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Inexplicable Multi-Annual Astral Action on the Precession of Allais Pendulum: An Influence of the Solar System (and Especially of Jupiter?)

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Abstract

Between 1954 and 1961, Allais conducted 6 one-month observations of the azimuth of the plane of oscillation of a pendulum installed in his laboratory. That of 1958 also implemented a second pendulum, identical to the first, located 6 km away in an underground quarry. Although, over these 6 years, the average azimuth of each observation, the amplitude of the 24 h 50 min and 24 h waves, as well as certain other quantities, have evolved considerably, in 1958 their values were very close to those of the second pendulum. The analysis shows that these evolutions could only result from an action external to the pendulum, that no classical phenomenon seems to be able to explain, and which appears, at least mainly, to be an astral action. The evolution of the average azimuth of the pendulum and of the amplitudes of the 24 h and 24 h 50 min components can be decomposed into a component associated with the annual revolution of the Earth around the Sun, and a multi-annual component, whose harmonic 1 has a period which was estimated to 5.74 years. An action of Jupiter is an excellent candidate to explain a large part of the multi-annual action: everything happens as if there were an important action of the modulus of its declination on the multi-annual component, and an important daily action of its hour angle on the azimuth of the pendulum. We cannot exclude an action of the solar cycle, whose period was then about 11 years. The main results were obtained by Allais himself, but this was only published in his book "The Anisotropy of Space", and remained very little known. Starting from the raw data of Allais, the author of this article found them again, and completed them on certain points.

Keywords

Allais Effect, Pendulum, Lunisolar Influence, Jupiter Influence

1. Introduction

- Maurice Allais (1911-2010), who had studied mathematics and physics at a very high level and received the Nobel Prize in economics in 1988, did not stop, throughout his life, being interested in physics. In particular, from 1954 to 1960, six one-month long observations, carried out day and night without interruption in his laboratory in Saint-Germain (the pendulum being restarted every 20 minutes), allowed him to study the precession anomalies of that instrument. The 1958 observation was particularly significant, since a second pendulum has been set up in the very stable environment of an underground quarry. In the evolution of the azimuth of the plane of oscillation of his pendulum, Allais highlighted indeed several periodic components which, taking their amplitude into account, could absolutely not be explained by classical gravitation nor by general relativity. Furthermore, an in-depth analysis has shown that, as far as one can judge, at least one of them, whose period was 24 h 50 min, could not be explained by any classical phenomenon. As it is nearly the period of the wave M1 in tidal theory and the average duration between 2 successive passages of the Moon at the meridian (in fact the exact value is 24.84 h, that is 24 h 50 min 24 s; hence, in this article, we have subsequently considered the 24.84 h wave), it is generally considered to be a signature of the influence of the Moon, and this is how Allais interpreted it in most of his publications. In [1] [2], however, he pointed out that it could also come from the Sun rotation, which is another monthly phenomenon (hence has a possible connection with the sunspots and the solar cycle).

The existence of this 24.84 h wave, and the apparent impossibility of explaining it by classical phenomena, were confirmed by observations conducted in Romania in 2019 [3].

Incidentally, while his first observation was under way, Allais also noticed a strikingly abnormal behavior of the oscillation azimuth during the total solar eclipse of June 30, 1954. From 1957 to 1959, these observations were the subject of publications in several journals, including in particular the "*Comptes rendus de l'Académie des Sciences*" [4]-[12]. They even earned Allais two scientific prizes in 1958[1]. A publication in "*Aerospace Engineering*" [1] [2], at the request of Wernher von Braun, director of the NASA, did much to make them known. In a summary book published in 1997 ([13] or [14]), Allais presented all his research in experimental physics together with the analysis that he made afterwards.

- The overall analysis that Allais made of the 6 observations carried with the Saint-Germain pendulum from 1954 to 1960 was published only in this summary book ([13] or [14], chap. V.A and V.B).

It showed important variations of the amplitude of the 24.84 h wave and of the average azimuth of the plan of oscillation of the pendulum. Allais hypothesized that they were the sum of a component related to the annual Earth's revo-

[1]The 1959 Galabert Prize from the *Société Française d'Astronautique* and the 1959 Gravity Research Foundation Prize.

lution around the Sun and of a multi-annual component. The fact that, for these two quantities, which are of different nature, the multi-annual components are sinusoids whose period is about 5.9 years, and which are almost in opposition of phase, strongly reinforces this hypothesis.

Allais pointed out that a period of 5.9 years, which is close to very significant periods of the solar system, and in particular of the 1/2 periods of the orbital revolution of Jupiter and of the solar cycle, could be interpreted as resulting from the action of the planets considered as a whole.

- Starting again from the raw data of the 6 observations allowed, besides to confirm Allais' results, a more in-depth exploitation of them.

This exploitation involves several steps:

- The first one highlights that the evolution, from 1954 to 1960, of certain elements of the precession of the Saint-Germain pendulum could only result from external actions acting both on this pendulum and on that of Bougival.
- The second one highlights that no classical phenomenon can explain the above.
- The third demonstrates that, with a high degree of probability, all this can, for the most part, be explained by the existence of an action of astral origin decomposable into a semi-annual action and a multi-annual action whose harmonic 1 has a period of approximately 6 years.
- A detailed analysis shows that Jupiter is an excellent candidate to explain most of the multi-annual action. However, an influence of the solar cycle cannot be excluded.

2. The Observations Concerned

Remark: angle units:

Allais used as an angle unit the "grade", which was then the official unit in France (100 grades = 90 degrees). Hence, all publications relating to his work, old or recent, has used also the term "grade" (or "grad" when they were in English: for example [1] [2] [14]).

This unit being since a long time very little used, and therefore very little known, in this article we have continued to use "grad", instead of "gon", which is the international term which was subsequently attributed to this unit.

2.1. Calendar and Procedures

The two observation sites were Saint-Germain en Laye (48.8989˚N; 2.0937˚E) and Bougival (48.8667˚N; 2.1333˚E). Table 1 provides the dates of observations.

The procedure used (entirely manual) was as follows:

- One release of the pendulum every 20 minutes, the pendulum being stopped after 14 minutes. The azimuth of the oscillation plane (measured between 0 and 200 grads in the anti-clockwise sense from the north) was then noted (measurement accuracy: 0.1 grads). The next release was from this last observed azimuth. In this article, we will call "run" the whole of this 20 minutes operation.

Table 1. Allais observations with his anisotropic pendulum.

periods	median date	St Germain	Bougival
9/6/1954-9/7/1954	24/06/54	x	
16/11/1954-22/12/1954	04/12/54	x	
7/6/1955-7/7/1955	22/06/55	x	
2/7/1958-1/8/1958	16/07/58	x	x
20/11/1959-15/12/1959	02/12/59	x	
16/3/1960-16/4/1960	31/03/1960	x	

- The pendulum was released by burning a wire, with an angular amplitude of 0.11 rad.
- The Saint-Germain pendulum was installed in a basement laboratory. The Bougival one was set up in an abandoned underground chalk quarry, overlaid with 57 meters of clay and chalk. The horizontal distance from the open surface was about 800 m. It has purposefully been built for that occasion, using the same plans as those of the Saint-Germain pendulum, which has remained in a fixed location from 1954 to 1960. The two pendulums were about 6.5 km apart.

2.2. The Pendulum

- A general picture of the Saint-Germain pendulum is provided by Figure 1, details being provided by Figure 2 and Figure 3 (these figures are extracted from [14]).

It consisted of:

- a fixed part, which at Saint-Germain remained in place from 1954 to 1960.
- a moving part: rigid assembly consisting of the stirrup, the rod, and the bob. It rests, through a steel cone trunk, on a steel ball that can roll in all directions on a removable plate S (cobalt and tungsten carbide), which itself rests on a cutaway aluminium circular support S'. With the exception of the ball, the cone trunk, the plate S and the support S', all the other elements were made of bronze. The ball was changed with each run, and the plate S was changed once a week. With the exception of the first observation, during which the bob consisted of a vertical disc and two horizontal discs (see [13] or [14], p. 90), all the others used a 7.5 kg vertical disc.

- Other information:
- The distance between the ball and the lower end of the pendulum is 105 cm. The length of the equivalent simple pendulum is 83 cm.
- The machining had been carried out to 1/100th of a mm.

2.3. General Tendency of the Oscillation Plane to Be Recalled towards the Azimuth 171 Grads ("Intrinsic Anisotropy" of the Pendulum)

This results from the fact that, due to an asymmetry in the fixed part of the

Figure 1. Overview of the Saint-Germain pendulum.

Figure 2. Suspension system.

pendulum, its elasticity (and therefore the restoring force which, at each oscillation, calls back the bob of the pendulum towards its rest position) is not the same in all directions. It follows (see 3.4) that the plane of oscillation of the pendulum tends to be called back towards the direction corresponding to the

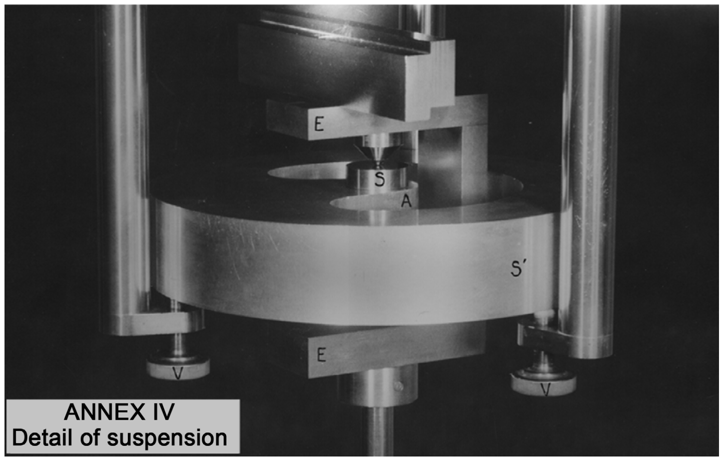

Figure 3. Details of the suspension.

minimum of the restoring force (the "direction of anisotropy" of the pendulum). In fact, this direction results both from the dissymmetry of the pendulum itself ("intrinsic" anisotropy), which is a priori constant, or at least slowly variable, and from external perturbing actions, which are varying with time, and therefore should be eliminated by averaging sufficiently spaced runs.

Several experiments were made by Allais to measure the intrinsic direction of anisotropy: a one-week continuous experiment in March 1955, followed by one-day continuous experiments on January 4, 1956, May 21, 1958 and August 13, 1958)[2]. The first experiment found the value of 170.67 grads, which appeared to be also the azimuth (171.16 grads) of the perpendicular to the direction of the suspension beam QP (see **Figure 1**): the origin of the intrinsic anisotropy was therefore perfectly identified. The results of the 3 experiments that followed (183.58 grads, 171.54 grads and 165.51 grads) did not reveal any significant drift of this azimuth: the difference of some grads from 171 grads can be explained by the fact that they lasted only one day.

Built from the same plans, the Bougival pendulum was also oriented in the same azimuth. The result of the same experiment, carried out on 13 August 1958, was 169.24 grads.

3. The Important Changes from 1954 to 1960 in Certain Elements of the Precession of the Saint-Germain Pendulum Can Only Be Explained by External Actions Which Would Have in 1958 Acted on Both the Two Pendulums

3.1. General Remark: This Study Started Again from Allais's Raw Data

Unless otherwise indicated, the results presented in this article have been completely recalculated from the raw data, which made it possible to go beyond the investigations published by Allais. As **Table 2** shows, the differences with the

[2]See [13] or [14], chap. IE3, and especially table X (p.180).

