Journal of Modern Physics, 2022, 13, 509-517
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

The Dispersion Effect in Photonic Crystal Fibers

Mingxiang Gui[1], Jing Huang[2]

[1]School of Information Science and Engineering, Hunan University, Changsha, China
[2]Physics Department, South China University of Technology, Guangzhou, China
Email: huanggesheng@tom.com

How to cite this paper: Gui, M.X. and Huang, J. (2022) The Dispersion Effect in Photonic Crystal Fibers. *Journal of Modern Physics*, **13**, 509-517.
https://doi.org/10.4236/jmp.2022.134033

Received: March 3, 2022
Accepted: April 23, 2022
Published: April 26, 2022

Copyright © 2022 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/
Open Access

Abstract

Based on the Harmonitor theory of field in PCFs in which the second order differential to the transmission distance is included, and by the Darboux solution, the dispersion effect on the field is re-discussed. The results can be used in the dispersion parameter design of photonics crystal fibers. The field will expand and split faster than that in common fibers so that (as the description of Harmonitor theory) the higher-order dispersion should be taken into account. The high-order dispersion can also induce pulse compression while its pre-order dispersion values are zeroes. The dispersion coefficient changes with distance.

Keywords

Harmonitor Theory, Darboux Solution, Dispersion

1. Introduction

With the development of nanophotonic technologies, it is available to engineer waveguides with the complex dispersion profiles. This is not adequately described by a simple second-order term in the expansion of the wave number with frequency [1]. Similarly, the nonlinear Schrodinger equation cannot describe fast modulations adequately because it is based on a Taylor expansion in the frequency domain and only several dispersion items are taken into account [2]. Therefore, the influence of the modulation of optical fiber dispersion on soliton propagation has attracted a lot of attentions recently.

More dispersion items have been included [2]. In [3], authors studied the effects of the entire modal dispersion curve and the frequency dependence of the nonlinear coefficients on the formation of modulation instabilities by using the harmonic analysis. New regions of modulation for the dispersion-flattened fibers were found and characterized by this approach [4]. In the strongly dispersion fiber, the chirped Gaussian pulse, as in a linear superposition of the Hermite-Gaussian harmonics and as the zeroth harmonic, presented a periodically varying modulation width [5]. The multipeak modulation instability spectrum was observed in the dispersion oscillating optical fibers [6].

In the fiber with a periodic dispersion, by the variation approach, the stochastic decay of pulses was predicted [7]. Also, the finite-energy Airy waves may endlessly retain their shapes and follow the pre-engineered temporal trajectories [8].

With an appropriately designed dispersion profile, the chirped pulses can be retrieved through the chirp reversal [9] [10]. In this case, of the nonlinear Schrödinger equation with designed group velocity dispersion, variable nonlinearity, and gain or loss, had been demonstrated that the chirp reversal is crucial for the pulse reproduction [9]. A wave suffered the strong modulation of dispersion and then it resulted in a significant chirp development. The periodic modulation of dispersion led to the radiation damping [11] [12], the existence of vibrating solitons [13] and splitting of solitons [14].

Pulse evolution is highly sensitive to the input pulse profile [15]. It was found that light pulses with Gaussian input profile produced less radiation in the fiber system than that hyperbolic-secant or raised-cosine pulses could do.

In this paper, based on our established theory on the field in PCFs[16] which contained both the second order differential to the transmission distance ($\frac{\partial^2 A}{\partial z^2}$) and the higher-order dispersion items, we will re-discuss the dispersion effect on pulse. These discussions include the various pulses with profiles of Gauss, chirped Gauss, super-Gauss and hyperbolic functions (the common features of pulse profile). It also presents a Darboux state description for the field in a dispersion medium [17].

