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Conceptual Content of the Generalized Theory of Gravitation of Jefimenko

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Abstract

In this work, we make a brief exposition of the Jefimenko's generalized theory of gravitation, describe its conceptual content, explain the mathematical apparatus used for the formulations of the theory and present the fundamental equations of the theory. We elucidate the main difference between Newton's original theory of gravitation and the generalized theory of gravitation.

Keywords

Cogravitation, Gravitational Waves, Gravikinetic Field

1. Introduction

It is obvious that the reader will quickly and easily perceive any new scientific theory not from a monograph, but from an article published in a readable scientific journal. Gravitational interaction of celestial bodies is a *very mysterious phenomenon*. It is traditionally attributed (without any further explanation) to the action of forces of "universal gravitation". But where are the threads, the ropes, the chains or the springs that pull celestial bodies one to the other? How does the Earth "know" that it needs to revolve around the Sun? How does it "feel" where the Sun is located? As far as we know there exists no material connection between celestial bodies. But if there is no material connection, does it not mean that gravitational interactions are not a manifestation of the action of forces, but a manifestation of the existence of some heretofore overlooked agent or mechanism? The Jefimenko's generalized theory of gravitation answers this question with perfect clarity.

Therefore, we decided to present the conceptual content of the Jefimenko's generalized theory of gravitation in a possibly short article.

The Jefimenko's generalized theory of gravitation arose from the analogy between the laws of gravitation and electromagnetism; that is, there existed a second gravitational field called cogravitational field, analogous to the magnetic field. Such analogy was proposed for the first time by Heaviside in a paper "A gravitational and electromagnetic Analogy" [1] published more than a century ago, where he supposed there must exist a second field due to moving masses and acting over moving masses only, called by Jefimenko, cogravitational field (sometimes this field is called Heaviside's field). The Heaviside paper was forgotten for a long time until Jefimenko returned his work and made improvement to the Heaviside's work in two books published and reissued since the 90's decade [2] [3]. Although there are, detractors of the Jefimenko's theory of gravitation[1] (see for example [5]).

The gravitodynamical theory [4] assumes that gravitational interactions are mediated by gravitational and cogravitational force fields.

A gravitational field is a region of space where a mass experience a gravitational force. Quantitatively, a gravitational field is defined in terms of the gravitational field vector $\boldsymbol{g}$ by the same equation by which it is defined in Newton's theory:

$$\boldsymbol{g} = -\boldsymbol{F}/m_t, \tag{1}$$

where $\boldsymbol{F}$ is the force exerted by the gravitational field on a stationary test mass m_t.

A cogravitational field is a region of space where a mass experience a cogravitational force. Quantitatively, a cogravitational field is defined in terms of the cogravitational field vector $\boldsymbol{K}$ by the equation

$$\boldsymbol{F} = m_t(\boldsymbol{v} \times \boldsymbol{K}), \tag{2}$$

where $\boldsymbol{F}$ is the force exerted by the cogravitational field on a stationary test mass m_t, moving with velocity $\boldsymbol{v}$. As noted in Chapter 1 of [3], cogravitational fields are created by moving masses only and act upon moving masses only. It should be noted that the cogravitational field $\boldsymbol{K}$ has not yet been actually observed. However, it is very likely that it can be revealed by the *Gravity Probe B* launched in 2004 by NASA in a polar orbit around the Earth. For the various theoretical considerations demanding the existence of the cogravitational field see O. Jefimenko [2] pp. 80-100.

It is assumed that both gravitational and cogravitational fields propagate in space with finite velocity. This velocity is not yet known, but is believed to be equal to the velocity of light. However, the generalized theory of gravitation is compatible with a propagation velocity of gravitation different from the velocity of light and is not affected by the actual speed with which gravitation propagates. Although we say that gravitational and cogravitational fields "propagate," it is not entirely clear what physical entity actually propagates, since by definition gravitational and cogravitational fields are "region of space". It is conceivable that what actually propagates is some particles that somehow create gravitational

[1]Called by us in a previous work *gravitodynamical theory [4]*.

and cogravitational fields. It is possible that these particles have already been described (see [6]), and it is possible that some of their effects have already been observed (see [7] pp. 137-223). Yet, there is not enough information about these particles for making any definite statement about their existence, nature, or properties.

The generalized theory of gravitation agrees with the principle of causality because, as we shall presently see, in this theory the gravitational and cogravitational fields are expressed in terms of retarded integrals whose integrands are the causative sources of the fields.

The generalized theory of gravitation agrees also with the law of conservation of momentum because according to this theory, gravitational-cogravitational fields are repositories of gravitational-cogravitational field momentum, and because mechanical momentum of a body moving in a gravitational-cogravitational field can be converted into the field momentum and the field momentum can be converted into the mechanical momentum of the body. As the result of this conversion, the sum of the mechanical and field momentum of the combined field-body system is always the same, and the total momentum of the system is thus conserved (see Chapter 8 in [3]).

According to the generalized theory of gravitation, gravitational-cogravitational fields are also repositories of field energy. Kinetic energy of a body moving in a gravitational-cogravitational field can be converted into the energy of the field, and the energy of the field can be converted into kinetic energy of the body. As a result of this conversion, the sum of the mechanical and field energy of the combined field-body system is always the same, and the total energy of the system is thus conserved (see Chapter 8 in [3] for a general proof of energy conservation in such systems).

Obviously, there are not derivations of the formulas presented in this text, because this is a review about the work made by Jefimenko. Also it is important to note that we can obtain all the results by replacing all variables and constants presented in **Table 1** in Maxwell equations. Too, it is important to note that this theory is developed from two standpoints, one of them is to postulate the retarded solutions and making use of the identities from vectorial calculus, we get the Jefimenko equations, and equivalently, we can potulate the system of Jefimenko equations and we get the retarded solutions given by (3) and (4).

2. Fundamental Equations of the Generalized Theory of Gravitation

The two principal equations of the generalized theory of gravitation are the equations for the gravitational field and $\boldsymbol{g}$ the cogravitational field $\boldsymbol{K}$:

$$\boldsymbol{g} = -G\int\left\{\frac{[\varrho]}{r^3}+\frac{1}{r^2c}\left[\frac{\partial\varrho}{\partial t}\right]\right\}\boldsymbol{r}\mathrm{d}V' + \frac{G}{c^2}\int\frac{1}{r}\left[\frac{\partial(\varrho\boldsymbol{v})}{\partial t}\right]\mathrm{d}V' \tag{3}$$

And

Table 1. Corresponding electromagnetic and gravitational-cogravitational symbols and constants.