Table 2. Allais values and recalculated values (angles in grads).

observation	1954 June-July	1954 Nov-Dec	1955	1958 Bo	1958 SG	1959	1960	average
median date (cf nota) ALLAIS	174.5	337.5	537.8	1658.5	1658.5	2161.75	2282	1258.65
median date (cf nota) RECALC	174.6	337.3	537.5	1658	1658	2161.9	2281.8	1258.43
average azimuth ALLAIS	164	161	150	161	164	171	174	163.57
average azimuth RECALC	163.8	160.6	150	161.4	164.3	171.34	174.25	163.66
2x ampl 25 h ALLAIS	3.2	12.9	14	2.2	2.1	1.3	1.5	5.31
2x ampl 24.84 h RECALC	4.0	12.2	9.1	2.4	1.9	1.1	1.2	4.57
2x ampl 24 h ALLAIS	2	10.3	11.7	1.4	0.8	2.5	1.8	4.36
2x ampl 24 h RECALC	2.2	11.2	11.7	1.5	0.7	2.4	1.9	4.52

Note: Dates expressed in days from 1/1/1954.

values published by Allais are small. The main differences relate to the lunar component (Allais considered 25 h instead of 24.84 h because, in the absence of computers, calculations were easier).

3.2. Quantities Selected to Be Studied

These are, for each observation:

1) The average azimuth, the evolution of which is given by Figure 4.

2) The standard deviation of the azimuth after deducing the trend during the observation, as an indicator of the influence on each run of all disruptive actions whatever their origin (Figure 5).

3) The amplitudes (Figure 6 and Figure 7) of periodic components of 24 h, 24.84 h, 25.82 h and 12 h, which were selected because they appear a priori to be the most representative of the influence on the Earth of the positions of the celestial bodies.

Let us remember first that the calculations were made from observations of 1 month: in the vicinity of 24 h one cannot therefore completely distinguish waves

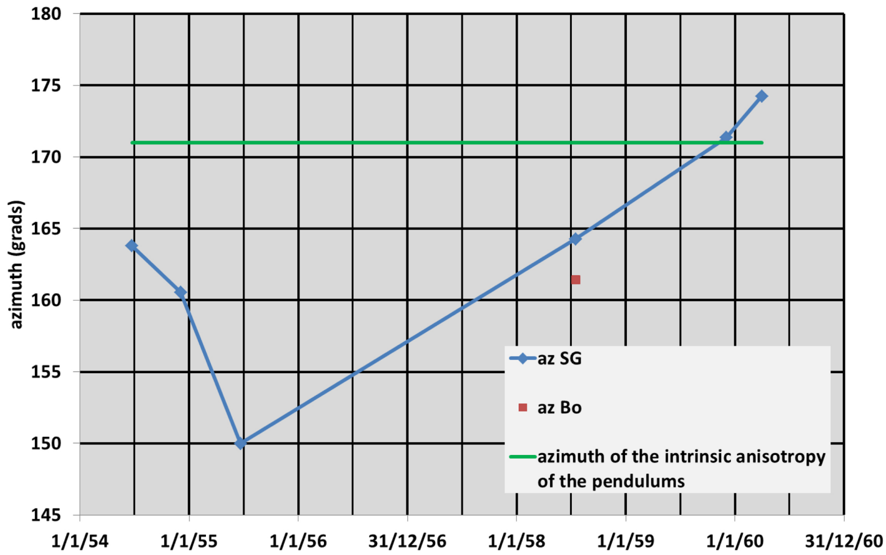

Figure 4. Average azimuths of the Saint-Germain and Bougival pendulums.

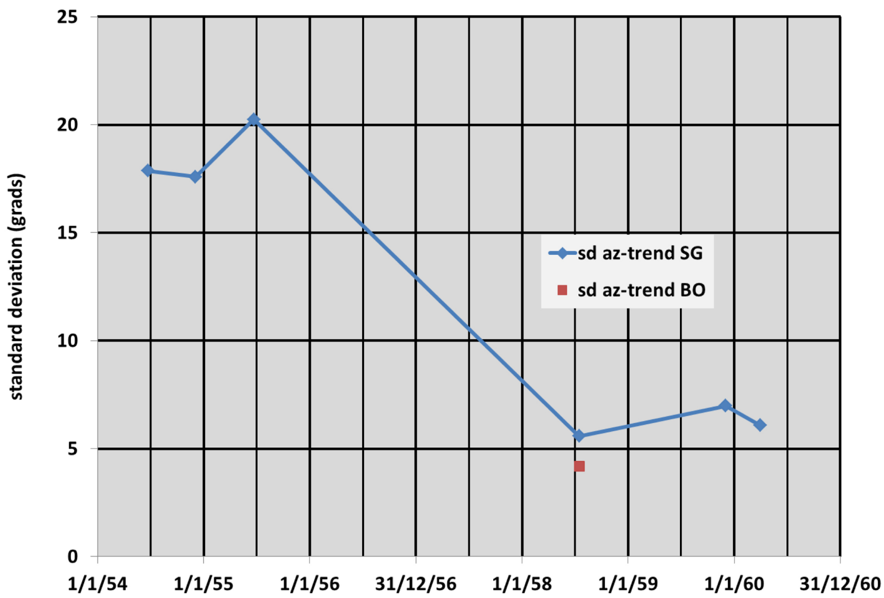

Figure 5. Standard deviation of the azimuths of Saint-Germain and Bougival pendulums (after substraction of the trend during the concerned observation).

separated by less than 50 min (and 25 min in the vicinity of 12 h). Each of the 3 waves therefore corresponds in fact to a group of lines.

The group of lines around 24 h results either directly from the anti-clockwise rotation around its axis of the Earth relative to the rest of the Universe in a sidereal day (23.98 h), or from the composition of this rotation with astral long term phenomena, the main one of them being the annual revolution of the Earth around the Sun (which gives the 24 h line), with possibly its first harmonics: 6 months, 4 months. An influence of the position of celestial bodies other than the Sun and the Moon, if it exists, affects only this group of lines: their angular velocities in the equatorial system being nil, or very small, only a range of few

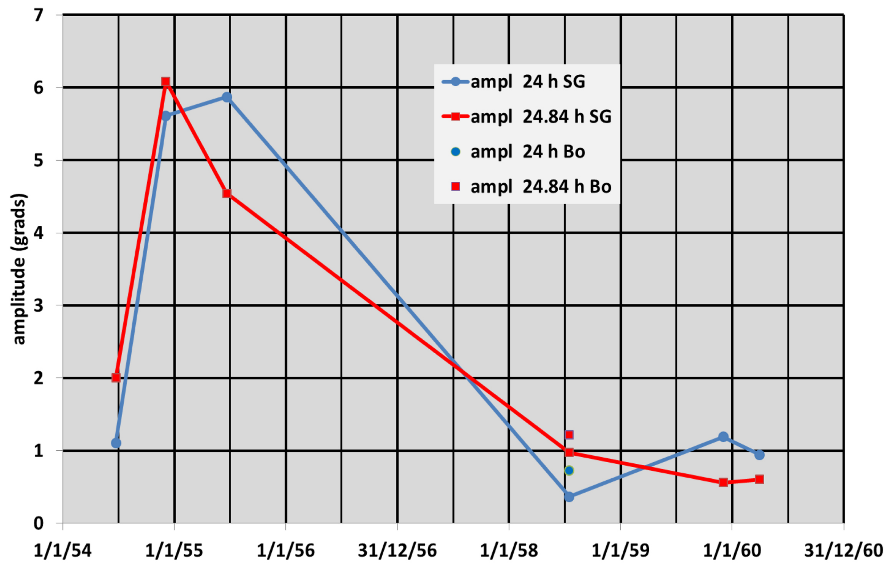

Figure 6. Amplitudes of 24 h and 24.84 h waves.

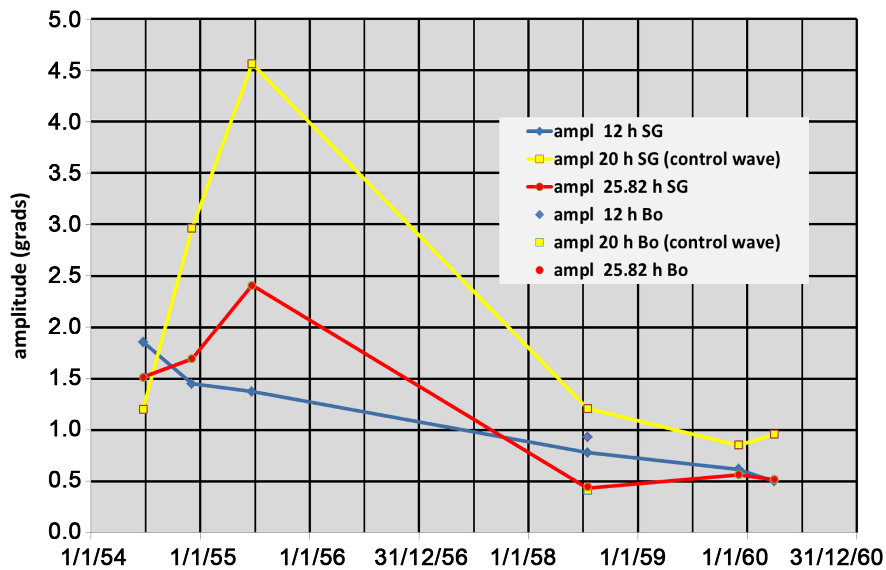

Figure 7. Amplitudes of 12 h, 20 h and 25.82 h waves.

minutes around the 23.98 h period is concerned.

In the case of the Moon, a 24.84 h wave results from the composition of the rotation of the Earth around it axis and of the revolution of the Moon around the Earth in a sidereal month, which are in the same anticlockwise sense.

In fact, this wave regroups the result of the composition of the rotation of the Earth with all the phenomena whose period is about 1 month. So it groups all the possible influences of the Moon (sidereal month, synodic month, …). Apart from the influence of the Moon, there does not seem to be anything other than the rotation of the Sun around its axis (as the Sun is not a solid system, the period varies with the latitude, from 25 days in the equator, to 36 days in the poles).

The 24 h and 24.84 h waves correspond to harmonics 1. Due to non-linearities, there are also harmonics, beats, harmonics of beats, etc. By limiting ourselves to harmonics 2, we obtain the period of 12 h and the period of 25.82 h (composition of the rotation of the Earth with the harmonic 2 of the revolution of the Moon around the Earth).

4) The ratios between the amplitude of the 24 h wave and the amplitudes of all other waves (Figure 8).

In addition, we have studied the amplitude of the 20 h wave (Figure 7), which was taken as a control wave, to verify that what is observed for the 24 h, 24.84 h, 25.82 h and 12 h waves is not at all a general phenomena.

Each value was calculated from more than 2000 azimuth measurements, and therefore has a good statistical significance.

3.3. Results

- In all cases the variations in the quantities studied are important.
- In all cases, with the exception of the 20 h control wave, the absolute value of differences between the Saint-Germain and Bougival values is small before the total amplitude of the variations: see Table 3.

