, self/rest mass of ARP must be none while self/rest mass of constituent particles of ARP is and must be massive. Therefore, *self/rest mass* must not be an unsigned attribute. Field energy of ARP is none. Therefore, under the law of energy conservation, *field energy* must not be an unsigned attribute.

ARP is neutral in electrics and gravitation and massless hence momentumless in kinematic motion but polarizable under any field of any nonzero strength. Therefore, electrogravitation vacuum, *i.e.*, physical space or region therein filled with ARPs, is massless in weight and inertia, neutral in electrics and gravitation, and polarizable under any field of any nonzero strength, hence matches perfectly with all known properties of physical vacuum. Accordingly, number density of ARPs in physical vacuum is estimated up to 5×10^{19} kilomoles per cubic meter.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. Electrogravitation Interaction

Consider an electron-positron pair at rest in $\mathbf{S}^3$ in symmetric particle configuration. Since interaction between charges of the particles of the pair is attraction while that of the masses is repulsion [24], total force experienced by a particle of an electron-positron pair in self field of the pair is, under infinite space approximation,

$$\begin{matrix} \boldsymbol{F}_{+,x} = -\dfrac{E_{e,\boldsymbol{i}}}{r_e}\left(\beta - \dfrac{\eta}{\beta^6}\right)\dfrac{\hat{\boldsymbol{s}}}{s^2} \\ \boldsymbol{F}_{-,x} = +\dfrac{E_{e,\boldsymbol{i}}}{r_e}\left(\beta - \dfrac{\eta}{\beta^6}\right)\dfrac{\hat{\boldsymbol{s}}}{s^2} \end{matrix}, \quad \begin{matrix} \beta \equiv \dfrac{c_x}{c_{\boldsymbol{i}}} \\ s \equiv \dfrac{2r}{r_e} \end{matrix}, \quad \begin{matrix} r_e \equiv \dfrac{\alpha_e \hbar c_{\boldsymbol{i}}}{E_{e,\boldsymbol{i}}} \\ r_g \equiv \dfrac{G_{\boldsymbol{i}} m_{e,\boldsymbol{i}}^2}{E_{e,\boldsymbol{i}}} \end{matrix}, \quad \begin{matrix} \eta \equiv \dfrac{r_g}{r_e} \\ x = f, 0, eg_{\text{EPP}} \end{matrix} . \tag{A1}$$

$\boldsymbol{F}_{\mp}$: Force experienced by particle of an electron-positron pair at rest in self field of the pair. β : Referred to as Planck Factor, for electrogravitation interaction. c_x : SLV as measured at rest at particle location in self field of the electron-positron pair. r : Distance between a particle of an electron-positron pair and symmetry center of the pair. s : Distance between particles of an electron-positron pair, in reduced unit. $\hat{s}$: Unit vector of s at positron location.

Thus,

$$\frac{d\beta}{ds} = \frac{1}{2s^2}\left(\beta - \frac{\eta}{\beta^6}\right) \rightarrow \beta = \left(\eta + (1-\eta)\exp\left[-\frac{7}{2s}\right]\right)^{1/7} . \tag{A2}$$

Therefore,

$$\begin{matrix} \epsilon_{\pm,x} \equiv \dfrac{E_{\pm,x}}{E_{e,\boldsymbol{i}}} = \beta \\ \mu_{\pm,x} \equiv \dfrac{m_{\pm,x}}{m_{e,\boldsymbol{i}}} = \dfrac{1}{\beta} \end{matrix}, \quad \begin{matrix} \lim\limits_{s\to 0} \epsilon_{\pm,x} = \kappa \\ \lim\limits_{s\to 0} \mu_{\pm,x} = \dfrac{1}{\kappa} \end{matrix}, \quad \kappa \equiv \eta^{1/7} . \tag{A3}$$

$E_{\mp,x}$: Self energy of particle of an electron-positron pair at rest in self field of the pair. $m_{\mp,x}$: Self mass of particle of an electron-positron pair at rest in self field of the pair. $\epsilon_{e,x}$: Self energy of particle of an electron-positron pair at rest in self field of the pair, in reduced unit. $\mu_{e,x}$: Self mass of particle of an electron-positron pair at rest in self field of the pair, in reduced unit.

That is, there exists a physically permissible configuration of an electron-positron pair, in which, particles of the pair is allowed to merge completely with each other, *i.e.*, occupying one and the same set of spacial points local simultaneously. Such configuration is referred to as particle amalgamation configuration, and an electron-positron pair in such configuration is referred to as ARP.

Appendix B. Oscillation of Annihilation Residue Particle

For linear motion of constituent particles of an ARP, from Equation (A1) and (A2),

$$d\ln\frac{\beta}{\beta_u} = 0,\ \beta_u \equiv \sqrt{1-u^2},\ u \equiv \frac{d\rho}{d\tau} \rightarrow \frac{\beta}{\beta_u} = \epsilon \rightarrow \epsilon_{\text{ARP}} = 2\epsilon . \tag{A4}$$

β : Planck Factor of constituent particle of ARP. β_u : Lorentz Factor of constituent particle of ARP. u : Velocity of constituent particle of ARP in Rest State, in reduced unit. ρ : Distance between constituent particle of ARP and symmetry center of same, in reduced unit. τ : Rest Time [22] in reduced unit. ϵ : Integration constant, total energy of constituent particle of ARP in Rest State, in reduced unit. ϵ_{ARP} : Total energy of ARP in Rest State, in reduced unit.

Therefore,

$$u^2 = 1 - \left(\frac{\beta}{\epsilon}\right)^2 \rightarrow u_0 \equiv u\big|_{s=0} = \pm\sqrt{1 - \left(\frac{\kappa}{\epsilon_x + \kappa}\right)^2}, \quad \begin{array}{c} \epsilon_x \equiv \epsilon - \kappa \\ 0 \le \epsilon_x \le 1 - \kappa \end{array}. \tag{A5}$$

ϵ_x : Excess energy of constituent particle of ARP, in reduced unit.

