Journal of Modern Physics, 2025, 16(4), 613-626
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Derivation and Fits of Fermion Masses from the Higgs Sector

Richard B. Holmes

Innoven Energy, Inc., Solana Beach, USA
Email: rholmes001@aol.com

How to cite this paper: Holmes, R.B. (2025) Derivation and Fits of Fermion Masses from the Higgs Sector. *Journal of Modern Physics*, **16**, 613-626.
https://doi.org/10.4236/jmp.2025.164033

Received: March 16, 2025
Accepted: April 22, 2025
Published: April 25, 2025

Abstract

The masses of the fundamental fermions are fit using properties of the minimal Higgs sector of the standard model and also using insights from an anomaly-free quantum field theory (QFT) with permutational symmetry. With this approach, three generations for each family of fermions arise due to the quartic potential of the Higgs fields and the details of their coupling to ghosts and gauge functions. A similar procedure allows calculation of the mass parameters including the hop amplitudes of the mass matrices of the QFT with permutational symmetry. With both approaches there are two free parameters per family to fit the masses. The latter QFT and the Higgs-based approach lead to related physical interpretations. This paper further reinforces the notion that fundamental fermions are composite particles, comprising "preons" within the minimal Higgs sector.

Keywords

Higgs Fields, Quantum Field Theory, Fermion Masses

1. Introduction

The minimal Higgs sector has been a fixture of the standard model for roughly 50 years [1]-[4]. A recent nonperturbative study of the Higgs sector indicates that there are persistent, purely oscillatory solutions that have properties that can be related to the observed fundamental fermions [5]. Such solutions involve Faddeev-Popov ghosts and gauge functions that oscillate in rings [2] [3], utilizing the ghost Lagrangian density of [2]. Assuming that there are three potential wells in a ring as in [6], the results indicate that there should be four and only four families of fundamental fermions due to the anticommuting nature of the ghost particles as well as their electrical charge properties. The equations derived for the ghosts and Higgs fields of [5] are here used to fit the masses of each of the three generations of masses precisely for each of the 4 families (sample neutrino masses are used for that family). The form of the solutions requires no additional assumptions outside of the minimal Higgs sector with a family-dependent modification to the Higgs potential. The Higgs sector provides oscillatory solutions for ghosts and gauge functions that propagate in loops. The latter is often noted in the literature [2] [3]. The results of this paper match with those of [6]. This enables a straightforward, orderly interpretation of the results herein. It should be noted that [5] and [6] have already provided a means for computing the sum of the masses in each family (except possibly the neutrino family) from the parameters of the Higgs sector, so computation of this sum is not addressed here.

The formulae here will precisely fit the most recent estimates of the masses [7], as well as older textbook values [8], and support limited running of the masses. The approach and results of this paper reinforce the notion of preons [9]-[13] using nonperturbative techniques drawn from nonlinear optics [14] [15].

Section 2 describes the computational approach. Section 3 presents the results. Section 4 discusses the results in relation to other approaches. Section 5 provides a summary along with implications.

2. Computational Approach

The overall approach is to write down a polynomial eigenvalue equation for $M = 3$ masses in each family, and then to derive an equation involving Fourier coefficients of identical form from the nonlinear Higgs equations. Matching coefficients then provide the needed result. The eigenvalue equation for a mass matrix $\boldsymbol{M}$ of order M is

$$\det\left(\lambda \boldsymbol{I} - \boldsymbol{M}\right) = \prod_{i=1}^{M}\left(\lambda - m_i\right) = 0, \tag{1}$$

where λ is the eigenvalue and m_i are the masses. A dimensionless version of this equation for $M = 3$ is

$$\lambda'^3 - \lambda'^2 + K\lambda' - \Lambda = 0, \tag{2}$$

where

$$\lambda' = \lambda \Big/ \sum_{i=1}^{3} m_i, \tag{3a}$$

$$K = \left(m_1 m_2 + m_2 m_3 + m_3 m_1\right) \Big/ \left(\sum_{i=1}^{3} m_i\right)^2, \text{ and} \tag{3b}$$

$$\Lambda = m_1 m_2 m_3 \Big/ \left(\sum_{i=1}^{3} m_i\right)^3. \tag{3c}$$

For the mass matrices of ([6], Ch. 2), the left-hand side of Equation (1) can be written in dimensionless form as

$$\begin{aligned} &\det\left(\lambda \boldsymbol{I} - \boldsymbol{M}\right) \Big/ \left(\sum_{i=1}^{3} m_i\right)^3 \\ &= \lambda'^3 - \lambda'^2 + \left[\left(1-d^2\right)/3\right]\lambda' - \left[1 - 3d^2 + 2d^3\cos\left(3\phi\right)\right]/27, \end{aligned} \tag{4}$$

where $d\mathrm{e}^{i\phi}$ is the dimensionless complex hop amplitude of [6].

