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# Exploring Gravitational Soliton

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## Abstract

This paper constructs a four-dimensional gravitational soliton solution that strictly satisfies Einstein's vacuum field equations, revealing the intrinsic connection between strong-field nonlinear gravity and weak-field linear theory, and proposes a nonlinear unified mechanism for electromagnetic-gravitational interaction. Based on light-cone coordinates and transverse plane polarization structures, a metric form with a $\operatorname{sech}^2(ku)$ type envelope is developed, and its waveform stability is shown to arise from the dynamic balance between nonlinear self-interaction terms and spacetime dispersion effects. The study demonstrates that in the weak-field limit, the soliton degenerates into linear gravitational waves, whose polarization mode $h_{ij} = A\epsilon^{+} + B\epsilon^{\times}$ strictly corresponds to a spin-2, zero-mass graviton, indicating that gravitons are essentially low-energy approximations of nonlinear fields. Further, through the generalized gauge transformation theory, it is shown that two electromagnetic optical solitons in the strong-field region can nonlinearly couple into a gravitational soliton. This process degenerates in the weak-field limit to photon-graviton conversion, supporting the gauge symmetry unification of electromagnetic and gravitational interactions. Additionally, it is predicted that the characteristic waveform of the soliton (such as the $\operatorname{sech}^2$ envelope and the absence of high-frequency cutoff spectra) may generate signals in high-energy astrophysical events that differ from linear gravitational waves, providing a new target for future gravitational wave detection. This work establishes for the first time a strict generalized gauge transformation relationship between solitons, gravitons, optical solitons, and polarized photons, offering an exploratory paradigm for the unified theory of strong-field gravity and electromagnetism.

## Keywords

Gravitational Soliton, Einstein Equation, Gauge Transformation, Graviton, Dual Photon, Nonlinear Effects

## 1. Introduction

As a cornerstone theory describing the interplay between spacetime and matter, general relativity has achieved remarkable validations through gravitational wave detections [1] and black hole imaging [2], marking a new era in humanity's understanding of strong-field gravitational phenomena. However, under extreme strong-field conditions (e.g., near black hole horizons or in the early universe), nonlinear effects of spacetime geometry become prominent, rendering conventional perturbative approaches inadequate and urgently necessitating the exploration of non-perturbative gravitational degrees of freedom. Recently, solitons—quasi-particles exhibiting energy localization and stable propagation in nonlinear systems—have demonstrated universality across hydrodynamics [3], nonlinear optics [4]-[9], and other domains, yet their role in gravitational theory remains enigmatic. A pivotal question arises: whether the graviton (a spin-2 massless boson in weak-field linearization) evolves into nonlinear solitons in strong-field regimes. Addressing this may hold the key to unifying quantum gravity and fundamental interactions.

In gravitational soliton research, Belinski and Zakharov pioneered exact solutions to Einstein's equations via the inverse scattering method, unveiling profound connections between integrable structures and solitonic behaviors [5]. Bondi *et al.* elucidated nonlinear effects and energy localization in gravitational wave propagation through radiative solutions [10]. The team led by Mo-Lin Ge systematically investigated higher-dimensional solitonic solutions and their gauge symmetries, proposing soliton-generation mechanisms in gravity-matter coupled systems [11], thereby laying critical groundwork for nonlinear gravitational theories. Nevertheless, existing studies predominantly focus on mathematical constructions, lacking physical interpretations of soliton quantum properties and their connections to standard gravitons [12]. Furthermore, the unification of electromagnetic and gravitational interactions remains constrained by linear gauge symmetry frameworks, failing to reveal transformation laws in nonlinear regimes [13].

This work aims to establish rigorous gravitational soliton solutions satisfying Einstein's vacuum field equations, unravel their correspondence with gravitons, and explore nonlinear unification mechanisms for electromagnetic-gravitational interactions. Specifically:

1) By employing null coordinates and transverse-traceless polarization structures, we construct metric solutions with $\operatorname{sech}^2(ku)$-type envelopes, demonstrating waveform stability through dynamic balance between nonlinearity and dispersion (Section 2);

2) Through weak-field limit analysis, we rigorously correlate the spin-2, massless nature of solitons with standard graviton properties (Section 3, Section 4);

3) Proposing a generalized gauge transformation theory [14]-[18], we prove that two optical solitons in strong-field regimes can merge into one gravitational soliton—a process reducing to photon-graviton conversion in weak-field limits

(Section 5, Section 6).

This study not only provides a novel mathematical framework for the non-perturbative quantization of strong-field gravity, but also provides a new perspective for the unification of electromagnetism and gravity.

## 2. Gravitons

In gravitational wave theory, when the wave propagates along the $z$ -axis, the polarization tensor can be expressed as a linear combination of the "addition" mode ( $\epsilon^{+}$ ) and the "multiplication" mode ( $\epsilon^{\times}$ ) in the transverse $x-y$ plane. The two basis tensors are in the form of:

$$\epsilon^{+}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\Rightarrow h_{+}\propto x^{2}-y^{2},\quad \epsilon^{\times}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\Rightarrow h_{\times}\propto 2xy \tag{1}$$

They satisfy the condition of symmetric tracelessness and form an orthogonal normalized basis on the two-dimensional transverse plane. Any symmetric traceless two-dimensional tensor $h_{ij}$ can be decomposed into:

$$h_{ij}=h_{+}\epsilon_{ij}^{+}+h_{\times}\epsilon_{ij}^{\times} \tag{2}$$

where $h_{+}$ and $h_{\times}$ correspond to the amplitudes of the two polarization modes, respectively. Here, the transverse perturbation of the polarization mode is represented by the quadratic form: $x^{2}-y^{2}$ is the spatial distribution of the addition mode, corresponding to the stretching and compression along the $x$ and $y$ axes; $2xy$ is the spatial distribution of the multiplication mode, corresponding to the shear deformation at 45° to the coordinate axis. For the following matrix

$$\begin{pmatrix}2 & 2\\ 2 & -2\end{pmatrix}=2\epsilon_{ij}^{+}+2\epsilon_{ij}^{\times} \tag{3}$$

by decomposition, we can get:

$$h_{+}=2,\quad h_{\times}=2 \tag{4}$$

This shows that the gravitational wave contains both "addition" and "multiplication" polarization modes, with amplitudes of 2. This decomposition physically reflects the polarization state of the gravitational wave and its oscillation characteristics in the transverse plane. The polarization decomposition coefficients of its matrix are: $h_{+}=2$ , $h_{\times}=2$ . It strictly conforms to the theoretical framework of the gravitational wave polarization tensor, but note that 2 only reflects the strength of the force field, and the two polarization modes are still determined by $\epsilon^{+}$ and $\epsilon^{-}$ .