Electric	Gravitational
q (charge)	m (mass)
ϱ (volume charge density)	ϱ (volume mass density)
σ (surface charge density)	σ (surface mass density)
λ (line charge density)	λ (line mass density)
φ (scalar potential)	φ (scalar potential)
$\boldsymbol{A}$ (vector potential)	$\boldsymbol{A}$ (vector potential)
$\boldsymbol{J}$ (convection current density)	$\boldsymbol{J}$ (mass-current density)
I (electric current)	I (mass current)
$\boldsymbol{E}$ (electric field)	$\boldsymbol{g}$ (gravitational field)
$\boldsymbol{B}$ (magnetic field)	$\boldsymbol{K}$ (cogravitational field)
ε_0 (permittivity of space)	$-1/4\pi G$
μ_0 (permeability of space)	$-4\pi G/c^2$
$-1/4\pi\varepsilon_0$ or $-\mu_0 c^2/4\pi$	G (gravitational constant)

$$\boldsymbol{K} = -\frac{G}{c^2}\int\left\{\frac{[\varrho\boldsymbol{v}]}{r^3} + \frac{1}{r^2 c}\left[\frac{\partial[\varrho\boldsymbol{v}]}{\partial t}\right]\right\}\times\boldsymbol{r}\,\mathrm{d}V', \tag{4}$$

where $\boldsymbol{g}$ is the gravitational field created by the mass m distributed in space with density ϱ, $r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$ is the distance from the *source point* (x', y', z'), where the volume element of integration $\mathrm{d}V'$ is located, to the field point (x, y, z), where $\boldsymbol{g}$ is been observed or computed, $\boldsymbol{r}$ is the radius vector directed from $\mathrm{d}V'$ to the field point, $\boldsymbol{v}$ is the velocity with which the mass distribution ϱ moves (the product $\varrho\boldsymbol{v}$ constitutes the "mass-current density), and c is the velocity of the propagation of gravitation (usually assumed to be the same as the velocity of light). The square brackets in these equations are the retardation symbol indicating that the quantities between the brackets are to be evaluated for the "retarded" time, $t' = t - r/c$, where t is the time for which $\boldsymbol{g}$ and $\boldsymbol{K}$ are evaluated. The integration in the integrals of Equations (3) and (4) is over all space.

According to Equations (3) and (4), the gravitational field has three causative sources: the mass density, the time derivative of ϱ, and the time derivative of the mass-current density $\varrho\boldsymbol{v}$; cogravitational field has two causative sources: the mass-current density $\varrho\boldsymbol{v}$ and the time derivative of $\varrho\boldsymbol{v}$.

In addition to Equations (3) and (4) for the gravitational and cogravitational fields, the following equations constitute the mathematical foundation of the generalized theory of gravitation:

1) *The mass conservation equation* ("*continuity equation*")

$$\nabla\cdot(\varrho\boldsymbol{v}) = -\frac{\partial\varrho}{\partial t}, \tag{5}$$

or, in the integral form,

$$\oint \varrho v \cdot \mathrm{d}\boldsymbol{S} = -\frac{\partial}{\partial t}\int \varrho \mathrm{d}V. \tag{6}$$

According to these equations, whenever a mass contained in a region of space diminishes or increases, there is an outflow or inflow of mass from or into this region.

2) *Force acting on the mass distribution of density* ϱ

$$\boldsymbol{F} = \int \varrho\left(\boldsymbol{g} + \boldsymbol{v}\times\boldsymbol{K}\right)\mathrm{d}V, \tag{7}$$

where $\boldsymbol{v}$ is the velocity of ϱ and the integral is extended over the region of space containing the mass under consideration.

3) *Density of the field energy contained in the gravitational-cogravitational field*

$$U_v = -\frac{1}{8\pi G}\left(\boldsymbol{g}^2 + c^2\boldsymbol{K}^2\right), \tag{8}$$

it is important to note that the gravitational-cogravitational field energy is negative. This means that *no energy can be extracted from the gravitational-cogravitational field by destroying the field. In the contrary, energy must be delivered to the field in order to destroy the field.*

4) *Field energy contained in a region of the gravitational-cogravitational field*

$$U = -\frac{1}{8\pi G}\int\left(\boldsymbol{g}^2 + c^2\boldsymbol{K}^2\right)\mathrm{d}V, \tag{9}$$

where the integration is extended over the region under consideration.

5) *Energy flow vector in the gravitational-cogravitational field* ("*gravitational Poynting vector*")

$$\boldsymbol{P} = \frac{c^2}{4\pi G}\boldsymbol{K}\times\boldsymbol{g}. \tag{10}$$

This vector represents the direction and rate of gravitational-cogravitational energy flow per unit area at a point of space under consideration. Equation (10) together with Equation s. (3), (4), (5) and (8) ensures the conservation of energy in gravitational-cogravitational interactions.

6) *Density of the field momentum contained in the gravitational-cogravitational field*

$$\boldsymbol{G}_{vf} = \frac{1}{4\pi G}\boldsymbol{K}\times\boldsymbol{g}. \tag{11}$$

7) *Field momentum contained in the gravitational-cogravitational field*

$$\boldsymbol{G}_f = \frac{1}{4\pi G}\int \boldsymbol{K}\times\boldsymbol{g}\mathrm{d}V, \tag{12}$$

where the integration is extended over the region under consideration.

8) *Correlations between the mechanical momentum,* $\boldsymbol{G}_m$, *and the gravitational-cogravitational field*

$$\frac{\mathrm{d}\boldsymbol{G}_M}{\mathrm{d}t} = -\frac{1}{4\pi G}\int\frac{\partial}{\partial t}(\boldsymbol{K}\times\boldsymbol{g})\mathrm{d}V + \frac{1}{4\pi G}\left[\frac{1}{2}\oint\left(\boldsymbol{g}^2 + c^2\boldsymbol{K}^2\right)\mathrm{d}\boldsymbol{S} - \oint\boldsymbol{g}(\boldsymbol{g}\cdot\mathrm{d}\boldsymbol{S}) - c^2\oint\boldsymbol{K}(\boldsymbol{K}\cdot\mathrm{d}\boldsymbol{S})\right], \tag{13}$$

where $\boldsymbol{g}$ and $\boldsymbol{K}$ are the gravitational and cogravitational fields in the system under consideration. In this equation, the derivative on the left represents the rate of change of the momentum of a body located in a gravitational-cogravitational field, the volume integral represents the rate of change of the field momentum in the region of the field where the bode is located, and the surface integrals represent the flux of the field momentum through the surface enclosing the region under consideration. Together with Equations (3), (4), (5), (7) and (11) this equation ensures the conservation of momentum in gravitational-cogravitational interactions.

3. Gravitational and Cogravitational Forces According to the Generalized Theory of Gravitation

One of the most important differences between Newton's original theory of gravitation and the generalized theory of gravitation is in the interpretation of the mechanism of gravitational interactions. Whereas in Newton's original theory of gravitation gravitational interaction between two bodies involves one single force of gravitational attraction, in the generalized theory of gravitation gravitational interaction between two bodies involves an intricate juxtaposition of several different forces. Mathematically, these forces result from Equations (3), (4) and (7). When Equations. (3) and (4) are written as five separate integrals, they become, using $\boldsymbol{J}$ for $\varrho\boldsymbol{v}$,

$$\boldsymbol{g} = -G\int\frac{[\varrho]}{r^3}\boldsymbol{r}\mathrm{d}V' - G\int\frac{1}{r^2c}\left[\frac{\partial\varrho}{\partial t}\right]\boldsymbol{r}\mathrm{d}V' + \frac{G}{c^2}\int\frac{1}{r}\left[\frac{\partial\boldsymbol{J}}{\partial t}\right]\mathrm{d}V' \tag{14}$$

and

$$\boldsymbol{K} = -\frac{G}{c^2}\int\frac{[\boldsymbol{J}]}{r^3}\times\boldsymbol{r}\mathrm{d}V' - \frac{G}{c^2}\int\frac{1}{r^2c}\left[\frac{\partial[\boldsymbol{J}]}{\partial t}\right]\times\boldsymbol{r}\mathrm{d}V'. \tag{15}$$

Each of these integrals represents a force field. Therefore, according to the generalized theory of gravitation, gravitational interaction between two bodies involve at least five different forces. Let us consider the physical sources of these forces.