That is, if excess energy of a constituent particle of ARP is nonzero then velocity of the particle at amalgamation configuration shall be nonzero. Therefore, the particle shall continue to move along its direction until reaching a turning point, at which, velocity of the particle becomes zero,

$$\rho_\epsilon \equiv \rho\big|_{u=0} = \frac{7}{4}\left(\ln\left[1-\eta\right] - \ln\left[\left(\epsilon_x + \kappa\right)^7 - \eta\right]\right)^{-1}. \tag{A6}$$

ρ_ϵ : Distance between turning point of constituent particle of an ARP in Rest State and symmetry center of same, in reduced unit, at which distance, the particle reverses its direction of motion.

Therefore, constituent particles of an ARP shall oscillate between turning points if excess energy of the particles is nonzero. Temporal period of such oscillation is

$$T_\epsilon = 4\int_0^{\rho_\epsilon} \frac{d\rho}{\sqrt{1-\left(\beta/\epsilon\right)^2}}. \tag{A7}$$

T_ϵ : Temporal period of linear oscillation of ARP, in reduced Rest Time unit.

Therefore, temporal period of the oscillation is a function of excess energy alone. Some of the temporal profiles of linear oscillation of ARP are illustrated in Figure A1. As can be seen from the plot, if excess energy is sufficiently high or sufficiently low then the higher or lower the excess energy the longer the temporal

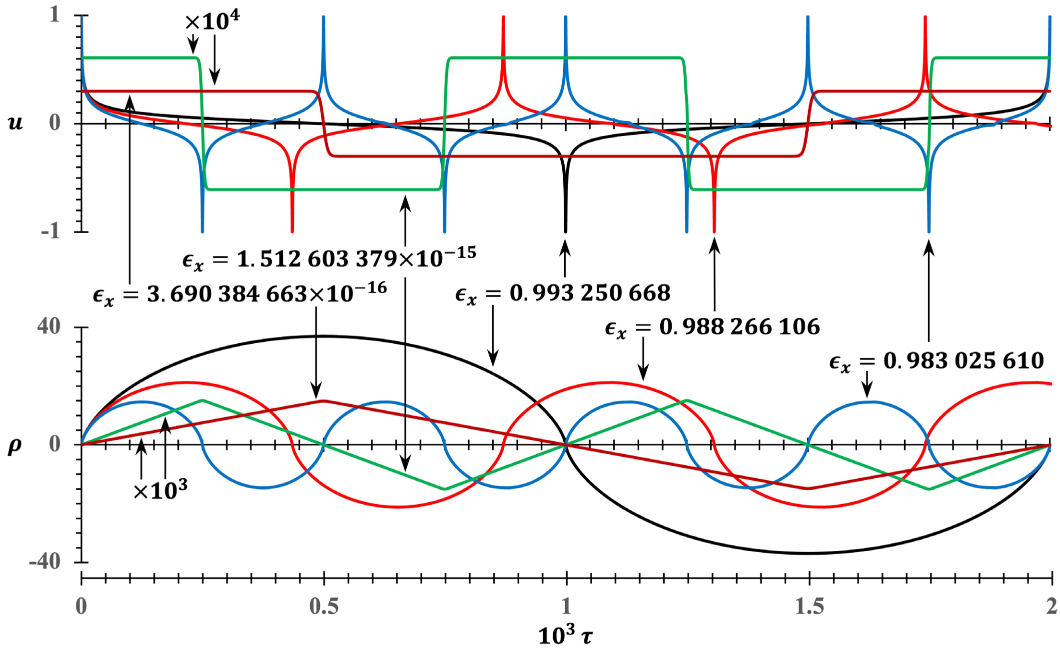

Figure A1. Some temporal profiles of linear oscillation of ARP in rest state.

period will be. The shortest temporal period is found at $T_\epsilon \approx 0.085\,453\,712$, corresponding to $\epsilon_x \approx 1.985\,751\,38 \times 10^{-6}$.

According to Maxwell electrodynamics, oscillatory motion of constituent charges of an ARP shall cause creation of an alternating electromagnetic field of same frequency in its surrounding. If the alternating field causes further creation of alternating fields in further surrounding, and so on and so forth, then an electromagnetic wave of same frequency is created and propagated. In other words, a photon of same frequency is transmitted, along transverse direction of the direction of the polarization of the ARP. Oscillation of an ARP shall also create an alternating gravitation field of same frequency in its surrounding. If the alternating field is propagated then a graviton is transmitted, but in direction parallel to the direction of the polarization of the ARP.

According to the law of Planck on photon energy, energy of photon associated with an electromagnetic wave is, in reduced unit,

$$\epsilon_p \equiv \frac{E_p}{E_{e,i}} = \frac{h\nu_p}{E_{e,i}} = \frac{2\pi\hbar\alpha_e}{\alpha_e E_{e,i} T_p \left(r_e/c_i\right)} = \frac{2\pi}{\alpha_e T_p}. \tag{A8}$$

ϵ_p: Energy of a photon, in reduced unit. E_p: Energy of a photon. ν_p: Frequency of the photon. T_p: Temporal period of electromagnetic wave associated with the photon, in reduced unit. α_e: Fine structure constant.

By the law of energy conservation, the photon emission process must satisfy

$$\epsilon_p = \epsilon_{x,initial} - \epsilon_{x,final},\ T_p = T_{\epsilon,initial} \ \rightarrow\ \left(\epsilon_{x,initial} - \epsilon_{x,final}\right) T_{\epsilon,initial} = \frac{2\pi}{\alpha_e}. \tag{A9}$$

$\epsilon_{x,initial}$ and $\epsilon_{x,final}$: Excess energy of constituent particle of an oscillating ARP before and after emitting a photon, in reduced unit. $T_{\epsilon,initial}$: Temporal period of linear oscillation of a constituent particle of ARP before emitting a photon.