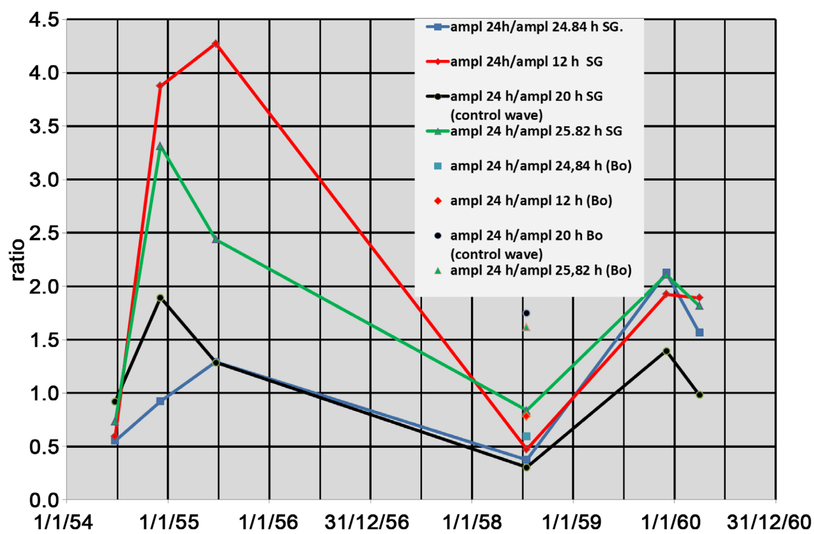

Figure 8. Ratios amplitudes (24 h/24.84h; 24 h/12h; 24 h/25.82h; 24 h/20h).

Table 3. |value SG 1958 − value Bo 1958|/(max SG − min SG).

azimuth	11.7%
SD azimuth-trend	9.5%
ampl 12 h	11.5%
ampl 20 h (control wave)	21.4%
ampl 24 h	6.6%
ampl 24.84 h	4.5%
ampl 25.82 h	1.2%

- Case of 12 h, 24 h, 24.84 h and 25.82 h waves.

The concordance of the amplitudes between Saint-Germain and Bougival is good, but remains relatively insignificant since, from 1958, all the amplitudes are small (which is consistent with the fact that the same is true of the standard deviation of the azimuth, which is an indicator of the activity of the disruptive actions considered as a whole). But it is remarkable that, in addition, except for the 25.82 h wave, which corresponds to a harmonic 2, there is also a quite good concordance between the ratios of their amplitudes and that of the 24 h wave, which also varies greatly from 1954 to 1960 (see Table 4 and Figure 8).

All the above is not at all general, as the case of the 20 h control wave shows.

3.4. These Variations Can Only Result from Actions External to the Pendulum

3.4.1. General Analysis of the Movement of the Pendulum

- If the period of a pendulum has been a classic investigative tool for several centuries, it is not at all the same for its precession which, apart from Foucault's precession, is very little known. We have therefore gathered a number of essential elements for its analysis in **Appendix A**, which also studies in detail the "anisotropy" of a pendulum.
- A major point is that the precession of a pendulum is the sum of a precession resulting directly from the various perturbing actions ("direct precession"), and of a precession resulting indirectly from their action on the ovalization of the trajectory (onset of elliptic orbits). This ellipticity causes in turn a nonlinear effect, the so-called Airy precession (**Appendix A.1**). This precession is all the more important as the pendulum is short and the angular amplitude important. Allais had deduced from preliminary experiments that the unknown perturbing action he was looking for most probably acted mainly through ovalization[3]. So, he decided to make them more evident by using a pendulum less than 1 m long released at the angular amplitude of 0.11 rad.
- As a general rule, the behavior of the pendulum is anisotropic: its trajectory depends on the starting azimuth. This results from the fact that the symmetry of the device is never perfectly revolutionary, on the one hand, and, on the other hand, that many external perturbing actions are directional.

Indeed a number of them make that the restoring force which, at each oscillation, calls back the bob of the pendulum towards its rest position, is not exactly the same in 2 perpendicular directions.

Table 4. |ratio SG 1958 − ratio Bo 1958|/(max ratio SG − min ratio SG).

ampl 24 h/ampl 24.84 h	12.5%
ampl 24 h/ampl 12 h	8.1%
ampl 24 h/ampl 25.82 h	32.4%
ampl 24 h/ampl 20 h	90.9%

[3]This has been confirmed by the recent observations in Romania [3].

The restoring force is then a periodic function of the azimuth of the major axis, the period of which is 180˚. The "linear anisotropy" is the simplest case: the one where this periodic function reduces to its harmonic 1. This anisotropy model, which makes it possible to account for a lot of perturbing actions, is studied in detail in **Appendix A.2**. In particular, it accounts quite well for the anisotropy of the suspension of the Allais pendulum[4], which we have seen to tend to bring the plane of oscillation towards the azimuth 171 grads. It is characterized by the azimuth θ_A of the "direction of anisotropy" (direction in which the restoring force is minimum, and which is also that towards which the oscillation plane tends to be called back), and by the "coefficient of anisotropy" η, which corresponds to the importance of the anisotropy.

The linear anisotropy acts mainly by creating ovalization, and therefore Airy precession.

- The azimuth at the end of a run results in first approximation from the starting azimuth and from 4 main actions:

1) Random perturbations that we can eliminate.

Allais attributed them to defects of the balls, but they are doubtless also due to the initial ellipticity (the pendulum is never strictly immobile when the wire is burnt), and to the resulting Airy precession. For a given run, they can be important, but their influence on the average azimuth, which results from more than 2000 measurements, is very strongly weakened.

2) Foucault's precession (in 14 min, -0.55×10^{-4} rad/s, *i.e.* −2.94 grads). This is a "direct" precession.

3) The restoring action towards the azimuth of the direction of intrinsic anisotropy of the pendulum (171 grads).

In the absence of any other perturbing action, the azimuth of the pendulum would settle at the value for which, over 14 minutes, the precession due to the restoring action would be 2.94 grads, which would cancel the Foucault precession. This equilibrium azimuth is then found to be 162.6 grads (see **Appendix A.6**), which is quite consistent with the average of the 6 observations (163.6 grads: see **Table 2**).

4) An unknown external action, whose observations by Allais showed that it tended to call back the plane of oscillation towards an azimuth variable over time. This second anisotropy is composed with the intrinsic anisotropy, and it is the anisotropy resulting from this composition which acts.

- Ultimately, Foucault's precession being constant, and the influence of ball defects and the initial ellipticity negligible, this resulting total anisotropy is the main cause of the variations, from one observation to another, of the average azimuth over a one-month observation.

3.4.2. It Is Impossible to Explain the Important Variations in the Quantities Studied by Variations in the Characteristics of the Saint-Germain Pendulum

First of all, it would indeed be quite extraordinary if, in 1958, this resulted in

[4]See the predominance of the harmonic 1: [13] or [14], chap. IE3, graph XXXIV, p. 181.

values very near of the corresponding values of the Bougival pendulum.

Anyway, if the average azimuth over a month, instead of being fixed at 163 grads, varies from 164 grads to 150 grads between June 1954 and June 1955, then from 150 grads to 174 grads in March 1960, this cannot result from variations in the intrinsic anisotropy. Indeed, the direction of the latter, which depends on that of the suspension beam, remained fixed (and Allais actually verified this experimentally). Only can therefore vary the coefficient of anisotropy, that is to say the elasticity of the beam. The analysis carried out in **Appendix A.6** (Table A1) shows that the magnitude that its fluctuations should have had, in one sense before June 1955, and in the other sense after, is not realistic at all.

4. As Far as Can Be Judged, No Classical Phenomenon Can Explain What Was Observed

Remark: we can note also that the corrections to classical mechanics made by general relativity (order of magnitude 10^{-9} in relative value) are on the Earth much too small to be able to explain the amplitudes observed.

4.1. To Be Excluded Because without Any Possible Perceptible Action on the Precession

4.1.1. Direct Gravitational Action of the Celestial Bodies

The forces of attraction of the celestial bodies at a point on the Earth, often called "tidal forces", have been very well known theoretically for a long time, and the consistency with what is observed by means of static devices such as gravimeters is excellent. Only the action of the Sun and Moon is not negligible. Calculating the influence of the lunisolar forces on the precession of a pendulum is therefore the first question that arose for Allais, who was the first one to have exploited the analysis of the precession as an investigative tool. He made this calculation in 1957, in the *Comptes rendus de l'Academie des Sciences* [7], and reminded it in 1958, in an article of *Aerospace Engeneering* [1] [2]. It is also reminded, with much more details, in [13] or [14] (chap. IB2). The result is that the influence on the precession, which is about six orders of magnitude smaller than the amplitude of the observed periodic components, can absolutely not explain them. In fact, this influence is completely undetectable over the duration of a run, and thus cannot, moreover, explain an eclipse effect.

4.1.2. Indirect Gravitational Action of the Celestial Bodies

1) General information.

These actions are those that result from the displacement of elements of the Earth under the tidal forces: ocean tides, atmospheric tides, Earth tides.

- **Preliminary remark:** influence on the precession of external accelerations acting on the bob, when they are very small compared to the acceleration of gravity *g*.
- The influence on the precession of a vertical component can be neglected. It results in a change in the apparent value of *g*. As regards precession, when

there is ellipticity, this results in a very small change (it is proportional to the ratio vertical acceleration/*g*) in the speed of Airy precession: cf Formula (3)).

- Hence it can be considered that only horizontal accelerations may act. They always deviate slightly the apparent vertical line given by the pendulum (tilt). This tilt, which modifies the angle between the apparent vertical given by the pendulum and the axis of rotation of the Earth, modifies the velocity of Foucault precession in the 1st order. This velocity is indeed given by the formula $\theta'_F = \Omega_T \sin(\lambda + \Delta\lambda)$, where Ω_T is the rotational speed of the Earth, λ the latitude, and $\Delta\lambda$ the value of the tilt, expressed in radians. Hence $\Delta\theta'_F = \Omega_T \cos(\lambda)\Delta\lambda$ rad/s.

With $\lambda \approx 45$ deg, $\Delta\theta'_F \approx 3.28\times10^{-3}\Delta\lambda$ grads/s.

With for example $|\Delta\lambda| \le 10^{-4}$ rad, $|\Delta\theta'_F| \le 3.28\times10^{-7}$ grads/s, which is very small, and more than 10^2 times smaller than the observed precession velocities (9.25×10^{-5} grads/s)[5].

The tilt has also an action in the 2nd order, which can only be negligible, on the apparent gravity exerted on the pendulum, and therefore on its pulsation, which intervenes in the Airy precession (Formula (3)).

- A constant horizontal acceleration has no other effect on the precession, and no effect on the ellipticity *e* (the action on one half cycle is exactly cancelled by the effect on the next one).
- Only variations in tilt during the run might therefore have an effect. The calculation confirms[6] that, when the tilt, which is very small, moreover varies slowly (which is the case for diurnal variations), the effects on the precession and the ellipticity can only be absolutely negligible: it was verified that it is the case for a diurnal tilt the amplitude of which is $<10^{-4}$ rad.

- **Whether the attractive mass is an element linked to the Earth or an element linked to a celestial body, the gravitational action acts by 2 ways on the precession of the pendulum:**

a) By the tilt due to direct attraction of the attractive mass: cf above "Preliminary remark".

The main consequence is a variation of the velocity of Foucault precession proportional to the tilt, which has no effect on the ellipticity. The value of the tilt is:

[5]The observed values of twice the average amplitudes of the 24 h and 24.84 h waves (Table 2) are > 4 grads, which corresponds over one 1/2 period to an average precession speed of $4/(12\times3600) = 9.25\times10^{-5}$ grads/s.

[6]The variation of the horizontal acceleration during the run results in a deviation of SG, G being the center of gravity of the pendulum and S its suspension point, and therefore, by gyroscopic effect, in a rotation of the almost horizontal axis around which the pendulum swings. Hence a gyroscopic couple, etc.

It is easy to see that only acts the component γ_y of the horizontal acceleration perpendicular to the major axis of the ellipse. Finally the analysis shows:

-That γ_y acts on the ellipticity, which creates an Airy's precession, which can be calculated knowing the law of variation of γ_y.