2. Theory on the Dispersion Effect in PCFs

In Micro-fibers (the PCFs in **Figure 1**), the transmission equation of field should include the second-order differential to transmission distance. It is [16]:

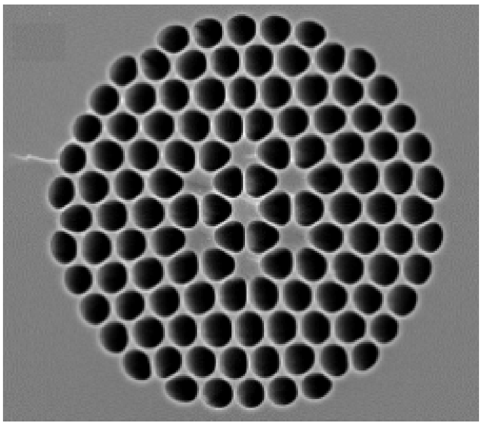

Figure 1. Scanning electron microscope image of the cross-section of the PCFs [18].

$$\frac{\partial^2 A}{\partial z^2}+2i\beta\frac{\partial A}{\partial z}=-\left(\Delta\beta^2-2i\beta\Delta\beta\right)A \tag{1}$$

Generally, for the discussion of field in fibers, people adopt the nonlinear Schordinger equation which ignores the item $\frac{\partial^2 A}{\partial z^2}$ because of the suppose of the slow-varying envelope. But in PCFs, the envelope of field varies very fast. Thus, the field equation should be (1).

And the dispersion item β should include many orders of its Taylor expansion or be the entire β coefficient curve.

Then only taking the dispersion effect into account ($\Delta\beta=0$), we get:

$$\frac{\partial^2 A}{\partial z^2}+2i\beta\frac{\partial A}{\partial z}=0 \tag{2}$$

The field is:

$$A(z,\omega)=A(0,\omega)\exp\left[-2iz\left(\beta_0+\beta_1\omega+\beta_2/2\,\omega^2+\cdots\right)\right] \\ \Big/\left[-2i\left(\beta_0+\beta_1\omega+\beta_2/2\,\omega^2+\cdots\right)\right] \tag{3}$$

From the view of the quantum theory, in the phase-amplitude representation, the system's Hamiltonian operator is [17]:

$$\hat{H}_S=\omega a'^{+}a'-i\beta\tanh\left(\beta\beta\right) \tag{4}$$

In [18], we have derived the (4). In the Darboux space, the lax pair is:

$$\beta^2=\beta_0^2+2\frac{\partial^2(\ln A)}{\partial z^2} \tag{5}$$

$$A=\frac{\partial A\left(\beta^2\right)}{\partial z}-\frac{\partial A\left(\beta_0\right)/\partial z}{A\left(\beta_0\right)}A\left(\beta\right) \tag{6}$$

The simulation results based on (6) are the same as those on (3).

3. Validation

In this section, we will compare the simulations of this theory with those of [19] [20] [21] [22] to prove the correction of our theory. The simulation procedure is shown in **Figure 2**.

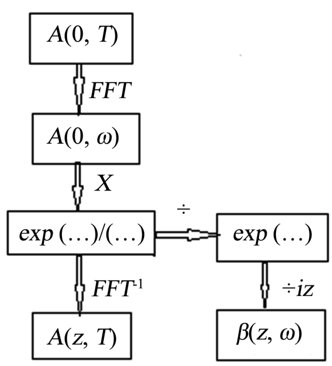

Figure 2. The simulation procedure.

For the Gauss pulse, there is:

$$A(0,T)=\exp\left[-(1+C)T^2/2/T_0^2\right] \tag{7}$$

$$A(0,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}A(0,T)\mathrm{e}^{-i\omega T}\mathrm{d}T \tag{8}$$

$$\begin{aligned}A(z,T)=\int_{-\infty}^{+\infty}\mathrm{d}\omega\Big\{&A(0,\omega)\exp\left[-2iz\left(\beta_0+\beta_1\omega+\beta_2/2\,\omega^2+\cdots\right)\right]\\&\Big/\left[-2zi\left(\beta_0+\beta_1\omega+\beta_2/2\,\omega^2+\cdots\right)\right]\exp\left[i\omega T\right]\Big\}\end{aligned} \tag{9}$$

We simulate the Gauss pulse evolution in **Figure 3** to prove the corrections of (3) and (6).