Conversely, if two photons are of same energy and in suitable polarization/entry direction then the photons may be absorbed local simultaneously by an ARP in Rest State via resonance absorption process if total energy of the process is conserved and frequency matched,

$$\epsilon_{x,final} - \epsilon_{x,initial} = \epsilon_p,\ T_{\epsilon,final} = T_p \ \rightarrow\ \left(\epsilon_{x,final} - \epsilon_{x,initial}\right) T_{\epsilon,final} = \frac{2\pi}{\alpha_e}. \tag{A10}$$

Therefore, for states of ARP permissible for photon emission/absorption, ϵ_x has to be a discrete attribute. **Table A1** lists solutions of the equations above. As a consequence, in the examples shown in **Figure A1**, only the one having $\epsilon_x = 0.988\,266\,106$ shall satisfy the energy and frequency requirements, which corresponds to state number $n = 1$ and is allowable to transit between its neighboring states, *i.e.*, $n = 0$ and $n = 2$, via the process of photon emission/absorption. Note that none of the temporal profile of linear oscillation of ARP is purely sinusoidal, as can be seen in **Figure A1**. Therefore, integer multiples of base frequency must exist in such oscillation. Further note that other states of ARP not satisfying Equation (A9) and/or (A10) are possible but photon emission/absorption of such states is forbidden or violation of the law of energy conservation inevitable. On the other hand, prohibition of photon emission/absorption does not prevent an ARP from oscillating, nor creating electromagnetic field in association with such motion but only propagation of the alternating fields.

Table A1. Oscillation states of ARP permissible for photon emission/absorption. Herein, energy is expressed in reduced unit, *i.e.*, 510,998.950 69 electron volts, and temporal period is also expressed in reduced unit, *i.e.*, 9.399 637 133 3 × 10^{-24} s.

n	ϵ	T_ϵ
∞	1	∞
⋮	⋮	⋮
9	0.999 999 473 105 68	2,904,153,363.704 380
8	0.999 999 176 625 96	1,486,651,025.269 860
7	0.999 998 597 456 70	668,698,438.877 868
6	0.999 997 309 847 03	251,732,403.543 574
5	0.999 993 889 458 69	73,533,407.043 831
4	0.999 982 180 187 88	14,765,503.473 083
3	0.999 923 867 067 84	1,672,005.683 393
2	0.999 408 903 162 43	77,277.326 015
1	0.988 266 921 148 04	871.245 681
0	0.000 000 815 594 44	∞

If an ARP is not in ground state, *i.e.*, $\epsilon_x \neq 0$, then, in addition to their linear oscillations, constituent particles of the ARP may have rotation about symmetry center of the ARP. From Equation (21), with Equation (A2),

$$\begin{matrix} \epsilon = \dfrac{\beta}{\beta_u} \\ \gamma = \dfrac{\rho^2\omega}{\beta_u} \end{matrix} \to \begin{matrix} q^2 = 1 - \dfrac{\beta^2}{\epsilon^2}\left(1 + \dfrac{\gamma^2}{\rho^2}\right) \\ \omega^2 = \dfrac{\gamma^2\beta^2}{\epsilon^2\rho^4} \end{matrix} \to \begin{matrix} \epsilon^2 = \dfrac{\left(a^2 - b^2\right)\beta_a^2\beta_b^2}{a^2\beta_b^2 - b^2\beta_a^2}, \\ \gamma^2 = \dfrac{a^2b^2\left(\beta_a^2 - \beta_b^2\right)}{a^2\beta_b^2 - b^2\beta_a^2} \end{matrix} \quad \begin{matrix} q \equiv \dfrac{d\rho}{d\tau} \\ \omega \equiv \dfrac{d\theta}{d\tau} \\ \beta_x \equiv \beta[x] \end{matrix} . \tag{A11}$$

γ : Integration constant, angular momentum of constituent particle of an ARP in Rest State in reduced unit, positive if direction of the associated rotation is assigned as positive direction. a, b : Maximal and minimal distance between constituent particle of the ARP and symmetry of same, in reduced unit. ρ, θ : Radius and angular variable of polar coordinates in plane of motion of the ARP.

That is, total energy and angular momentum of the system is determined completely by the distance extremes a and b. In the simple case of circular motion,

$$\begin{matrix} \epsilon\big|_{b\to a} = \dfrac{\beta_a^{9/14}}{\sqrt{\eta + \left(1 - \dfrac{1}{4a}\right)(1-\eta)e^{-\frac{7}{4a}}}}, \ \gamma\big|_{b\to a} = \dfrac{a}{\sqrt{4a\left(\dfrac{\eta}{1-\eta}e^{\frac{7}{4a}} + 1\right) - 1}} \to \\ a_{\mathrm{TB}} = \dfrac{7}{4\left(7 - W_{-1}^{\lambda}\right)}, \ a_{\mathrm{RB}} = \dfrac{7}{4\left(7 - W_0^{\lambda}\right)}, \ \lambda \equiv -\dfrac{7e^7\eta}{1-\eta}, \ \begin{matrix} \lim\limits_{a\to 0, a_{\mathrm{TB}}^-, a_{\mathrm{RB}}^+, \infty} \epsilon = \kappa, \infty, \infty, 1 \\ \lim\limits_{a\to 0, a_{\mathrm{TB}}^-, a_{\mathrm{RB}}^+, \infty} \gamma = 0, \infty, \infty, \infty \end{matrix} \end{matrix} . \tag{A12}$$

W_k^λ : Lambert W function of branch k.

