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Einstein-Local Counter-Arguments and Counter-Examples to Bell-Type Proofs

Karl Hess

Center for Advanced Study, University of Illinois, Urbana, USA
Email: karlfhess@gmail.com

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Abstract

J. S. Bell's well-known proofs of inequalities (and related work) are shown to be invalidated by two counter-arguments (-examples) that are based on Einstein-local propositions: Bell-type inequalities of Einstein-Podolsky-Rosen experiments must include, as virtually all physical theories do, elements of physical reality and their mathematical representations that relate to continua as opposed to exclusively finite numbers. Furthermore, Bell-type inequalities must be valid for all possible experimental geometries that lead to the quantum result. Based on these propositions, violations of Bell-type inequalities are demonstrated without violating Einstein locality, without conspiracy type theories and even for the case that all known "loopholes" are closed.

Keywords

Bell's Inequalities, Quantum Entanglement, EPR Experiments

1. Introduction

Einstein-Podolsky-Rosen (EPR) Gedanken-experiments [1] are at the core of the Einstein-Bohr debate about the nature of physical reality. Einstein intended to prove the existence of elements of physical reality that bring about distant correlations and provide an explanation for the "entanglement" apparent in the formalism of quantum theory. Variations of these Gedanken-experiments have been suggested by Bohm. These EPRB experiments have actually been implemented and performed mostly by use of photons and measurements of photon-polarization. The detailed history of these experiments and measurements has been described by Gilder [2].

Kocher and Commins [3] have been the first to introduce entangled photon pairs and to perform with them EPRB-type measurements. (I use the word entangled exclusively in the operational sense of being prepared in a certain way). They have confirmed correlations of the measurements with polarizers and detectors over macroscopic distances. Their measurements emphasize the very significant correlation of pair-detector-outcomes when equal polarizer directions are used in both stations. Einstein had linked such significant correlations to the existence of elements of physical reality based on his separation principle and elementary logic. Kocher and Commins [3] had, thus, contributed to the Einstein-Bohr debate in Einstein's favor.

Subsequently, Bell [4] and later Clauser, Horne, Shimony and Holt (CHSH) [5] have presented a theory and model-inequalities for the distant EPRB correlations that use (at least two) different and specifically chosen polarizer directions in each wing of the experiment. Their model results contradict the results of quantum theory as well as the statistical results of additional actual measurements [6] for these polarizer directions. Ordinarily, such a theory and model would have been dismissed out of hand. However, Bell-CHSH have claimed that their results derive directly from Einstein's hypothesis of elements of physical reality and the strict enforcement of Einstein's separation principle (Einstein's local realism), which is only based on the limiting speed of light in vacuum. I would like to emphasize that Bell and later CHSH claim emphatically that their theory and model of EPRB experiments contains only Einstein-type physics [1].

Einstein's specific world-views of physical reality were, thus, put in jeopardy. More recent experiments (see [6] [7] and references therein) have indeed proven that the Bell-CHSH model-results are in plain contradiction to these new measurements and the majority of today's physicists, therefore, maintains that Einstein's views indeed need to be revised and have accepted that Bell-type theories together with modern EPRB experiments prove this fact.

It is the purpose of this paper to present counter-arguments and counter-examples to the Bell-CHSH-type theories that claim to have derived a limit to what may be "classically" observed (*i.e.*, follow in essence from Einstein's physics based on his separation principle). I show that the Bell-CHSH limitations of the correlations of the entangled-pair measurements follow instead from considerations regarding the mathematical cardinality of the sets of elements of physical reality as compared to the cardinality of the set of measurements that are involved, as well as from other factors. The disagreement of the Bell-CHSH theorems with the experiments (e.g. [6]) gives, therefore, no indication that Einstein's world view related to EPRB experiments has any deficiencies and the original measurements of Kocher and Commins [3] appear to have correctly supported Einstein's views.