-That only the second derivative of γ_y begins to act (in average over one oscillation the acceleration and its primary derivative have no effect at all).

$$\mathrm{Tilt}\left(\mathrm{rad}\right)=\frac{GM\sin z}{gd^{2}} \tag{1}$$

where G is the gravitational constant, g the acceleration of gravity, d the distance of the attractive body from the point of rest of the pendulum, M its mass, and z its zenith distance.

We can summarize by saying that a diurnal tilt whose amplitude is $\leq 10^{-4}$ rad could not explain the observed precession, and anyway would have no perceptible effect on the ellipticity *e*. We can note that daily variations of the tilt of 10^{-4} rad would be easily detectable by accelerometers, and is quite enormous[7].

b) By the variation in the restoring force which calls back the bob to its rest position, due to variations in the force of attraction in the space swept by the pendulum at each oscillation. This perturbation acts in resonance with the pendulum oscillations.

In the end, the tilt being, as we will see, too small to have a non-negligible action, the variation of the restoring force in the space swept by the pendulum remains the only way of action of gravitational forces to be considered.

If the lines of force of the gravitational field are parallel in this space, which is the case with a sufficiently distant attractive body, one finds, by calculating the restoring force, that this creates a linear anisotropy. This time, there is an action mainly on ellipticity, and therefore on Airy precession.

The direction of anisotropy, which is the direction of the anisotropy vector, is the direction of the attractive body, and we find that, in the case of an attractive body linked to the Earth[8], the coefficient of anisotropy, which is its modulous, is given by the formula:

$$\eta=\frac{GMl\sin^{2}z}{2gd^{3}} \tag{2}$$

where l is the length of the pendulum (considered as a simple pendulum).

The important point is that η, which is inversely proportional to the cube of *d*, decreases very rapidly with *d*.

According to Formula (13) in **Appendix A.2**, the rate of precession resulting over a run from a value of $\eta \leq 10^{-10}$ is $\ll 1.12\times 10^{-7}$ grads/s.

This value being more than hundred times smaller than the observed value (9.25×10^{-5} grads/s: see above), an anisotropy variation vector whose modulus η would be $<10^{-10}$ could not explain the observed waves.

Remark: In the case of the gravitational action of celestial bodies, which are very far, we have always a linear anisotropy. From calculations of Allais, who evaluated an upper bound of the action of the Moon on the precession of a pendulum, one can deduce (see [3], appendice B.5.c, formula (16)) an upper bound of the coefficient of anisotropy associated to this action: $\eta \leq 1.3\times 10^{-13}$. We again find that the effect of the direct gravitational action of the Moon is at least

[7]We find easily with accelerometers the tilt resulting directly from the tidal forces, which is at most 10^{-7} rad.

[8]The calculation for a celestial body was made by Maurice Allais: the formula is not exactly the same.

10^5 weaker than the observed effect.

2) Gravitational action of ocean and atmospheric tides. Saint-Germain is more than 150 km away from the sea (English Channel). It follows from Formula (2) that, to obtain an anisotropy whose coefficient is 10^{-10}, it would be necessary to place at 150 km a mass of 12×10^{13} tons, which corresponds to 12×10^{13} m^3 of water. With a tide height of 5 m, this corresponds to an area of 2.39×10^7 km^2, what is about half of the surface of the Earth. From Formula (1) we deduce that it would be necessary, to obtain a tilt of 10^{-4} rad, to have a mass of water of 3.31×10^{13} tons, which is even more important. We are absolutely not in the necessary orders of magnitude…

The same is true of the influence of atmospheric density variations resulting from atmospheric tides[9].

3) Gravitational action of Earth tides.

- There results from lunisolar forces not only ocean tides and atmospheric tides, but also diurnal deformations of the Earth's itself ("Earth tides"), which are not negligible at all (several tens of centimeters per day at the surface of the Earth). In principle, these deformations are vertical, which has effects on gravimeters, but no effect on the precession. Nevertheless, it cannot be ruled out it might occur locally horizontal motions. For a horizontal deformation of 1 m peak to peak of period 24 h, the maximum variation of tilt would remain $<2.7 \times 10^{-10}$ rad, which can only have a negligible influence.
- More generally, for there to be a detectable action, it is necessary:

• That the horizontal attraction of the whole of the Earth on the pendulum is not zero, that is to say that there is an anisotropy in the environment of the place of observation. Hence a tilt, which can be measured, as well as a linear anisotropy of the pendulum, the coefficient of which can be calculated by Formula (2).

• That, moreover, due to tidal forces (deformation of the solid part of the Earth or variations in the density of the magma), this anisotropy varies sufficiently (modulous of the vector variation of anisotropy at least $>10^{-10}$).

Saint-Germain not being in a mountainous area, this seems unlikely, but due to lack of data no calculation could be made.

However, one can think that the variations under the effect of tidal forces in the azimuth of the anisotropy of the pendulum environment cannot be large. It is not compatible at all with the results of Allais, whose procedure (pendulum started from the final azimuth of the previous run) made it possible to follow in a certain extent the direction of the anisotropy. The change in the azimuth could be almost 100 grads: see for example [13] or [14] §I.A.2, p. 89, Graph II.

4.1.3. Variations in the Earth's Magnetic Field

Whether due to their direct action (except the cone trunk and the ball, which are

[9]In their analysis of the effect on the motion of a pendulum of the displacement of large air masses in the upper atmosphere, Van Flandern and Yang [15] also calculated the influence of the gravitational attraction of these masses to be totally negligible.

in steel, the Allais's pendulum is totally non-magnetic: see §2.2), or to their action via induced eddy currents (the action during one half-oscillation cancels the action during the previous one: eddy currents change direction with every half-oscillation).

4.2. Are Also to Be Excluded, Due to the Stable and Protected Environment of the Bougival Quarry, the Following Phenomena

- The Earth's electric field.
- Temperature and humidity.
- Human activities.

4.3. Atmospheric Pressure Cannot Explain What Was Observed, Due to a Harmonic Structure Very Different from That of the Azimuths of the Pendulum

Indeed, during the 1955 experiment, this was verified by Allais [13] or [14] (Table II, p. 99). The ratio of the 24 h and 24.84 h components equals 1.24 for the azimuths, and 2.7 for the atmospheric pressure. Moreover, during a year, the extrema of the pressure are at the solstices, while they are at the equinoxes for the azimuth (see below §5.2).

4.4. Remark about the 24.84 h Wave

In the analysis carried out from observations of 1 month considered separately, the 24.84 h wave is the only one whose in-depth analysis has really shown that it cannot be explained, at least in a very large part, by classical phenomena. This is the case for both the Allais's observations and the Horodnic 2019 observations [3]. Moreover, given its very particular value, it can only result from the composition of the rotation of the Earth around its axis with a phenomenon whose period is approximately one month, which very strongly circumscribes the field of research.

It therefore holds an absolutely essential place.

The above:

- explains why its amplitude varies greatly over time.
- confirms, by a different approach from that used in the analysis of one-month observations, that it cannot be explained by classical phenomena.

5. All the Above Can Be Explained, at Least for the Most Part, by the Existence of an Astral Action Decomposable into a Semi-Annual Action and a Multi-Annual Action Whose Harmonic 1 Has a Period of Approximately 6 Years

5.1. Allais's Analysis

For all the known geophysical factors, we find a very important influence of the Earth's revolution around the Sun: it would be very extraordinary if it were not the same for those who remain unknown. Allais ([13] or [14], Chapter V.B, p.

432-439) thus made the hypothesis that the precession of the pendulum was subject to both an annual action (which in the general case results in both an annual and a semi-annual component), and a multi-annual action. He first considered the average azimuth, and found that it was the semi-annual influence which was preponderant, with extrema near the equinoxes and solstices. The fitting by a sinusoid of the residue of the fitting by a semi-annual wave of the average azimuth then revealed a sinusoid the period of which was 5.9 years, and the minimum of which was on August 10, 1956.

Then the fitting by a 5.9 year sinusoid of the residue of the fitting by a semi-annual wave of the amplitude of the 25 h[10] wave showed a maximum on 31 July 1956 ([13] or [14], p. 439): the multi-annual actions on the average azimuth and the 25 h wave amplitude are almost in phase opposition.

This very remarkable coincidence strongly enforces the hypothesis of both of an influence of the Earth's revolution and of a multi-annual action, of which the period of the harmonic 1 would be about 6 years[11].

5.2. Complementary Analysis

- This analysis started again from raw Allais's data: all the calculations have been redone.
- The analysed quantities were the average azimuth, the standard deviation of the azimuth after deducing the trend during each observation, and the amplitudes of the 24.84 h, 24 h, 25.82 h and 12 h waves. Each of these quantities was analysed separately.
- **Figure 9** and **Figure 10** show the results of the fitting of the average azimuth and the amplitude of the 24.84 h wave by an annual and a semi-annual wave. The results of the fitting of the average azimuth[12] are very close to those of Allais ([13] or [14], §V.B, p. 435, Graph II) as shown in **Table 5**.

As shown in **Figure 9**, the annual and semi-annual components of the average azimuth are of similar importance (more than 6 observations would have been needed to determine them both at once).

It is remarkable that the extrema of the annual component are at the equinoxes, and not at the solstices. Of all known geophysical factors, only the Earth's magnetic field has this particularity.

- **Figure 11** shows the fittings of the residues by a sinusoid (harmonics 1 of these residues). The periods are not identical, but remain close (5.53 years for the average azimuth, 6.11 years for the amplitude of the 24.84 h wave). We are always near a phase opposition (the difference is 42 days).

As regards the azimuth, there is a significant difference between 5.53 years

[10]We remind that Allais had taken 25 h instead of 24.84 h to simplify the calculations (**Table 2**).

[11]Without the hypothesis of the existence of an action partially linked to the annual revolution of the Earth, that is to say if we adjust directly by a sinusoid the evolution over the 6 years, and not the residues of the fitting of this evolution by a semi-annual wave, one finds for the azimuth and the 24.84 h wave periods which are still close (approximately 7 years this time), but there is no longer the coincidence of phase: this time the difference is 6 months.

[12]As regards the amplitude of the 24.84 h wave, they were not published.

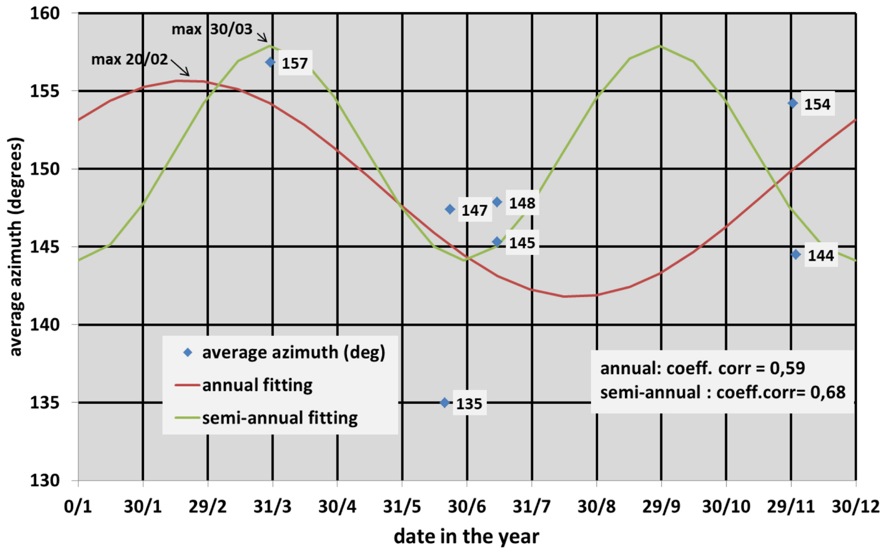

Figure 9. Fitting of the average azimuth with annual and semi-annual waves.