From the Lagrangian densities of the Appendix, including the ghost Lagrangian of [2], one can obtain an equation for neutral Higgs fields. Assume a solution for a linear combination $H_4 - iH_3$ of the neutral Higgs fields H of the form

$$H_4 - iH_3 = H_{34,0}^{+}(t)\cos\left(\boldsymbol{p}_{H34}\cdot\boldsymbol{x}/\hbar + \vartheta\right). \tag{5}$$

One finds the same results as below if the cosine is replaced by complex exponential. From Equation (5), one obtains the following equation for the temporal dependence, when the electroweak bosons W and Z are not present:

$$\begin{aligned}\partial_t^2 H_{34,0}^{+} = {} & (c/\hbar)^2\left[-\left|\boldsymbol{p}_{H34}\right|^2 + \hbar^2\left(\mu - \lambda\left|H_{34}^{+}\right|^2\right)\right]H_{34}^{+} - c^2\sigma_f\left[H_{34}^{+} - (\mu/\lambda)^{1/2}\right] \\ & + 0.5(c/\hbar)^2\,\xi g M_Z\left[-\cos(\theta_W)\eta^{-}\omega^{+} + \eta_Z\omega_Z/\cos(\theta_W)\right].\end{aligned} \tag{6}$$

Note the dimensionless gauge factor, ξ. Here, this factor is a positive real number which is nominally set to 1. All the variables in this equation that are not defined here are defined in the Appendix. The equation for H_{34}^{-} is very similar; the only difference is that superscript pluses and minuses are exchanged. The various terms in Equation (6) are assumed to be matched in 3-momentum, with particles that are co-propagating or counter-propagating in a ring geometry. Next, Equation (6) is re-written in dimensionless form by dividing by the vacuum expectation value of the Higgs, $(\mu/\lambda)^{1/2}$, and also separating out variations δh about the expectation value:

$$H_{34,0}^{+}\Big/(\mu/\lambda)^{1/2} = h_o - \delta h. \tag{7}$$

The resulting equation can be separated into an equation for the zeroth-order term h_o and an exact expression for the higher-order terms:

$$\partial_t^2 h_o = -\left(\left|\boldsymbol{p}_{H34}\right|c/\hbar\right)^2 h_o \quad \text{and} \tag{8a}$$

$$\begin{aligned}\partial_t^2\delta h = {} & -\left(\left|\boldsymbol{p}_{H34}\right|c/\hbar\right)^2\delta h + \left[\left(M_H c^2/\hbar\right)^2\Big/2\right]\left[\delta h h_0^{*} + \delta h^{*} h_o - \left|\delta h\right|^2\right]\left[h_o - \delta h\right] \\ & + \xi\left(M_Z c^2/\hbar\right)^2\left[-\cos^2(\theta_W)\eta'^{-}\omega'^{+} + \eta'_Z\omega'_Z\right] - c^2\sigma_f\delta h.\end{aligned} \tag{8b}$$

The coefficients of the latter terms have been simplified using $\cos(\theta_W) = M_W/M_Z$ and $g(\mu/\lambda)^{1/2} = 2M_W c^2$, as well as using $\eta' = \eta\big/(\mu/\lambda)^{1/2}$ and $\omega' = \omega\big/(\mu/\lambda)^{1/2}$.

Equation (8a) can be solved trivially, with solution

$$h_o = h_{oo}\exp\left(\pm i\left|\boldsymbol{p}_{H34}\right|ct/\hbar + i\vartheta\right). \tag{9}$$

Here h_{oo} is a constant which is equal to 1 based on the definition of h_o in the case of interest in which $H_{34,0}^{+} \approx (\mu/\lambda)^{1/2}$. The global phase ϑ is set to zero and the positive frequency is used in the following. Equation (9) will be substituted into Equation (8b) shortly. Note that Equation (8b) is a cubic equation in δh. Writing this equation in the same form as Equation (2), one finds that

$$\begin{aligned}&\left|\delta h\right|^2\delta h - 2h_o\left|\delta h\right|^2 - h_0^{*}\delta h^2 + \left\{1 - \left[c^2\sigma_f + \left(\left|\boldsymbol{p}_{H34}\right|c/\hbar\right)^2 + \partial_t^2\right]\Big/\left[\left(M_H c^2/\hbar\right)^2\Big/2\right]\right\}\delta h \\ & + h_o^2\delta h^{*} + 2\xi\left(M_Z/M_H\right)^2\left[-\cos^2(\theta_W)\eta'^{-}\omega'^{+} + \eta'_Z\omega'_Z\right] = 0.\end{aligned} \tag{10}$$

Here δh is a function, but we wish to obtain an equation of the form of Equation (2) involving a cubic of just numbers and not functions. This motivates writing δh as a Fourier series in time. The Fourier series should have at least three terms because the cubic term in δh will have three times higher frequencies than the lowest frequency in δh. Hence, a trial form for δh is

$$\delta h = A_1 \mathrm{e}^{i\Omega t} + A_2 \mathrm{e}^{2i\Omega t} + A_3 \mathrm{e}^{3i\Omega t} + \text{negative frequencies}, \tag{11}$$

where the coefficients A_i and the base frequency Ω are to be determined. One can tentatively set the base frequency to $\alpha |\boldsymbol{p}_{H34}| c/\hbar$ with $\alpha \approx 1$ based on how the zeroth-order solution h_o appears in Equation (10), and that there should be a requirement for frequency matching between the different terms. That this base frequency Ω should not be zero follows from the Bohr-like condition of [5]. It should also be noted that the use of negative frequencies gives the same result as the following.