Although the form of (3) is completely different from the equations in [19] [20], the pulse evolution is the same in the following points:

1) The time axis values minus $z\beta_1$. It is $T=t-z\beta_1$. The whole pulse envelope is moved at the velocity of $1/\beta_1$;

2) The second order dispersion β_2 causes the pulse broaden;

3) The compressed chirped pulse (when C = 0.5 and Z = 0.1L_D, L_D is the dispersion length [23]) is shown in subfigure (c).

And differences as follow:

1) The second order dispersion β_2 causes the symmetrical fluctuations at the pulse edges (high frequency brims);

2) When the chirped pulse is compressed, as subfigure (c), the edge fluctuations turn severer;

3) The item $2zi\left(\beta_0+\beta_1\omega+\beta_2/2\,\omega^2+\cdots\right)$ in phase will cause the fluctuations in the envelopes of pulse.

These differences occur in our simulation, are not apparently shown in [19] [20].

Figure 4 is the demonstration of hyperbolic Secant pulse,

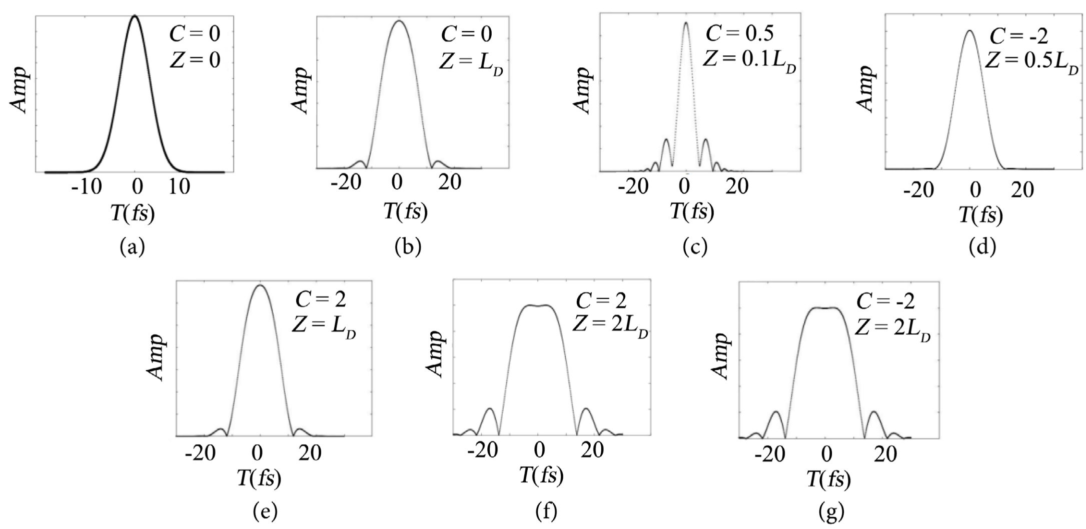

Figure 3. The evolution of Gauss pulse, only including the zero-, first- and the second-order dispersion items. $P_P=1(\mathrm{mW})$, $L=70(\mathrm{cm})$, $T_0=300(\mathrm{fs})$, $\gamma=0.1(\mathrm{W/m})$, $\beta_2=-4.22(\mathrm{ps}^2/\mathrm{km})$, $\beta_3=9.9\times10^{-2}(\mathrm{ps}^3/\mathrm{km})$, $\lambda=780(\mathrm{nm})$.