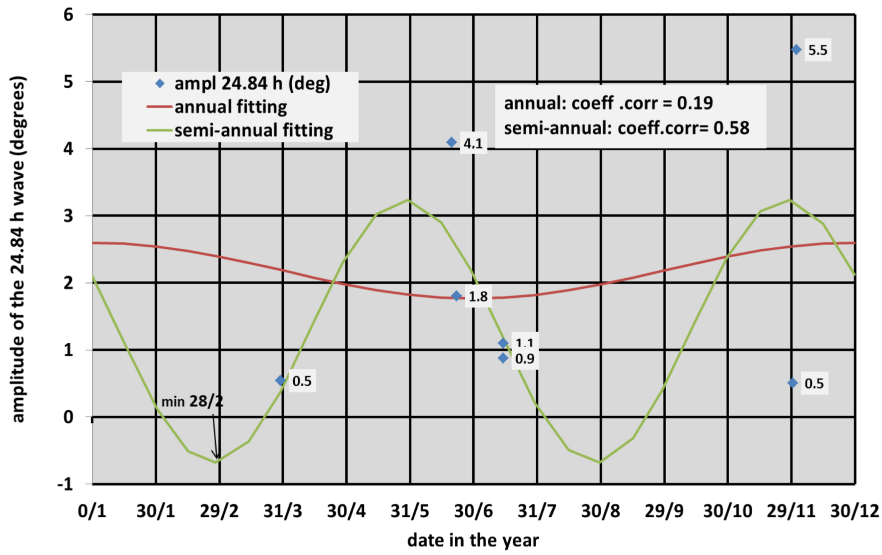

Figure 10. Fitting of the amplitude of the 24.84 h wave with annual and semi-annual waves.

and the period of 5.9 years found by Allais, though the starting data were almost the same (Table 2), and that were also almost the same the results of the fitting by a semi-annual wave. That can be explained by a less good fitting in the case of Allais's calculations (correlation coefficient of 0.91, instead of 0.96).

- Table 6 also gives the result of the decomposition of the other studied quantities:

- the amplitude of the 24 h wave which, as the 24.84 h wave, corresponds to a harmonic 1;
- the amplitudes of the 12 h and 25.82 h waves, which correspond to harmonics 2;
- the standard deviation of the azimuth.

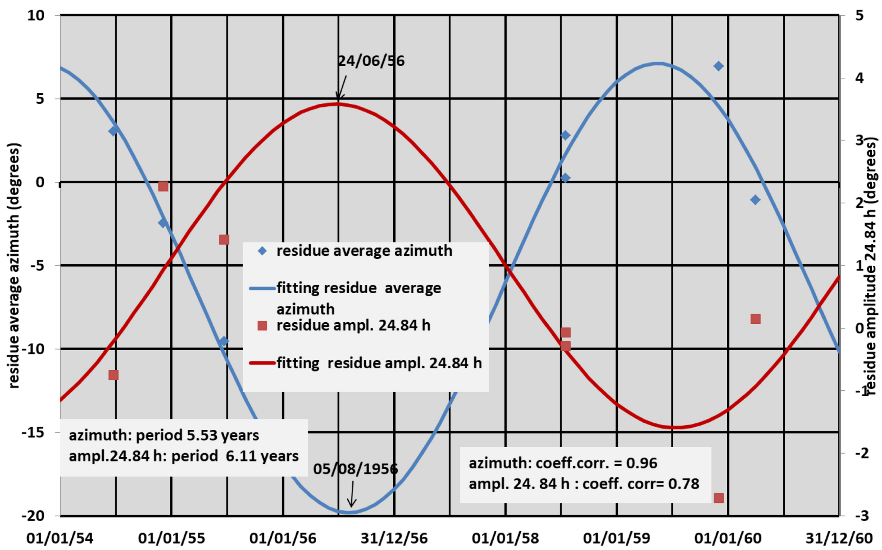

Figure 11. Fitting by a sinusoid of the residues of the fitting by a semi-annual wave of the average azimuth and the amplitude of the 24.84 h wave.

Table 5. Semi-annual fitting of the average azimuth.

	amplitude (deg)	date 1st maximum
Allais	6.77	0 h on April 2
Recalculated	6.87	13 h on March 30

Table 6. Multi-annual component.

	coeff. corr	period (years)	date minimum	date maximum
sd azimuth (grads)	0.80	6.71		06/12/1955
average azimuth (grads)	0.96	5.53		05/08/1956
amplitude 24 h wave (grads)	0.92	5.57	10/08/1856	
amplitude 24.84 h wave (grads)	0.78	6.11	24/06/1956	
amplitude 25.82 h wave (grads)	0.87	5.85	24/05/1956	
amplitude 12 h wave (grads)	0.8	31.28		20/01/1943

It shows that:

- The decomposition of the amplitude of the 24 h wave is remarquably consistent with the one of the average azimuth: the multi-annual components have almost exactly the same periods, and are almost exactly in phase opposition.

- As regards the amplitude of the 25.82 h wave, its decomposition is consistent with those of the average azimuth and of the amplitude of the 24.84 h wave.
- As regards the standard deviation of the azimuth, there is little consistency. This can be explained by the fact that the noise due to the initial ovalization and the defects of the balls, which do not result at all from the unknown action, are significant.
- As regards the amplitude of the 12 h wave, there is no exploitable decomposition. One explanation can be that the amplitude of the variations of the amplitude of the 12 h wave is quite small (1.3 grads). Besides, it is only an harmonic 2.

• Estimate of the period of the harmonic 1 of the multi-annual component:

We can take, as an estimate of it, the average of the periods relating to the most significant quantities: the average azimuth, and the amplitudes of the 24 h and 24.84 h waves, which are the two harmonics 1.

Hence a value of (5.53 + 5.57 + 6.11)/3 = 5.74 years.

5.3. Probability that Such a Decomposition of Both the Average Azimuth and the Amplitude of the 24.84 h Wave in a Semi-Annual Wave and a Multi-Annual Wave Is the Result of Chance

The observations taken into account are few and irregularly distributed. It is thus particularly important to check that, at least for the main quantities (average azimuth and amplitudes of the 24 h and the 24.84 h waves), it is very unlikely that, from random initial data, we can find the above decomposition, with coefficient of correlation at least as good.

To simplify, we will only consider the average azimuth and the amplitude of the 24.84 h wave: if we also take into account the amplitude of the 24 h wave, or if we consider the latter instead of the first, the probability of finding a fake decomposition can only be smaller (the coefficient of correlation is better for the 24 h wave: 0.92, instead of 0.76).

We therefore consider that the measurements of the average azimuth and of the amplitude of the 24.84 h wave are random variables, which are respectively $a = a(t_i)$ and $b = b(t_i)$, with $i = 1,2,\cdots,7$. Each of these 2 random variables is without memory, and they are independent.

N draws of (a,b) were made. For each draw was calculated, for $a(t_i)$ and for $b(t_i)$, the harmonic 1 of the residue of the fitting by a semi-annual wave. Hence the sinusoids $S_a(t)$ and $S_b(t)$. We therefore counted the number of draws that resulted in:

- a coefficient of correlation relative to $S_a(t) \geq 0.96$;
- a coefficient of correlation relative to $S_b(t) \geq 0.78$;
- a difference between the periods ≤0.58 years;
- a difference between the dates of extremums ≤42 days.

Two probability laws were considered: the normal law and the uniform law.

Table 7 shows that the estimate probability of getting a fake decomposition is $\leq 4 \times 10^{-4}$.

Table 7. Probability of fake decomposition.

law average azimuth	normal	uniform	normal	uniform
law amplitude 24.84 h wave	normal	uniform	uniform	normal
number of draws	10,000	10,000	20,000	20,000
number of fake decompositions	3	4	3	7
probability of fake decomposition	3×10^{-4}	4×10^{-4}	1.5×10^{-4}	3.5×10^{-4}

5.4. This Unknown External Action Is Almost Certainly of Astral Origin

From what precedes, it follows that it is very probably the same unknown action which is the main cause both of the variations of the average azimuth, and of those of the amplitudes of the 24 h and 24.84 h waves.

This action is therefore almost certainly of astral origin. Indeed it does not seem that, in a range of few hours around 24 h, what is the case of the 24 h and 24.84 h waves, there is any known phenomenon which would be a phenomenon only terrestrial, that is to say which would have nothing to do with the rotation of the Earth relative to the rest of the Universe.

6. From What Results the Multi-Annual Component of This Astral Action? Jupiter Is an Excellent Candidate to Explain an Important Part of It

The period of 5.74 years is very close to half the period of revolution of Jupiter (11.86/2 = 5.93 years)[13]. It is therefore quite natural to think of an action of Jupiter, since it is by far the main planet of the solar system, and that it is relatively close to the Earth (radius of its orbit: 5 AU).

We cannot exclude an action of the solar cycle, whose period was then about 11 years. Then the concerned solar cycle would be the solar cycle 19 (April 1954-October 1964).

6.1. Everything Is as If There Was an Important Influence of the Modulus of Jupiter's Declination

The graph in Figure 12 is indeed quite compatible with that: the shift is of about 1 year, for a period of about 6.2 years, and this difference can be explained by the fact that there is no reason why Jupiter would be the only planet to act on the pendulum.

[13]Jupiter being 5 times further from the Sun than the Earth, its orbital period intervenes strongly in its trajectory in relation to the Earth.

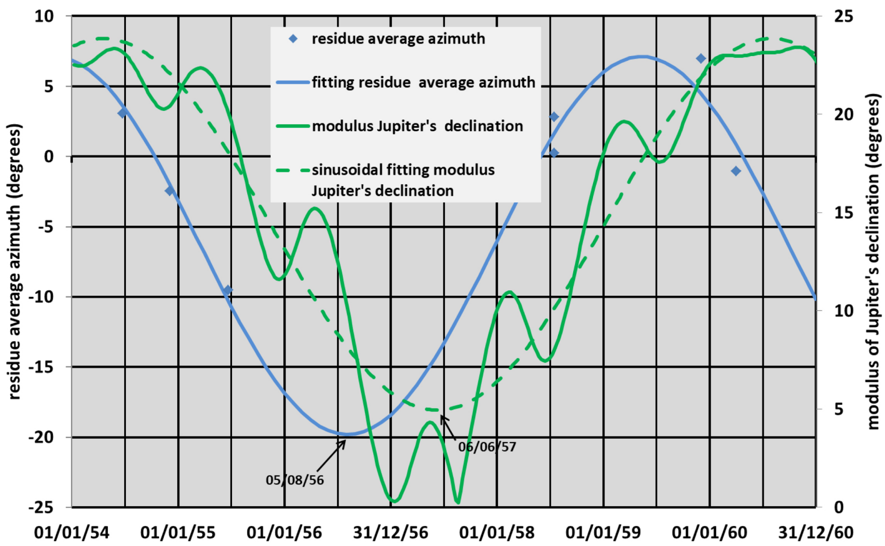

Figure 12. Modulus of the Jupiter declination and multi-annual evolution of the average azimuth.

6.2. Everything Is Also as if There Was an Important Influence of the Hour Angle of Jupiter on the Azimuth of the Pendulum

For more details: see **Appendix B**.

- Between 1954 and 1960, the average duration between two successive passages of Jupiter at the meridian varied between 0.9970 day and 0.9980 day. A daily action of Jupiter on the pendulum azimuth would therefore result in a periodic component of slowly variable period in the range (0.9970 day - 0.9980 day). Over 30 days, it is impossible to distinguish such a component from a 24 h component, as well as from all components close to 24 h, and in particular from a sidereal diurnal component (0.9973 day).