Frequency matching between the various terms implies that the products $\eta'^{-}\omega'^{+}$ and $\eta'_Z \omega'_Z$ also have the same frequency content. It was shown that these two sets of terms can be mutually exclusive in [5], depending on whether the fermion family is charged or not charged. For the uncharged case, it was shown that η_Z and ω_Z are both present. Focusing on the un-charged case ($\eta'^{-}\omega'^{+} = 0$) for the moment, one may write

$$\eta'_Z \omega'_Z = B_1 \mathrm{e}^{i\Omega t} + B_2 \mathrm{e}^{2i\Omega t} + B_3 \mathrm{e}^{3i\Omega t} + \text{c.c.}. \tag{12}$$

Here again, the Fourier coefficients B_i are to be determined. Note that the product $\eta'_Z \omega'_Z$ of pure real variables has both the sum and difference frequencies of the two individual frequencies. Because the difference frequency should be non-zero in order to avoid long-term, non-oscillatory changes in the Higgs fields, the frequencies of η_Z and ω_Z must therefore be distinct, so one option is to set one frequency to Ω and the other to 2Ω, so that the sum and difference frequencies are ±Ω and ±3Ω. With this choice, one may set B_2 and A_2 to zero. Next, substituting Equations (9)-(12) into Equation (10), one then obtains the following equations for the temporal Fourier components, $n\Omega$, for n = 1, 3, 5, 7, and 9, respectively:

$$\begin{gathered}\left\{1 - \left[\hbar^2 c^2 \sigma_f + \left(|\boldsymbol{p}_{H34}| c\right)^2 - \hbar^2 \Omega^2\right] \Big/ \left[\left(M_H c^2\right)^2 / 2\right]\right\} A_1 \\ + 2\xi \left(M_Z / M_H\right)^2 B_1 + A_1^* - A_1^2 - 2A_1^* A_3 + \left(|A_1|^2 + |A_3|^2\right)\left(3A_1 - 2\right) = 0\end{gathered} \tag{13a}$$

$$\begin{gathered}A_1^3 - A_1^2 + A_{-1}^* + \left\{1 - \left[\hbar^2 c^2 \sigma_f + \left(|\boldsymbol{p}_{H34}| c\right)^2 - 9\hbar^2 \Omega^2\right)\right]\right\} \Big/ \left[\left(M_H c^2\right)^2 / 2\right] A_3 \\ + 2\xi \left(M_Z / M_H\right)^2 B_3 - A_1^2 - 2A_1 A_3 - 4A_1^* A_3 + 3\left(|A_1|^2 + |A_3|^2\right) A_3 = 0\end{gathered} \tag{13b}$$

$$3A_1^2 A_3 - 4A_1 A_3 - A_3^2 + A_{-3}^* \approx 0 \tag{13c}$$

$$3A_1 A_3^2 - 2A_3^2 \approx 0, \text{ and} \tag{13d}$$

$$A_3^3 \approx 0. \tag{13e}$$

Note that no frequencies appear with even multiples of Ω, because Equation

(10) always contains products of an odd number of functions of the form $\exp(in\Omega)$, and n is always odd, when $A_2 = 0$. The sum of an odd number of odd integers is always odd, so all frequencies are odd. Also, Equations (13c) to (13e) should be viewed as approximate because there may be higher-frequency terms in the Fourier series of Equation (11). Further, note that A_3 is a common factor in all terms in all of Equations (13c) to (13e), which indicates that $A_3 \approx 0$ is a feature of any solution. Equation (13a) is used to determine B_1 after the A_i are determined.

Equation (13b) is of particular interest because it has the desired form of a cubic polynomial in dimensionless A_1:

$$\begin{aligned} &A_1^3 - A_1^2 + A_{-1}^* + \left\{1 - 2\left[\hbar^2 c^2 \sigma_f - \left(9\alpha^2 - 1\right)\left(\left|\boldsymbol{p}_{H34}\right| c\right)^2\right] \Big/ \left(M_H c^2\right)^2\right\} A_3 \\ &+ 2\xi\left(M_Z/M_H\right)^2 B_3 - A_1^2 - 2A_1A_3 - 4A_1^* A_3 + 3\left(\left|A_1\right|^2 + \left|A_3\right|^2\right) A_3. \end{aligned} \tag{14}$$

The first two terms automatically match the form of Equation (2). The B_3 term can be associated with Λ in Equation (2). The terms after B_3 will be addressed shortly. The third and fourth terms in the first line can be associated with the $K\lambda'$ term in Equation (2). If so, one should have either $A_3 = \beta A_1$, if A_1 is much less than 1, or $A_3 \approx 0$ if A_1 is comparable to 1, in order to satisfy Equations (13c) to (13e). Hence one may write $A_3 \equiv f(A_1)A_1$, where $f(A_1)$ is approximately constant for small A_1 and tends to 0 as A_1 tends to 1. Unfortunately, this approach does not determine $f(A_1)$ in more detail than that outlined above. A trial form for A_3 is

$$A_3 \equiv \beta\left(1 - A_1\right) A_1 \tag{15}$$

The results of this effort should be insensitive to this particular choice for the form of A_3. Given this form for A_3, one may address the remaining terms at the end of Equation (14). One can choose β so that the coefficients of A_1^3 and A_1^2 are equal and opposite to maintain the form of Equation (2). Assuming A_1 is nearly pure real and significantly less than 1, one may rewrite the cubic and quadratic terms of Equation (14):

$$\begin{aligned} &-2A_1^2 - 2A_1A_3 - 4A_1^* A_3 + A_1^3 + 3\left(\left|A_1\right|^2 + \left|A_3\right|^2\right) A_3 \\ &= -\left(2 + 6\beta\right) A_1^2 + \left[1 + 3\left(1 + \beta^2\right)\beta\right] A_1^3. \end{aligned} \tag{16}$$