2. The Experiments

Modern versions of such experiments [7] use entangled (in the operational sense) photon-pairs that are guided by optical fibers and subsequently controlled by electro optical modifiers (EOMs) as well as by polarizers (that we describe, for reasons of clarity and illustration, by Wollaston prisms (cubes)). These polarizers partition the set of incident photons into two sets, one with horizontal and the other with vertical polarization. **Figure 1** shows details of such experiments and includes also some information about their usual mathematical description. The right-hand side of the figure shows the coordinate system with the z-axis in the direction of photon-propagation. The x-, y-axes are perpendicular to z and to each other, but otherwise may be chosen in arbitrary directions of ordinary three-dimensional space. The quantum theory of such experiments is invariant to rotations of the Wollaston-pair around the z-axis, which is confirmed by the actual measurements. The square Wollaston-cube-faces perpendicular to the z-axis have boundary-lines parallel to the x-axis (y-axis) that determine the vertical (horizontal) polarization carried in the red (green) channels.

The effects of the EOMs are equivalent to rotating the left-wing Wollaston by an angle j and the right-wing Wollaston by an angle j' around the z-axis. Note the clocks in each wing. They indicate that the photon-pair-parts emitted from the source are identified as correlated (entangled) by using the space-time system.

Detectors following the green and red channels click at a certain location and measurement time. These clicks are symbolized by values of +1 and −1 respectively in both wings.

The actual experiments à la Aspect [6] are performed by using one source of photon-pairs and include fast random switching between the EOM angles $j = a$, a' in one wing of the experiment as well as $j' = b$, b' in the other. The differences $(j - j')$ of the Wollaston angles are fixed, however, to the so-called Bell-test angles during all "random" switching procedures.

Because of the latter fact and for the sake of clarity and understanding, these experiments may be regarded as equivalent to experiments and measurements that use four different photon sources and the same four pairs of EOM angles, one for each source. This equivalence may require, in general, a number of additional more or less reasonable assumptions such as the lack of memory from past measurements. However, all the reasoning presented below is valid for both, experiments that use four sources (denoted by $i = 1, 2, 3, 4$) and for experiments that use only one source, as the actual Aspect-type experiments do.

If we consider only one photon source, we imagine a time axis sub-divided into 4 non-intersecting sequences of time intervals, each related to one of the 4 EOM angle-pairs and we use $i = 1, 2, 3, 4$ for the 4 sequences of time intervals. The non-intersection of the time intervals is justified by the inertia related to the switching of the EOM angle pairs.

3. Connecting Experiments to Set-Theoretic Probability

The measurements and their outcomes need to be brought into a one-to-one correspondence with the model-outcomes of Bell's functions. We are exclusively interested in the experiment-types that are described in [3] [6] [7] and schematically in **Figure 1**. The detector clicks are modeled by functions $A = \pm 1$ in the left wing as well as $B = \pm 1$ in the right, respectively. The entangled photon pair is mathematically modeled by a given symbol λ_{in}, where $n = 1, 2, \ldots, N$ enumerates the measurement outcomes for time segment i. The two detector clicks for a given photon pair and the corresponding function outcomes (e.g. $A = -1$ and $B = +1$) are certified as a "whole" belonging causally together by the registration of measured clock times (see **Figure 1**). This fact is very important, because the properties of the λ_{in} are at this point not specified. The only concern of the Bell-CHSH work is whether the outcomes on the two sides for given λ_{in} are equal or not [8]. Therefore, the mathematical model-procedure dealing with these measurement-outcomes must also consider them an indivisible whole. In fact, the definition of the "indivisible elementary events" is the premise of any probability theory of the Kolmogorov type that attempts to describe actual events in space and time (see discussion in [8]). Different indivisible elementary events describe, in general, different experiments and different experiments have different Bell-CHSH-type limitations (constraints).

The indivisible elementary model event that describes measurements of entangled pairs is, thus, represented by the function-pair $A(j,\lambda_{in})B(j',\lambda_{in})$ or simply by the product of this pair that is defined after letting the function assume values of +1 or −1 in the left wing and the function B the values +1 or −1 in the right wing of the EPRB experiment shown in **Figure 1**. The symbols j and j' describe the possible EOM angles and λ_{in} represents, as mentioned, the properties of the given prepared photon-pair that causes the particular outcomes. Notice that, these products are never partitioned or "reduced" in any of my considerations below, in contrast to the procedures of Bell-CHSH. Such partitioning or reduction always involves necessarily assumptions about the properties of λ_{in} as we will see below.