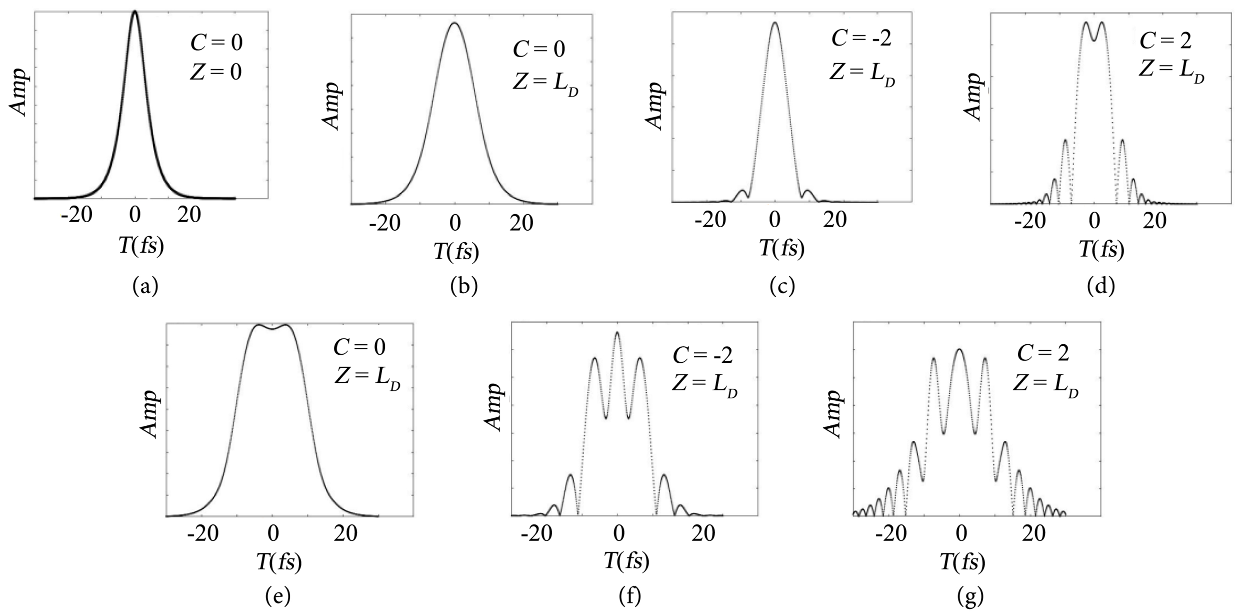

Figure 4. The evolution of the hyperbolic Secant pulse. Parameters are the same as **Figure 3**. (a) The initial pulse; (b)-(d) The results of [19], the fields in common fibers; and (e)-(f) The results of (3), the fields in PCFs.

$$A(0,T) = \text{sech}\left(\frac{T}{T_0}\right)\exp\left(-\frac{iCT^2}{2T_0}\right).$$

By a transmission, this pulse is broadened due to the second-order dispersion according to the Formula (3). The fluctuations are more evident than those in **Figure 3**, and even the pulse splittings occur. The simulations based on our theory exhibit more severe pulse expandings and splittings (e)-(g) than those in [19] [21]. This is consistent with the experiment result that in PCFs [2], and we have observed strong super-continuum spectrum which is not induced in the common fibers.

The same results are obtained from **Figure 5** for the super-Gauss pulse. $A(0,T) = \exp\left[-\frac{1+iC}{2}\left(\frac{T}{T}\right)^{2m}\right]$. There are fluctuations in the pulse edges and the pulses will split with increase of the distance [19] [22]. These phenomena are more evident than those occurred in a Gauss-pulse.

4. Expanding Results by the Darboux State Description

4.1. The Compression of Pulse

We can get a very interesting result that from (3). The Harmonitor is reproduced at some situations and at some frequency components. Actually, it is very difficult to observe this state (reproduced pulse) for the pulse expands very fast. This can be realized by limiting the second-order dispersion (regarded as 0), while the third-order dispersion is taken into account. Then the symmetry is broken. We simulate this case in **Figure 6**. In the figure, the forward of "C" pulse is compressed and the backward keep unchanged.