Setting $2 + 6\beta$ equal to $\left[1 + 3\left(1 + \beta^2\right)\beta\right]$ gives a cubic equation in β which can be solved to give

$$\beta = \beta_0 = 1.137,\ \ 2 + 6\beta_0 = 1 + 3\left(1 + \beta_0^2\right)\beta_0 = 8.82. \tag{17}$$

Hence, the result of this paragraph is that Equation (14) may be written:

$$\begin{aligned} &8.82A_1^3 - 8.82A_1^2 + A_1 + f\left(A_1\right)\left\{1 - 2\left[\hbar^2 c^2 \sigma_f - \left(9\alpha^2 - 1\right)\left(\left|\boldsymbol{p}_{H34}\right| c\right)^2\right] \Big/ \left(M_H c^2\right)^2\right\} A_1 \\ &+ 2\xi\left(M_Z/M_H\right)^2 B_3 = 0. \end{aligned} \tag{18}$$

One finds that Equation (18) also has a form that is similar to Equation (2)—it is a cubic polynomial in dimensionless parameters, with two coefficients as free parameters. The above then provides a solution procedure in which Equations (2) and (14) to (18) are all satisfied exactly with the known masses as roots, as required. First divide Equation (18) by 8.82. Then the procedure is: 1) first pick a value of A_1 which corresponds to a desired root; 2) use Equation (15) to find a value of A_3 given A_1; 3) insert that value of A_3 into Equation (18); and 4) find the unique values of $f(A_1)\left\{1-\hbar^2c^2\sigma_f-\left(9\alpha^2-1\right)\left|\boldsymbol{p}_{H34}c\right|^2\right\}$ and ξB_3 that result in a match to Equation (2). The parameter α is set to 1 for this process, as might be expected. Since the last step does not uniquely specify σ_f and $\left|\boldsymbol{p}_{H34}c\right|^2$, the latter is tentatively set to the value of $m_ic^2/30$, where m_i is the mass of fermion *i*. This then uniquely determines σ_f. The factor of 1/30 is arbitrary and is chosen solely to ensure that the kinetic energy of all the constituents is a fraction of the fermion's total rest energy.

One other complication arises in using this approach, which occurs when A_1 is approximately one, which occurs for the largest mass in the family for at least three of the four families. In this case, Equation (15) gives $A_3 \approx 0$, by construction. One can still obtain a root to Equation (18) for $A_1 \approx 1$, but the Higgs momentum is not determinable from Equation (18) because $f(A_1) \approx 0$. To obtain zero for the eigenvalue equation, one must then have

$$2\xi\left(M_Z/M_H\right)^2 B_3 \approx 1. \tag{19}$$

More precise values for this case are given in the next section.

A similar approach above applies when $\eta'^-\omega'^+ \neq 0$ and $\eta'_Z\omega'_Z = 0$, *i.e.*, when the states are charged. There are some important differences in this case. In particular, Equation (A5a) shows that the nominal oscillation frequency of ω'^+ is

$$\Omega_\omega = \pm\left(2\xi M_W^2c^4+\left|\boldsymbol{p}_{\omega12}\right|^2c^2\right)^{1/2}\Big/\hbar, \tag{20}$$

Here M_W is the mass of the W boson. A similar expression applies for the oscillation frequency of $\eta'^\pm$. When $\left|\boldsymbol{p}_{\omega12}\right| \ll M_Wc$, the nominal Higgs frequency, which is the difference frequency in $\eta'^-\omega'^+$ is approximately

$$\Omega = \left|\boldsymbol{p}_{H34}\right|c/\hbar \approx \pm\left(\left|\boldsymbol{p}_{\omega12}\right|^2-\left|\boldsymbol{p}_{\eta12}\right|^2\right)\Big/\left[2(2\xi)^{1/2}M_W\hbar\right]. \tag{21}$$

The leading sign on the right hand side is chosen so that Ω is positive, in accord with the conventions above. The corresponding sum frequency is quite large, about $2(2\xi)^{1/2}M_Wc^2/\hbar$, so the arguments surrounding Equation (12) need to be modified. In this case, the $\delta H_{34}^{\mp}\omega^{\pm}$ term in Equation (A5a) parametrically generates other frequencies that are multiples of $\left|\boldsymbol{p}_{H34}\right|c/\hbar$. With this insight, the generic result associated with Equations (11) to (18) still apply. Two other differences are that the sign is reversed for the $\eta'^-\omega'^+$ term compared to the $\eta_Z\omega_Z$ term and there is a $\cos^2(\theta_W)$ factor.

The issue of opposite charges in the product $\eta'^-\omega'^+$ in Equation (8b) is addressed by realizing that only the imaginary parts of this term are purely oscillatory, so one may set the real parts to zero for this steady-state analysis. This results in a pure real product and also decouples the charged states from the uncharged states as shown in [5]. This also seems to cause a loss of charge identity, which might be addressed by conservation of charge. However, this issue is not further considered here. The key result of this section is that Equations (15) and (18) can be used to precisely fit the measured, published three masses in three of the four fermion families, and to fit example masses for the neutrino families. One should note that this approach also allows some flexibility in this fit, so that it can accommodate a limited running of the masses as well.