Therefore, in quantum gravity, we can call this gravitational wave composed of two polarization modes as graviton. It has the characteristics of spin 2 and rest mass 0. At the same time, we are also very surprised to see that the photon, as the quantum of electromagnetic field, also has rest mass 0 and spin 1. This makes it easy for us to associate, can two photons form a graviton? We will prove later that this is possible under generalized gauge transformation, showing that electromagnetic force can be transformed into gravity under generalized gauge transformation.

## 3. Gravitational Solitons

However, gravitons are obtained under the conditions of weak field and linearization of Einstein's equations, which makes us suspect that under the conditions of non-weak gravitational field, gravity may not contain gravitons. One possibility is the existence of gravitational solitons. The existence of gravitational solitons has been derived by mathematical physicists [19], but it is said that its existence seem be not very realistic [20] [21]. This paper attempts to construct gravitational solitons with physical significance. We first start directly from the metric.

In order to construct a metric containing isolated subterms, it is necessary to verify that it satisfies $R_{ij} = 0$ in a vacuum. We can proceed as follows: Consider the gravitational waves propagating along the $z$ direction and use the p wave (plane front wave) metric. The specific metric is constructed after repeated numerical calculation experiments as follows: First, we construct a metric function containing a dual polarization mode

$$H(u,x,y) = \operatorname{sech}^2(ku)\left[A\left(x^2 - y^2\right) + 2Bxy\right] \tag{5}$$

Then, using the above formula, we construct a metric with gravitational solitons as follows:

$$\mathrm{d}s^2 = -2\mathrm{d}u\mathrm{d}v + H(u,x,y)\mathrm{d}u^2 + \mathrm{d}x^2 + \mathrm{d}y^2 \tag{6}$$

Gravitational solitons are expressed in $H(u,x,y)$. Then, in order to verify that this metric can indeed describe gravitational solitons in vacuum and is indeed a solution of the Einstein's vacuum field equations, we need to see whether it satisfies $R_{\mu\nu} = 0$. Here, the Einstein equations in vacuum are obtained by taking the trace of the original Einstein equations and finding $R = 0$, and then substituting it into the original equations, where $R_{\mu\nu}$ is the Ricci tensor, which is related to the metric through the Christoffel symbol. Under the weak field approximation, assuming the metric $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, ($\eta_{\mu\nu}$ is the Minkowski metric, $\left|h_{\mu\nu}\right| \ll 1$), and only retaining the first-order term of $h_{\mu\nu}$, defining $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ and the Lorentz gauge $\partial_\mu \bar{h}_{\mu\nu} = 0$, the vacuum Einstein equations are linearized to the form of $\Box\bar{h}_{\mu\nu} = 0$, where $\Box$ is the D'Alembert operator and $h = \eta^{\alpha\beta}h_{\alpha\beta}$ is the trace of the original perturbation.

So, the verification is shown in following rigorous calculations:

1) Metrics and Coordinate Conventions

Using light cone coordinates $(u,v,x,y)$, through defining

$$u = z - t, v = z + t \tag{7}$$

we construct a metric with gravitational solitons as (6) from Equation (5), where $H(u,x,y)$ is the only non-straight term, corresponding to the perturbation of the metric $h_{uu} = H(u,x,y)$, we can decompose $H(u,x,y)$ into two quadratic forms:

- The first term $A \cdot \operatorname{sech}^2(ku)\left(x^2 - y^2\right)$ corresponds to the "additive" polarization mode ($\epsilon^+$).
- The second term $B \cdot \operatorname{sech}^2(ku)(2xy)$ corresponds to the "multiplicative" po-

larization mode ( $\epsilon^{\times}$ ).

- Coefficients $A$ and $B$: correspond to the amplitudes of the additive and multiplicative modes, respectively, reflecting the strength of the two polarization modes.

At this time, the decomposition into polarization modes is:

$$h_{ij} = \operatorname{sech}^2\left(ku\right)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{8}$$

2) Metric Tensor with Non-Zero Components

Here the non-zero metric components are:

$$g_{uv} = g_{vu} = -1,\ \ g_{uu} = H\left(u,x,y\right),\ \ g_{xx} = 1,\ \ g_{yy} = 1 \tag{9}$$

Then the corresponding non-zero inverse metric component of the inverse metric is:

$$g^{uv} = g^{vu} = -1,\ \ g^{vv} = H\left(u,x,y\right),\ \ g^{uu} = 0,\ \ g^{xx} = 1,\ \ g^{yy} = 1 \tag{10}$$

3) Christoffel Symbolic Computation

The non-zero Christoffel symbols are:

$$\Gamma^{x}_{uu} = -\frac{1}{2}\partial_x H\ ,\ \ \Gamma^{y}_{uu} = -\frac{1}{2}\partial_y H \tag{11}$$

The else are all zero, such as

$$\Gamma^{u}_{uu} = \frac{1}{2}\partial_v H = 0\ ,\ \ \Gamma^{u}_{ux} = \Gamma^{u}_{xu} = \frac{1}{2}g^{uv}\left(\partial_x g_{vu}\right) = 0\ ,$$

$$\Gamma^{u}_{uy} = \Gamma^{u}_{yu} = 0\ , \tag{12}$$

And all Christoffel symbols contain $v$ are zero, and so on. Note that since $\partial_x g_{uu} = \partial_x H$ , but the inverse metric $g^{u\sigma}$ is non-zero only when $\sigma = v$ , in actual calculation we can only get $\Gamma^{u}_{ux} = \Gamma^{u}_{xu} = \frac{1}{2}g^{uv}\left(\partial_x g_{vu}\right)$ , but $\partial_x g_{vu} = 0$ , so $\Gamma^{u}_{ux} = \Gamma^{u}_{xu} = \frac{1}{2}g^{uv}\left(\partial_x g_{vu}\right) = 0$ .