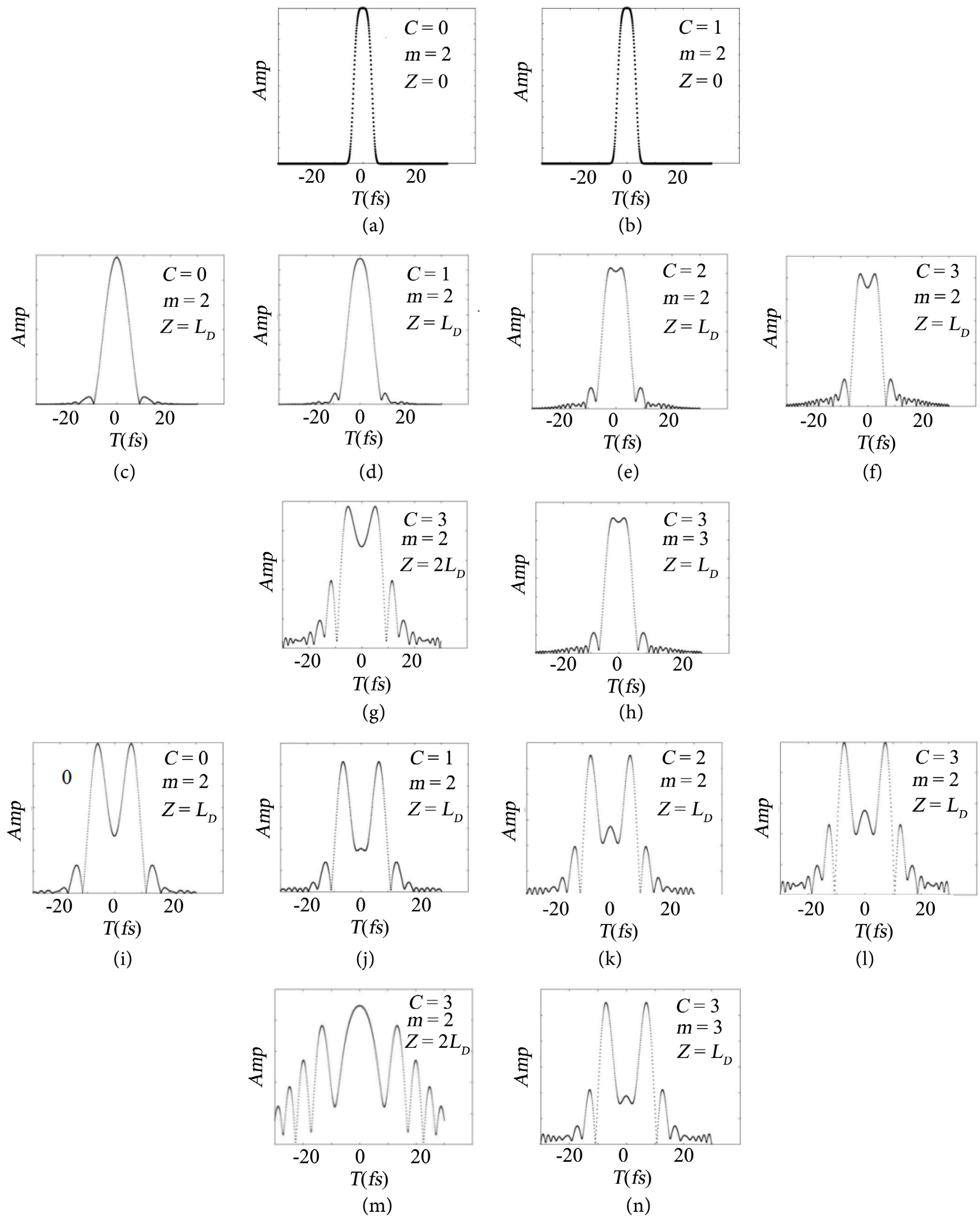

Figure 5. The evolution of super-Gauss pulse. (a) and (b) The initial pulses; (c)-(h) The results of [19] [22], the fields in common fibers; and (i)-(n) The results of (3), the fields in PCFs. Parameters are the same as Figure 3.

4.2. The Dispersion Coefficient $\beta(\omega)$

In Figure 7, we plot the entire dispersion coefficient. It complete differs from the Fourier serious results [19]. At the different places, it has different values, and the discrepancies are apparent. These exhibit another property of PCFs [4].

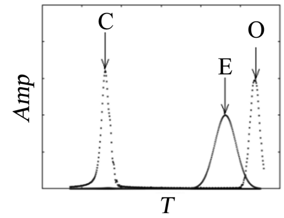

Figure 6. The pulse compression by the β_3. "O", "C" and "E" refer to the origin pulse, the compressed pulse, and the expanded pulse respectively.