That is, an ARP in self rotation shall have two classes of states, tight bound states in $\left[0, a_{\mathrm{TB}}\right)$ and regular bound states in $\left(a_{\mathrm{RB}}, \infty\right)$. For regular bound states, there is also a minima in energy as well as angular momentum,

$$\begin{gathered} a_{\epsilon_{\min}} = a_{|\gamma|_{\min}} \equiv a_m \\ a_m \approx \frac{1}{2} - 74.510\eta \end{gathered}, \quad \epsilon_{a_m} = \frac{3}{\sqrt{32 - 7/a_m}} \left(\frac{576\eta}{1 - 2a_m} \right)^{1/7}, \quad \gamma_{a_m} = \frac{a_m \sqrt{4a_m + 7}}{\sqrt{32a_m - 7}}. \quad \text{(A13)}$$

Figure A2 plots out energy and angular momentum as function of radius of an ARP in pure circular motion.

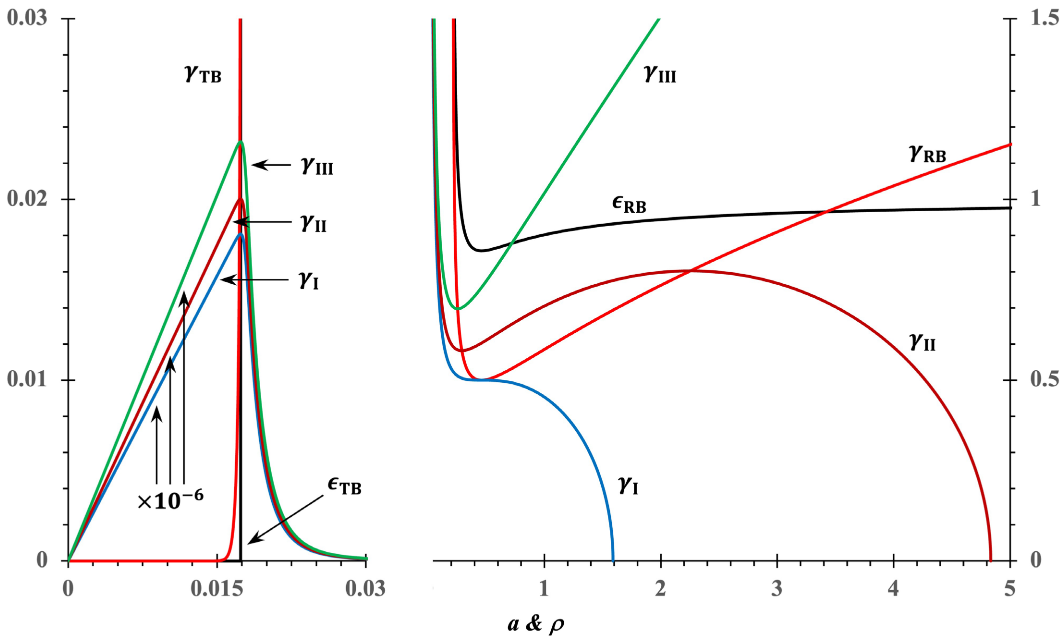

Figure A2. Total energy and angular momentum as function of radius of an ARP in circular motion.

From Equation (A11),

$$\omega_a = \pm \frac{\sqrt{1-\eta}}{2a^{3/2}\beta_a^{7/2}} e^{-\frac{7}{8a}} \rightarrow T_a = 4\pi a^{3/2} \sqrt{1 + \frac{\eta}{1-\eta} e^{\frac{7}{4a}}}. \quad \text{(A14)}$$

ω_a : Angular velocity of constituent particle of an ARP in circular motion of radius a with respect to symmetry center of the ARP, in reduced unit. T_a : Temporal period of the rotation in Rest Time in reduced unit.

Therefore, self rotation of an ARP shall cause creation of an electric field and a gravitation field, both alternating in sinusoidal form, in rotation plane of the ARP. If the alternating electric field causes photon emission/absorption then

$$\begin{gathered} \left(\epsilon_{initial} - \epsilon_{final}\right) T_{a,initial} = \frac{2\pi}{\alpha_e} \\ \left(\epsilon_{final} - \epsilon_{initial}\right) T_{a,final} = \frac{2\pi}{\alpha_e} \end{gathered} \rightarrow \begin{gathered} \left(\epsilon_{initial} - \epsilon_{final}\right) a_{initial}^{3/2} \sqrt{1 + \frac{\eta}{1-\eta} e^{\frac{7}{4a_{initial}}}} = \frac{1}{2\alpha_e} \\ \left(\epsilon_{final} - \epsilon_{initial}\right) a_{finial}^{3/2} \sqrt{1 + \frac{\eta}{1-\eta} e^{\frac{7}{4a_{finial}}}} = \frac{1}{2\alpha_e} \end{gathered}. \quad \text{(A15)}$$

Solutions of the above equation are listed in Tables A2-A4.

Table A2. Tight bound states of ARP in circular motion, all in reduced units.

n_{TB}	ϵ	γ $\times 10^{-8}$	$a_{\mathrm{TB}} - a_{n>0}$ $\times 10^{24}$	$T_{a,1} - T_{a,n>1}$ $\times 10^{21}$
⋮	⋮	⋮	⋮	⋮
9	70,990.092	14.966 985	0.025 244	0.575 626
8	63,102.304	13.303 987	0.031 949	0.573 714
7	55,214.516	11.640 988	0.041 729	0.570 927
6	47,326.728	9.977 990	0.056 798	0.566 632
5	39,438.940	8.314 992	0.081 789	0.559 508
4	31,551.152	6.651 993	0.127 795	0.546 395
3	23,663.364	4.988 995	0.227 192	0.518 063
2	15,775.576	3.325 997	0.511 181	0.437 116
1	7,887.788	1.662 998	2.044 724	0.109 159
0		0	0	∞

Table A3. Regular bound states of ARP in compact circular motion in reduced units.

