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Transforming Electromagnetic Tensor to Weyl Tensor for Curvature Drive

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Abstract

Based on the theory of generalized gauge transformation unifying the four fundamental interactions, this paper explores the theoretical foundation of the conversion from the electromagnetic tensor to the Weyl tensor and its application in curvature-driven propulsion. Starting from the gauge similarity transformation mechanism between the electromagnetic field and spacetime geometry, we derive the mathematical form of the transformation from the electromagnetic tensor to the Weyl tensor, and analyze its practical application in dynamic curvature-engine-driven spacecraft, wormhole exploration, and flying saucer-like curvature engines. By comparing the energy requirements of traditional negative energy models with electromagnetic drive models, this paper highlights the unique advantages of electromagnetic propulsion technology in superluminal propulsion and interstellar travel. So in the paper, a novel theoretical framework has been constructed, through generalized gauge transformation, we found a clear quantitative relationship between the electromagnetic tensor and the Weyl tensor, through this formula, the electromagnetic tensor can directly control the Weyl tensor, and then control the curvature. This enables manipulation of the curvature in front of the spacecraft, producing propulsion similar to that of the Alcubierre curvature engine, but without the need for negative energy or exotic matter. The calculations show that for a spacecraft with a volume of 10^3 m^3, reaching twice the speed of light requires a total energy of 1.8×10^{14} J and a magnetic field strength of 680 T. For a flying saucer-like curvature engine, the required magnetic field strength is 2.15×10^3 T, with an energy density of 1.8×10^{12} J/m^3 and a total energy requirement of 7.36×10^{15} J. Although these demands exceed current laboratory capabilities, they represent a significant advancement compared to negative energy technologies by bypassing the challenges associated with exotic matter. This significantly increases the feasibility of interstellar travel, suggesting that humanity's entry into the era of interstellar exploration could potentially come sooner than expected.

Keywords

Controlling Curvature, Warp Drive Ship, Electromagnetic Tensor and Weyl Tensor

1. Introduction

In recent years, with the deepening interdisciplinary research between general relativity and electromagnetism, the technology of controlling spacetime curvature using electromagnetic fields has gradually become a frontier topic in theoretical physics and aerospace engineering. The core goal of this research is to explore the profound connection between electromagnetic fields and spacetime geometry, particularly how electromagnetic field energy can directly drive spacetime curvature, thereby providing theoretical support for novel propulsion systems such as curvature-drive engines and wormhole construction. This research direction not only breaks the traditional reliance on negative energy or exotic matter in curvature-drive theories but also opens up new possibilities for interstellar travel.

Traditional curvature-drive theories, such as the one proposed by Alcubierre (1994), initially introduced the concept of a theoretical model for superluminal travel through spacetime curvature. However, this model relies on the existence of negative energy or exotic matter, which has not been experimentally verified, and its theoretical feasibility is severely restricted by energy conditions [1]. In response to this challenge, more recent research has explored new approaches, such as directly using electromagnetic field energy to drive spacetime curvature, thus avoiding the dependence on negative energy. The core idea of this approach is the coupling mechanism between the electromagnetic field and spacetime curvature, where the energy-momentum distribution of the electromagnetic field induces local spacetime geometric changes [2]. White and Davis (2021) further investigated the feasibility of warp drives, suggesting the possibility of reducing energy requirements by optimizing spacetime geometry [3].

In the study of the coupling between electromagnetic fields and spacetime curvature, Thorne and Blandford (2017) systematically described the interaction between the electromagnetic field and the gravitational field, providing a theoretical framework for the conversion of the electromagnetic tensor to the Weyl tensor [4]. Misner, Thorne, and Wheeler (1973) in their classic work *Gravitation* discussed in detail the coupling between spacetime geometry and matter fields, laying an important foundation for subsequent research [5]. Additionally, Mashhoon (2008) and Hehl and Obukhov (2003) explored the coupling mechanisms between the electromagnetic field and the gravitational field from the perspectives of nonlocal theories and classical electrodynamics, providing new insights for electromagnetic-driven curvature technologies [6] [7].