4. Strategies for Proofs of Bell-CHSH Model-Inequalities

The Bell-CHSH inequalities are numerical constraints for each of the following quadruples of the model-outcome products $A(j,\lambda_{in})B(j',\lambda_{in})$:

$$Q_n = A(a,\lambda_{1n})B(b,\lambda_{1n}) + A(a,\lambda_{2n})B(b',\lambda_{2n}) + A(a',\lambda_{3n})B(b,\lambda_{3n}) - A(a',\lambda_{4n})B(b',\lambda_{4n}). \quad (1)$$

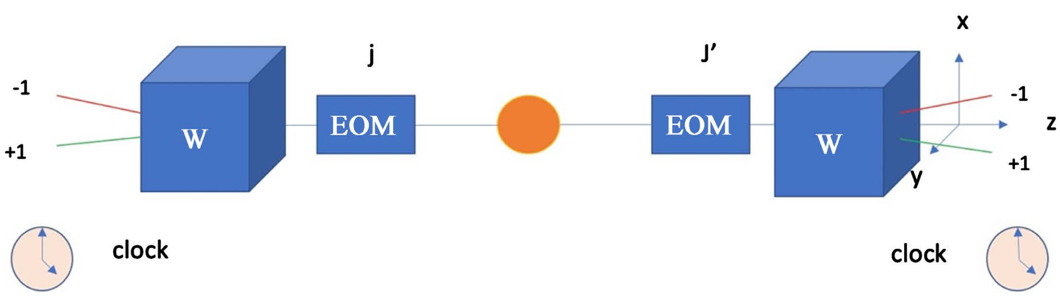

Figure 1. Schematics of EPRB experiments with appropriately prepared photon-pairs being emitted from a source (orange) toward electro optical modifiers (EOMs) and Wollaston prisms (W). The mathematical symbols are explained in the text.

As explained, subscript i of λ_{in} indicates correspondence to the photon pair from photon-source $i = 1, 2, 3, 4$ (or equivalent time-segments). The subscript $n = 1, 2, 3, \ldots, N$ enumerates different sets of such fourfold model-outcomes, where N is a large number. Equation (1) represents, thus, a very large matrix or table with rows consisting of 4 pair products of possible function outcomes AB, which also form columns of N such pair products. The Bell-CHSH inequalities corresponding to Q_n are given by the constraint $|Q_n| \leq 2$ for the absolute values of about all quadruples of the matrix (1).

In order to explain the origin of the Bell-CHSH inequality and other inequalities and constraints of similar type, we reduce generality in a plausible way. We define the λ_{in} as somehow randomly chosen symbols.

If indeed the λ_{in} correspond to random photon pair emanations of 4 different experiments and measurements, there appears to be a lack of any direct physical reason for such a constraint and we must ask ourselves why such a constraint or any constraint may exist. Intuitively one may suspect that the constraint exists, because the model-quadruples have a mathematical structure: each of the EOM angles appears twice in each quadruple. Certain additional elementary mathematical or physical characterizations of the domain and range of the functions may then constrain the numerical value of each quadruple. This fact also means that the sum of all columns as well as their expectation values must follow a constraint, because one must obtain the same result whether one adds all N quadruples or all 4 columns, particularly if we deal with a countable number of matrix elements. Even for an uncountable number, the interchangeability of row and column summations is guaranteed by Fubini's theorem.

The question of such constraints has indeed been dealt with in very general topological-combinatorial terms by Vorob'ev [9], not known, however, to Bell-CHSH. The relevance and importance of Vorob'ev's findings for EPRB discussions, is probably best illustrated by an example.