n_{RBC}	ϵ	γ	$a_{n>0} - a_{\mathrm{RB}}$ $\times 10^{7}$	$T_{a,1} - T_{a,n>1}$ $\times 10^{6}$
⋮	⋮	⋮	⋮	⋮
9	4,934.153	3,353.105	0.013 897	1.044 878
8	4,386.009	2,980.602	0.017 588	1.041 400
7	3,837.865	2,608.100	0.022 971	1.036 327
6	3,289.721	2,235.597	0.031 263	1.028 511
5	2,741.577	1,863.095	0.045 014	1.015 551
4	2,193.433	1,490.592	0.070 324	0.991 698
3	1,645.289	1,118.090	0.124 988	0.940 178
2	1,097.145	745.587	0.281 075	0.793 069
1	549.001	373.085	1.122 548	1.570 797
0	0.857 764	1/2	1/2	4.442 883

Table A4. Regular bound states of ARP in circular motion, in reduced units.

n_{RB}	ϵ	γ	a	T_a
∞	1	∞	∞	∞
⋮	⋮	⋮	⋮	⋮
9	0.999 999 874	497.823 533	991,312.829	$1.240\,298 \times 10^{10}$

Continued

8	0.999 999 804	399.791 774	639,333.599	$6.423\,935 \times 10^{9}$
7	0.999 999 670	307.939 002	379,305.465	$2.935\,577 \times 10^{9}$
6	0.999 999 377	223.991 427	200,688.388	$1.129\,778 \times 10^{9}$
5	0.999 998 615	150.212 013	90,254.346	$3.407\,313 \times 10^{8}$
4	0.999 996 088	89.377 552	31,953.137	$7.177\,614 \times 10^{7}$
3	0.999 984 092	44.322 099	7,857.544	$8.752\,660 \times 10^{6}$
2	0.999 885 719	16.536 790	1,093.612	$4.544\,693 \times 10^{5}$
1	0.997 991 152	3.946 126	62.037	$6.140\,194 \times 10^{3}$
0	0.857 763 885	1/2	1/2	4.442 882 938

From Table A2, for tight bound states, except the ground state, energy levels of an ARP are much higher than that for complete separation of the constituent particles. Therefore, ARP in tightly bounded self rotation is a high energy particle. Since the ARP is unbreakable due to infinity of the energy barrier at a_{TB}, such particle can withhold tremendous amount of energy. The energy differences between the neighboring tight bound states are essentially constant, which is a consequence of the sharp rising of the energy near a_{TB}, as illustrated in Figure A2. Therefore, two photons of ~4 GeV each would excite the ARP one level up if configuration of the system conserves momentum of same. Conversely, an ARP in excited tight bound state could step down by one level via emission of two such photons, provided that total momentum of the system is conserved during the process. Note also that, for all the excited states, radius of the ARP is essentially constant, $\sim a_{\text{TB}}$, therefore T_a is nearly constant regardless of energy levels of the states.

Regular bound states of ARP in self rotation can be classified into two categories: one is in compact motion with radius of the ARP shorter than that in the ground state, and the other one is in regular motion with the radius longer than that in the ground state. As can be seen from Table A3, features of compact states are similar to that of tight bound states, except that compact excited states can lead to dissociation of the ARP hence are unstable in nature. In comparison, the ground state and all states of regular motion are stable.

In general, an ARP shall have combined motion of rotation and oscillation. Therefore, in general, $a \neq b$, and the extreme distances a and b satisfy the equation

$$\frac{\beta_{\rho_m}^2}{\epsilon^2}\left(1+\frac{\gamma^2}{\rho_m^2}\right)=1, \quad \begin{matrix} \rho_m \equiv a,b \\ \beta_{\rho_m} \equiv \beta\left[\rho_m\right] \end{matrix} \rightarrow \rho_m\sqrt{\frac{\epsilon^2}{\beta_{\rho_m}^2}-1}-\gamma=0. \tag{A16}$$

That is, given ϵ and γ, a and b are determined completely by solving Equation (A16). As can be seen in Figure A2, in range $\kappa < \epsilon < \epsilon_{a_m}$, referred as zone I, γ has maxima. Accordingly, two solutions exist in the range $0 < \gamma < \gamma_{\text{I,max}}$, corresponding to a and b of combined motion of the ARP. Characteristic pattern of such motion is exemplified in Figure A3(a) and Figure A3(b). If $\gamma = \gamma_{\text{I,max}}$, only one solution exists, corresponding to pure circular motion of the ARP. In general, $\gamma = \gamma_{\text{extreme}}$ corresponds to circular motion, regardless of ϵ. If $\epsilon = \epsilon_{a_m}$ & $\gamma = \gamma_{a_m}$, Equation (A16) yields two solutions, b and a_m. If the corresponding motion starts from a_m then it is pure circular. However, if starting from b then motion of the ARP shall be spirally approaching the circle of radius a_m while ϵ and γ being invariant during the entire process, as illustrated in Figure A3(e). In $\epsilon_{a_m} < \epsilon < 1$, noted as zone II, γ has two maxima and one nonzero minima. Therefore, four solutions of ρ_m can be found in $\gamma_{\text{II,min}} < \gamma < \gamma_{\text{II,2nd max}}$, corresponding to two modes of combined motion with identical ϵ and γ, as illustrated in Figure A3(c) and Figure A3(d), in which, pattern of compact motion (**c**) is indifferent in essence from that of (**a**) and (**b**). In $\epsilon \ge 1$, noted as zone III, γ has one maxima and one nonzero minima. If $\gamma_{\text{III,min}} < \gamma < \gamma_{\text{III,max}}$, three solutions exist, corresponding to compact motion and deflection, as seen in Figure A3(f). If $\gamma > \gamma_{\text{III,max}}$, the only motion is deflection. If $\gamma < \gamma_{\text{III,min}}$, pattern of the motion is open spiral, which is also an unbound state, as illustrated in Figure A3(f).