3. Results

Table 1 shows masses for all the known fundamental fermions. These masses are from the 2024 Particle Data Group publication [7], except for the neutrino family, for which example masses of 0.05, 0.01, and 0.0505 eV/c^2 are assumed for the three known generations of neutrinos. These neutrino masses satisfy the recent measured differences of the square of neutrino masses, as described in [5] assuming the normal hierarchy. The *u*-, *d*-, and *s*-quark masses are the $\overline{MS}$ (minimal subtraction) masses at the energy scale 2 GeV. The *c*- and *b*-quark masses are the $\overline{MS}$ masses renormalized at the $\overline{MS}$ mass. The *t*-quark mass is extracted from event kinematics. Also, to recover the individual masses using this approach, the sum of the three masses within a family must be known, as seen from Equations (2) and (3). The sums of the masses of each family are known experimentally but can also be computed without their prior knowledge using the approach outlined in [5]. This can be done for the electron, up-quark, and down-quark families but not accurately for the neutrino family, based on the assumed and now measured input parameters of the Higgs sector.

Two columns of Table 1 show the hop magnitude d and the hop phase ϕ. These are computed from Equations (2) to (4):

$$d = \left(1 - 3K\right)^{1/2}, \tag{22}$$

$$\phi = (1/3)\operatorname{acos}\left[\left(27\Lambda - 1 + 3d^2\right)/\left(2d^3\right)\right]. \tag{23}$$

A column shows the dimensionless parameter K_1, the normalized Higgs momentum. This quantity is defined as follows:

$$K_1 \equiv \left[\hbar^2 c^2 \sigma_f - \left(9\alpha^2 - 1\right)\left(\left|\boldsymbol{p}_{H34}\right| c\right)^2\right] / \left(M_H c^2\right)^2 . \tag{24}$$

This is a family property, as desired, since it is related to the family parameter K by

$$\left[1 + \beta_0\left(1 - 2K_1\right)\right]/8.82 = K . \tag{25}$$

when A_1 is much less than 1 (as it is for all but the highest masses in each family).

The results in Table 1 show that ξB_3 is the same for the lowest two masses in each family and so can be viewed as a family property. It has a positive value except for the neutrino family, which

Table 1. Fit parameters for the 12 fundamental fermions of the standard model. Neutrino masses are example masses as described in text. K, Λ, d, ϕ, K_1, and ξB_3 are dimensionless. "Indet" denotes indeterminate, as discussed in text in Section 2.

Family or Particle	Particle Mass m (GeV/c^2)	K	Λ	d	ϕ	K_1	ξB_3
up family		0.0073	9.028×10^{-8}	0.9890	0.006400		
up quark	2.16×10^{-3}					0.9115	7.504×10^{-7}
charm quark	1.273					0.9146	7.504×10^{-7}
Top quark	172.6					Indet	0.8753
down family		0.0224	2.343×10^{-5}	0.9658	0.018600		
down quark	4.70×10^{-3}					0.8531	1.947×10^{-4}
strange quark	9.35×10^{-2}					0.8606	1.947×10^{-4}
bottom quark	4.183					Indet	0.7387
electron family		0.0532	1.437×10^{-5}	0.9167	0.052776		
electron	5.110×10^{-4}					0.7334	1.194×10^{-4}
muon	1.0566×10^{-1}					0.7472	1.194×10^{-4}
tau	1.776.93					Indet	0.4719
neutrino family		0.1882	8.985×10^{-3}	0.6598	0.100362		
electron neutrino	0.50×10^{-11}					0.1857	−0.075
muon neutrino	1.00×10^{-11}					0.1574	−0.075
Tau neutrino	5.05×10^{-11}					Indet	0.4049

indicates that the ghost and gauge functions are out of phase for the neutrino family. For the highest mass in each family, ξB_3 is roughly equal to 1 based on Equation (19) and the related discussion. The column for K_1 shows that the normalized Higgs momentum is roughly constant for each family for the two lowest masses and changes mildly between families. This is perhaps expected. Also, this normalized momentum is not defined for the highest mass in each family, based on the discussion preceding Equation (19), since $f(A_1) \approx 0$.

Another aspect of this approach is that ξB_3 is not formally determined because ξ is a free parameter. However, with any specific choice of free parameters, there is a unique solution for the more physical parameter B_3. These results can provide a physical picture for the various masses. For the highest mass in each family (except possibly the neutrino family), the ghost and gauge fields are marginally bound in the quartic potential of the Higgs and have indefinite momentum. In this case all the energy resides in the Higgs, ghost and gauge fields. For the lower two masses in each family, there is a definite momentum and energy corresponding to oscillatory states, and in this case the energy and mass resides both in Higgs kinetic energy as well as in the ghost and gauge fields. This discussion of the distribution of momentum and energy between Higgs, ghosts, and gauge fields may seem superficial, but such explanations are common and much more concrete in nonlinear optics [14] [15]. Analogous papers in nonlinear optics also provide a more rigorous quantum-mechanical formulation in that context.

Next consider the dimensionless "hop" parameters of **Table 1**, d and ϕ. These parameters show orderly trends as one progresses down the table, and these trends are more readily interpretable. For example, when the hop amplitude is zero, there is no hop at all. On the other hand, when this dimensionless parameter is nearly 1, the constituent particles are hopping constantly, indicating a higher momentum and energy which should correspond to higher-mass states, which is the case for the up-quark family in particular, and also for the down and electron families to a lesser degree.