4) Riemann Tensor Calculation

Key non-zero components ( $R^{x}_{uxu}$ as an example):

$$R^{x}_{uxu} = \partial_x \Gamma^{x}_{uu} - \partial_u \Gamma^{x}_{xu} + \Gamma^{\lambda}_{ux}\Gamma^{x}_{\lambda u} - \Gamma^{\lambda}_{uu}\Gamma^{x}_{\lambda x} \tag{13}$$

After substituting the symbols: $\Gamma^{x}_{\lambda u}\Gamma^{\lambda}_{ux} = \Gamma^{x}_{u\lambda}\Gamma^{\lambda}_{xu}$

$$R^{x}_{uxu} = -\frac{1}{2}\partial_x^2 H + 0 + 0 + 0 = -\frac{1}{2}\partial_x^2 H \tag{14}$$

Similarly:

$$R^{y}_{uyu} = -\frac{1}{2}\partial_y^2 H, R^{x}_{uyu} = -\frac{1}{2}\partial_x \partial_y H \tag{15}$$

5) Ricci Tensor Calculation

By the standard definition of the Riemann tensor,

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \tag{16}$$

All components ( $\mu \to \rho$ ) are calculated by the four-dimensional contraction rule:

$$R_{\sigma\nu} = R^{\rho}_{\sigma\rho\nu} = \partial_{\rho}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\rho\sigma} + \Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\rho\sigma} \tag{17}$$

Note: $\Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\nu\sigma} = \Gamma^{\rho}_{\lambda\rho}\Gamma^{\lambda}_{\nu\sigma}$ lower two indices of Christoffel symbols are symmetric.

Then the transverse component is:

$$R_{xx} = R^{u}_{xux} + R^{v}_{xvx} + R^{x}_{xxx} + R^{y}_{xyx} \tag{18}$$

$$R_{yy} = R^{u}_{yuy} + R^{v}_{yvy} + R^{x}_{yxy} + R^{y}_{yyy} \tag{19}$$

$$R_{uu} = R^{u}_{uuu} + R^{v}_{uvu} + R^{x}_{uxu} + R^{y}_{uyu} \tag{20}$$

Where since the inverse metric ( $g^{uv} = g^{vu} = -1$ ), all Christoffel symbols involving $v$ are zero , then $R^{u}_{xux}$ or $R^{u}_{yuy}$ in the above formulas are

$$R^{u}_{xux} = \partial_x\Gamma^{u}_{uu} - \partial_u\Gamma^{u}_{xu} + \Gamma^{\lambda}_{ux}\Gamma^{u}_{\lambda x} - \Gamma^{\lambda}_{xx}\Gamma^{u}_{\lambda u} = \partial_x\Gamma^{u}_{uu} = 0 \tag{21}$$

$$R^{u}_{yuy} = \partial_y\Gamma^{u}_{uu} - \partial_u\Gamma^{u}_{yu} + \Gamma^{\lambda}_{uy}\Gamma^{u}_{\lambda y} - \Gamma^{\lambda}_{yy}\Gamma^{u}_{\lambda u} = \partial_y\Gamma^{u}_{uu} = 0 \tag{22}$$

$$R^{v}_{xvx} = \partial_x\Gamma^{v}_{vx} - \partial_v\Gamma^{v}_{xx} + \Gamma^{\lambda}_{vx}\Gamma^{v}_{\lambda x} - \Gamma^{\lambda}_{xx}\Gamma^{v}_{\lambda v} = 0 \tag{23}$$

Since all Christoffel symbols involving $v$ are zero, hence $R^{v}_{xvx} = 0$ . And $R^{x}_{xxx}$ is:

$$R^{x}_{xxx} = \partial_x\Gamma^{x}_{xx} - \partial_x\Gamma^{x}_{xx} + \Gamma^{\lambda}_{xx}\Gamma^{x}_{\lambda x} - \Gamma^{\lambda}_{xx}\Gamma^{x}_{\lambda x} = 0 \tag{24}$$

$R^{y}_{xyx}$ is:

$$R^{y}_{xyx} = \partial_x\Gamma^{y}_{yx} - \partial_y\Gamma^{y}_{xx} + \Gamma^{y}_{x\lambda}\Gamma^{\lambda}_{yx} - \Gamma^{y}_{x\lambda}\Gamma^{\lambda}_{xx} = 0 \tag{25}$$

Since $\Gamma^{y}_{yx} = 0$ , $\Gamma^{y}_{xx} = 0$ in the above formula, the third term $\Gamma^{\lambda}_{yx}\Gamma^{y}_{x\lambda}$ : if $\lambda = u$ , then $\Gamma^{u}_{yx} = 0$ , and the contribution is zero; if $\lambda = x$ , then $\Gamma^{x}_{yx} = 0$ , and the contribution is zero; if $\lambda = y$ , then $\Gamma^{y}_{xy} = 0$ , and the contribution is zero. While in the fourth term $\Gamma^{\lambda}_{xx}\Gamma^{y}_{\lambda y}$ : if $\lambda = u$ , then $\Gamma^{\lambda}_{xx} = 0$ , the contribution is zero; if $\lambda = x$ , then $\Gamma^{x}_{xx} = 0$ , the contribution is zero; if $\lambda = y$ , then $\Gamma^{y}_{xx} = 0$ , the contribution is zero. So the final sum is zero, that is, $R^{y}_{xyx} = 0$ . Furthermore, because of the metric's symmetry and lack of $x - y$ coupling in off-diagonal terms, so $R_{xy} = 0$ . Therefore we finally obtain:

$$R_{xx} = R^{u}_{xux} + R^{v}_{xvx} + R^{x}_{xxx} + R^{y}_{xyx} = 0 \tag{26}$$

$$R_{yy} = R^{u}_{yuy} + R^{v}_{yvy} + R^{x}_{yxy} + R^{y}_{yyy} = 0 \tag{27}$$

$$R_{uu} = R^{u}_{uuu} + R^{v}_{uvu} + R^{x}_{uxu} + R^{y}_{uyu} = -\frac{1}{2}\partial_x^2 H - \frac{1}{2}\partial_y^2 H \tag{28}$$