First let us consider Equation (14). The field represented by the first integral of this equation is the ordinary Newtonian gravitational field created by the mass distribution ϱ corrected for the finite speed of the propagation of the field, as indicated by the square brackets (the retardation symbol) in the numerator. The field represented by the second integral is created by a mass whose density varies with time. Like the ordinary Newtonian gravitational field, these two fields are directed toward the masses, which create them. The field represented by the last integral in Equation (14) is created by a mass current whose magnitude and/or direction varies with time. The direction of this field is parallel to the direction along which the mass current increases. All three fields in Equation (14) act on stationary as well as on moving masses.

Consider now Equation (15). The first integral in this equation represents the cogravitational field created by the mass current. The direction of this field is normal to the mass current vector. The second integral represents the field created by a time variable mass current. The direction of this field is normal to the direction along which the mass current increases. By Equation (7), both fields in Equation (15) act on moving masses only.

If the mass under consideration does not move and does not change with time, then there is no retardation and no mass current. In this case, both integrals in Equation (15) vanish and only the first integral remains in Equation (14). As a result, one simply obtains the integral representing the ordinary Newtonian gravitational field. Thus, the ordinary Newtonian gravitational theory is a special case of the generalized theory, as it should be.

As far as the gravitational interaction between two masses is concerned, the meaning of the five integrals discussed above can be explained with the help of **Figure 1**. The upper part of **Figure 1** shows the force, which the mass m_1 experiences under the action of the mass m_2 according to the ordinary Newtonian theory. The lower part of **Figure 1** shows five forces which the same mass m_1 experiences under the action of the mass m_2 according to the generalized theory. The time for which the positions of the two masses and the force experienced by m_1 are observed is indicated by the letter t. Let us not first of all that, according to the ordinary Newtonian theory, the mass m_1 is subjected to one single force directed to the mass m_2 at its present location, that is, to its location at the time t. However, according to the generalized theory, all forces acting on the mass m_1 are associated not with the position of the mass m_2 at the time of observation, but with the position of m_2 at an earlier time $t' < t$. Therefore, the magnitude of the mass m_2, its position and its state of motion at the present time t have no effect at all on the mass m_1.

The subscripts identifying the five forces shown in the lower part of **Figure 1** correspond to the five integrals in the Equations (14) and (15). The force $\boldsymbol{F}_1$ is associated simply with the mass m_2 and differs from the ordinary Newtonian gravitational force only insofar as it is it is directed not to the mass m_2 at its present position, but to the place where m_2 was located at the past time t'. The force $\boldsymbol{F}_2$ is associated with the variation of the density of the mass m_2 with time; the direction of thus force is the same as that of $\boldsymbol{F}_1$. The force $\boldsymbol{F}_3$ is associated with the time variation of the mass current produced by m_2; tis force is directed along the acceleration vector $\boldsymbol{a}$ (or along the velocity vector $\boldsymbol{v}_2$) which the mass m_2 had at the time t'. The three forces are produced by the gravitational field $\boldsymbol{g}$ (if m_2 is a point mass moving at constant velocity, $\boldsymbol{g}$ and the resultant of the three forces are directed toward the *present position* of m_2; see Chapter 5 in [3]).

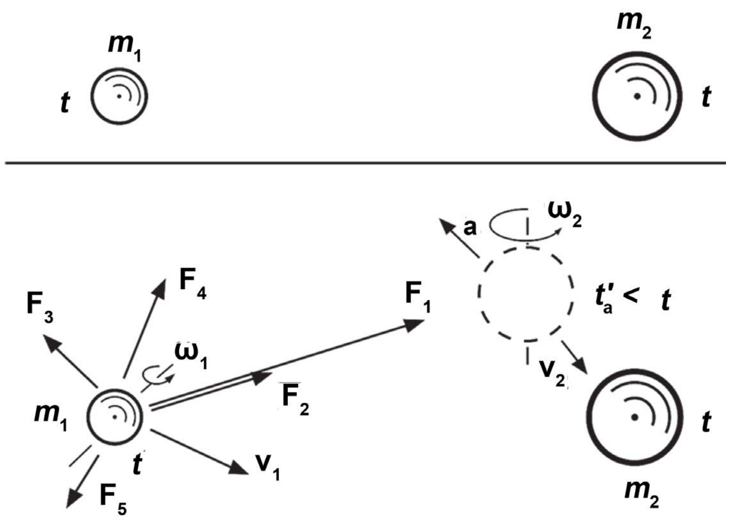

Figure 1. The upper part of this figure shows the force that the mass m_1 experiences under the action of the mass m_2 according to the ordinary Newtonian theory. Lower part shows five forces, which the same mass m_1 experiences under the action of the mass m_2 according to the generalized Newtonian theory.

The forces $\boldsymbol{F}_4$ and $\boldsymbol{F}_5$ are due to the cogravitational field $\boldsymbol{K}$. The force $\boldsymbol{F}_4$ is associated with the mass current created by the mass m_2 and with the velocity of the mass m_1. Its direction is normal to the velocity vector $\boldsymbol{v}_2$ which the mass m_2 had at the time t' and normal to the velocity vector $\boldsymbol{v}_1$ which the mass m_1 has at the present time t. The force $\boldsymbol{F}_5$ is associated with the velocity of the mass m_1 and with the variation of the mass current of the mass m_2 with time; the direction of this force is normal to the acceleration vector (or to the velocity vector) that the mass m_2 had at the time t' and normal to the velocity vector that the mass m_1 has at present time t. Although not shown in Figure 1, additional forces associated with the rotation of m_1 and m_2 (angular velocities ω_1 and ω_2) are generally involved in the interaction between t5he two masses (see Chapters 14 and 15 in [3]).

The forces $\boldsymbol{F}_2$, $\boldsymbol{F}_3$, $\boldsymbol{F}_4$ and $\boldsymbol{F}_5$ are usually much weaker than the force $\boldsymbol{F}_1$ because of the presence of the speed of gravitation c (usually assumed to be the same as the speed of light) in the denominators of the integrals representing the fields responsible for these four forces. This means that only when the translational or rotational velocity of m_1 or m_2 is close to c, are the forces $\boldsymbol{F}_2$, $\boldsymbol{F}_3$, $\boldsymbol{F}_4$ and $\boldsymbol{F}_5$ dominant. Of course, the cumulative effect of these forces in long-lasting gravitational systems (such as Solar system, for example) may be significant regardless of the velocities of the interacting masses.

4. The Relationship between the Generalized Theory of Gravitation and the Special Relativity Theory

Until recently it was believed that the analogy between electromagnetic and gravitational equations could not apply to fast moving systems, because the electric charge is not affected by velocity, but the mass of a moving body was though to vary with velocity. It is now generally accepted that mass, just like the electric charge, does not depend on velocity. For a discussion of the history and use of the concept of relativistic mass, see C. Adler, "Does mass really depend on velocity, dad?" [8], also L. Okun, "The concept of mass" [9] and the letter in response to these articles by T. Sandin, "In defense of relativistic mass" [10]. This also means that transformation equations of the special relativity theory developed for electromagnetic systems (see [11] pp. 148-206) have their gravitational and cogravitational counterparts.

Thus there is no need to *derive* relativistic gravitational-cogravitational transformation equations, because we can easily obtain them by replacing symbols and constants appearing in relativistic electromagnetic equations by the corresponding gravitational-cogravitational symbols and constants with the help of the following table [3].