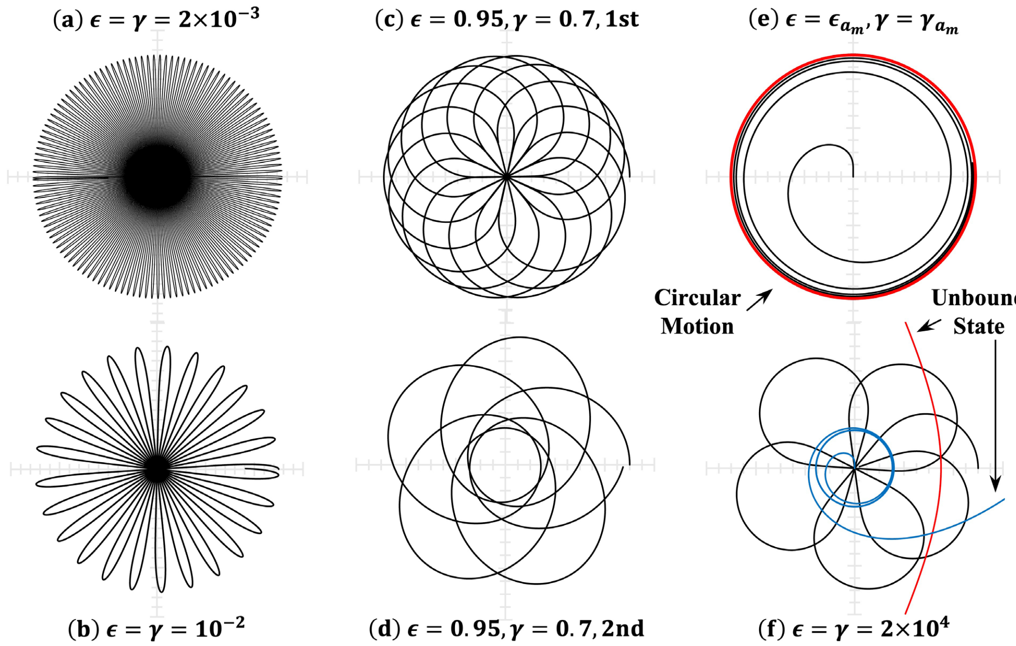

Figure A3. Pattern of ARP motion. Semi major axes of bound states are normalized to 1.

In any case,

$$\begin{aligned} T_{ab} &= 2\int_b^a \frac{d\rho}{\sqrt{q^2}} = 2\int_b^a \left(1 - \frac{a^2-\rho^2}{a^2-b^2}\frac{b^2\beta_\rho^2}{\rho^2\beta_b^2} - \frac{\rho^2-b^2}{a^2-b^2}\frac{a^2\beta_\rho^2}{\rho^2\beta_a^2}\right)^{-1/2} d\rho \\ \theta_{ab} &= 2\int_b^a \sqrt{\frac{\omega^2}{q^2}}d\rho = 2\int_b^a \left(\frac{\beta_a^2-\beta_\rho^2}{\beta_a^2-\beta_b^2}\frac{\rho^2\beta_b^2}{b^2\beta_\rho^2} + \frac{\beta_\rho^2-\beta_b^2}{\beta_a^2-\beta_b^2}\frac{\rho^2\beta_a^2}{a^2\beta_\rho^2} - 1\right)^{-1/2} \frac{d\rho}{\rho} \end{aligned} \quad \text{(A17)}$$

T_{ab}: Temporal period of oscillation of an ARP in Rest State, in reduced unit. θ_{ab}: Angular advancement of the ARP in rotation plane during T_{ab}.

For compact motion of ARP, *i.e.*, $b < a_{\text{TB}}$, T_{ab} is always shorter than rotation period. Therefore, oscillation frequency of the ARP is always higher than rotation frequency of same. For regular bounded motion,

$$\theta_{ab} \simeq 2\pi + \frac{\pi}{4}\left(\frac{1}{a}+\frac{1}{b}\right)\left(1+\frac{3}{16}\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{1}{192}\left(\frac{7}{a^2}+\frac{16}{ab}+\frac{7}{b^2}\right)+\cdots\right). \quad \text{(A18)}$$

That is, angular advancement of ARP always exceeds 2π during one oscillation period unless $b \to \infty$. Therefore, rotation frequency of the ARP is always higher than oscillation frequency of same. Either way, bounded ARP in noncircular motion shall have two base frequencies. Further, temporal profile of ARP oscillation is not sinusoidal, hence integer multiples of the base frequency exist in such oscillation. Consequently, ARP in noncircular bounded motion shall have infinite variety of discrete resonance frequencies, at which, photon emission/absorption is permissible.

Appendix C. Polarization of ARP under External Field

Consider an ARP immersed in a static electric field, assuming the field is along *z*-axis with constant strength of the field in region containing the ARP. Then, total force experienced by a constituent particle of the ARP is, under infinite space approximation,

$$\begin{gathered}\boldsymbol{\mathcal{F}}_{\Sigma,\pm} = \pm\beta^{1/2}\boldsymbol{\mathcal{E}}_{\text{ext}} \mp \frac{\beta}{4\rho^2}\left(1-\frac{\eta}{\beta^7}\right)\boldsymbol{k},\ \boldsymbol{\mathcal{E}}_{\text{ext}} \equiv \mathcal{E}_0\boldsymbol{k} \\ \mathbb{U}_{\mathcal{E}} \equiv \frac{E_{e,i}}{\boldsymbol{e}r_e} \simeq 1.813\,374\,589\times10^{20}\ \text{V m}^{-1}\end{gathered}. \quad \text{(A19)}$$

$\mathcal{F}_\Sigma$: Total force experienced by a constituent particle of an ARP in Rest State, in reduced unit. $\boldsymbol{\mathcal{E}}_{\text{ext}}$: External static electric field, in reduced unit. $\mathcal{E}_0$: Strength of external static electric field, in reduced unit. $\boldsymbol{k}$: Unit vector of *z*-axis. $\mathbb{U}_\varepsilon$: Unit of strength of electric field.