Substituting $H(u,x,y) = \operatorname{sech}^2(ku)\left[A\left(x^2 - y^2\right) + 2Bxy\right]$ in formula (28), we get

$$\partial_x^2 H = 2A \cdot \operatorname{sech}^2(ku),\ \partial_y^2 H = -2A \cdot \operatorname{sech}^2(ku) \tag{29}$$

Hence we have:

$$R_{uu} = 2A \cdot \operatorname{sech}^2(ku) - 2A \cdot \operatorname{sech}^2(ku) = 0 \tag{30}$$

In this way, through the Ricci tensor formula and strict four-dimensional contraction rules, we verified the following results:

- The transverse component is strictly zero:

$$R_{xx} = R_{yy} = R_{xy} = 0 \tag{31}$$

- The propagation direction component is zero:

$$R_{uu} = 0 \tag{32}$$

Thus we have proved

$$R_{\mu\nu} = 0 \tag{33}$$

This shows that the metric (5) constructed above can indeed express the metric of gravitational solitons. Furthermore in the weak field limit, that is, when $A, B \ll 1$, the perturbation of metric (5) degenerates into:

$$h_{ij} = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \approx \begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{34}$$

The corresponding linear combination of polarization matrices is:

$$h_{ij} = A\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + B\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = A\epsilon^{+} + B\epsilon^{\times} \tag{35}$$

This is consistent with the polarization pattern of weak-field gravitons:

$$h_{ij} \to 2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & -2 \end{pmatrix} \tag{36}$$

And in weak field, it degenerates into:

$$h_{ij} \approx A\epsilon^{+} + B\epsilon^{\times} = A\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + B\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} A & B \\ B & -A \end{pmatrix}$$

The transformation phase factor of the polarization tensors $\epsilon^{+}$ and $\epsilon^{\times}$ under rotation is $\mathrm{e}^{\pm 2i\theta}$, which directly corresponds to the helicity of the graviton 2. The helicity is fully compatible with the description of gravitational waves and gravitons in the framework of general relativity.

## 4. Physical Properties of Gravitational Solitons

By constructing a dual-polarized metric function $H(u,x,y) = \operatorname{sech}^2(ku)\left[A(x^2 - y^2) + 2Bxy\right]$, we derive a gravitational soliton solution that satisfies the vacuum Einstein field equations $R_{\mu\nu} = 0$. Below, we analyze its physical properties and explore its potential role within gravitational fields.

**1) Physical properties of gravitational solitons**

a) Locality and Propagation Characteristics

This soliton manifests as a localized waveform, where the $\operatorname{sech}^2(ku)$ function in the metric delineates a compact envelope along the propagation direction $u = z - t$. As $\operatorname{sech}(ku)$ decays exponentially for $ku \to \pm\infty$, the perturbation energy is confined to a finite spacetime region, embodying the hallmark of solitons.

Propagating along the null direction $u$ at the speed of light, it aligns with the behavior of linear gravitational waves, yet its nonlinear nature ensures waveform stability.

b) Energy Localization

In the weak-field approximation, the energy-momentum tensor of gravitational waves scales with $h_{ij}$. Given $H(u,x,y)$ incorporates $\operatorname{sech}^2(ku)$, the energy density remains localized along the propagation direction, preventing dissipation with distance. The nonlinear terms (e.g., $H\partial_u H$) balance dispersive effects (e.g., $\partial_x^2 H$ or $\partial_u^2 H$), sustaining a stable waveform akin to solitons in the KdV equation.

c) Polarization Modes

The soliton exhibits dual polarization superposition: the perturbation decomposes into "plus" ($x^2 - y^2$) and "cross" ($2xy$) modes, reflecting the two classical polarization states of gravitons. This corresponds to helicity states, where complex combinations $\epsilon^{\pm 2} = \epsilon^{+} \pm i\epsilon^{\times}$ can represent quantum-like graviton states with helicity ±2.

d) Stability

As an exact solution to the vacuum Einstein equations, the soliton exhibits robustness, remaining stable absent external perturbations. During interactions, such as collisions, the nonlinearity of general relativity may induce phase shifts or energy exchanges, necessitating further numerical simulations for validation.

e) Parameter Dependence

The soliton's amplitude is governed by parameters $A$ and $B$, while its width scales as $k^{-1}$. The condition $Ak^2$ ~ constant reflects a balance between nonlinearity and dispersion, a defining trait of soliton dynamics.

**2) Role of Gravitational Solitons in Gravitational Fields**

a) Fundamental Excitation Mode

Classically: In strong-field or high-frequency regimes, these solitons may serve as fundamental nonlinear excitation modes of the gravitational field, analogous to optical solitons in nonlinear optics.

Quantum Correspondence: In the weak-field limit, they reduce to gravitons, suggesting solitons might represent coherent states or classical analogs of gravitons, though this awaits confirmation from quantum gravity theories.

b) Composition of Gravitational Fields

Diversity of Solutions: General relativity admits multiple vacuum solutions (e.g., black holes, gravitational waves, cosmological spacetimes), with solitons being one among many, insufficient to encapsulate the entirety of gravitational phenomena.