The basic relativistic gravitational-cogravitational transformation equations obtained in this way are listed below. It is important to note that these equations can be derived directly, without using the analogy between electromagnetic and gravitational-cogravitational systems (see O. Jefimenko, "Derivation of Relativistic Transformation for Gravitational Fields from Retarded Field Integrals" [12]). In these equations, the unprimed quantities are those measured in the stationary reference frame Σ ("laboratory"), and the primed quantities are those measured in the moving reference frame Σ'.

Transformation equations correlating quantities measured in Σ with quantities measured in Σ':

a) *Equations for space and time coordinates*

$$x = \gamma\left(x' + vt'\right), \tag{4.1}$$

$$y = y', \tag{4.2}$$

$$z = z', \tag{4.3}$$

$$t = \gamma\left(t' + vx'/c^2\right). \tag{4.4}$$

b) *Equations for the gravitational field*

$$g_x = g'_x, \tag{4.5}$$

$$g_y = \gamma\left(g'_y + vK'_z\right), \tag{4.6}$$

$$g_z = \gamma\left(g'_z + vK'_y\right), \tag{4.7}$$

c) *Equations for the cogravitational field*

$$K_x = K'_x, \tag{4.8}$$

$$K_y = \gamma\left(K'_y - vg'_z/c^2\right), \tag{4.9}$$

$$K_z = \gamma\left(K'_z - vg'_y/c^2\right), \tag{4.10}$$

d) *Equations for the mass and mass-current densities*

$$\varrho = \gamma\left[\varrho' + \left(v/c^2\right)J'_x\right], \tag{4.11}$$

$$J_x = \gamma\left(J'_x + v\varrho'\right), \tag{4.12}$$

$$J_y = J'_y, \tag{4.13}$$

$$J_z = J'_z. \tag{4.14}$$

e) *Equations for gravitational and cogravitational potentials*

$$\varphi = \gamma\left(\varphi' + vA'_x\right), \tag{4.15}$$

$$A_x = \gamma\left[A'_x + \left(v/c^2\right)\varphi'\right], \tag{4.16}$$

$$A_y = A'_y, \tag{4.17}$$

$$A_z = A'_z. \tag{4.18}$$

Transformation equations correlating quantities measured in Σ' with quantities measured in Σ:

a) *Equations for space and time coordinates*

$$x' = \gamma\left(x - vt\right), \tag{4.19}$$

$$y' = y, \tag{4.20}$$

$$z' = z, \tag{4.21}$$

$$t' = \gamma\left(t - vx/c^2\right). \tag{4.22}$$

b) *Equations for the gravitational field*

$$g'_x = g_x, \tag{4.23}$$

$$g'_y = \gamma\left(g_y - vK_z\right), \tag{4.24}$$

$$g'_z = \gamma\left(g_z - vK_y\right). \tag{4.25}$$

c) *Equations for the cogravitational field*

$$K'_x = K_x, \tag{4.26}$$

$$K'_y = \gamma\left(K_y + vg_z/c^2\right), \tag{4.27}$$

$$K'_z = \gamma\left(K_z + vg_y/c^2\right). \tag{4.28}$$

d) *Equations for the mass and mass-current densities*

$$\varrho' = \gamma\left[\varrho - \left(v/c^2\right)J_x\right], \tag{4.29}$$

$$J'_x = \gamma\left(J_x - v\varrho\right), \tag{4.30}$$

$$J'_y = J_y, \tag{4.31}$$

$$J'_z = J_z. \tag{4.32}$$

e) *Equations for gravitational and cogravitational potentials*

$$\varphi' = \gamma\left(\varphi - vA_x\right), \tag{4.33}$$

$$A'_x = \gamma\left[A_x - \left(v/c^2\right)\varphi\right], \tag{4.34}$$

$$A'_y = A_y, \tag{4.35}$$

$$A'_z = A_z. \tag{4.36}$$

Quite clearly, transformation equations for physical quantities not involving electric and magnetic fields (such as velocity, acceleration, force, etc.) remain valid for gravitational-cogravitational systems as well. However, the constant c appearing in the conventional relativistic transformation equations represents the velocity of propagation of electromagnetic fields in a vacuum, which is the same as the velocity of light. The velocity of propagation of gravitational and cogravitational fields is not known, although it is generally believed to be equal to the velocity of light. If the velocity of propagation of gravitational fields is not the same as the velocity of light, our relativistic transformation equations for gravitation would still remain correct, but the constant c appearing in them would be different from c appearing in the corresponding electromagnetic equations. Therefore, the behavior of rapidly moving bodies involved in gravitational interactions would be different from the behavior of rapidly moving bodies involved in electromagnetic interactions. In effect there would be two different mechanics: the "gravitational-cogravitational mechanics," and the "electromagnetic mechanics" involving different effective masses, different effective momenta, and different rest energies.

A possibility exists that our gravitational relativistic transformation equations are not entirely correct. According to Einstein's mass-energy equation, any energy has a certain mass. But a mass is a source of gravitation. Therefore the gravitational field of a mass distribution may be caused not only by the mass of the distribution as such, dut also by the gravitational energy of this distribution (for a detail discussion of this effect, including the possibility of antigravitational mass distributions arising from it, see Chapter 19 in [3]). If this effect is taken into account, the equation for the divergence of the gravitational field (see Eq. (7-1.1) in [3])

$$\nabla \cdot \boldsymbol{g} = -4\pi G\varrho \tag{16}$$

becomes only approximately correct, and all equations derived with the help of Equation. (16) also become only approximately correct. It is important to note, however, that this energy effect, if it exists, is extremely small[2]. In connection with the foregoing, we recommend that the reader become familiar with the work "Binormal Motion of Curves of Constant Curvature and Torsion. Generation of Soliton Surfaces." [13].

5. Covariant Formulation of the Generalized Theory of Gravitation

Covariant formulation of physical formulas and equations is considered by some authors to be the most appropriate formulation for expressing the laws of phys-

[2]Contrary to the prevailing belief, equations relativistic electrodynamics and the entire theory of special relativity is also only approximately correct, since it is valid only for inertial systems ("inertial frames of reference"). In reality such systems do not exist, because everywhere in the Universe there is a gravitational force field, making all systems and locations in the Universe non-inertial.

ics in a frame-independent form. It is also believed by some authors to be more concise and occasionally more informative than the conventional formulation. Since any equation invariant under relativistic transformations should be expressible ib a covariant form, and since the principle of relativity is considered to be a fundamental law of nature, the laws of physics that cannot be expressed in a covariant form are considered by some authors to be incomplete or incorrect. This view is *unquestionably wrong*, since according to it, even Maxwell's equations in their vector form should be classified as "incomplete" or "incorrect." Note also that covariant formulation changes the form of equations but does not create new physical laws and thus is of very limited utility.

Newton's gravitational law is an example of a physical law that *cannot be expressed in a covariant form*. The problem of finding an invariant form of the law of gravitation was first considered be Poincaré, but without success (see his article "Sur la dynamique de L'Électron" [14]). It is interesting to note that Poincaré attempted to solve the problem on the basis of just one gravitational field (the gravitational analog of the electrostatic field). But even if the theory of gravitation is built upon two fields, a covariant theory of gravitation is not possible unless the gravitational mass, just like the electric charge, does not depend on the velocity with which the mass moves.