4.1. Illustration of the Origin of Bell-CHSH Constraints

Assume that all λ_{in} are randomly chosen from a set S_λ containing a countable number of elements, such as $S_\lambda = [H, V]$ that are related to the physics of correlated photon pairs. Then we may reorder the matrix (table) of Equation (1) in the following way: the columns of the model-pair products AB with given EOM angle-pair and given source (or set of time segments), may be reordered arbitrarily without changing the column average or expectation value. We may, therefore, reorder the 4 columns of the matrix (1) such that about all rows corresponding to the 4 different EOM angle-pairs contain exclusively either H or V and add up to either +2 or −2, because we have assumed that $A, B = \pm 1$. This numerical limitation of the single rows limits the value of the N-row averages and, thus, also the value of the 4 column averages. It is important to realize that virtually all of the Bell-CHSH type proofs deal with countable sets S_λ. For these sets, the Bell-CHSH constraint $|Q_n| \leq 2$ is guaranteed, as can be proven by the reordering process (see also page 1803 of [8]); the best-known examples being

Bertlmann's socks in one of Bell's own proofs, or the eight combinations of green and red flashes in Mermin's proof for a broad public (Physics Today). These proofs are, therefore, in a limited way correct and indeed may correspond to certain actual experiments that are commensurate with countable sets S_λ. The question whether these model-results indeed represent "impossibility proofs" and whether they apply to actual EPRB experiments is a main topic of this paper.

4.2. Discussion of the Bell-CHSH Constraints

In all of these just discussed cases the quadruples in the rows of the matrix corresponding to Equation (1) may be reordered into:

$$\begin{aligned}&\left|A(a,\lambda_n)B(b,\lambda_n)+A(a,\lambda_n)B(b',\lambda_n)\right.\\&\left.+A(a',\lambda_n)B(b,\lambda_n)-A(a',\lambda_n)B(b',\lambda_n)\right|\le +2,\end{aligned} \tag{2}$$

if N is just large enough. We have, by the reordering process, demonstrated that matrix (2) follows from any possible matrix (1), because S_λ is countable, a condition that enforces what Vorob'ev [9] called a cyclicity: Each EOM angle appears twice and each product of a given quadruple contains the same λ_n and, thus, is fully determined if the values of the three other products are chosen. Insert all possible values of +1 and −1 for the functions A, B and the inequality $|Q_n| \le 2$ is indeed fulfilled for all quadruples with given row-number $n = 1, 2, 3, \ldots, N$. (For further details regarding Vorob'ev's cyclicity and reordering see the Appendices of [10] [11]).

(Remark: Given matrix (1), it may be possible to reveal agreement with (2) in a variety of ways if and only if matrix (1) contains already (for whatever reason) a Vorov'ev cyclicity and a corresponding Bell-CHSH constraint. Lambare [12] has discussed equivalence classes of the λ_{in} whose definitions are based on properties of the functions A and B, which frequently must assume identical values for different λ_{in}. The use of equivalence classes must, however, under all circumstances conserve the sum of all columns (and rows) of the original matrix (1) and can, therefore, not introduce any Bell-CHSH constraint that is not a property of matrix (1) to start with. This fact will be an important point when discussing counter-argument (i) below.)

The Bell-CHSH constraint of Equation (2) is, thus, a consequence of possible topological-combinatorial patterns that the model-functions AB as well as their domains may assume in the matrix (1). As we saw, the fact that S_λ represents a countable set of discrete values of elements λ_{in} is important to guarantee the CHSH constraint. Note also the additional necessary assumption: the λ_{in} of the set S_λ that represent the emanated photon pairs, must be stochastically independent from the possible EOM settings j, j', because we have used S_λ for all measurements independently of any EOM settings (see [13] for a detailed elaboration on the latter fact).

Physics, however, does by no means guarantee that S_λ represents a countable set of discrete values of elements λ_{in}. If the physics requires a different constraint or no constraint, then we need to change the properties of S_λ as well as the functions A, B and hope that such change permits the model to agree with the physics of the actual measurement outcomes.

An alternative, very radical, path is to revise Einstein's local realism altogether [13]. Therefore, those who consider the proofs of Bell-CHSH as "impossibility-theorems", must show that all possible sets S_λ that are legitimate within the physics and mathematics of Einstein's local realism must lead to matrix (2) and that each quadruple of the rows must encompass identical or equivalent λ_n. Then and only then, does Einstein's local realism contradict the actual experiments and quantum mechanics, which are not bound by +2 but rather by $2\sqrt{2}$ for the Aspect test angles. (We stipulate for the present purpose that the actual experiments have closed all loopholes and exhibit values close to the quantum result.)