In the field of wormhole theory and construction, Morris and Thorne (1988) first proposed a mathematical model of traversable wormholes and pointed out that maintaining wormhole stability requires negative energy or exotic matter [8]. Visser (1995) further studied the geometric structure and energy conditions of wormholes, suggesting the possibility of reducing energy requirements by optimizing wormhole geometry [9]. Recently, researchers have used numerical simulations to verify the potential of rotating electromagnetic fields in wormhole exploration, offering a new theoretical framework for interstellar travel [10].

Based on the generalized gauge transformation theory that unifies the four fundamental interactions proposed by the author [11]-[15], this paper systematically explores the theoretical basis of the conversion of the electromagnetic tensor to the Weyl tensor and its application in curvature-driven propulsion. Starting from the coupling mechanism between the electromagnetic field and spacetime geometry, we derive the mathematical form of the conversion from the electromagnetic tensor to the Weyl tensor and analyze its practical applications in dynamic curvature engine-driven spacecraft, wormhole exploration, and flying saucer-like curvature engines. By comparing the energy demands of traditional negative energy models with those of electromagnetic drive models, this paper aims to highlight the unique advantages of electromagnetic drive technology in superluminal propulsion and interstellar travel. Through this research, we hope to provide a systematic theoretical framework for electromagnetic-driven curvature technology and lay the theoretical foundation for its application in future aerospace engineering and interstellar exploration.

2. Gauge Similarity Transformation from Electromagnetic Tensor to Weyl Tensor

The electromagnetic tensor $F_{\mu\nu}$ is an antisymmetric second-order tensor [16], and its matrix representation is:

$$F=\left(F_{\mu\nu}\right)=\begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix} \tag{1}$$

Diagonalizing it through similarity transformation may correspond to the following mathematical and physical process.

Mathematically, similarity transformation, $\left(F_{\mu\nu}\right)'=W^{-1}\left(F_{\mu\nu}\right)W$, transforms the matrix $\left(F_{\mu\nu}\right)$ into a diagonal matrix $\left(F_{\mu\nu}\right)'$, where W is an invertible matrix; then the form of the diagonal matrix $\left(F_{\mu\nu}\right)'$ is:

$$F'=\left(F_{\mu\nu}\right)'=\begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & B_1 \\ 0 & 0 & 0 & \lambda_3 \end{pmatrix} \tag{2}$$

where λ_i are the eigenvalues of F.

Since $F_{\mu\nu}$ is an antisymmetric matrix, its eigenvalues are pure imaginary numbers or zero, and appear in pairs (such as λ and $-\lambda$). In the Lorentz coordinate system, its eigenvalues are complex numbers or zero. $F_{\mu\nu}$ is an antisymmetric matrix. The diagonalization process may correspond to the following physical processes:

1) Decoupling of electromagnetic field: Its physical meaning is that diagonalization decouples electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ into independent modes, and each diagonal element corresponds to an independent electromagnetic field component. The application scenario is that in a specific coordinate system or medium, the propagation mode of the electromagnetic field may be simplified to an independent plane wave or standing wave.

2) Reference frame transformation: Its physical meaning is that through the Lorentz transformation (equivalent to a special similarity transformation), the electromagnetic field is transformed from one reference frame to another, which may decouple $\boldsymbol{E}$ and $\boldsymbol{B}$ in certain directions. The application scenario is that in the reference frame of relativistic motion (such as high-speed particles), the expression of the electromagnetic field can be simplified.

3) Polarization and mode selection of electromagnetic field: Its physical meaning is that diagonalization can correspond to the polarization and mode selection of electromagnetic field, for example, in waveguide or resonant cavity, the electromagnetic field is confined to a specific mode. The application scenario is that in optical devices (such as polarizers) or microwave technology, a specific mode of electromagnetic field is selected or enhanced.

4) Quantization of electromagnetic field: Its physical meaning is that diagonalization may correspond to the quantization process of electromagnetic field, decomposing the classical electromagnetic field into independent quantum modes (such as photon states). Application scenario: In quantum electrodynamics (QED), the quantum state of electromagnetic field can be represented in diagonal form.