However, it is now generally accepted that mass does not depend on the velocity with which a body moves (see C. Adler, "Does mass really depend on velocity, dad?" [8], also L. Okun, "The concept of mass" [9] and the letter in response to these articles by T. Sandin, "In defense of relativistic mass" [10]). Therefore a covariant formulation of the theory of gravitation based on gravitational-cogravitational fields is not only possible but can be constucted straightaway from the covariant theory of electromagnetism by a mere substitution of symbols and constants in accordance with the list above.

In particular, from electromagnetic equations (see [11] pp. 284-292). We can directly obtain for the covariant "position 4-vector"

$$\boldsymbol{r} = (x_1, x_2, x_3, x_4) = (x, y, z, ict). \tag{17}$$

From the 4-vector electric current [11] we obtain by substitutions the the covariant expressions for the 4-vector mass current

$$\boldsymbol{J} = (J_1, J_2, J_3, J_4) = (J_x, J_y, J_z, ic\varrho), \tag{18}$$

where J_x, J_y and J_z are *x*, *y* and *z* components of mass current density. From the electromagnetic field tensor [11] we obtain the gravitational-cogravitational field tensor by replacing the *x*, *y* and *z* components of $\boldsymbol{E}$ by the corresponding components of $\boldsymbol{g}$ and the *x*, *y* and *z* components of $\boldsymbol{B}$ by the corresponding components of $\boldsymbol{K}$

$$F_{\mu\nu} = \begin{bmatrix} 0 & K_z & -K_y & -ig_x/c \\ -K_z & 0 & K_x & -ig_y/c \\ K_y & -K_x & 0 & -ig_z/c \\ ig_x/c & ig_y/c & ig_z/c & 0 \end{bmatrix}, \tag{19}$$

or

$$F^{\mu\nu} = \begin{bmatrix} 0 & K_z & -K_y & ig_x/c \\ -K_z & 0 & K_x & ig_y/c \\ K_y & -K_x & 0 & ig_z/c \\ -ig_x/c & -ig_y/c & -ig_z/c & 0 \end{bmatrix}, \tag{20}$$

where the subscript μ indicates the row (1, 2, 3, 4 top to bottom) and the subscript ν indicates the column (1, 2, 3, 4 left to right). Finally, in the same manner, we obtain covariant expressions of the present-time differential equations for gravitational-cogravitational fields:

$$\sum_{\nu=1}^{4} \frac{\partial F_{\mu\nu}}{\partial x_\nu} = -\frac{4\pi G}{c^2} J_\mu \tag{21}$$

and

$$\frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} + \frac{\partial F_{\lambda\mu}}{\partial x_\nu} = 0. \tag{22}$$

It should be note, however, that c in the gravitational-cogravitational equations stands for the speed of propagation of gravitational-cogravitational fields, which is generally assumed to be the same as the speed of light, but has never been actually measured. In 2002 Fomalont Kopeikin tried indirectly to measure the speed of gravitation and reported in the paper "The measurement of the light deflection from Jupiter: Experimental results" [15] that the velocity of gravitation was found to be equal to the velocity of light.

6. The Gravikinetic Field

As we have already shown, one of the main differences between the generalized theory of gravitation and Newton's gravitational theory is that in the generalized theory of gravitation there is especial force field – the cogravitational, or Heaviside's field. The cogravitational field is produced by all moving masses, and it acts ob all moving masses. In this Section we shall learn that in the generalized theory of gravitation there is yet *another* force field prodiced by moving masses. However, in contrast with the cogravitational field, thid field is produced only by masses whose velocity changes in time and, again in contrast with the cogravitational field, it acts on *all masses*, moving as well as stationary.

As we already know, the principal gravitatonal field equation of the generalized theory of gravitation is

$$\boldsymbol{g} = -G\int\left\{\frac{[\varrho]}{r^3} + \frac{1}{r^2 c}\left[\frac{\partial \varrho}{\partial t}\right]\right\}\boldsymbol{r}\mathrm{d}V' + \frac{G}{c^2}\int\frac{1}{r}\left[\frac{\partial \boldsymbol{J}}{\partial t}\right]\mathrm{d}V', \tag{23}$$

where $\boldsymbol{J} = \varrho\boldsymbol{v}$ is the mass current density produced by a moving mass distribution ϱ. The first term on the right in Equation (23) represents the retarded Newtonian gravitational field. Just like the ordinary Newtonian field, this field originates at any mass distribution ϱ and is responsible for the gravitational attraction. However, the last term on the right of Equation (23) represents a gravitational field very different from the Newtonian field. As can be seen from Equation (23), this new field is produced by a time-variable mass current $\partial \boldsymbol{J}/\partial t$ and it differs in two in two importabt respects from the Newtonian gravitational field: it is directed along the mass-current (more accurately, along its partial time derivative) rather than along a radius vector, and it exists only as long as the current is changing in time. Therefore the gravitational force caused by this field is also different from the ordinary Newtonian force. This force (designated as $\boldsymbol{F}_3$ in **Figure 1**) is directed along $\partial \boldsymbol{J}/\partial t$ and it lasts only as long as the mass current is changing. Unlike the Newtonian gravitational force, which is always an interaction between graviting masses, the force due to the time-variable $\boldsymbol{J}$ is basically a *dragging* force. If only the magnitude but not the direction $\boldsymbol{J}$ changes, this force is directed parallel or antiparallel (if $\partial \boldsymbol{J}/\partial t$ is negative) to $\boldsymbol{J}$, causing a mass subjected to this force to move parallel or antiparallel to (rather than toward) the mass distribution forming the mass current. However, like the Newtonian force, the force due to the time-variable $\boldsymbol{J}$ acts upon all masses. It is important to note that unlike that unlike the cogravitational field, the field produced by $\partial \boldsymbol{J}/\partial t$ usually is not created by masses moving with constant velocity $\boldsymbol{v}$,

Since the cogravitational field created by time-variable mass currents is very different from the Newtonian field and from the cogravitatational field, a special name should be given to it. Taking into account that the cause of this field is a motion of masses, we can call it the *gravikinetic field*, and we may call the force which this field exerts on other masses the *gravikinetic force*. We shall designate the gravikinetic field by the vector $\boldsymbol{g}_k$. From Equation (23) we tus have

$$\boldsymbol{g}_k = \frac{G}{c^2}\int \frac{1}{r}\left[\frac{\partial \boldsymbol{J}}{\partial t}\right]\mathrm{d}V'. \tag{24}$$

Because of the c^2 in the denominator in Equation (24) the gravikinetic field cannot be particularly strong except when the mass-current responsible for it changes very fast. On the othr hand, taking into account that the time scale in gravitational interractions taking place in the Univerce may be very long, ultimate effect of the gravikinetic field in such interactions may be very considerable regardless of the rate at which the mass current changes.