(Remark: Bell-CHSH have incorrectly assumed the general possibility of reordering based only on the stochastic independence of the λ_{in} and the EOM angles. They also have not considered the indices i, and n in their original works [4] and [5]. However, as we will show by counter-argument (i) below, the Bell-CHSH inequalities cannot be derived for the row-sums of model-outcome pairs AB, as soon as S_λ represents a continuum out of which the λ_{in} are randomly and uniformly chosen. As a consequence, the inequalities cannot be derived using integrals that are defined on the basis of such sums. A further problem of the original Bell-CHSH proofs is revealed by the algebraic step explicitly shown in [5]:

$$\left|A(a,\lambda)B(b,\lambda)-A(a,\lambda)B(b',\lambda)\right|=\left|1-B(b,\lambda)B(b',\lambda)\right|,$$

which partitions the indivisible elementary events with catastrophic consequences: The product of the two B-functions on the right-hand side does not correspond to any elementary event and is either ill-defined or does not correspond to the actually performed experiments but instead to other experiments with elementary events given by the separate functions $A(a, \lambda)$, $B(b, \lambda)$ and $B(b', \lambda)$; all assumed to exhibit the same λ whenever they appear algebraically together.)

4.3. Early Counter-Arguments to Proofs of Bell-CHSH Inequalities

The contradictions of Bell-CHSH-type models to both quantum mechanics and the actual experiments have astounded the physics community and a sizeable majority has indeed considered sacrificing Einstein's separation principle to circumvent the stochastic independence of λ_{in} and the EOM angles j, j'. They did not consider the cardinality of S_λ and they did not acknowledge Vorob'ev's findings.

A large number of present-day physicists hold "quantum-nonlocalities" as inevitable to explain the violations of Bell-CHSH; they propose forms of "influences" faster than the speed of light that remove the stochastic independence of λ_{in} and j, j', while not violating the velocity of light as the limiting speed for information related signals, because of the randomness of the measurement outcomes.

Gerard't Hooft [14] has proposed the possibility to look more carefully at no-tions such as "super-determinism" and "conspiracy" and to invoke "…variables controlled by various types of conservation laws" that remove the stochastic independence. He also has hinted toward a lack of free will of the experimenters to choose j and j'.

Less general and often very complicated violations of the Bell-CHSH inequalities have also been devised. There exists a huge literature on so-called loopholes; ways around Bell-CHSH that do not use super-determinism or quantum non-localities but still work around the stochastic independence without violating Einstein's separation principle. These works often use rather complex reasoning (see [10] [11] and references therein), involving inter alia stochastic processes, nonergodicity, reasoning about dynamics and kinematics of particle-instrument interactions as well as questions of identification of the correlated (entangled) pairs. The following considerations are independent of and additional to all these investigations and even hold if all known loopholes are closed.

5. Counter-Arguments to Bell-CHSH Using Vorob'ev's Findings

There exists a straightforward path out of the Bell-CHSH conundrum that altogether circumvents Gerard't Hooft's invocation of super-determinism and "conspiracy" [14] as well as influences faster than the speed of light. The key of this approach is to remove the Vorob'ev cyclicity of Equation (2) and, thus, to remove the Bell-CHSH constraints on the correlations altogether, instead of attempting to strengthen them by some means. After the cyclicity of expression (2) is removed for the quadruples, correlations may be introduced by appropriate definition of the functions A, B that may originate from laws of physics that only govern the pair-outcomes.

The two counter-arguments to Bell-CHSH that are discussed below, show how the Bell-CHSH-Vorob'ev cyclicity may indeed be removed even though we specify that the λ_{in} and the EOM angles j, j' are stochastically independent. In that respect, the counter-arguments represent COUNTER-EXAMPLES to the common belief that said stochastic independence is, with straightforward additional assumptions [13], sufficient to validate the Bell-CHSH constraints. Because the stochastic independence is related to locality considerations, the reasoning below represents a COUNTER-EXAMPLE to the common belief that the Bell-CHSH constraints follow directly from Einstein's separation principle.