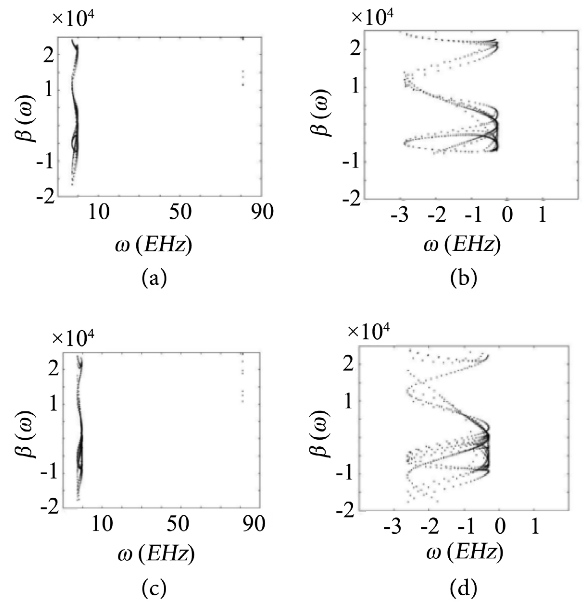

Figure 7. The dispersion curve (a) and (b) at $z = 0.2L_D$; and (c) and (d) at $z = 2L_D$. EHz: 10^{15} (Hz).

4.3. The Impacts of Birefringence and Confinement Loss in PCF Structure

If there are asymmetric and twist in the PCFs, these will result in the birefringence. The frequency component is much and will lead to more kinds of split and more induced waves [24] [25] [26].

The impact of confinement loss can be discussed by:

$$\frac{\partial^2 A}{\partial z^2} + 2i\beta\frac{\partial A}{\partial z} = \left(\frac{\alpha_{cl}}{\beta_0}\right)^2 A - 2\beta\alpha_{cl}/\beta_0 A \tag{10}$$

The first item of right side brings a gain or a loss on the field. The second item results in a phase shift along the transmission. The confinement loss value (<10^{-7} dB/km compared the fiber loss 10^{-3} dB/km) is very small, so both the impacts on amplitude and phase are enough small to be ignored.

5. Conclusions

By resolving the transmission equation of field in PCFs, the dispersion effect is re-discussed. Although the second order differential to the transmission distance

($\partial^2 A/\partial z^2$) is taken into account, the first-order (β_1) and second-order dispersion (β_2) effects have the same influences as those of the nonlinear Schordinger equation, which cause the envelope of pulse move and pulse broadened, respectively. Additionally, the second-order dispersion also brings the fluctuations in the pulse edges, and with the increase of distance, even results in the pulse splittings.

But by our calculation, the third-order dispersion (β_3) will also induce pulse compression while the second-order dispersion is assumed as zero. The dispersion coefficient $\beta(\omega)$ alters with the distance.

Conflicts of Interest

The authors declare no conflict of interests.

References

[1] Kruglov, V.I. and Harvey, J.D. (2018) *Physical Review A*, **98**, Article ID: 063811. https://doi.org/10.1103/PhysRevA.98.063811

[2] Berkovsky, A.N., Kozlov, S.A. and Shpolyanskiy, Y.A. (2005) *Physical Review A*, **72**, Article ID: 043821. https://doi.org/10.1103/PhysRevA.72.043821

[3] Yu, M., McKinstrie, C.J. and Agrawal, G.P. (1998) *Physical Review E*, **52**, 1072-1080. https://doi.org/10.1103/PhysRevE.52.1072

[4] Wang, W.C., Wang, N. and Jia, H.Z. (2021) *Optik*, **241**, Article ID: 166935. https://doi.org/10.1016/j.ijleo.2021.166935

[5] Lakoba, T. and Kaup, D. 1998) *Physical Review E*, **58**, 6728. https://doi.org/10.1103/PhysRevE.58.6728

[6] Droques, M., Kudlinski, A., Bouwmans, G., Martinelli, G. and Mussot, A. (2013) *Physical Review A*, **87**, Article ID: 013813. https://doi.org/10.1103/PhysRevA.87.013813