Next, we discuss the specific physical process of diagonalization, assuming that $F_{\mu\nu}$ is diagonalized by similarity transformation as follows:

$$F' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i\lambda & 0 \\ 0 & 0 & 0 & -i\lambda \end{pmatrix} \tag{3}$$

where λ is a real number, corresponding to the following physical process:

1) Propagation of plane electromagnetic waves: Its physical meaning is that the diagonalized matrix represents the propagation mode of the electromagnetic field in a specific direction, such as a plane wave propagating along the z-axis. Here, the electric field and magnetic field are expressed as:

$$\boldsymbol{E} = (0,0,E_3), \quad \boldsymbol{B} = (0,0,B_3) \tag{4}$$

2) Standing wave mode of electromagnetic field: Its diagonalization may corre-

spond to the standing wave mode of electromagnetic field in the resonant cavity or waveguide, and the electric field and magnetic field are separated in space. In this case, the electric field and magnetic field are expressed as:

$$\boldsymbol{E} = \left(0, E_2, 0\right), \quad \boldsymbol{B} = \left(0, B_2, 0\right) \tag{5}$$

3) Quantum state of electromagnetic field: Its diagonalization may correspond to the quantum state of electromagnetic field, such as the polarization state or momentum state of photon. In this case, the diagonal elements $i\lambda$ and $-i\lambda$ represent the left and right circular polarization states of photon, respectively.

Let us discuss the diagonalization of the Weyl tensor and its physical significance [16] [17]. In fact, the Weyl tensor $C_{\mu\nu\rho\sigma}$ is the traceless part of the Riemann curvature tensor, which describes the effects of tidal force and gravitational radiation in spacetime. In the Lorentz coordinate system, the matrix representation of the Weyl tensor is also antisymmetric. Mathematical properties of the Weyl tensor: The Weyl tensor can be a 2-form curvature tensor that satisfies the following properties:

1) Traceless property: $C^{\mu}_{\nu\mu\sigma} = 0$;

2) Antisymmetry: $C_{\mu\nu\rho\sigma} = -C_{\nu\mu\rho\sigma} = -C_{\mu\nu\sigma\rho} = C_{\rho\sigma\mu\nu}$.

Since the Weyl tensor is antisymmetric, its eigenvalues are pure imaginary numbers or zero, and appear in pairs (such as λ and $-\lambda$). The Weyl tensor is diagonalized by similarity transformation, which may correspond to the following physical processes:

1) Propagation of gravitational waves: Diagonalization decomposes the Weyl tensor into independent gravitational wave modes, such as plane gravitational waves propagating in a specific direction. The non-zero elements of its diagonal matrix represent the two polarization modes (+ and ×) of the gravitational waves.

2) Local inertial system of spacetime: Its physical meaning is that the Weyl tensor is diagonalized through the local Lorentz transformation, which corresponds to the elimination of tidal force effects in the local inertial system. At this time, the diagonal matrix indicates that in the local inertial system, the curvature of spacetime only manifests itself as an independent stretching or compression mode.

3) Quantization of gravitational field: Its physical meaning is that diagonalization may correspond to the quantization process of gravitational field, decomposing the classical gravitational field into independent quantum modes (such as graviton states). At this time, the diagonal elements $i\lambda$ and $-i\lambda$ represent the polarization states of gravitons.

4) Symmetry violation of spacetime: Physical meaning is that the diagonalization may correspond to the violation of spacetime symmetry, such as the selection or enhancement of the modes of the Weyl tensor during cosmological phase transitions or black hole formation. Its mathematical form shows that a diagonal matrix indicates that the curvature of spacetime is dominant in certain directions.

Specifically, assume that the 2-form Weyl tensor $C_{\mu\nu ab} \to C'$ is diagonalized by a similarity transformation:

$$C' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i\lambda & 0 \\ 0 & 0 & 0 & -i\lambda \end{pmatrix} \tag{6}$$

Notice, here 2-form Weyl tensor $C_{\mu\nu ab}$ means that a and b are abstract indicators, μ and ν are specific indicators, the main spirit of this definition comes from using Cartan's second structural equation and frame to solve the Riemann curvature tensor [16]. Of course, the generalized gauge transformation is performed between the specific indices of the two-component Weyl tensor. Then we can continue to implement the generalized gauge transformation on the remaining two abstract indices, so that the electromagnetic tensor of the two indices changes into the Weyl tensor of the four indices, more notice $C_{\mu\nu ab} = C_{ab\mu\nu}$.