4. Discussion

There have been many approaches used to explain the masses of the fundamental fermions over the years. In recent decades, the masses of the neutrinos have been included in some of these approaches. A sample of the more recent approaches are given here. In [16], one-family extended technicolor (ETC) models are used to predict quark masses. In [17], charged-fermion masses are obtained from an added Abelian family-dependent symmetry and infra-red stable fixed points of renormalization group equations. In [18], fermion masses, including neutrinos, are predicted using Dirac see-saw mechanisms implemented by the introduction a new set of $SU(2)_L$ weak singlet vector-like fermions. Reference [19] models fermion masses as analogues of Weyl curvature states. The spin-1/2 nature of the masses are attributed to curvature that emerges as a necessary condition for the relevant supergravity grand-unified theory realizations. In another approach, fermion masses are obtained from a holographic analysis using the holographic relation between the mass of the observable universe and the event horizon radius [20]. Fermions are spheres with a mass of 0.187 g/cm^2 multiplied by their surface area in [20]. Reference [6] obtains fits to the fermion masses using a permutationally-symmetric mass matrix for each family.

In particular, reference [6] is conceptually quite similar to the approach shown here. As discussed in [5], both use a similar physical construct, involving three potential wells in a ring geometry. Both constructs imply that some wave or particle should circulate in that ring. For both constructs, permutational symmetry is a natural fit because permutational symmetry is obeyed for particles in three identical potential wells in a ring. Both involve a polynomial equation for the 3 masses in a family. Both imply that the Higgs fields should be interpreted as a bond, as discussed in ([6], Ch. 11) and ([5], Sections 3 and 6). However, there is one key difference, which is that the bonds of [6] depend on the types of preons in the fermion. This is in turn linked to fermion masses by the family "mass parameter", which is the average of the measured masses of the family. The approach here also gives the proper coupling constant of the fermions to the Higgs, which is proportional to the particle mass, as evidenced by [21], for example. This can be seen from the definition of δh and A_1, which leads to a coupling constant g_i for fermion i that is equal to

$$g_i = m_i c^2 \Big/ \left(\mu/\lambda\right)^{1/2} = \left(\sum\nolimits_{j=1}^{3} m_j c^2\right) A_{1,i} \Big/ \left(\mu/\lambda\right)^{1/2}, \tag{26}$$

where m_i is the mass of fermion i. Hence this approach offers the same mass coupling constant for fermions as does the standard model.

One might ask if there are any concrete experimental tests that could test or distinguish this preon-based model from the existing standard model. Given that the closely related theory of [6] makes a number of such predictions that can be tested, it might be best to list some of them. These include predictions of a few dozen exotic quark states with energies in the vicinity of 4 to 7 GeV. There is already evidence for at least some of these, as detailed in ([6], Ch. 9). There are also predictions of 5 new boson resonances with energies ranging from about 7 GeV to 116 GeV. There is already some published evidence for at least three of these, as discussed in ([6], Ch. 11). These predictions reflect the most significant known differences for this specific preon model and could be tested in greater detail. Finally, these extensions of the standard model *do* yield masses for the neutrino family in a unifying framework.

5. Summary and Implications

This paper further reinforces the notion that the Faddeev-Popov ghost fields $\eta^{\pm}$ and η_Z of the standard model can be viewed as the "preon" constituents of the fundamental fermions. Based on Equation (8b), the local gauge functions $\omega^{\pm}$ and ω_Z can be viewed as the mediators between these preons and the Higgs field, and this coupling can be responsible for mass. The concept of preons has long been known [9-13] but has been shunned in many circles of particle physics. This paper shows that the ghosts along with the corresponding gauge functions of the electroweak sector can provide precise matches to the published masses of the known fundamental fermions. This is done in the context of the minimal Higgs sector, with simple, family-dependent modifications to the Higgs potential. There are three and only three generations of fermions within each family with this approach, due to the quartic potential of the Higgs field and the structure of the ghost Lagrangian. There are four and only four families of fermions with this approach due to the anticommuting nature of the ghost fields, which will only allow at most 3 ghost particles in the 3 potential wells in a ring that are necessitated by this approach. There are two free, dimensionless fit parameters, K_1 and ξB_3. The fit parameters that are found with this approach offer an interpretation in which the highest mass particles are merely bound preons in a quartic Higgs potential, whereas the lower-mass states have a defined momentum and energy (albeit with somewhat arbitrary scale factors).

A second interpretation can be obtained in the context of an anomaly-free QFT with permutational symmetry [6]. In this context, an orderly pattern appears as seen in **Table 1**. The approach of [6] is physically similar to the standard-model approach here, in which particles circulate in a ring with 3 potential wells. In this case, the kinetic energy of the preons is associated with the hop amplitude in a conceptually straightforward way. Moreover, with the theory of [6], one finds a three-fold mass degeneracy which leads to the well-known color states of quantum chromodynamics for the quark families. The results of this paper, taken together with [5] and [6], provide a more complete, quantitative explanation of the fermions in terms of the parameters and structure of the standard model. There is still uncertainty associated with the momentum of the Higgs fields, $|\boldsymbol{p}_{H34}|$, in these bound states that arise in this nonperturbative treatment. Finally, it should be noted that the nonperturbative treatment used here for the Higgs sector stems from approaches in nonlinear optics [14] [15].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Weinberg, S. (1967) A Model of Leptons. *Physical Review Letters*, **19**, 1264-1266. https://doi.org/10.1103/physrevlett.19.1264

[2] Taylor, J.C. (1976) Gauge Theories of Weak Interactions. Cambridge University Press.

[3] Weinberg, S. (1996). The Quantum Theory of Fields. Vol. 2, Cambridge University Press. https://doi.org/10.1017/cbo9781139644174

[4] Peskin, M. and Schroeder, D. (1995) Introduction to Quantum Field Theory. Perseus. https://doi.org/10.1201/9780429503559

[5] Holmes, R.B. (2025) Analysis and Reinterpretation of the Minimal Higgs Sector. *Journal of Modern Physics*.