Let us now show the correlation between the gravikinetic field and the cogravitational field. If we compare Equation (24) with the expression for the retarded cogravitational vector potential $\boldsymbol{A}_{ret}$ produced by a mass current $\boldsymbol{J}$ (seee, Section 3-3 Equation (3-3.2) in [4]),

$$\boldsymbol{A}_{ret} = -\frac{G}{c^2}\int \frac{[\boldsymbol{J}]}{r}\mathrm{d}V', \tag{25}$$

we recognize that the gravikinetic field is equal to the time derivative of retarded $\boldsymbol{A}_{ret}$:

$$\boldsymbol{g}_k = -\frac{\partial \boldsymbol{A}_{ret}}{\partial t}. \tag{26}$$

Observe that Equation (26) points out the possibility of a new defibition and interpretation of the cogravitational vector potential. Let us integrate Equation (26). We otain

$$\boldsymbol{A}_{ret} = -\int \boldsymbol{g}_k \mathrm{d}t + \text{const.} \tag{27}$$

Let us call the time integral of $\boldsymbol{g}_k$ the *gravikinetic impulse*. We then can say that the cogravitational vector potential created by a mass current at a point in space is equal to the negative of the gravikinetic impulse produced by this current at that point during the action of the mass current. Since the gravikinetic impulse is, in principle, a measurable quantity, we thus have an operational definition and a physical interpretation of the cogravitational vector potential (for a related interpretation of the magnetic vector potential see [2] pp. 30, 31).

A more direct relation between the gravikinetic field and the cogravitational field one can obtain as follows. Let as assume that an initially stationary mass current $\boldsymbol{J}(x', y', z')$ (an initially stationary rotating spherical mass, for example) moves as a whole with a constant velocity v toward a stationary observer located at the origin of coordinates. The mass current is then a function of $(x' - v_x t)$, $(y' - v_y t)$ and $(z' - v_z t)$, or

$$\boldsymbol{J} = \boldsymbol{J}\left(x' - v_x t, y' - v_y t, z' - v_z t\right). \tag{28}$$

The time derivative of the current is

$$\frac{\partial \boldsymbol{J}}{\partial t} = -\frac{\partial \boldsymbol{J}}{\partial x'} v_x - \frac{\partial \boldsymbol{J}}{\partial y'} v_y - \frac{\partial \boldsymbol{J}}{\partial z'} v_z = -(\boldsymbol{v} \cdot \nabla')\boldsymbol{J}. \tag{29}$$

The gravikinetic field caused by the moving mass current is then, by Equations. (24) and (29),

$$\boldsymbol{g}_k = -\frac{G}{c^2} \int \frac{\left[(\boldsymbol{v} \cdot \nabla')\boldsymbol{J}\right]}{r} \mathrm{d}V'. \tag{30}$$

The spatial derivative appearing in Equation (30) can be eliminated as follows. Using vector identity (V-6) from [3], which can be written as

$$\nabla'(\boldsymbol{v} \cdot \boldsymbol{J}) = (\boldsymbol{v} \cdot \nabla')\boldsymbol{J} + \boldsymbol{v} \times (\nabla' \times \boldsymbol{J}) + (\boldsymbol{J} \cdot \nabla')\boldsymbol{v} + \boldsymbol{J} \times (\nabla' \times \boldsymbol{v}), \tag{31}$$

and taking into account that $\boldsymbol{v}$ is a constant vector, we obtain

$$\boldsymbol{g}_k = -\frac{G}{c^2} \int \frac{\left[\cdot\nabla'(\boldsymbol{v} \cdot \boldsymbol{J})\right]}{r} \mathrm{d}V' + \frac{G}{c^2} \int \frac{\left[\boldsymbol{v} \times (\nabla' \times \boldsymbol{J})\right]}{r} \mathrm{d}V'. \tag{32}$$

If we compare Equation, (32) with Equation (3-1.2) from [3] for the cogravitational field,

$$\boldsymbol{K} = -\frac{G}{c^2} \int \frac{\left[\nabla' \times \boldsymbol{J}\right]}{r} \mathrm{d}V', \tag{33}$$

we find that Equation, (32) can be written as

$$\boldsymbol{g}_k = -\frac{G}{c^2} \int \frac{\left[\nabla'(\boldsymbol{v} \cdot \boldsymbol{J})\right]}{r} \mathrm{d}V' - \boldsymbol{v} \times \boldsymbol{K}, \tag{34}$$

where $\boldsymbol{K}$ is the cogravitational field created by the moving mass current $\boldsymbol{J}$.

7. Dynamic Effects of Gravikinetic Fields; Gravitational Induction

We shall now present one example (fore more examples see the Section 2-2 in [3]) demonstrating force effects of the gravikinetic field. For simplicity we shall use gravikinetic fields calculated in the Section 12-2 of [3].

The force effects that we shall show constitute the gravitational analogue of electromagnetic induction and of electromagnetic Lenz's law. As we now know, electromagnetic induction is caused by the electrokinetic field (see [3]). The gravikinetic field is the gravitational counterpart of the electrokinetic field, and their dynamic effects are similar, except that the gravikinetic force exerted on a mass by an increasing/decreasing gravikinetic field is parallel/antiparallel to the field, whereas the electrokinetic force exerted on a positive charge by an increasing/decreasing electrokinetic field is antiparallel/parallel to the field.

An example: A thin-walled cylinder of radius R_0, length $2L$ and wall thickness t has a uniformly distributed mass of density ϱ is initially at rest. A ring of mass m_r and radius R us placed around the cylinder coaxially with it. The cylinder is then suddenly set in motion along its axis and attains a velocity v_c (mass current $\boldsymbol{J}_c$). The gravikinetic force causes the ring to move along (follow) the cilinder (**Figure 2**). Assuming that no other forces act on the ring, and assuming that the ring stays near the middle of the cylinder during the time that the velocity of the cylinder changes, find the final velocity $\boldsymbol{v}_f$ of the ring.

According to our assumptions, the gravikinetic field through which the ring moves is a function of time only. Therefore we can use Equation (12-1.5) from [3] for finding the final momentum and velocity of the ring. When the gravikinetic force acts on a mass distribtion ϱ, it changes the mechanical momentum $\boldsymbol{G}_M$ of the mass distribution (see [3]), and if $\boldsymbol{g}_k$ is a function of time only, the momentum change is

$$\Delta \boldsymbol{G}_M = m\int \boldsymbol{g}_k \mathrm{d}t = -m\Delta \boldsymbol{A}, \tag{35}$$

where m is the total mass of the distribution, and $\Delta\boldsymbol{A}$ is the change in the vector potential during the time interval under consideration. From Equation (35) and Equation (12-2.3) (in [3])

$$\boldsymbol{g}_k = \frac{\partial I}{\partial t}\frac{2G}{c^2}\ln\frac{2L}{R}\boldsymbol{k}, \tag{36}$$

where $\boldsymbol{k}$ is a unit vector in the direction of the mass current I, we have

$$\Delta \boldsymbol{G}_M = m_r \boldsymbol{v}_f = m_r I_c \frac{2G}{c^2}\ln\frac{2L}{R}\boldsymbol{k}, \tag{37}$$

so that the final velocity of the ring is

$$\boldsymbol{v}_f = \frac{2G}{c^2} I_c \ln\frac{2L}{R}\boldsymbol{k}. \tag{38}$$

Substituting $\varrho v_c 2\pi R_0 t$ for I_c, where ϱ is the density of the cylinder, R_0 is its radius, and t is its thickness, we obtain

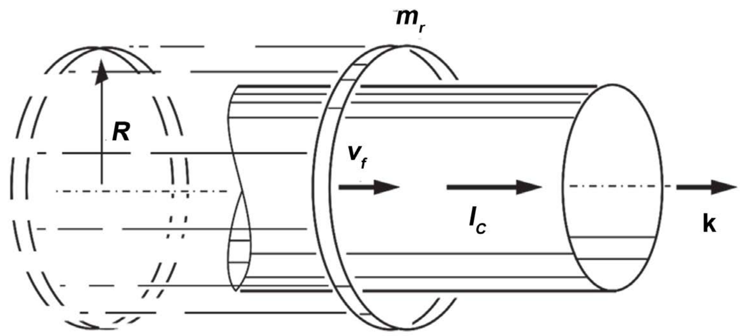

Figure 2. Accelerating cylinder drags the ring with itself.

$$\boldsymbol{v}_f = \frac{4G\varrho v_c \pi R_0 t}{c^2}\ln\frac{2L}{R}\boldsymbol{k}. \tag{39}$$

The cylinder *drags* the ring so that the ring moves in the direction of the moving cylinder. It is ineresting to not that the final velocity of the ring does not depend on its mass.