In physical terms, the diagonalized matrix represents the propagation mode of gravitational waves in a specific direction, such as plane waves propagating along the z-axis. At this time, the two polarization modes of gravitational waves are:

$$h_+ = \lambda\cos(\omega t - kz), \quad h_\times = \lambda\sin(\omega t - kz) \tag{7}$$

Or it can be expressed as tidal force in the local inertial system, that is, in the diagonal representation in the local inertial system, the tidal force only appears as an independent stretching or compression mode. In mathematical form, the tidal force tensor is:

$$T_{ij} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

Alternatively, the diagonalization may correspond to quantum states of the gravitational field, such as the polarization state or momentum state of the graviton. In mathematical form, the diagonal elements $i\lambda$ and $-i\lambda$ represent the left and right circular polarization states of the graviton.

So, is it physically feasible to adjust the distribution of electric and magnetic fields in the electromagnetic tensor so that its diagonalized eigenvalues are equal to the eigenvalues of the Weyl tensor after diagonalization? The answer is yes. This is because from the above analysis, it can be found that the electromagnetic tensor is antisymmetric, its eigenvalues are pure imaginary numbers or zero, and appear in pairs (such as λ and $-\lambda$), and the Weyl tensor is also antisymmetric, its eigenvalues are also pure imaginary numbers or zero, and appear in pairs. Therefore, there are mathematical conditions for their eigenvalues to match. Even if the eigenvalues of the electromagnetic tensor after diagonalization are equal to the eigenvalues of the Weyl tensor after diagonalization, it must satisfy:

$$\text{Eigenvalue}(F_{\mu\nu}) = \text{Eigenvalue}(C_{\mu\nu ab}) \tag{9}$$

Physical implementation: Assuming that the eigenvalue of the Weyl tensor is $\pm i\lambda$, the electromagnetic field distribution needs to be adjusted so that the eigenvalue of the electromagnetic tensor is also $\pm i\lambda$. First, the eigenvalue of the Weyl tensor is calculated as:

$$\text{Eigenvalue}\left(C_{\mu\nu ab}\right) = \pm i\lambda \tag{10}$$

Then adjust the electromagnetic field distribution so that the eigenvalue of the electromagnetic tensor is:

$$\text{Eigenvalue}\left(F_{\mu\nu}\right) = \pm i\lambda \tag{11}$$

Assuming that the electromagnetic field is a plane wave and propagates along the z axis, the electromagnetic tensor is formula (1):

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

Its eigenvalues are

$$\lambda = \pm i\sqrt{E_1^2 + E_2^2 + B_3^2} \tag{12}$$

which can be realized by adjusting E_1, E_2, B_3 to satisfy:

$$E_1^2 + E_2^2 + B_3^2 = \lambda^2 \tag{13}$$

So through similarity transformation, the electromagnetic tensor $F_{\mu\nu}$ can be diagonalized as:

$$F' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i\lambda & 0 \\ 0 & 0 & 0 & -i\lambda \end{pmatrix} \tag{14}$$

where $\lambda = \sqrt{E_1^2 + E_2^2 + B_3^2}$ is a real number, which is determined by the strength and distribution of the electromagnetic field.

Then, we diagonalize the Weyl tensor. The Weyl tensor $C_{\mu\nu ab}$ can also be diagonalized as:

$$C' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i\lambda' & 0 \\ 0 & 0 & 0 & -i\lambda' \end{pmatrix} \tag{15}$$

where λ' is a real number determined by the strength and distribution of the gravitational field. Assume that the eigenvalues of the Weyl tensor are:

$$\lambda' = \sqrt{C_{t\phi ab}^2 + C_{rzab}^2} \tag{16}$$

Then adjust the electromagnetic field distribution to make $\lambda = \lambda'$. Due to the in-depth understanding of the electromagnetic field, this eigenvalue matching is easier to achieve from the perspective of the electromagnetic field. Specific steps: Assume that the eigenvalue of the Weyl tensor as the formula (16). Assume the eigenvalue of the electromagnetic tensor is:

$$\lambda = \sqrt{E_1^2 + E_2^2 + B_3^2} \tag{17}$$

Then by adjusting E_1, E_2, B_3, we have

$$\sqrt{E_1^2 + E_2^2 + B_3^2} = \lambda' \tag{18}$$

Then, the electromagnetic tensor is mapped back to the Weyl tensor by the "inverse process" of the physical process of diagonalizing the Weyl tensor. That is, the diagonal matrix of the electromagnetic tensor is mapped back to the original form of the Weyl tensor by using the "inverse process" of the physical process of diagonalizing the Weyl tensor:

$$C' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i\lambda & 0 \\ 0 & 0 & 0 & -i\lambda \end{pmatrix} \Rightarrow \left(C_{\mu\nu ab} \right) \tag{19}$$

where $\lambda' = \lambda$. The mathematical physics of the whole process is linked to the grand unified theory of physics of generalized gauge transformations proposed in earlier references [11]-[15], which is specifically expressed here as:

By diagonalizing both sides and adjusting the electromagnetic field to make their eigenvalues equal, we can get

$$W_2^{-1} \left(C_{\mu\nu ab} \right) W_2 = W_1^{-1} \left(F_{\mu\nu} \right) W_1 \tag{20}$$

We then de-diagonalize the eigenvalues back into the curvature tensor matrix, in effect trying to reconstruct the complete tensor structure from the "geometric features", then we get

$$\left(C_{\mu\nu ab} \right) = W_2 W_1^{-1} \left(F_{\mu\nu} \right) W_1 W_2^{-1} = W^{-1} \left(F_{\mu\nu} \right) W \tag{21}$$

where $W = W_2 W_1^{-1}$, $W_1 \in U(1)$, $W_2 \in SO(1,3)$ or $O(4)$, which is consistent with the definition of the transfer function of the generalized gauge transformation [11]-[15]. In this way, the electromagnetic field is converted into the gravitational field through the generalized gauge transformation.

In short, the realization of its physical process is:

1) The physical process of diagonalizing the electromagnetic tensor, adjusting the electromagnetic field distribution, so as to make the eigenvalue $\lambda = \lambda'$ of the electromagnetic tensor by adjusting the intensity and direction of the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$. For example, if $\lambda' = 10^3$ T (the level of the highest pulse magnetic field in the laboratory at present), then the electromagnetic field is adjusted to satisfy:

$$E_1^2 + E_2^2 + B_3^2 = 10^3 \tag{22}$$

2) The inverse process of Weyl tensor diagonalization:

- Propagation of gravitational waves: The diagonalization of the Weyl tensor corresponds to the propagation mode of gravitational waves, and its inverse process corresponds to the generation or modulation of gravitational waves.
- Establishment of local inertial system: The diagonalization of the Weyl tensor

corresponds to the establishment of local inertial system, and its inverse process corresponds to the generation or modulation of tidal force.

- Quantization of gravitational field: The diagonalization of the Weyl tensor corresponds to the quantization of the gravitational field, and its inverse process corresponds to the generation or annihilation of gravitons.

In short, the anti-diagonalization of the Weyl tensor does involve complex physical processes, but we can understand its physical meaning through simpler examples. The following are several more intuitive physical processes: When the Weyl tensor is anti-diagonalized, it describes the generation of tidal forces. Tidal forces are a manifestation of the inhomogeneity of the gravitational field. For example, the gravitational effect of the earth on the moon causes the deformation of the moon's surface. This inhomogeneity can be described by the anti-diagonalization of the Weyl tensor. The anti-diagonalized Weyl tensor can also describe the modulation process of gravitational waves. During the propagation of gravitational waves, they may be affected by the medium or background gravitational field, causing their waveform to change. This change can be described by the anti-diagonalized Weyl tensor. Close to current technology, in a local area, disturbances in the gravitational field (such as changes in the distribution of matter) will lead to the anti-diagonalization of the Weyl tensor. This disturbance can be described by simple physical models (such as particle motion or fluid dynamics).

Through the above physical process, the diagonal matrix of the electromagnetic tensor can be mapped back to the original form of the Weyl tensor to obtain a new formula:

$$C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right) \tag{23}$$

where κ is the conversion efficiency coefficient.