If total force experienced by a constituent particle of the ARP is nonzero then the particle shall relocate along direction of the force until net force experienced by the particle is zero,

$$\begin{gathered}\boldsymbol{\mathcal{F}}_{\Sigma,\pm} = \boldsymbol{0} \to \mathcal{E}_{0,\rho_\mathcal{E}} = \frac{1-\eta}{4\rho_\mathcal{E}^2\beta_{\rho_\mathcal{E}}^{13/2}}\exp\left[-\frac{7}{4\rho_\mathcal{E}}\right] \to \\ \mathcal{E}_{0,\rho_\mathcal{E},\max} < \frac{64}{e^2},\ \rho_{\mathcal{E}_{0,\rho_\mathcal{E},\max}} = \frac{1}{16}+\delta,\ \delta \approx 2.821\times10^{-31}\end{gathered}. \quad \text{(A20)}$$

ρ_ε : Equilibrium distance between constituent particle of ARP in Rest State and symmetry center of same under external static electric field, in reduced unit. β_{ρ_ε} : Planck Factor of constituent particle of ARP in equilibrium under external static electric field.

That is, an ARP shall be polarized under external static electric field to become an electrogravitation dipole, and length of the dipole is a function of strength of the applied field below the dissociation threshold. Such state of polarization of ARP is referred to as polarization saturation state. If strength of applied static electric field $\mathcal{E}_0 \ge (8/e)^2 \approx 8.661$, or ~1.571 × 10^{21} volts per meter, at location of an ARP then the ARP shall be dissociated to individual particles, referred to as dissociation state. However, static electric field of such strength is unrealizable in practice. Therefore, dissociation of ARP by static electric field is infeasible in reality. For relatively weaker polarization,

$$\mathcal{E}_0 < 0.373 \;\to\; s_{\mathcal{E}} \simeq -\frac{7}{4}W_{-1}\left[-\frac{7\eta^{13/28}}{4\sqrt{1-\eta}}\mathcal{E}_0^{1/2}\right]^{-1} < -\frac{7}{2\ln\eta} \approx 0.035\,. \qquad \text{(A21)}$$

W_{-1}: Lambert W function of branch −1.

For an ARP initially in ground state, from Equation (A19) and (A2),

$$\frac{d}{d\rho}\ln\frac{\beta}{\beta_u} = \frac{\mathcal{E}_{\text{ext}}}{\beta^{1/2}},\; \mathcal{E}_{\text{ext}} = \begin{cases} 0, & \tau \le 0 \\ \mathcal{E}_0, & \tau > 0 \end{cases}. \qquad \text{(A22)}$$

Therefore, constituent particles of the ARP shall be set to motion in parallel to direction of the applied field at onset of the field. Figure A4 shows typical temporal profiles of such motion, as function of strength of the applied field. If strength of the static electric field applied is stronger than ~0.545 393 units, *i.e.*, ~9.890 × 10^{19} volts per meter, that shall lead to dissociation of the ARP. Comparing with that of Equation (A20), this dissociation threshold is lower by an order of magnitude, due to the excess energy gained by the particles during initial acceleration of the particles under the field applied. However, even such level of field strength is still unrealizable in real world. Therefore, ARP cannot be dissociated by external electric field.

Below the dissociation threshold, motion of constituent particles of an ARP shall be stopped at a separation distance $\rho_\epsilon > \rho_\mathcal{E}$. At such distance, internal electrogravitation force between the particles is stronger than the external electrostatic force and the particles shall reverse their respective directions of motion to move against the external field till complete merging with each other again. All excess

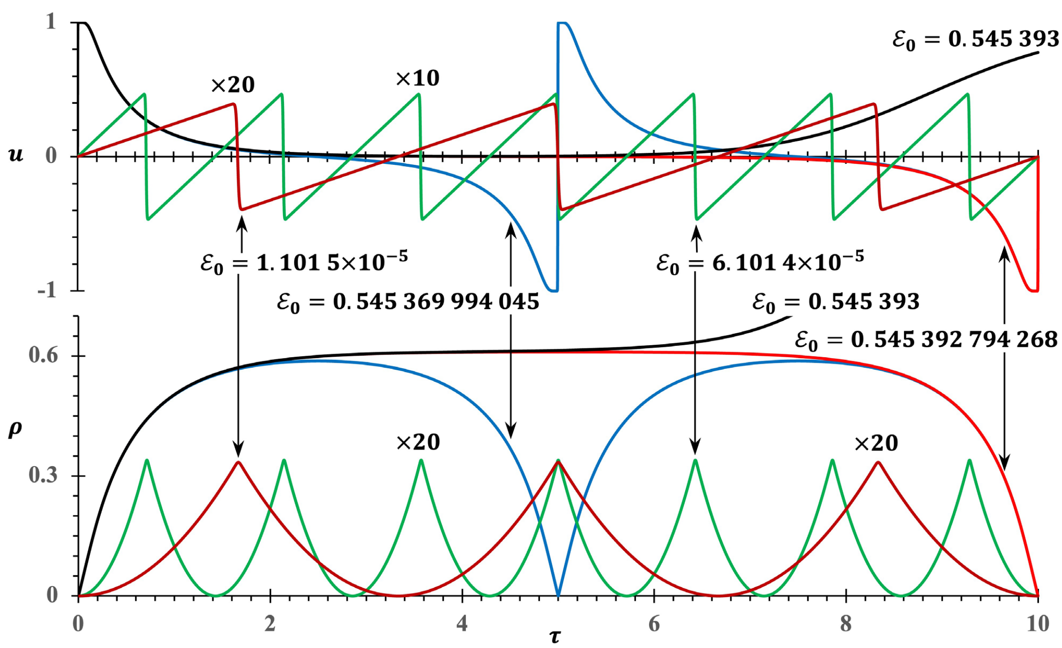

Figure A4. Motion profiles of constituent particle of ARP upon onset of external static electric field, as function of strength of the excitation field.

energies gained by the particles during initial acceleration are also returned to the external field during the reverse motion. The particles are then reaccelerated by the excitation field again, and so on, hence resulted in forced oscillation of constituent particles of ARP under static field.