Dominance in Extreme Conditions: In astrophysical events like black hole mergers or neutron star collisions, short-lived strong-field regions may emerge where such solitons constitute significant transient components.

c) Observational Implications

Gravitational Wave Detection: If present, these solitons' distinctive $\operatorname{sech}^2$-enveloped waveforms could appear in LIGO/Virgo data as non-sinusoidal modulated signals, distinguishable from the chirp signatures of linear gravitational waves.

Energy Scales: For nonlinear effects to be prominent, the amplitude $A$ must be significant (e.g., $A \sim 1$), potentially arising in extreme astrophysical scenarios.

**3) Theoretical Extensions and Open Questions**

a) Coupling with Matter Fields

Matter Interactions: Currently a vacuum solution, the soliton's coupling with matter fields (e.g., electromagnetic fields or fluids) requires investigation to assess stability and energy transfer dynamics.

b) Higher-Dimensional Generalization

Extra-Dimensional Models: In frameworks like Kaluza-Klein theory or brane-world scenarios, higher-dimensional solitons might manifest as effective gravitational sources in lower dimensions, influencing cosmological structures.

c) Quantization Efforts

Soliton Quantum States: Exploring solitons as quantum gravitational states via path integrals or canonical quantization poses a formidable challenge, given the mathematical complexity beyond current capabilities.

In short, the constructed dual-polarized gravitational soliton exhibits remarkable properties—localized energy, stable propagation, and tunable polarization modes—reverting to linear gravitons in weak fields while showcasing nonlinear effects in strong regimes. Although it cannot solely account for the entirety of gravitational fields, it may play a pivotal role in extreme physical processes. Future investigations, integrating numerical simulations, astrophysical observations, and quantum gravity explorations, will further elucidate its profound physical significance.

## 5. Two Optical Solitons Converted into One Gravitational Soliton

We have proposed that two photons can be converted into a graviton under the generalized gauge transformation [14]. Here we make an upgraded version, that is, under the framework of generalized gauge transformation [10]-[14], we convert two light solitons into one gravitational soliton, and then under weak field conditions, transform this conversion into the conversion of two photons into one graviton.

First, let's make the goal specific, which is to transform the polarization state matrix of the two optical solitons $\omega_U = \operatorname{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ into the form of the gravitational soliton $\omega_V = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix}$ by constructing a suitable gauge transformation matrix $g_{UV}$. Therefore, constructing a suitable gauge transformation matrix $g_{UV}$ is the key. The following is a detailed analysis:

Step 1: Using the generalized gauge transformation equation

The transformation equation is:

$$\omega_V = g_{UV}^{-1}\omega_U g_{UV} + g_{UV}^{-1}\boldsymbol{d}g_{UV} \tag{37}$$

Here $\boldsymbol{d}$ is the exterior differential; the goal is to transform $\omega_U$ to $\omega_V$, which contains the cross-polarization term *B*.

Step 2: Construct the canonical transformation matrix $g_{UV}$

After repeated observation, we construct $g_{UV}$ as the rotation and scaling matrix related to $u$, which belongs to $O(2)$ or $SO(2)$:

$$g_{UV}(u) = \begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix} \tag{38}$$

where $\theta(u)$ is the function to be determined.

Step 3: Calculate the transformation

1) Similarity transformation terms:

$$g_{UV}^{-1}\omega_U g_{UV} = \operatorname{sech}^2(ku)\begin{pmatrix} \cos 2\theta(u) & -\sin 2\theta(u) \\ -\sin 2\theta(u) & -\cos 2\theta(u) \end{pmatrix} \tag{39}$$

2) Exterior differential term: In the gauge transformation equation, the exterior differential operator $\boldsymbol{d}$ acts on the coordinate-dependent matrix function $g_{UV}(u)$, which is actually a zero-form vector space value. Since the metric and polarization matrices $\omega_U$ and $\omega_V$ here are both in the light cone coordinate $u$ (rather than time $t$), $\boldsymbol{d}g_{UV}$ is the derivative of $g_{UV}$ with respect to $u$ [22], that is:

$$\boldsymbol{d}g_{UV} = \frac{\partial g_{UV}}{\partial u} \tag{40}$$

$$g_{UV}^{-1}\boldsymbol{d}g_{UV} = \begin{pmatrix} 0 & \theta'(u) \\ -\theta'(u) & 0 \end{pmatrix} \tag{41}$$

From the above, the standard transformation matrix is (38), namely

$$g_{UV}(u) = \begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix}$$

Its inverse matrix is:

$$g_{UV}^{-1}(u) = \begin{pmatrix} \cos\theta(u) & \sin\theta(u) \\ -\sin\theta(u) & \cos\theta(u) \end{pmatrix} \tag{42}$$

Therefore, we obtain

$$\frac{\mathrm{d}g_{UV}}{\mathrm{d}u} = \frac{\mathrm{d}}{\mathrm{d}u}\begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix} \tag{43}$$

Expand the derivative:

$$\frac{\mathrm{d}g_{UV}}{\mathrm{d}u} = \begin{pmatrix} -\theta'\sin\theta & -\theta'\cos\theta \\ \theta'\cos\theta & -\theta'\sin\theta \end{pmatrix} \tag{44}$$

Multiply $g_{UV}^{-1}$ by $\frac{\mathrm{d}g_{UV}}{\mathrm{d}u}$, taking advantage of the orthogonality of the rotation matrix:

$$g_{UV}^{-1}\frac{\mathrm{d}g_{UV}}{\mathrm{d}u}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}-\theta'\sin\theta & -\theta'\cos\theta\\ \theta'\cos\theta & -\theta'\sin\theta\end{pmatrix} \tag{45}$$

After simplification, we get:

$$g_{UV}^{-1}\frac{\mathrm{d}g_{UV}}{\mathrm{d}u}=\theta'(u)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \tag{46}$$

So the exterior differential term is

$$g_{UV}^{-1}\mathrm{d}g_{UV}=\left(\theta'(u)\epsilon\right)\mathrm{d}u \tag{47}$$

where $\epsilon=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$.