It is not the same for observations spread over 6 years, *i.e.* approximately 2200 days: we can then, around the period of 1 day, discriminate between waves separated by less than 0.00045 day. It thus becomes possible to distinguish a specific action of Jupiter, and, more precisely, an action of its azimuth and its hour angle. For this purpose, the azimuth $az(t)$ of the pendulum has been expressed as a function of these two quantities, which is equivalent to use filterings matched to each of them.

As shown in Figure 13, as regards the hour angle the coefficient of correlation is quite remarkable, and much better than for the azimuth. In addition, the zero crossings are close (25 min) to Jupiter's meridian and anti-meridian crossings.

- By expressing the azimuth of the pendulum as a function of the solar hour and of the sidereal hour, we obtain also an estimate of the solar and sidereal diurnal influences on $az(t)$: see Figure 14.

The amplitude of the estimated influence of Jupiter (1.17 grads) is between that of the Sun (1.80 grads) and the sidereal diurnal influence (0.89 grads).

- We can exclude an explanation by noise of this estimated "jupiterian" wave. We made the assumption that $az(t)$ was only uncorrelated Gaussian noise

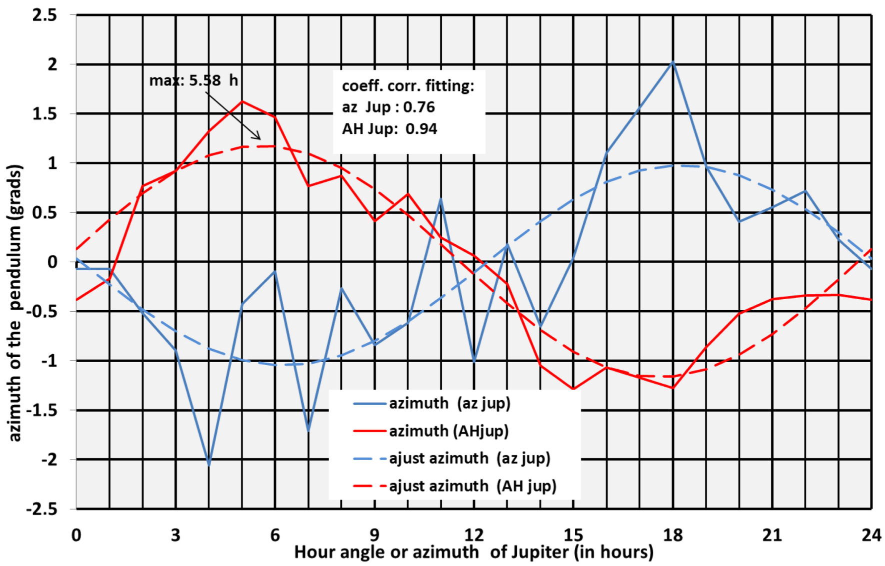

Figure 13. Azimuth of the Saint-Germain pendulum (minus the trend of every observation) versus hour angle and azimuth of Jupiter, and the fittings with sinusoids.

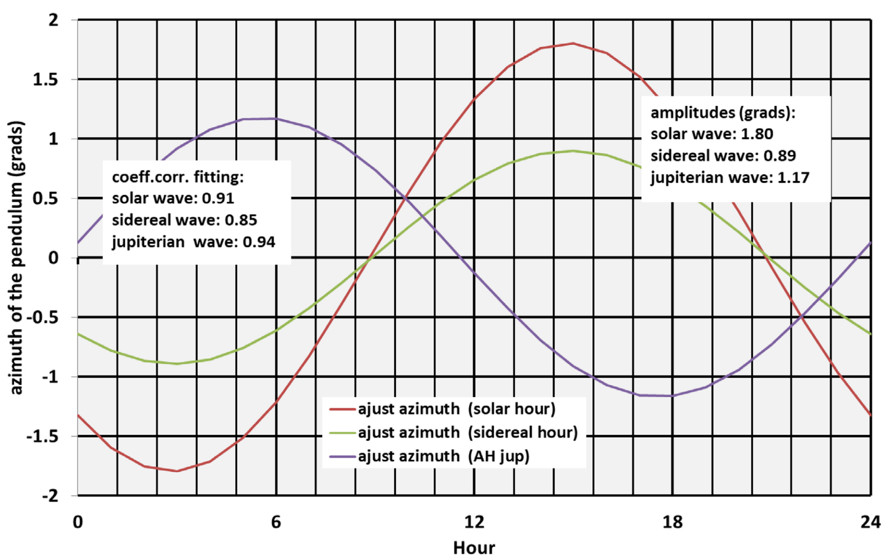

Figure 14. Azimuth of the Saint-Germain pendulum (minus the trend of every observation) as a function of hour angle of Jupiter, solar hour, and sidereal hour.

whose standard deviation is the average standard deviation (13.99 grads) of $az(t)$ after deduction of the trends. This assumption is obviously very pessimistic. A simulation of 1040 draws showed only 3 amplitudes slightly greater than 1 grad, and no one greater than or equal to 1.17 grads. The probability that a fake jupiterian wave results from noise is therefore very low.

- It remains to be examined to what extent this jupiterian wave would not come from other causes than the movement of Jupiter.

The problem is complicated by the fact that the measurements were not regularly distributed from 1954 to 1960: hence non-linearities, and therefore beats which create waves which did not exist in reality, and which might have created fake jupiterian waves.

It was verified that one could exclude an explanation by the solar diurnal influence and by the sidereal diurnal influence. The filter used to isolate the influence of Jupiter does not utterly eliminate them. But the amplitude of the solar wave (1.80 grads) is weakened by 70%, and that of the diurnal sidereal wave (0.89 grads) by 30%: they cannot explain the observed amplitude of 1.17 grads.

More generally, the analysis carried out in **Appendix B** showed that it would be necessary, to create a fake jupiterian wave having an amplitude of 1.17 grads, that there exist in the range 0.978 day - 1.018 day waves of about the same amplitude having very particular frequencies. This seems unlikely, but has not been verified.

7. Conclusions

7.1. Results

The re-use of Allais' raw data from his six one-month observations from 1954 to 1960 made it possible to confirm his results, as well as to complete them on certain points, to finally achieve the following results:

1) As far as can be judged, there is an unknown action of astral origin, apparently inexplicable by classical phenomena, and which includes, with a high probability, both a component related to the annual Earth revolution around the Sun (predominantly semi-annual), and a multi-annual component, the period of the harmonic 1 of which has been estimated to 5.74 years.

- This is indeed what emerges from the analysis of the following three quantities:

- the average azimuth on each observation.
- the amplitudes, for each observation, of the waves of 24 h and 24.84 h, which, on observations of 1 month, take into account the totality of the harmonics 1 of the actions of the motions of the celestial bodies, and correspond therefore to very remarkable periods.

Indeed, as one cannot over 1 month, around 24 h, distinguish waves separated by less than 0.83 h, the 24 h wave is representative of the group of lines that one find around 24 h, and which result from the action in a point of the Earth of the composition of the rotation of the Earth around its axis with the movements of the Sun and of all the celestial bodies other than the Moon.

The 24.84 h wave, which results from the composition of the rotation of the Earth around its axis with the revolution of the Moon around the Earth in one sidereal month, is, for its part, representative of the actions of the Moon (sidereal diurnal, synodal diurnal, …). It also takes into account the action, if it exists, of the composition of the rotation of the Earth around its axis with that of the Sun, the only other known phenomenon whose period is about 1 month. The multi-annual components of the amplitudes of the 24 h and 24.84 h waves are in phase, and in phase opposition with that of the average azimuth.

- With regard to the others remarkable periods of 12 h (harmonic 2 of the 24 h wave) and 25.82 h (composition of the rotation of the Earth with the harmonic 2 of the revolution of the Moon around the Earth), the corresponding waves cannot also be explained by classical phenomena. However, the decomposition of their amplitudes into a semi-annual wave and a multi-annual wave with a period near 6 years is only found for the 25.82 h wave. It is in phase with the multi-annual components of the 24 h and 24.84 h waves.

2) Jupiter, whose period of revolution is 11.86 years, which is by far the most important planet, and which is relatively close to the Earth, is an excellent candidate to explain a large part of the multi-annual component. The phase difference between the multi-annual component of the average azimuth and the modulus of Jupiter's declination is indeed only 0.83 years, and can be explained by the influence of other planets. Moreover, everything happens as if there were an important diurnal influence (of the same order of magnitude as that of the Sun) of the hour angle of Jupiter on the azimuth of the pendulum, this influence being minimum almost exactly at the passages of Jupiter at the meridian and the anti-meridian.

3) We cannot exclude an action of the solar cycle, whose duration was, for the concerned cycle, about 11 years (solar cycle 19: April 1954-October 1964).

4) As regards the annual action (action of the Earth's revolution around the Sun), it is mainly semi-annual, but the annual component is also important, at least for the average azimuth. It is remarkable that the extrema are at the equinoxes, and not at the solstices, which is the case for all the known geophysical factors, except variations in the Earth's magnetic field.

7.2. Many Questions Remain Regarding the Unknown Multi-Annual Action. In Particular:

- To what extent would the sunspots (and therefore the solar cycle) play a role? The explanations proposed in priority of the 24.84 h component and the multi annual component of about 6 years are respectively unconventional actions of the Moon and Jupiter. But in both cases, as we have seen, we cannot exclude an action of sunspots, the period of the solar cycle being close to that of Jupiter's orbit, and the period of rotation of the Sun on itself being close to that of the Moon's revolution around the Earth.
- If there is an action of Jupiter, there is no reason why there should be no action of the other planets. Can they be highlighted? Their angular velocity in the equatorial system being low, their movement in a local reference point appears, at least for distant planets, as a movement with a slowly variable period close to the duration of the sidereal day. To separate their action from the rest, and in particular from actions of the outside the solar system, of which there is no evidence that they do not exist, observations over several years are necessary.

7.3. About New Observations

This harvest of results shows the remarkable interest of observations of very long duration. Regarding new observations:

- They will have to be done continuously: this is absolutely essential for their overall harmonic analysis to be fully exploitable, which is very far from having been the case. Automation must therefore be total.
- It would be very interesting if, at least for part of the time, they used a second device identical to the first, if possible placed in a particularly protected environment: the fact that the observations of the summer of 1958 used a second pendulum, moreover implanted in an underground quarry, played an absolutely essential role in this study.
- It would also be very interesting for them to be coupled with very long-duration optical observations. Among the observations that Allais brought back from oblivion are those of Miller at Mont Wilson (1925-1926) [16], and of Esclangon at Strasbourg (1927-1928) [17] [18]. Spread over almost one year, and not over a few days, they revealed that, in what was thought to be noise, there was in fact a nearly diurnal sidereal component. Allais, by re-exploiting their data ([13] or [14], § V.C.[14] and § V.D.), showed that there was also an annual action which, as for the azimuth of the pendulum, had its extrema at the equinoxes, and not at the solstices.

Data Access

Digitised data is available on the Maurice Allais Foundation website:
http://www.fondationmauriceallais.org/the-physicist/maurice-allaiss-experimental-research-in-mechanics-his-observations-of-anomalies-in-the-movement-of-a-pendulum/?lang=en

Data before digitization is in the private archives of Maurice Allais. For any consultation request, please contact Mrs Christine Allais (christine.allais@fondationmauriceallais.org)

Conflicts of Interest

The author declares no conflict of interest regarding the publication of this paper.