If strength of external electric field is below the dissociation threshold but higher than about 1.986×10^{-6} then the weaker the excitation strength is, the higher the frequency of the oscillation will be, as exemplified in Figure A4. Further below, this relationship is reversed, *i.e.*, the lower the excitation strength is, the lower the oscillation frequency will be. In any case, temporal profiles of the forced oscillation are not sinusoidal. Since fields of nonzero strength are present in anywhere and at any time, it is thus clear that it would be impossible for an ARP to remain in amalgamation nor equilibrium configuration under static field, unless there exists damping mechanisms, such as gravitation retardation [8], for ARP to dissipate its excess energy.

The step-like onset of excitation field above is unnatural but only as an approximation, since any field of nonzero strength cannot be established in zero duration of time. A more realistic model of the excitation is

$$\frac{d}{d\rho}\ln\frac{\beta}{\beta_u}=\begin{cases}0, & \tau<0\\ \mathcal{E}_0\beta^{-1/2}\sin\left[2\pi x\right], & \tau\geq 0\end{cases},\quad x\equiv\frac{\tau}{T_e}. \tag{A23}$$

T_e : Temporal period of alternating field of electric excitation, in reduced unit.

Typical profiles of motion of ARP under such excitation are shown in Figure A5. Similar to that in step-like excitation, if strength of external electric field is nearing

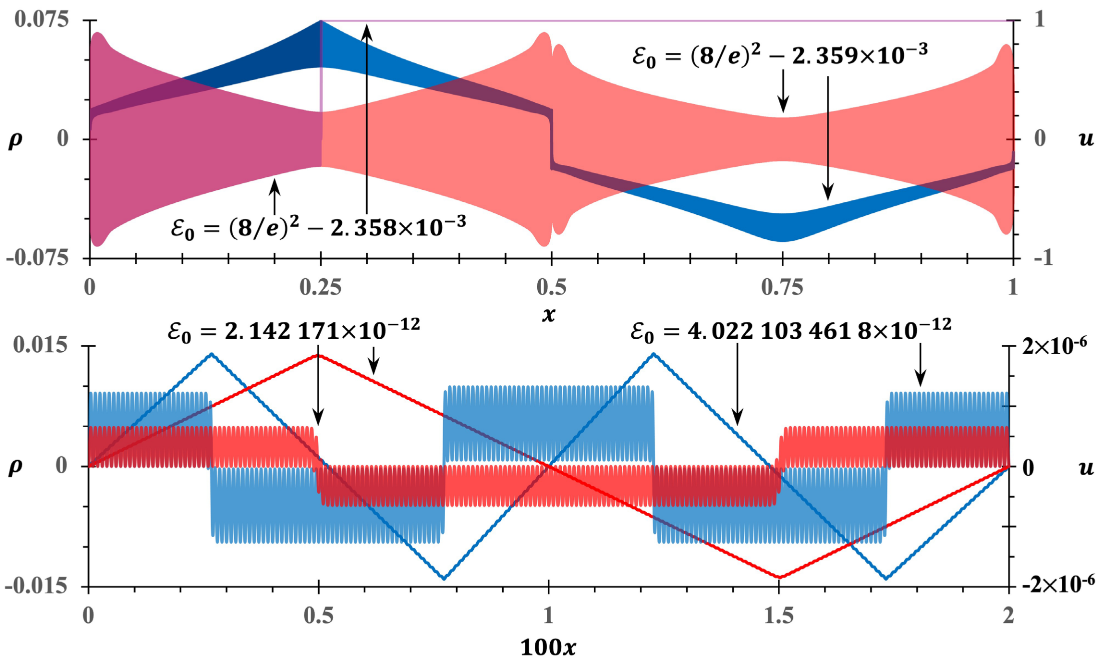

Figure A5. Motion profiles of constituent particles of ARP under sinusoidal electric excitation, wherein, T_e is taken as $T_e=\frac{2\pi}{(1-\kappa)\alpha_e}$.

the static threshold, an ARP in such field can be dissociated by such excitation. However, photons cannot dissociate an ARP in ground state regardless of frequencies hence energies of the photons, unless strength of the electromagnetic waves associated with the photons is near or above the threshold. Nevertheless, such strength of photon is unrealizable. Therefore, an excited ARP may transit to the ground state by emitting photons but the process is not reversible, *i.e.*, an ARP in the ground state cannot transit to an excited state by absorbing photons.

Below the dissociation strength, an ARP under sinusoidal excitation shall oscillate but not at the excitation frequency, as can be seen in **Figure A5**, and the weaker the excitation strength is, the lower the oscillation frequency will be, until excitation strength reaches a lower threshold, ~5×10^{-10}. Below such threshold, velocity profiles of the constituent particles of an ARP are essentially sinusoidal and in sync with the excitation but having phase lag and offset. Therefore, magnetic field induced by motion of constituent particles of the ARP under such excitation is sinusoidal and of the same frequency as that of the excitation. On the other hand, displacement of constituent particles of the ARP during each of the excitation cycle is small, as if the constituent particles are drifting in space, till ρ_ε. At such distance, the particles reverse their respective directions of motion and continue to drift, till reaching the opposite ρ_ε. Therefore, under sufficiently weak sinusoidal excitation, oscillation frequency of an ARP under such excitation is much lower than the excitation frequency, and approximately proportional to the excitation strength, and amplitude of the oscillation is essentially $\rho_\epsilon \cong \rho_\varepsilon$.