This example (along with other examples in the Section 2-2 in [3]) illustrates the phenomenon of *gravitational induction*, whereby a changing mass current induces a secondary mass current in neighboring bodies. The effect is similar to electromagnetic induction (for a detail analyses and novel interpretation of the phenomenon of electromagnetic induction see Oleg D. Jefimenko, "Presenting electromagnetic theory in accordance with the principle of causality", [16]), except that, in contrast to the latter, the direction of the induced current is the same as that of the original current if the original cerrent increases, and is opposite to the original current if the original current decreases. Thus the sign of the "gravitational Lenz's law" is opposite to that of the electromagnetic Lenz's law.

8. Instead of Conclusion: On the Truth of the Generalized Theory of Gravitation

The comparison of the generalized theory of gravitation with the general theory of relativity of Einstein (GR) involuntarily suggests.

As is known, the experimental confirmation of GR generally relies only on the fact that it allegedly explained the previously unexplained discrepancy between the theoretical (calculated) and observed displacement (precession) of the perihelion of the planet Mercury; all other predictions and conclusions of the general theory of relativity can either not be verified with sufficient accuracy, or can be explained without this theory.

Speaking about the problem of Mercury, it should be pointed out that the so-called discrepancy in the displacement of its perihelion is the difference between the observed and calculated, on the basis of the usual Newton theory, perihelion.

This difference, which is approximately 575'' per century attracted the attention of Urben Leverrier, who predicted the existence of the planet Neptune and accurately calculated its coordinates. Leverrier explained the difference in the precession of Mercury by the influence of near planets and, having calculated their effect on Mercury, found that these planets cause a precession of 532" per century. The remaining 43" he could not explain. These remaining 43" for centuries were, *as it is now believed*, explained by Einstein's general theory of relativity. But then a certain discrepancy is immediately evident. After all, the main divergence in 532" was calculated according to Newton's theory, and the remainder in 43" was calculated according to the general theory of relativity. It would be much more convincing if the entire discrepancy of 575" per century was calculated *on the basis of the same theory*. On this occasion J. Synge remarked [17]: "Such a mixture of Newtonian and Einstein's theories is psychologically unpleasant, since these theories are based on too different initial concepts." Until the *complete discrepancy* is calculated using the general theory of relativity, without invoking Newton's theory, the experimental verification of general relativity can not be considered valid.

But let us go back to the generalized theory of gravitation. We note that, as shown in [3], within the framework of the generalized theory of gravitation, one can obtain formulas according to which the perihelion precession *for all planets* is a necessary consequence of this theory. However, these formulas hardly can prove anything. The fact is that according to the generalized theory of gravitation, all celestial mechanics and its results should be revised. As it was shown above (see **Figure 1**), the action of the Sun on planets is expressed not by one force, but by five forces, and the action of each planet on each other planet is expressed not by one force, but by five forces. Therefore, as a matter of fact, all the information about our solar system, obtained on the basis of Newton's conventional theory, should be considered only approximately correct. So, from the point of view of the generalized theory, there is no point in trying to explain the 43"rd residue in the displacement of the perihelion of Mercury. After all, if we take into account all the forces that act on Mercury, including the forces associated with the movement of the Sun in relation to the Galaxy, the rotation of the Sun around its axis, the dependence of the forces acting on Mercury on the speed and acceleration of the planets, on the speed of Mercury itself, and , finally, the retardation in the action of gravitational and cogravitational forces, will we get this discrepancy at all in the displacement of the perihelion of Mercury, especially the discrepancy exactly in 43 seconds? It is clear that until all these calculations are performed, until the necessary corrections in celestial mechanics are amended, it is pointless to speak of testing the generalized theory of gravitation by analyzing the motion of Mercury.

So neither the generalized theory of gravitation of Jefimenko nor the general theory of relativity of Einstein can be verified in this way.

However, one very strong testimony to the truth of the generalized theory of gravitation can still be cited (although it is not as sensational as the explanation of the discrepancy in the displacement of Mercury's perihelion). Speech will go about one "white spot" associated with the phenomenon of gravitation. As is known, the motion of stellar bodies and the fall of bodies under the action of the gravitational field p associated with conversion of potential energy unto kinetic energy and vice versa. In particular, when a body is falling under the action of the gravitational of the Earth, its potential energy diminishes and its kinetic energy increases. But how, exactly, does this come about? How is this energy exchange actually accomplished? In the past this phenomenon was simply interpreted as a result of the energy conservation, but the process, or mechanisms, of the energy exchange remained unknown. As we shall now see, the generalized theory of gravitation explains this heretofore hidden process with perfect clarity.

Let a body of mass m fall under the action of Earth's gravitational field $\boldsymbol{g}$ (Figure 3). Note that the magnitude of $\boldsymbol{g}$ is equal to the acceleration of gravity g. Let the velocity of the body at the moment of observation be $\boldsymbol{v}$. Like all moving masses, the falling creates around itself a cogravitational field $\boldsymbol{K}$ lefthanded relative to the velocity vector of the body. Therefore, according to Equation (10) (gravotational Poynting vector equation)

$$\boldsymbol{P} = \frac{c^2}{4\pi G}\boldsymbol{K} \times \boldsymbol{g}. \tag{40}$$

there is a flow of gravitational energy U_{gr} at the surface of the falling body directed into the body. The rate at which the ghravitational energy enters the body is

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \frac{c^2}{4\pi G}\oint(\boldsymbol{K} \times \boldsymbol{g}) \cdot \mathrm{d}\boldsymbol{S}_{in} = \frac{c^2}{4\pi G}\oint(\boldsymbol{g} \times \boldsymbol{K}) \cdot \mathrm{d}\boldsymbol{S}, \tag{41}$$

where $\mathrm{d}\boldsymbol{S}_{in}$ is a surface element vector of the falling body directed into the body, and $\mathrm{d}\boldsymbol{S}$ is a surface element vector directed, as usually accepted in vector analysis, from the body into the surrounding space; the integration is over the entire surface of the falling body. Transposing in the integrand the cross and the dot and factoring out the constant vector $\boldsymbol{g}$ together with the dot from under the integral sign, we have

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \frac{c^2}{4\pi G}\oint \boldsymbol{g} \cdot (\boldsymbol{K} \times \mathrm{d}\boldsymbol{S}) = \frac{c^2}{4\pi G}\boldsymbol{g} \cdot \oint \boldsymbol{K} \times \mathrm{d}\boldsymbol{S}. \tag{42}$$

Converting now the last surface integtal into the volume integral, we obtain

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = -\frac{c^2}{4\pi G}\boldsymbol{g} \cdot \int \nabla \times \boldsymbol{K}\mathrm{d}V. \tag{43}$$