[6] Holmes, R. (2021) A Quantum Field Theory with Permutational Symmetry. 2nd Edition, Lambert Academic Press. https://doi.org/10.5281/zenodo.5047237

[7] Navas, S., Amsler, C., Gutsche, T., Hanhart, C., Hernández-Rey, J.J., Lourenço, C., *et al.* (2024) Review of Particle Physics. *Physical Review D*, **110**, Article ID: 030001. https://doi.org/10.1103/physrevd.110.030001

[8] Thomson, M. (2013) Modern Particle Physics. Cambridge University Press. https://doi.org/10.1017/cbo9781139525367

[9] Harari, H. (1979) A Schematic Model of Quarks and Leptons. *Physics Letters B*, **86**, 83-86. https://doi.org/10.1016/0370-2693(79)90626-9

[10] Harari, H. and Seiberg, N. (1982) The Rishon Model. *Nuclear Physics B*, **204**, 141-167. https://doi.org/10.1016/0550-3213(82)90426-6

[11] Shupe, M.A. (1979) A Composite Model of Leptons and Quarks. *Physics Letters B*, **86**, 87-92. https://doi.org/10.1016/0370-2693(79)90627-0

[12] Robson, B.A. (2024) The Generation Model of Particle Physics. *European Journal of Applied Sciences*, **12**, 1-17. https://doi.org/10.14738/aivp.123.16922

[13] Raitio, R. (2018) Supersymmetric Preons and the Standard Model. *Nuclear Physics B*, **931**, 283-290. https://doi.org/10.1016/j.nuclphysb.2018.04.021

[14] Shen, Y.R. and Bloembergen, N. (1965) Theory of Stimulated Brillouin and Raman Scattering. *Physical Review*, **137**, A1787-A1805. https://doi.org/10.1103/physrev.137.a1787

[15] Holmes, R. and Flusberg, A. (1988) Rotationally Invariant Theory of Stimulated Raman Scattering. *Physical Review A*, **37**, 1588-1596. https://doi.org/10.1103/physreva.37.1588

[16] Evans, N. (1995) Fermion Mass Predictions in a Generalized Extended Technicolor Scenario. *Physical Review D*, **51**, 1377-1385. https://doi.org/10.1103/physrevd.51.1377

[17] Ross, G.G. (1995) Fermion Mass Prediction from Infra-Red Fixed Points. *Physics Letters B*, **364**, 216-226. https://doi.org/10.1016/0370-2693(95)01052-1

[18] Hernandez-Galeana, A. (2011) Predictions for Fermion Masses and Mixing from a Low Energy SU(3) Flavor Symmetry Model with a Light Sterile Neutrino. https://doi.org/10.48550/arXiv.1111.7286

[19] Towe, J. (2013) On Approximating Fermion Masses in Terms of Stationary Super-String States. *Journal of Modern Physics*, **4**, 551-554. https://doi.org/10.4236/jmp.2013.44076

[20] Mongan, T.R. (2024) Standard Model Fermion Masses and Charges from Holographic Analysis. *Journal of Modern Physics*, **15**, 796-803. https://doi.org/10.4236/jmp.2024.156035

[21] Sirunyan, A.M., Tumasyan, A., Adam, W., Bergauer, T., Dragicevic, M., Erö, J., *et al*. (2021) Evidence for Higgs Boson Decay to a Pair of Muons. *Journal of High Energy Physics*, **2021**, Article No. 148. https://doi.org/10.1007/jhep01(2021)148

Appendix: Lagrangian Densities of the Higgs Sector

The well-known Lagrangian density L_H for the time evolution of the Higgs fields is given by [1]-[4]

$$L_H = 0.5\left|\left[\partial_\mu - i/(2\hbar c)\left(g\boldsymbol{t}\cdot\boldsymbol{W}_\mu + g'YB_\mu\right)\right]\begin{bmatrix} H_2 - iH_1 \\ H_4 - iH_3 \end{bmatrix}\right|^2 - V(H). \tag{A1}$$

Here the indices for the Higgs degrees of freedom use the conventions of Taylor [2] for the four real-valued components of the Higgs fields, H_1 to H_4, with the exception of the sign for H_1. In this equation $\boldsymbol{t}$ is a vector of three weak-isospin generators and Y is the weak hypercharge, $\boldsymbol{W}_\mu$ are the three corresponding SU(2) gauge bosons, and B_μ is the familiar U(1) gauge field. The coupling constants g and g' follow the standard definitions, with units of (energy×length)$^{1/2}$. The coupling constant g is equal to $e/\sin(\theta_W)$ where e is the charge of the electron and g' is equal to $e/\cos(\theta_W)$, where θ_W is the Weinberg angle (also known as the weak mixing angle). As is customary, $\hbar$ is Planck's constant divided by 2p, and c is the speed of light. To this Lagrangian density is added a modification to the Higgs potential well that depends on the fermion family:

$$L'_H = L_H + \sigma_f\left|\begin{bmatrix} H_2 - iH_1 \\ H_4 - iH_3 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}(\mu/\lambda)^{1/2}\right|^2, \tag{A2}$$

where σ_f is a family-dependent parameter that will be determined later and could be zero. This modification accounts for the possibility of family-dependent preon bonds. Equations (A1) and (A2) are manifestly invariant under SU(2) transformations before symmetry breaking, and this fact is often used to gauge away components of the Higgs fields [1]-[4] in presentations of the standard model. The potential term $V(H)$ has the form $-\mu/2H^HH + \lambda/4\left(H^HH\right)^2$ in the standard model, where μ and λ are positive real numbers, and "^{H}" denotes Hermitian conjugation. The nominal mass M_H of the Higgs is equal to $(2\mu)^{1/2}\hbar/c$ with the chosen conventions, after dividing the energy by c^2. The nominal vacuum expectation value is equal to $(\mu/\lambda)^{1/2}$ with the above notation.