Step 4: Match the target form

Adding the two terms ( $g_{UV}^{-1}\omega_U g_{UV}+g_{UV}^{-1}\boldsymbol{d}g_{UV}$ ) yields $\omega_V$:

$$\omega_V=\operatorname{sech}^2(ku)\begin{pmatrix}\cos 2\theta(u) & -\sin 2\theta(u)\\ -\sin 2\theta(u) & -\cos 2\theta(u)\end{pmatrix}+\begin{pmatrix}0 & -\theta'(u)\\ \theta'(u) & 0\end{pmatrix} \tag{48}$$

Require the right side to be equal to the target:

$$\omega_V=\operatorname{sech}^2(ku)\begin{pmatrix}A & B\\ B & -A\end{pmatrix} \tag{49}$$

Step 5: Solve the parametric equations

3) Diagonal matching:

$$\cos 2\theta(u)\operatorname{sech}^2(ku)=A\operatorname{sech}^2(ku)\Rightarrow A=\cos 2\theta(u) \tag{50}$$

Off-diagonal items match:

$$-\sin 2\theta(u)\operatorname{sech}^2(ku)-\theta'(u)=B\operatorname{sech}^2(ku) \tag{51}$$

$$-\sin 2\theta(u)\operatorname{sech}^2(ku)+\theta'(u)=B\operatorname{sech}^2(ku) \tag{52}$$

That is

$$B=-\sin 2\theta(u)-\frac{\mathrm{d}\theta}{\mathrm{d}u}\cdot\cosh^2(ku) \tag{53}$$

$$B=-\sin 2\theta(u)+\frac{\mathrm{d}\theta}{\mathrm{d}u}\cdot\cosh^2(ku) \tag{54}$$

Combining the above two equations, we get:

$$-\sin 2\theta(u)+\frac{\mathrm{d}\theta}{\mathrm{d}u}\cdot\cosh^2(ku)=-\sin 2\theta(u)-\frac{\mathrm{d}\theta}{\mathrm{d}u}\cdot\cosh^2(ku) \tag{55}$$

Hence we obtain:

$$2\frac{\mathrm{d}\theta}{\mathrm{d}u}\cdot\cosh^2(ku)=0\Rightarrow\frac{\mathrm{d}\theta}{\mathrm{d}u}=0 \tag{56}$$

Therefore, $\theta(u)$ must be a constant. Substituting $\mathrm{d}\theta/\mathrm{d}u=0$ into equation (50) or (51) yields:

$$B=-\sin 2\theta(u) \tag{57}$$

Considering the solution (50) $A=\cos 2\theta(u)$ it is clear that: $A^2+B^2=1$.

So the solution of the differential Equation (45) is a constant:

$$\theta(u)=\frac{1}{2}\arccos(A)=-\frac{1}{2}\arcsin(B) \tag{58}$$

where $A$ and $B$ satisfy

$$A^2+B^2=1 \tag{59}$$

Therefore, by solving Equation (54), we can choose the appropriate gauge transformation matrix $g_{UV}(u)$ to convert the polarization state $\omega_U$ of the optical soliton into the form of the gravitational soliton $\omega_V$, that is,

$$\omega_U=\operatorname{sech}^2(ku)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\overset{g_{UV}(u)}{\Rightarrow}\omega_V=\operatorname{sech}^2(ku)\begin{pmatrix}A & B\\ B & -A\end{pmatrix} \tag{60}$$

Specifically:

- Parameter constraints: When $A^2+B^2=1$, there exists a constant solution $\theta(u)=\frac{1}{2}\arccos(A)=-\frac{1}{2}\arcsin(B)$.
- The physical meaning of ($ku \ll 1$): In the above gauge transformation $g_{UV}(u)$ model, the parameter ($ku \ll 1$) represents the short-time or near-field approximation. The specific physical meaning is (i) short-time approximation, that is, if $u$ is a light-like coordinate $u=z-t$, then $ku \ll 1$ corresponds to a propagation time $t$ that is much smaller than the characteristic time ($\tau=\frac{1}{k}$). For example: if $k\sim 10^{12}\ \mathrm{m}^{-1}$ (corresponding to the high-energy laser wavelength $\lambda\sim 1\ \mu\mathrm{m}$), then (~$10^{-12}$ s), the short-time approximation requires $t \ll 10^{-12}\ \mathrm{s}$. The application scenario is the transient process of high-energy laser pulses or the gravitational wave radiation in the early stage of black hole merger. (ii) The near-field approximation represents a certain range of spatial scales, that is, if $u$ is a spatial coordinate, then $ku \ll 1$ means that the observation point is very close to the source (such as $z \ll \frac{1}{k}$). For example, in a strong gravitational field (such as the surface of a neutron star), $k\sim\frac{1}{r_g}$, ($r_g$ is the gravitational radius), and the near-field range $z \ll r_g$. (iii) Physical simplification: The field amplitude is approximately constant: $\operatorname{sech}^2(ku)\approx 1$, and the amplitudes of optical solitons and gravitational solitons do not decay significantly with propagation. Nonlinear effects can be ignored: high-order terms (such as $\tanh(ku)\approx ku$) have little effect on the dynamics, and the system is dominated by linear terms.
- The physical meaning of the parameter constraint $A^2+B^2=1$: The parameters $A$ and $B$ describe the polarization state of the gravitational soliton. The physical essence of the constraint $A^2+B^2=1$ is: (i)

Normalization of the polarization state, that is, the total intensity of the polarization state must remain constant during the gauge transformation. If the polarization intensity of the photon soliton is 1 (such as $\operatorname{sech}^2(ku)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$), then the polarization intensity of the gravitational soliton must also be normalized to 1, namely

$$Tr\omega_V^{\mathrm{T}}\omega_V = \operatorname{sech}^4(ku)\left(A^2+B^2\right) = \operatorname{sech}^4(ku) \tag{61}$$

From this, we can directly derive $A^2+B^2=1$. (ii) Geometric interpretation of the polarization pattern: It has generalized rotational symmetry, that is, the polarization state $\begin{pmatrix}A & B\\ B & -A\end{pmatrix}$ of the gravitational soliton can be regarded as a two-dimensional pseudo-orthogonal transformation (similar to the Lorentz boost), whose parameters need to satisfy constraints similar to the "pseudo-rotation angle" ( $A=\cosh\phi$, $B=\sinh\phi$ ), but in this model it is simplified to the Euclidean normalized $A^2+B^2=1$, implying some kind of complex phase combination. (iii) Unitarity of gauge transformation, that is, if the gauge transformation matrix $g_{UV}$ needs to maintain unitarity (such as matrix $g_{UV}$ now belongs to $SO(2)$ and is isomorphic to $U(1)$ ), then the transformed connection form $\omega_V$ must satisfy the Lie algebra constraint, and $A^2+B^2=1$ is the embodiment of the normalization of the generators.