By Equation (7-1.4) in [3]

$$\nabla \times \boldsymbol{K} = -\frac{4\pi G}{c^2}\boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{g}}{\partial t}, \tag{44}$$

since $\boldsymbol{g}$ is not a function of time,

$$\nabla \times \boldsymbol{K} = -\frac{4\pi G}{c^2}\varrho\boldsymbol{v}. \tag{45}$$

Therefore Equation (43) reduces to

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \boldsymbol{g} \cdot \int \varrho\boldsymbol{v}\mathrm{d}V. \tag{46}$$

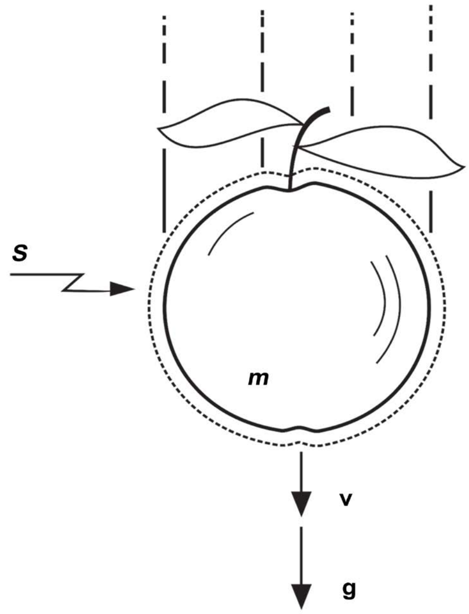

Figure 3. The generalized theory of gravitation provides a clear explanation of the mechanism of energy exchange involved in gravitational interactions: the increase of the kinetic energy of a body moving under the action of a gravitational field occurs as a consequence of the influx of gravitational field energy into the body via the gravitational Poynting vector.

Factoring out constant vector $\boldsymbol{v}$ from under the integral sign, we obtain

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \boldsymbol{g} \cdot \boldsymbol{v} \int \varrho \mathrm{d}V. \tag{47}$$

Thus, since $\boldsymbol{g}$ and $\boldsymbol{v}$ are parallel, and since the last integral in Equation (47) represents the mass of the falling body, we find that when the body is falling, there is an influx of the gravitational field energy (potential energy) into the body at the rate

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \boldsymbol{g} \cdot \boldsymbol{v} m = mvg\,. \tag{48}$$

Let us now consider the kinetic energy. The kinetic energy of a falling body increases at the rate

$$\frac{\mathrm{d}U_{gr}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{mv^2}{2}\right) = mv\frac{\mathrm{d}v}{\mathrm{d}t} = mvg, \tag{49}$$

where g is the acceleration of the falling body. However, as was mentioned above, g in Equation (48) is the same acceleration, and therefore the rate at which the kinetic energy of the falling body increases is equal to the rate of influx of the gravitational field energy into the body. Note that a less general case of the gravitational and kinetic energy exchange was previously considered by D. Bedford and P. Krumm in "The gravitational Poynting vector and energy transfer" [18].

Thus the generalized theory of gravitation provides a clear explanation of the mechanism of the energy exchange involved in gravitational interactions: the increase of the kinetic energy of the body moving under the action of a gravitational field occurs as a consequence of the gravitational field energy influx into the body via the gravitational Poynting vector. Essentially the same considerations apply to the case when a body moves against the gravitational field, in which case its kinetic energy diminishes due to outflow of energy from the body into surrounding space again via the gravitational Poynting vector.

The simplicity of the above calculation tends to hide the utmost significance of the obtained results. The fact is that no gravitational theory can be considered definitive if it cannot provide a clear explanation of the mechanism of conversion of "gravitational potential energy" into the kinetic energy of falling bodies. Therefore, in spite of their simplicity, the above calculations constitute an exceptionally important proof of the validity of the generalized theory of gravitation and, at the same time, reveal the true nature of the "gravitational potential energy."

Finally, we will talk about the *existence of gravitational and cogravitational waves*. From the theoretical point of view, it is especially important that the Equations (3) and (4) can be transformed into the following differential equations in the present time:

$$\nabla \cdot \boldsymbol{g} = -4\pi G\varrho, \tag{50}$$

$$\nabla \cdot \boldsymbol{K} = 0, \tag{51}$$

$$\nabla \times \boldsymbol{g} = -\frac{\partial \boldsymbol{K}}{\partial t} \tag{52}$$

and

$$\nabla \times \boldsymbol{K} = -\frac{4\pi G}{c^2}\boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{g}}{\partial t}. \tag{53}$$

We shall now show by direct calculation how Equations. (50)-(53) predict the existence of gravitational and cogravitational waves.

We start with Equation (52). Taking the curl of this equation, we obtain

$$\nabla \times \nabla \times \boldsymbol{g} = -\frac{\partial}{\partial t}\nabla \times \boldsymbol{K}. \tag{54}$$

Substituting Equation (53) into Equation (54), we obtain

$$\nabla \times \nabla \times \boldsymbol{g} = \frac{4\pi G}{c^2}\frac{\partial \boldsymbol{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{g}}{\partial t^2}, \tag{55}$$

Or

$$\nabla \times \nabla \times \boldsymbol{g} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{g}}{\partial t^2} = \frac{4\pi G}{c^2}\frac{\partial \boldsymbol{J}}{\partial t}. \tag{56}$$

Similarly, taking the curl of Equation (53), we have

$$\nabla \times \nabla \times \boldsymbol{K} = -\frac{4\pi G}{c^2}\nabla \times \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \nabla \times \boldsymbol{g}}{\partial t}, \tag{57}$$

and, substituting Equation (52) into Equation (57), we obtain

$$\nabla \times \nabla \times \boldsymbol{K} = -\frac{4\pi G}{c^2} \nabla \times \boldsymbol{J} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{K}}{\partial t^2}, \tag{58}$$

or

$$\nabla \times \nabla \times \boldsymbol{K} + \frac{1}{c^2} \frac{\partial^2 \boldsymbol{K}}{\partial t^2} = -\frac{4\pi G}{c^2} \nabla \times \boldsymbol{J}. \tag{59}$$

Equations (56) and (59) are mathematical expressions for waves propagating in space with velocity *c*. In the present case they represent waves carrying with themselves the gravitational field $\boldsymbol{g}$ and the cogravitational field $\boldsymbol{K}$, respectively.

Furthemover, by Equations. (50) and (51), in a region of space where there are no masses and no mass currents, Equations. (56) and (59) become the more familiar "wave equations"

$$\nabla^2 \boldsymbol{g} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{g}}{\partial t^2} = 0 \tag{60}$$

and

$$\nabla^2 \boldsymbol{K} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{K}}{\partial t^2} = 0. \tag{61}$$

Studing in [3] (pp. 303-304) energy relations in gravitational and cogravitational waves O. Jefimenko found that the energy density in these waves is *negative*.

$$U_v = -\frac{1}{8\pi G} \boldsymbol{g}^2 - \frac{1}{8\pi G} c^2 \boldsymbol{K}^2. \tag{63}$$

An important consequence of the negative energy density in gravitational-cogravitational waves is that in contrast to the electromagnetic waves, a gravitational-cogravitational wave striking a body pulls toward the wave, that is, exerts a negative rather than a positive pressure on the body. The calculations of the negative pressure by gravitational-cogravitational waves are similar to the corresponding calculations of the positive pressure by electromagnetic waves (see [2] pp. 132-133).

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