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[14]Regarding the Esclangon observations, Allais actually only mentions the existence of a semi-annual component. However, as the analysis of the data he provides shows ([13] or [14] § V.C.2, p. 450), there is both a semi-annual component and an annual component, and in fact it is even the latter which is predominant. Its minimum is March 6, 1927.

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Appendices

A. Some Theoretical Elements about Pendulum

If the period of a pendulum has been a classic investigative tool for several centuries, it is not at all the same for its precession which, apart from Foucault's precession, is very little known. In addition, the authors who have studied it theoretically have used different notations, and sometimes different coordinate axis.

We have therefore gathered here a number of essential elements for its analysis, using the same notations and coordinate axes as in the article.

We have also explicitly introduced the notion of "anisotropy" of a pendulum, and more particularly of "linear anisotropy", which is a disturbance model making it possible to account for the effective action on the pendulum of a lot of disturbing phenomena.

We also verify that, concerning the Saint-Germain pendulum, it is impossible to explain by a change in its intrinsic anisotropy the changes from 1954 to 1960 in the average azimuth during each observation.

The formulas used are approximate formulas, which are valid only for very small ellipticities, which is possible only if the pendulum is sufficiently frequently stopped and restarted.

A.1. Two Kinds of Precession May Result from a Given Perturbing Action, in the More General Case

They are:

- the "direct" precession, which results from a direct action of the perturbing action on the trajectory,
- the Airy precession, which results from an indirect action: the perturbation acts on the ovalization and, as the pendulum is a spherical pendulum, the ovalization makes it to precess.

The rate of that precession is given by Airy's formula [19] [20] [21]:

$$\theta' = \frac{3}{8}\alpha\beta\sqrt{\frac{g}{l}} \tag{3}$$

where θ is the azimuth (defined modulo 180˚) of the major axis of the ellipse described by the pendulum, g the acceleration of gravity, l the length of the equivalent simple pendulum, and α and β the angular half major axis[15] and half minor angular axis of the ellipse (α and $\beta \ll 1$).

We can observe that the Airy precession is all the more important as α is large, and l small.

Whether you have a direct action or an indirect action depends on the phase of the perturbation [21]. For example, if you have a lateral impulsion exactly when the pendulum is at the extremity of its trajectory (or, more generally, in phase with the oscillation), you have only ovalisation. If you have a lateral impulsion exactly when the pendulum is at the middle of its trajectory (or at 90 degrees with the oscillation), you have only direct precession. The Coriolis force

[15] Angular half major axis = half major axis/physical length of the pendulum.

induces only a direct precession, which is the well-known Foucault precession.

Formula (3) is only an approximate formula. It was verified, by numerical integration, that for Horodnic pendulums [3], and an ellipticity of less than 0.01 (which is significantly higher than the measured ellipticities, which are of the order of 0.001), the difference with the formula (3) was only a few per thousand.

A.2. Anisotropy of a Pendulum; Notion of "Linear Anisotropy"

- As a general rule, the behavior of the pendulum is anisotropic: its trajectory depends on the starting azimuth. This results from the fact that the symmetry of the device is never perfectly revolutionary, on the one hand, and, on the other hand, that many external perturbing actions are directional.

Indeed a number of them make that the restoring force which calls back the bob of the pendulum to its rest position is not exactly the same in 2 perpendicular directions: hence a disturbance that acts in resonance with the oscillations of the pendulum. In particular, this is the case when the elasticity of the suspension varies with the azimuth of the plane of oscillation, which is a classic flaw of the pendulums [22]. It is also the case when the pendulum is in a field of forces which varies in the space swept during each oscillation.

Hence the interest of studying this kind of anisotropy. It is the simplest model (called "linear anisotropy"[16]) that has been considered:

- For a given azimuth the restoring force is proportional to the distance of the gravity center of the pendulum from the equilibrium point. The restoring coefficient is a function of the azimuth, which is periodic, with a period of 180˚.
- This 180˚ periodic function is a sinusoid.
- We consider that the oscillations remain small, which makes it possible to assimilate the movement of the pendulum to that of a plane oscillator.

The movement of the projection of the center of gravity of the pendulum on a straight line Ox of a given azimuth is then a solution of the differential equation $x'' + kx = 0$, k being the restoring coefficient. The solutions are sinusoids of pulsation $\omega = \sqrt{k}$. The period $T = 2\pi/\omega$.

Considering that the variations in the restoring coefficient are very small in relative value, we have, for the azimuth θ, as regards the period and the restoring coefficient:

$$T = T_0\left[1 + \eta\cos 2\left(\theta - \theta_A\right)\right], \tag{4}$$

$$k = k_0\left[1 - 2\eta\cos 2\left(\theta - \theta_A\right)\right], \tag{5}$$

where $\eta > 0$ and $\ll 1$. The period is maximum and the restoring force minimum in the direction θ_A.

η is called the "coefficient of anisotropy", and θ_A the "direction of aniso-

[16]This designation results from the fact that an analogy can be drawn between this kind of anisotropy affecting the behaviour of a plane oscillator and the notion of "linear polarization" of a field perpendicular to its propagation direction [22].

tropy".

- We can demonstrate[17] that it results from (4) or (5), with the additional hypothesis that the ellipticity $e \ll 1$:

1) A direct precession, the rate of which is θ'_d:

$$\theta'_d = 2\eta\omega\frac{\beta}{\alpha}\cos 2(\theta-\theta_A) = 2\eta\omega e\cos 2(\theta-\theta_A), \tag{6}$$

with $\omega = 2\frac{\pi}{T} = \sqrt{\frac{g}{l}}$. Hence the direct action calls back the major axis towards the azimuth $\theta_A + \frac{\pi}{2}$.

2) An ovalization $e = \beta/\alpha$ of the trajectory, which is such that:

$$e' = \mathrm{d}\left(\frac{\beta}{\alpha}\right)\Big/\mathrm{d}t = -\eta\omega\sin 2(\theta-\theta_A). \tag{7}$$

Hence:

$$e(t) = -\eta\omega\int_0^t \sin 2(\theta-\theta_A)\,\mathrm{d}t + e_L = -t\eta\omega\overline{\sin 2(\theta-\theta_A)} + e_L, \tag{8}$$

where $\overline{\sin 2(\theta-\theta_A)}$ = average value of $\sin 2(\theta-\theta_A)$ on $[0,t]$, and e_L is the initial ellipticity.

Hence, from (3) and (8), the speed θ'_i of the Airy precession (we can consider that α remains constant, as the pendulum is frequently relaunched):

$$\theta'_i = \frac{3}{8}\omega\alpha^2 e = -\frac{3}{8}t\eta\omega^2\alpha^2\overline{\sin 2(\theta-\theta_A)} + \frac{3}{8}\omega\alpha^2 e_L. \tag{9}$$

Therefore the indirect effect tends to call back the plane of oscillation towards the direction of anisotropy θ_A.

The average speed of Airy precession over a run (duration Δt) is:

$$\overline{\theta'_i} = \frac{1}{\Delta t}\int_0^{\Delta t}\theta'_i\,\mathrm{d}t = \frac{3}{16}\eta\omega^2\alpha^2\Delta t\,\overline{\sin 2(\theta-\theta_A)} + \frac{3}{8}\omega\alpha^2 e_L. \tag{10}$$

The initial ellipticity e_L is purely random, and its average is nil. Its average over a complete observation can therefore be neglected. Hence:

$$\overline{\theta'_i} \approx \frac{3}{16}\eta\omega^2\alpha^2\Delta t\,\overline{\sin 2(\theta-\theta_A)}. \tag{11}$$

Hence:

$$\left|\overline{\theta'_i}\right| \le \frac{3}{16}\omega^2\alpha^2\Delta t\eta \tag{12}$$

The value of α to be taken into account is its average value $\bar{\alpha}$ over a run, which is equal to 0.097 rad in the case of the Allais's pendulum ([13] or [14], § I.A.4 graphs III and IV, p. 94-95).

Hence, in the case of the Allais's pendulum ($\omega^2 = 11.82$; $\bar{\alpha} = 0.097$ rd; $\Delta t = 14$ min):

$$\left|\overline{\theta'_i}\right| \le 17.5\eta \text{ rad/s} = 1.12\times10^3\eta \text{ grad/s} \tag{13}$$

[17]Cf, with different notations, [21] p. 83 (with different coordinate axes), or [20].

A.3. Composition of Several "Linear Anisotropies"

All these perturbations are very small: we are in the linear area. Thus, if we consider for example the period (Equation (4)), and two linear anisotropies, we have:

$$T = T_0\left[1+\eta_1\cos 2\left(\theta-\theta_{A_1}\right)+\eta_2\cos 2\left(\theta-\theta_{A_2}\right)\right] = T_0\left[1+\eta_{tot}\cos 2\left(\theta-\theta_{A_{tot}}\right)\right] \quad (14)$$

Indeed the sum of two sinusoids of period 180˚ is a sinusoid of period 180˚. The composition of several linear anisotropies is therefore also a linear anisotropy.

If we define an "anisotropy vector", the modulus of which is η and the argument $2\theta_A$, it is easy to verify that the anisotropy vector of the composition of several anisotropies is the sum of the anisotropy vectors.

A.4. Measurement of the Intrinsic Anisotropy of the Allais Pendulum

- On a given run, we can only measure the total anisotropy, which is the sum of the intrinsic anisotropy of the pendulum, which is constant, and of a variable external anisotropy: this makes it necessary to measure a sufficiently large number of runs over a sufficiently long period. This is what Allais did, according to a procedure described in [13] or [14] § I E 3, the results of which are given in table X, page 180. It consisted in measuring the average precession velocity over the run for azimuths 0, 10, ⋯, 190 grads. For every azimuth several measurements were made to reduce random influences. The curve obtained looks like a 200 grads period sinusoid. By limiting itself to its harmonic 1, we obtain, with Allais's notations:

$$\bar{\Phi}' = a_0 + a_1\sin 2\left(\Phi-\Sigma_1\right) \quad (15)$$

where Φ is the starting azimuth of the run, and $\bar{\Phi}'$ the average precession speed over the run.

For the longest measurement (March 4 to 10, 1955), which is taken as the reference, $a_1 = -0.838$ grads/min = −2.19 × 10^{-4} grads/s, and $\Sigma_1 = 170.67$ grads[18].

- This value is almost exactly the azimuth of the perpendicular to the suspension beam QP (see **Figure 1**), which is 171.16 grads: the cause of the intrinsic anisotropy is perfectly identified.
- Over the duration of the run, θ varies only of few grads. Hence we can confuse $\overline{\sin 2\left(\theta-\theta_A\right)}$ and $\sin 2\left(\theta_L-\theta_A\right)$. Hence, from Equation (11):

$$\bar{\theta}_i' = \frac{3}{16}\eta\omega^2\bar{\alpha}^2\sin 2\left(\theta_L-\theta_A\right) \quad (16)$$

- By identifying Equations (15) and (16), with $a_0 = 0$, $\theta_L = \Phi$, and $\theta_A = \Sigma_1 + 100$ grads modulo 200 grads, we have $\theta_A = 170.67$ grads, rounded to 171 grads, and

$$-a_1 = \frac{3}{16}\omega^2\bar{\alpha}^2\Delta t\eta \quad (17)$$

[18]In Allais' table the value is 370.67 grads, because angles are counted from south, and not from north.

Hence, with $a_1 = -2.19 \times 10^{-4}$ grads/s, $\omega = 3.44$, $\bar{\alpha} = 0.097$ rad and $\Delta t = 840$ s, $\eta = 1.25 \times 10^{-5}$.