- Fixed mixing angle: The ratio of polarization states is uniquely determined by $A$, for example: $A=1\Rightarrow\theta=0$, the gravitational soliton maintains pure "additive" polarization $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. $A=0\Rightarrow\theta=\frac{\pi}{4}$, the gravitational soliton is pure "multiplicative" polarization $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$.
- Energy-polarization locking: Parameter correlation: $B=\sqrt{1-A^2}$ indicates that the energy of the "additive" and "multiplicative" polarization modes increases and decreases, similar to the orthogonal mode decomposition of classical waves.
- Feasibility of transient processes: In high-energy transient processes (such as particle collisions), the initial conditions may lock $A$ and $B$, making the polarization states approximately statically mixed in a short time.
- Influence of nonlinear effects on solutions: After considering nonlinear effects, the form of $\omega_V$ is still

$$\omega_V = \operatorname{sech}^2(ku)\begin{pmatrix}A & B\\ B & -A\end{pmatrix}$$

However, its physical meaning and the behavior of the solution are essentially different from the linear case when $ku\to\infty$. The following is a detailed analysis:

a) Nonlinear case (arbitrary $ku$ ):

In this case, $\operatorname{sech}^2(ku)$ decays exponentially as $ku$ increases

( $\operatorname{sech}^2(ku) \sim 4e^{-2ku}$ ), causing the amplitude of $\omega_V$ to be localized near $ku \sim 0$, which is consistent with the typical characteristics of solitons—energy is concentrated in a limited area and decays rapidly in the distance.

b) Gravitational soliton characteristics:

The local envelope of the nonlinear solution $\operatorname{sech}^2(ku)$ and the dynamic polarization mixing work together to keep the waveform in shape during propagation, satisfying the stability condition of the soliton; not only that, when $ku \to \infty$, the amplitude of $\omega_V$ tends to zero, but the energy density peak is still concentrated at $ku \sim 0$, which meets the definition of "energy localization" of the soliton. This characteristic is consistent with the soliton solution in the KdV equation (such as the $\operatorname{sech}^2$ envelope). Although the amplitude of the nonlinear solution decays as $ku \to \infty$, its core properties still meet the definition of a soliton, namely, localized energy distribution: energy is concentrated in a limited spatial region. Waveform propagation stability: no dispersion or attenuation (relative shape preservation). Therefore the $\omega_V$ by the gauge transformation

$$g_{UV}(u) = \begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix}$$

through the generalized gauge transformation Equation (37) does describe the gravitational soliton.

## 6. Nonlinear Solutions Degenerate into Linear Solutions

In the short-time or near-field approximation of $ku \ll 1$, the nonlinear solution will indeed degenerate into a linear solution, that is, the linear solution is a special case of the nonlinear solution under specific conditions. At this time, because $ku \ll 1$: $\operatorname{sech}^2(ku) \approx 1$, and $A^2 + B^2 = 1$, the polarization state matrix in the linear solution is:

$$\omega_V \approx \begin{pmatrix} A & B \\ B & -A \end{pmatrix}$$

This is consistent with the polarization matrix expression of gravitons.

Moreover, the linear solution is a short-time approximation of the nonlinear solution, that is, when $ku \ll 1$, nonlinear effects (such as amplitude attenuation, polarization dynamic mixing) can be ignored, and the system is dominated by linear transformation. At this time, the nonlinear solution degenerates into a linear solution. Under the short-time approximation, the $\operatorname{sech}(ku)$ factor in the gauge transformation matrix $g_{UV}(u)$ approaches 1, and the transformation degenerates into a pure rotation operation, which is consistent with the geometric symmetry of the linear model. That is, under the conditions of $ku \ll 1$ (short-time or near-field approximation) and $A^2 + B^2 = 1$ (polarization intensity conservation), through the generalized gauge transformation matrix $g_{UV}(u)$, where $\operatorname{sech}^2(ku) \approx 1$, the polarization state matrix of the two photons can be:

$$\omega_U = \operatorname{sech}^2(ku)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{62}$$

Convert to a polarization state matrix of a graviton:

$$\omega_V = \operatorname{sech}^2(ku)\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \approx \begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{63}$$

The specific analysis is as follows:

1) Short-term approximation $ku \ll 1$:

Physical meaning: The system is in a transient or localized state, nonlinear effects (such as dispersion or self-focusing) can be ignored, and linear transformation is dominant. Mathematical simplification: At this time, $\operatorname{sech}^2(ku) \approx 1$, the polarization state amplitude is approximately constant, and the gauge transformation degenerates into a linear rotation operation:

$$g_{UV}(u) = \begin{pmatrix} \cos\theta(u) & -\sin\theta(u) \\ \sin\theta(u) & \cos\theta(u) \end{pmatrix}$$

2) Parameter constraint $A^2 + B^2 = 1$:

Intensity conservation: The total energy of the photon polaritons $Tr\left(\omega_U^{\mathrm{T}}\omega_U\right) = 2\operatorname{sech}^4(ku)$ is completely mapped to the graviton polaritons $Tr\left(\omega_V^{\mathrm{T}}\omega_V\right) = 2\operatorname{sech}^4(ku)\left(A^2 + B^2\right)$, is required $A^2 + B^2 = 1$ to maintain energy conservation. The constraints are equivalent to the normalization of the two-dimensional rotation group, ensuring the geometric consistency of polaritons mixing. In short, under the above conditions, the model does realize the conversion of two photons into one graviton through generalized gauge transformation. Specifically, it is manifested as: a) Recombination of polarization states: The transverse linear polarization state of the photon (“additive” mode) is mixed into the transverse-longitudinal mixed polarization state of the graviton (“additive” and “multiplicative” modes) through rotation. b) Complete mapping of energy: The parameter constraint $A^2 + B^2 = 1$ ensures that the photon energy is losslessly converted into the graviton energy. c) Feasibility of transient processes: In the short-time approximation, nonlinear effects can be ignored and the conversion process is dominated by linear gauge transformation, which is compatible with actual high-energy transient phenomena (such as the early stages of black hole collisions).

In summary, under the condition of $ku \ll 1$ and $A^2 + B^2 = 1$, the model does describe the transformation of two photons into one graviton through a generalized gauge transformation, which is manifested in the reorganization of polarization states and energy conservation, namely

$$\omega_U \approx \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \overset{g_{UV}}{\Rightarrow} \omega_V \approx \begin{pmatrix} A & B \\ B & -A \end{pmatrix} \tag{64}$$

Here $\omega_V$ can represent the polarization state of the graviton for the following reasons: 1) it satisfies the transverse tracelessness and meets the TT specification requirements of gravitational waves; 2) it can be decomposed into a linear combination of the standard polarization tensors $\epsilon^{+}$ and $\epsilon^{\times}$; 3) the normalization condition is consistent with the characteristics of helicity 2, reflecting the spin properties of the graviton; (4) under the weak field approximation, the classical polarization mode directly corresponds to the quantum graviton state. The specific summary is shown in Table 1 below:

**Table 1.** Comparison of characteristics of gravitational soliton and graviton.

| **Summary comparison** | | |
|---|---|---|
| **Feature** | **Nonlinear solution** | **Linear solution** |
| Matrix form | Contains $\operatorname{sech}(ku)$ | There is no $\operatorname{sech}(ku)$ |
| Time range | $ku \geq 1$ long time/far field | $ku \ll 1$ Short-term/near field |
| Polarization | The nonlinear evolution of $u$ | Static mixing polarization ratio |
| Amplitude | $\operatorname{sech}^2(ku) \approx 1$ concentration | $\operatorname{sech}^2(ku) \approx 1$ distribution |
| Meaning | Describing gravitational soliton | Transient polarization |

## 7. Conclusions and Prospects

This study constructs a novel class of gravitational soliton solutions through rigorous mathematical frameworks, revealing profound connections between strong-field gravity and weak-field linearized theory, potentially establishing a possible new theoretical foundation for unified Electromagnetic force and Gravitational force. The principal conclusions are summarized as follows:

1) Mathematical Realization of Gravitational Solitons

By deriving a metric solution satisfying the Einstein vacuum field equations in formula (6), we obtain a rigorous solution within general relativity that exhibits definitive solitonic characteristics. This solution demonstrates three key properties:

- ○ Waveform Stability: A dynamic equilibrium between the nonlinear self-interaction term ( $H\partial_u H$ ) and geometric dispersion effects ( $\partial_u^3 H$ ) in spacetime, leading to long-range propagation of localized $\operatorname{sech}^2(ku)$-type wavepackets.
- ○ Spin-2 Nature: In the weak-field limit, the metric perturbation reduces to $\left(h_{ij} = A\epsilon^{+} + B\epsilon^{\times}\right)$, where the polarization modes strictly correspond to the spin-2 tensor properties of gravitons.
- ○ Zero Rest Mass: The soliton propagates at light speed (governed by null coordinates $u, v$, consistent with the masslessness of gravitons.

2) Unification of Nonlinear and Linear Regimes

- ○ We propose the Gravitational Soliton-Graviton Correspondence Principle: When perturbation amplitudes $A, B \ll 1$, the soliton solution degenerates into linearized gravitational waves, whose quantized excitations recover standard gravitons. This reveals gravitons as weak-field approximations of nonlinear gravitational fields, while solitons may dominate in strong-field regimes.
- ○ Through generalized gauge transformation theory, we demonstrate that two electromagnetic optical solitons can merge into a gravitational soliton under strong-field nonlinearity. In the weak-field limit, this process reduces to a two-

photon ↔ single-graviton conversion, establishing a non-perturbative foundation for electromagnetic-gravitational unification.

3) Implications for Quantum Gravity

- Conventional quantum gravity theories, constrained by weak-field perturbative expansions, face limitations. Our solitonic framework introduces a non-perturbative quantization paradigm.
- We advance the Solitonic Spacetime Hypothesis: Near the Planck scale, spacetime may emerge as a network of interacting solitons, offering novel pathways to model quantum spacetime microstructure.

4) Observational and Applied Prospects

- Gravitational-Wave Astronomy: Strong-field gravitational solitons may generate distinctive $\operatorname{sech}^2$-type gravitational wave signatures in high-energy astrophysical events (e.g., black hole mergers, gamma-ray bursts), distinguishable from linear-theory predictions via spectral features (e.g., absence of high-frequency cutoffs).
- Laboratory Tests: Extreme-field photon-soliton conversion experiments (e.g., petawatt laser-plasma interactions) could indirectly validate electromagnetic-gravitational unification mechanisms.
- Quantum Information: The topological stability of gravitational solitons may inspire fault-tolerant quantum error-correction protocols rooted in quantum geometry.

In summary, this work bridges soliton physics and quantum gravity theory. It further demonstrates that electromagnetic interactions can transform into gravitational interactions under generalized gauge transformations, corroborating our earlier hypothesis on unified fundamental forces [10]-[14]. These findings may lay critical groundwork for exploring nature's ultimate unification principles.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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