In a self-consistent treatment of the Higgs fields and Faddeev-Popov ghosts, the above Lagrangian density L_H must be supplemented by a ghost Lagrangian L_{Gh} that includes the coupling of Higgs and ghost particles [2] [5]:

$$\begin{aligned} L_{Gh} = & -\eta^+\Box\omega^- - \eta^-\Box\omega^+ - \eta_Z\Box\omega_Z - \eta_\phi\Box\omega_\phi \\ & + 1/(\hbar c)^2\Big[-\xi M_W^2c^4\left(\eta^+\omega^- + \eta^-\omega^+\right) - \xi M_Z^2c^4\eta_Z\omega_Z \\ & + g(\hbar c)\boldsymbol{\eta}\cdot\left(\partial_\mu\boldsymbol{\omega}\times\boldsymbol{W}_\mu\right) - 0.5\xi gM_Wc^2\left(H_{34}^+ - H_{34}^-\right)\left(\eta^+\omega^- - \eta^-\omega^+\right)/2 \\ & + 0.5\xi gM_Zc^2\eta_Z\left(H_{12}^+\omega^- + H_{12}^-\omega^+\right) \\ & - 0.5\xi gM_Wc^2\left(H_{34}^+ + H_{34}^-\right)\left(\eta^+\omega^- + \eta^-\omega^+\right)/2 \\ & + 0.5\xi\left(g^2 + g'^2\right)^{1/2}M_Zc^2\left(H_{34}^+ + H_{34}^-\right)\eta_Z\omega_Z/2 \\ & - 0.5\xi gM_Zc^2\omega_Z\left(H_{12}^+\eta^- + H_{12}^-\eta^+\right)\Big]. \end{aligned} \tag{A3}$$

M_Z and M_W are the masses of the Z and W bosons, respectively. ξ is the dimensionless gauge factor, a positive real number here. The ghosts η and gauge functions ω of Equation (A3) are related to those of [2] via the following equations:

$$\eta_Z = \cos(\theta_W)\eta_4 - \sin(\theta_W)\eta_3, \tag{A4a}$$

$$\omega_Z = \cos(\theta_W)\omega_4 - \sin(\theta_W)\omega_3, \tag{A4b}$$

$$\eta_\phi = \sin(\theta_W)\eta_4 + \cos(\theta_W)\eta_3, \tag{A4c}$$

$$\omega_\phi = \sin(\theta_W)\omega_4 + \cos(\theta_W)\omega_3, \tag{A4d}$$

$$\eta^\pm = (\eta_1 \pm i\eta_2)/\sqrt{2}, \tag{A4e}$$

$$\omega^\pm = (\omega_1 \pm i\omega_2)/\sqrt{2}, \tag{A4f}$$

$$H_{12}^\pm = (H_2 \mp iH_1)/\sqrt{2}, \tag{A4g}$$

$$H_{34}^\pm = (H_4 \mp iH_3). \tag{A4h}$$

$$H_{12}^\pm = (\mu/\lambda)^{1/2} + \delta H_{12}^\pm \text{ and} \tag{A4i}$$

$$H_{34}^\pm = (\mu/\lambda)^{1/2} + \delta H_{34}^\pm. \tag{A4j}$$

There is no $1/\sqrt{2}$ is in the definition of $H_{34}^\pm$ because it is not a charged state. This choice does not affect the final resulting equations of motion. The equations for the gauge functions are derived in [5]. Near the vacuum expectation value sample momentum-matched equations are given by

$$\begin{aligned}\partial_t^2\omega_0^\pm = -\xi(c/\hbar)^2\Big[&\left(2M_W^2c^2 + 0.5M_W g\delta H_{34}^\mp + |\boldsymbol{p}_{\omega 12}|^2/\xi\right)\omega^\pm \\ &+\left(M_W M_Z c^2 + 0.5M_Z g\,\delta H_{12}^\pm\right)\omega_Z\Big],\end{aligned} \tag{A5a}$$

and

$$\begin{aligned}\partial_t^2\omega_{Z0} &= +\xi(c/\hbar)^2\Big\{\left(M_W M_Z c^2 + 0.5M_Z g\delta H_{12}^-\right)\omega^+ \\ &\quad+\left(M_W M_Z c^2 + 0.5M_Z g\delta H_{12}^+\right)\omega^- \\ &\quad+\left[-|\boldsymbol{p}_{\omega Z}|^2/\xi + 0.5M_Z g\delta H_4/\cos(\theta_W)\right]\omega_Z\Big\} \\ &= +\xi(c/\hbar)^2\Big\{Re\left[\left(M_W M_Z c^2 + 0.5M_Z g\delta H_{12}^-\right)\omega^+\right] \\ &\quad+\left[-|\boldsymbol{p}_{\omega Z}|^2/\xi + 0.5M_Z g\delta H_4/\cos(\theta_W)\right]\omega_Z\Big\}.\end{aligned} \tag{A5b}$$

For the ghost fields η one also finds a similar set of equations. These are given in [5], Equations (13).