[7] Abdullaev, F.K. and Caputo, J.G. (1998) *Physical Review E*, **58**, 6637-6648. https://doi.org/10.1103/PhysRevE.58.6637

[8] Wang, S.F., Fan, D.F., Bai, X.K. and Zeng, X.L. (2014) *Physical Review A*, **89**, Article ID: 023802. https://doi.org/10.1103/PhysRevA.89.039901

[9] Atre, R. and Panigrahi, P.K. (2007) *Physical Review A*, **76**, Article ID: 043838. https://doi.org/10.1103/PhysRevA.76.043838

[10] Kumar, S. and Hasegawa, A. (1997) *Optics Letters*, **22**, 372. https://doi.org/10.1364/OL.22.000372

[11] Gordon, J.P. (1992) *Journal of the Optical Society of America B*, **9**, 91. https://doi.org/10.1364/JOSAB.9.000091

[12] Abdullaev, F.Kh., Caputo, J.G. and Flytzanis, N. (1994) *Physical Review E*, **50**, 1552. https://doi.org/10.1103/PhysRevE.50.1552

[13] Malomed, B.A., Parker, D.F. and Smyth, N. (1993) *Physical Review E*, **48**, 1418-1425. https://doi.org/10.1103/PhysRevE.48.1418

[14] Mak, W.C.K., Malomed, B.A. and Chu, P.L. (2004) *Journal of Modern Optics*, **51**, 2141-2158. https://doi.org/10.1080/09500340408232519

[15] Ngabireng, C.M., *et al.* (2005) *Physical Review E*, **72**, Article ID: 036613. https://doi.org/10.1103/PhysRevE.72.036613

[16] Huang, J. (2016) *Journal of Nano-Photonics*, **10**, Article ID: 016004. https://doi.org/10.1117/1.JNP.10.016004

[17] Gui, M.X. and Huang, J. (2018) *IEEE Photonics Journal*, **10**, Article ID: 2400408.

[18] Luo, Z.Z., Yu, H.H., Zheng, Y., Zheng, Z., Li, Y.J. and Jiang, X. (2020) *Journal of Lightwave Technology*, **38**, 4560-4571. https://doi.org/10.1109/JLT.2020.2989926

[19] Agrawal, G.P. (2007) Nonlinear Fiber Optics. 4th Edition, Academic, San Diego.

[20] Nair, A.A., Boopathi, C.S., Jayaraju, M. and Mani Rajan, M.S. (2019) *Optik*, **179**, 718-725. https://doi.org/10.1016/j.ijleo.2018.11.021

[21] Shaheen, S., Gris-Sánchez, I. and Gasulla, I. (2020) *Journal of Lightwave Technology*, **38**, 6237-6246. https://doi.org/10.1109/JLT.2020.3011548

[22] Suresh, M.I., Hammer, J., Joly, N.Y., Russell, P.St.J. and Tani, F. (2021) Extension of Supercontinuum via Tapered Single-Ring PCF. *Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference* (*CLEO*/*Europe-EQEC*), Virtual, 21-25 June 2021, Paper cd_5_6.

[23] Huang, J., Gui, M.X. and Man, W.Q. (2020) *Journal of Nano-Photonics*, **14**, Article ID: 026010.

[24] Zhang, J.-X. (2022) *Optik*, **251**, Article ID: 168425. https://doi.org/10.1016/j.ijleo.2021.168425

[25] Wong, G.K.L., Roth, P., Frosz, M.H. and Philip, St.J. (2020) Russell, Cross-Phase Modulational Instability of Vortex Modes in a Twisted Three-Core Photonic Crystal Fibre. *Conference on Lasers and Electro-Optics*/*Pacific Rim* (*CLEOPR*), Virtual, 2-6 August 2020, C8B_3.

[26] Guo, X., Han, L.Y., Liu, F. and Li, S.T. (2020) *Optik*, **218**, Article ID: 164796. https://doi.org/10.1016/j.ijleo.2020.164796