A.5. Verification that, in the Case of Allais Pendulum, We Can Neglect the Direct Effect

The ratio between the direct effect and the indirect effect can be deducted from (6) and (9) (we neglect in (9) the term dependent of e_L, which can only increase the indirect precession, and which is random and whose average is nil).

$$\frac{\theta_i'}{\theta_d'} = \frac{3\alpha^2}{16\eta\cos 2(\theta - \theta_A)} \geq \frac{3\alpha^2}{16\eta} \tag{18}$$

Hence the indirect effect is dominant if the coefficient of anisotropy η remains very small.

In the case of Allais's pendulum (see above), α is about 0.1 rd, and η about 10^{-5}. Hence $\frac{\theta_i'}{\theta_d'} \geq 180$.

A.6. Average Azimuth Resulting from Total Anisotropy

It is the starting azimuth θ_L for which the Airy precession, apart from the random term resulting from the initial ellipticity, exactly compensates the Foucault precession (-5.5×10^{-5} rad/s) at the end of the run.

It is assumed that the anisotropy of external origin is also, to a first approximation, a "linear anisotropy". The relevance of this hypothesis could be verified for the observations conducted in Romania in 2019: see [3], Appendix B.5, c).

From Equation (11), we have:

$$\bar{\theta}_i' = \frac{3}{16}\eta\omega^2\bar{\alpha}^2 \sin 2(\theta_L - \theta_A) = -5.5 \times 10^{-5} \text{ rad/s} \tag{19}$$

With the intrinsic anisotropy of the pendulum acting only ($\theta_A = 171$ grads, and $\eta = 1.25 \times 10^{-5}$)), the equilibrium starting azimuth is 162.6 grads, which is quite consistent with the average of the 6 observations (163.6 grads: see **Table 2**).

The direction of intrinsic anisotropy not having moved, only could have evolved η, *i.e.* the elasticity of the beam.

Table A1 shows that it would be not realistic at all: very large variations of the coefficient of anisotropy would have been necessary to explain the large variations of the average azimuth observed from 1954 to 1960, moreover in one direction,

Table A1. Anisotropy coefficient and azimuth of the equilibrium position.

anisotropy coefficient η	0.5×10^{-5}	1×10^{-5}	1.5×10^{-5}	2×10^{-5}	2.5×10^{-5}
azimuth θ_L corresponding to the equilibrium position (grads)	148.4	160.4	164	165.8	166.8

and then in the other (from 164 grads to 150 grads between June 1954 and June 1955, then from 150 grads to 174 grads in march 1960). Besides, it would have been an extraordinary coincidence that, in 1958, there would have been concordance with the average azimuth of Bougival pendulum.

B. Search for a Diurnal Action of Jupiter on the Pendulum Azimuth

B.1. Nature of This Search: Filtering to Be Performed

- $s(t)$ is the signal to be analysed. It consists of 4463 measurements spread over 6 observations of about 1 month, from $t_{ini} = 8/06/1954$ at 8 h to $t_{end} = 16/04/1960$ at 5 h, with 1 measurement per hour (which is itself the average of 3 measurements made every 20 minutes). We have:

$$s(t) = az(t)\sum_{i=1}^{6} p_{Ti}(t - t_i), \tag{20}$$

with:

$az(t)$: azimuth of the pendulum, the pendulum being considered as released without interruption from t_{ini} to t_{end}.

$p_{T_i}(t - t_i) = 1$ in $[-T_i/2, +T_i/2]$, and =0 elsewhere.

t_i = midpoint date of the *i*th experiment

T_i = duration of the *i*th experiment.

What interests us is $az(t)$. In fact, what we have is $s(t)$, which is deduced from $az(t)$ by an operation that introduces non-linearities, as shown by Equation (20): there are therefore in $s(t)$ waves that do not really exist in $az(t)$.

- The purpose is to investigate, from the observation of $s(t)$, to what extent Jupiter's daily revolution around the Earth has an influence on $az(t)$. Two quantities were examined:

- the azimuth of Jupiter $az_{jup}(t)$ (we are in the local horizontal coordinate system).
- its hour angle $AH_{jup}(t)$ (we are in the equatorial coordinate system).

If these two quantities were exactly periodic with a frequency f_{jup} characteristic of this revolution, it would be a simple problem of harmonic analysis: a band-pass filter of the input signal around this frequency would provide the amplitude and the phase of the wave of frequency f_{jup}. A classical method would then consist in expressing the input signal as a function of $AH_{jup}(t)$ (or of $az_{jup}(t)$, expressed in hours), then in calculating the amplitude and the phase of the fitting by a sinusoid of period 24 h, the correlation coefficient with the fitting being an indicator of the credibility of the wave found.

But $AH_{jup}(t)$ and $az_{jup}(t)$ are in fact sinusoids with a slowly varying period (between 0.9970 day and 0.9980 day). However, the previous approach still allows, with the hypothesis that the input signal contains, with a possible phase shift, a wave proportional to $AH_{jup}(t)$ (or to $az_{jup}(t)$, to calculate its amplitude and its phase. We have therefore a filter matched to $AH_{jup}(t)$ (or to $az_{jup}(t)$).

B.2. Result: Everything Is as if There Was an Important Influence of $AH_{jup}(t)$

This is what emerges from the results of filtering $s(t)$ by filters matched to $AH_{jup}(t)$ and $az_{jup}(t)$): see § 6.2, Figure 13.

Figure 14 shows that the amplitude of the wave linked to $AH_{jup}(t)$ (the "jupiterian" wave) is 1.17 grads, which is of the same order of magnitude as the diurnal solar and diurnal sidereal waves (respectively 1.80 grads and 0.95 grads), which were obtained by expressing $s(t)$ in solar hour and sidereal hour.

It shall be noted that the analyzes were made after, on each experiment, deduction of the trend, so as to eliminate the slow variations of the azimuth of the pendulum, which are due to factors other than the diurnal movement of Jupiter, and of which we saw that they were very important (several dozen of grads: see Figure 4 in § 3.2).

B.3. It Remains to Be Examined to What Extent This Jupiterian Wave Would Not Come from Other Causes than the Movement of Jupiter

- Let us study the filter matched to $AH_{jup}(t)$, which is in fact very close to a band-pass filter around the average period of $AH_{jup}(t)$ (0.9975 day). To determine the extent to which the amplitude found corresponds to a real jupiterian wave, and not to an ordinary periodic wave, we search for the response of this filter when $az(t)$ is a wave of a given period T (the amplitude is taken equal to 1, and the phase to zero).

If there were no irregularity in the sampling, $s(t)$ would be this wave. As shown in Figure B1 or Figure B2 (red curves), only waves very close to the average period of 0.9975 day could create large fake jupiterian wave. As there are irregularities, it is quite different (blue curves).

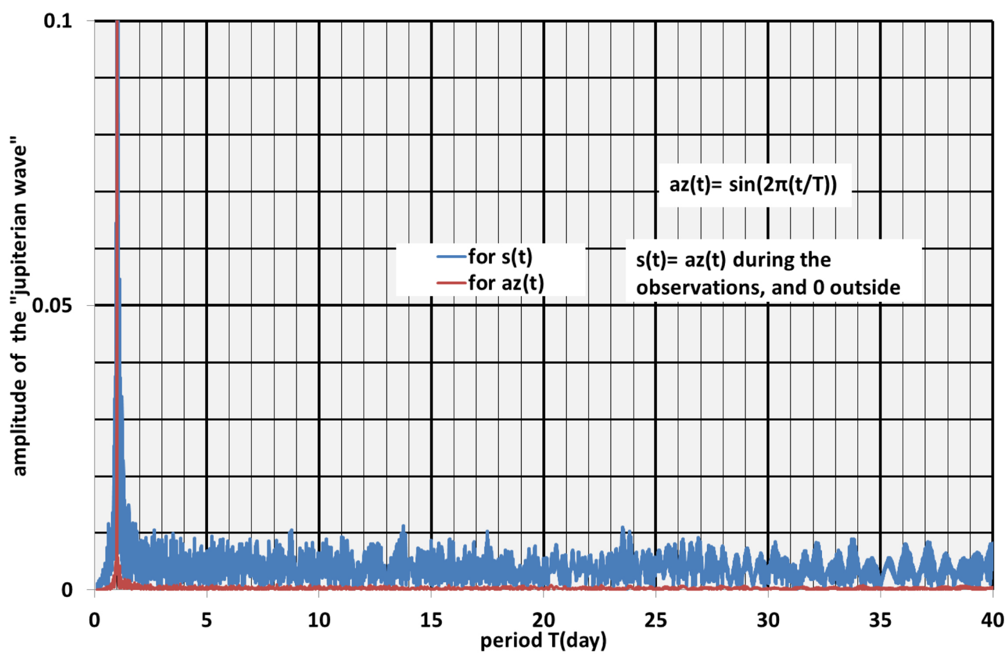

Figure B1. Transmission coefficient of the "AHjup filter".

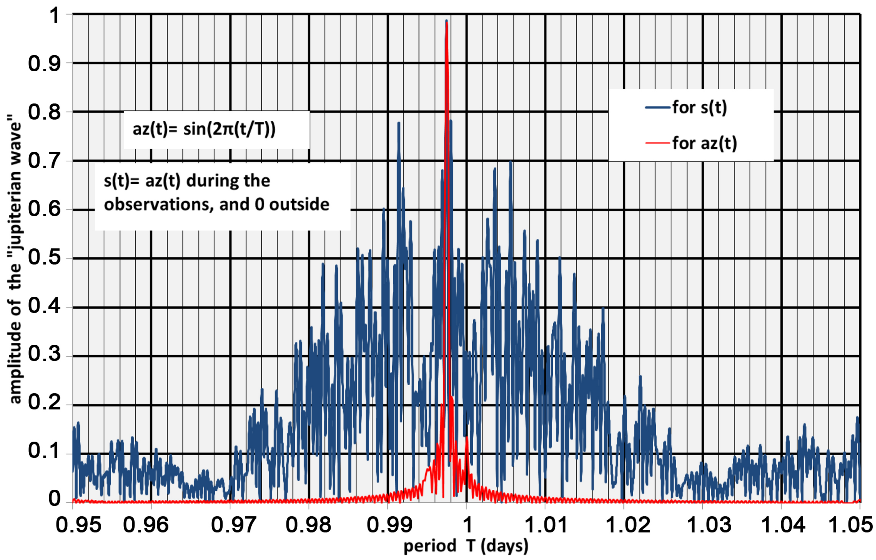

Figure B2. Transmission coefficient of the "AHjup filter"-Range 0.95 day-1.05 day.

However **Figure B2** shows that, outside the range 0.978 day - 1.018 day, the attenuation is at least 70%, which seems to exclude that a wave could result in a fake jupiterian wave whose the amplitude is 1.17 grads (the amplitude of this wave should be at least 1.17/0.3 = 3.9 grads, which is twice the amplitude of the solar wave).

In particular that excludes a lunar action: 24.84 h = 1.035 day is very clearly outside this range.

- In this range 0.978 day - 1.018 day, it was verified that one could exclude an explanation by the solar diurnal influence and by the sidereal diurnal influence. The filter used to isolate the influence of Jupiter does not utterly eliminate them. But the amplitude of the solar wave (1.80 grads) is weakened by 70%, and that of the diurnal sidereal wave (0.89 grads) by 30%: they cannot explain the observed amplitude of 1.17 grads.

More generally, in this range, it would be necessary, to create a fake jupiterian wave, that there exist in the reality waves of great amplitude and whose frequency is exactly that of one of the peaks of the blue curve of **Figure B2**. It is not the most probable, but we cannot say that there is none.