My counter-arguments are based on the fact that a particular Vorob'ev cyclicity must be demonstrated to enforce the Bell-CHSH constraints and that such cyclicity cannot be demonstrated if the following postulates hold:

(i) The mathematical representations of Einstein's elements of physical reality also relate to continua as opposed to exclusively countable sets.

(ii) The Bell-CHSH-type inequalities must be valid for all EPRB equipment-configurations corresponding to the same EOM angle-differences.

(Remark: Postulate (ii) does not have the mathematical capacity that postulate

(i) has and represents rather a requirement of proper modeling and proper comparison to quantum theory as well as the expectation values of actual experiments that both depend exclusively on the EOM angle-differences. Nevertheless, if (ii) is not fulfilled, it is possible to find for any quantum result at least one experimental configuration for that the Bell-CHSH-type inequalities are invalid and, thus, no corresponding constraint exists.)

5.1. Counter-Argument and Counter-Example Using Postulate (i)

Consider using a continuum for the determination of the values of λ_{in}. For example, choose for the λ_{in} a random number from the unit interval [0, 1] of the reals (or different sets of such numbers). Then as shown by set theoretic probability theory, a countable number of measurements cannot be reordered into quadruples with identical λ_{in}, because the probability to obtain by random choice one quadruple of identical λ_{in} is zero and a fortiori so the probability for each and all quadruples. Therefore, the existence of a Vorob'ev cyclicity is not guaranteed for any physical or mathematical reason and, thus, the Bell-CHSH constraint for the rows of matrix (1) is bereft of a mathematical basis. (Note that an early criticism of Bell-CHSH involving continua has been published by Marian Kupczynski [15]. The importance of both continuum and discreteness in quantum mechanical description was already emphasized by Bohr's profound work on complementarity.)

5.1.1. Illustration of the Counter-Argument

Just for the sake of exploring mathematical possibilities, we may choose virtually arbitrary outcomes of ±1 for the products of the model-functions AB, because they all contain different λ_{in}. Therefore, we also may choose arbitrary ratios of outcomes +1 and −1 for the function products A B in each column of the matrix (1) (meaning for each given EOM angle-pair). In particular, we may choose the ratios that are typical for violations of the Bell-CHSH inequalities: Select for the first column (with EOM-angles a, b) 85% of the possible model-outcome-products equal to +1 and 15% equal to −1, resulting in an average of 0.7. Similarly, we may create an average of 0.72 for the second column and of 0.705 for the third, while we select for the fourth and last column the value −1 for 0.845% of the outcomes and +1 for 0.155%, resulting in an average of −0.69. Thus, the averages of the columns of all quadruples add to 2.815, which is close to the maximal quantum result of $2\sqrt{2}$ for the Bell-test angles.

Instead of just freely choosing the AB—outcomes, we also may obtain outcomes that violate Bell-CHSH by invoking a Malus-type law as I did in [11].

5.1.2. Questions Regarding Counter-Argument (i)

As mentioned, Lambare [12] has discussed conditions that permit the use of equivalent λ_{in} that lead to identical outcomes for the functions A, B. However, any use of such equivalence classes must not change the column sums of matrix (1). Therefore, that method cannot introduce any constraints for these sums that were not there in the first place, meaning the following: if we start working with a matrix (1) that violates (for whatever reason) the Bell-CHSH constraints, then the use of equivalence classes cannot undo that violation. This fact is the basis of the reductio ad absurdum presented below.

Some might also object and reason that the real numbers chosen from the unit interval become increasingly close to each other for large N.

These questions and objections to the counter-argument based on (i) are easily disproven by the method of reductio ad absurdum that also permits us to formulate my counter-argument in form of a lemma:

LEMMA: Given four columns of pair products AB as in matrix (1), each for a different EOM angle pair, given further that these columns may exhibit arbitrary ratios of product outcomes equal to +1 or −1, respectively (because all λ_{in} are different with probability 1), given further that these arbitrary ratios are chosen such that E_N (defined as the sum of the first three columns minus the sum of the fourth column) obeys the inequality $E_N > 2N$ that violates the Bell-CHSH constraint, then the following statement holds:

There exists no possibility of manipulating (reordering) the columns such that about all rows have a value $|Q_n| \le 2$.