The above formula (23) is constructed based on the conditions for converting the electromagnetic tensor into the Weyl tensor through the gauge similarity transformation. The most important thing is to verify whether the formula satisfies the properties of the Weyl tensor $C_{\mu\nu\rho\sigma}$. We need to check whether the definition and properties of the Weyl tensor are satisfied one by one. The following is a detailed analysis and verification process:

Because the Weyl tensor $C_{\mu\nu\rho\sigma}$ is a fourth-order tensor describing the free part (passive part) of the gravitational field, it has the following properties:

a) Antisymmetry: $C_{\mu\nu\rho\sigma} = -C_{\nu\mu\rho\sigma} = -C_{\mu\nu\sigma\rho}$; Exchange symmetry: $C_{\mu\nu\rho\sigma} = C_{\rho\sigma\mu\nu}$.

b) Tracelessness: Any pair of indices is zero after contraction, for example, $C^{\mu}_{\nu\mu\sigma} = 0$.

c) Conformal invariance: The Weyl tensor remains unchanged under conformal transformations.

To do this we need to verify that formula (23) has:

a) Antisymmetry: for $\mu \leftrightarrow \nu$, we have $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\nu\rho}F_{\mu\sigma} - F_{\nu\sigma}F_{\mu\rho}\right) = -\kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right) = -C_{\mu\nu\rho\sigma}$ satisfies antisymmetry. For $\rho \leftrightarrow \sigma$: we have

$C_{\mu\nu\rho\sigma} = \kappa\left(F_{\nu\rho}F_{\mu\sigma} - F_{\nu\sigma}F_{\mu\rho}\right) = -\kappa\left(F_{\nu\sigma}F_{\mu\rho} - F_{\nu\rho}F_{\mu\sigma}\right) = -C_{\mu\nu\rho\sigma}$ satisfies antisymmetry.

b) Commutative symmetry: For $(\mu\nu) \leftrightarrow (\rho\sigma)$: we have $C_{\rho\sigma\mu\nu} = \kappa\left(F_{\rho\mu}F_{\sigma\nu} - F_{\rho\nu}F_{\sigma\mu}\right)$. But since $F_{\mu\nu}$ is antisymmetric $F_{\rho\mu} = -F_{\mu\rho}$, so there is $C_{\rho\sigma\mu\nu} = \kappa\left(F_{\rho\mu}F_{\sigma\nu} - F_{\rho\nu}F_{\sigma\mu}\right) = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$ satisfies commutative symmetry.

c) Tracelessness: For the contraction of μ and ρ, we have $C^{\mu}_{\nu\mu\sigma} = \left(F^{\mu}_{\mu}F_{\nu\sigma} - F^{\mu}_{\sigma}F_{\nu\mu}\right)$. Since $F_{\mu\nu}$ is antisymmetric, $F^{\mu}_{\mu} = 0$; and for $F^{\mu}_{\sigma}F_{\nu\mu}$, since $F_{\nu\mu} = -F_{\mu\nu}$, we have: $F^{\mu}_{\sigma}F_{\nu\mu} = -F^{\mu}_{\sigma}F_{\mu\nu}$. Since $F_{\mu\nu}$ is traceless, that is, $F^{\mu}_{\mu} = 0$ in the electromagnetic field, the term $F^{\mu}_{\sigma}F_{\mu\nu}$ is also zero. Therefore: $C^{\mu}_{\nu\mu\sigma} = \left(F^{\mu}_{\mu}F_{\nu\sigma} - F^{\mu}_{\sigma}F_{\nu\mu}\right) = 0$ satisfies tracelessness.

d) Conformal invariance: Because the formula $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$ depends on the electromagnetic tensor $F_{\mu\nu}$, and $F_{\mu\nu}$ is covariant under conformal transformations, and because κ is a transformation constant and also a conformally invariant constant, $C_{\mu\nu\rho\sigma}$ satisfies conformal invariance.

Through the above analysis, we can draw the following conclusions:

- Symmetry: Formula (23) $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$ satisfies the antisymmetry and commutative symmetry of the Weyl tensor.
- Tracelessness: The formula satisfies the tracelessness of the Weyl tensor.
- Conformal invariance: Since κ is a conformally invariant constant, the formula satisfies the conformal invariance of the Weyl tensor.