PROOF: Any manipulation or reordering of columns of matrix (1) based on reasons of equivalence or closeness of the λ_{in}, which results in a constraint $|Q_n| \le 2$ for about all rows, leads also to a constraint for E_N:

$$E_N = \sum |Q_n| \le 2N$$

Then, however, we have a contradiction, because we have chosen the ratios of the product outcomes such that $E_N > 2N$. This contradiction holds for arbitrary N.

As a consequence of the lemma, postulate (i) and the corresponding counter-argument permit arbitrary violations of all Bell-CHSH-type inequalities.

5.2. Counter-Argument and Counter-Example Using Postulate (ii)

This postulate provides a counter-argument against Bell's theorem that claims that it is impossible to obtain all quantum results for EPRB experiments by a model that uses Einstein-local physics. I show below that all quantum results may indeed be obtained by Einstein local physics, if we restrict ourselves to EOM-angle configurations that leave the EOM angle difference $(j' - j)$ unchanged but remove the Vorob'ev cyclicity of the model. That such configurations do indeed abundantly exist, may be shown as follows.

As in some Bell-type proofs [13], the properties of the λ_{in} are not nearer specified for the present purpose and the set S_λ may even be empty (thus denying from the start Einstein's elements of physical reality). Then, Equation (1) reduces to about N quadruples that all fulfill

$$\left|A(a)B(b)+A(a)B(b')+A(a')B(b)-A(a')B(b')\right| \le +2, \tag{3}$$

just like inequalities (2) do for any given row index n. Inequality (3) appears obvious and is incontrovertible. However, (3) does not cover the model inequalities for all possible actual equipment-configurations that exhibit identical EOM angle-differences. In fact, one can find for all quadruples of quantum mechanical expectation values corresponding model-inequalities based on Einstein-local physics that are not subject to the Bell-CHSH constraints, provided one considers exclusively a subset of suitable actual-equipment configurations.

To show this latter fact, we rotate the EOM-angle-pairs of the actual quadruple measurement-pairs such that the left EOM angle of **Figure 1** is being identical for all four pairs and is equal to the value a. The difference of the angles of any given EOM-pair is being left unchanged. Therefore, we have for the model-quadruples:

$$\left|A(a)B(b)+A(a)B(b')+A(a)B(b+a-a')-A(a)B(b'+a-a')\right|\leq +4. \quad (4)$$

Inequality (4) is not limited to +2, because the Vorob'ev cyclicity has been removed: all functions B exhibit a different EOM angle-value in their function-domain. The Bell-test angle-differences ($b-a$, $b'-a$, $b-a'$, $b'-a'$), however, are unchanged and so are the corresponding expectation values of quantum theory as well as the actual experiments. Yet, the Bell-CHSH model does not lead to any constraints for this case.

Thus, we have the following situation: Bell-CHSH and many textbooks demand that Einstein-local theories must obtain all possible results of quantum-theory for EPRB experiments and they claim that this is impossible. However, the Bell-CHSH inequalities are not valid for the experimental configurations leading to inequality (4). For these configurations (that cover all possible Bell-CHSH test-angles), all quantum results may indeed be obtained. The Bell-CHSH claim that it is impossible to obtain all quantum results needs, therefore, to be qualified by the addition that for certain experimental configurations that cover all Bell-CHSH angles and all possible expectation values for EPRP pairs, all the quantum results may indeed be obtained by Einstein local theories.

6. Summary

I am convinced that the counter-arguments related to postulates (i) and (ii) confront us with the serious fact that the Bell-CHSH inequalities and corresponding experimental Bell-tests are insufficient to provide any claims that Einstein's local realism needs revision. They do suggest interesting possibilities for the mathematical properties of S_λ as well as caution when comparing the Bell-CHSH model with quantum mechanics and actual experiments. They also emphasize the correctness and importance of the original work by Kocher and Commins [3].

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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