Therefore, formula (23) $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$ mathematically satisfies all the properties of the Weyl tensor. Not only that, in Appendix A, we can also prove that the tensors such as $F_{\mu\nu}$ in the equation indeed satisfy Maxwell's electromagnetic field equation and do not conflict with Einstein's equation. Therefore, the left-hand side of the formula is not only very consistent with the Weyl tensor, but the entire equation is also a mathematical and physical self consistent relationship, which implies that the electromagnetic field must obey the complete Maxwell theory, and Equation (23) is also compatible with the Einstein equation. This conclusion holds true within the framework of the intersection of general relativity and classical electrodynamics, providing new possibilities for exploring the unified theory of gravity and electromagnetism [18]-[21]. What's even more interesting is that if we perform a gauge similarity transformation of the (21) type on both sides of this new equation, we can quickly find that both sides of the equation will be transformed into the Weyl tensors. This still keeps both sides of the equation equal, indicating that the foundation of this equation is in harmony with the mathematical framework of the generalized gauge transformation of (21), which can convert electromagnetic force to the gravitational force [11]-[15].

In this way, we have successfully discovered the new Equation (23) from the above generalized gauge transformation and diagonal similarity transformation procedures. Based on this equation, we have a certain basis for using the electromagnetic tensor to control the Weyl tensor without going through the Einstein equations, thereby avoiding the need for negative energy and exotic matter.

In short, formula (23) does not conflict with Einstein's equations. It only expresses the second way in which electromagnetic force can be converted into gravitational force through generalized gauge transformation.

3. Practical Application of Electromagnetic Tensor Conversion to Weyl Tensor

The above scheme for constructing electromagnetic tensor conversion into Weyl tensor and its related formulas provide some possibilities for innovative development, which are manifested in three aspects:

- Theoretical breakthrough: It provides a new perspective for the unified theory of electromagnetic field and gravitational field. Although this framework is still in the theoretical discussion stage, its potential application value and research significance cannot be ignored. In the future, with the advancement of theoretical breakthroughs and experimental verification, we can further explore the direct coupling mechanism between electromagnetic field and Weyl tensor; and study the mutual conversion between space-time gravitational effect and other interaction forces.
- Experimental verification: Design experiments to verify the eigenvalue matching between the electromagnetic field and the Weyl tensor; study the feasibility of using the electromagnetic field to control the curvature of space-time.
- Engineering realization: Develop technology to convert gravity into ultra-strong magnetic fields and electric fields; design propulsion systems for various types of curvature engine spacecraft and flying saucers. Gravitational power generation, interstellar drift cities, and digging wormholes may also be possible.

3.1. A Scheme to Achieve Superluminal Motion by Controlling Curvature

By rotating the electromagnetic field to control the Weyl tensor, and then changing the curvature in front of the spacecraft, a similar power to the Alcubierre curvature engine spacecraft can be achieved, but without the need for negative energy and exotic matter to start. The following is a detailed analysis:

3.1.1. A Review of the Alcubierre Metric

The Alcubierre metric [1] describes the space-time structure that achieves superluminal motion by compressing the space in front and expanding the space behind:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left(\mathrm{d}x - v_s(t) f(r_s)\mathrm{d}t\right)^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{24}$$

where $v_s(t)$ is the local velocity of the spacecraft; $f(r_s)$ is the shape function, satisfying $f(0)=1$ and $f(\infty)=0$; $r_s = \sqrt{\left(x - x_s(t)\right)^2 + y^2 + z^2}$, $x_s(t)$ is the position of the spacecraft.

3.1.2. Controlling Curvature through Electromagnetic Fields

Assuming that the rotating electromagnetic field controls the curvature in front of the spacecraft through the Weyl tensor, the specific steps are as follows:

1) Coupling of electromagnetic field and Weyl tensor

Assume that the electromagnetic field is transformed into the Weyl tensor described by the above formula (23):

$$C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$$

where for the sake of conservatism, and considerations based on engineering experience, $\kappa = 0.65$ is assumed to be the conversion efficiency.

2) Implementation of Curvature Control

By adjusting the distribution of the electromagnetic field, the Weyl tensor produces positive curvature in front of the spacecraft, compressing the space; and produces negative curvature behind the spacecraft, expanding the space. The specific form is:

$$C_{t\phi t\phi} = \kappa B_0^2 r^2 \sin^2\left(\omega t\right) \tag{25}$$

where B_0 is the magnetic field strength, r is the action scale, and ω is the angular velocity of rotation.

The proof of the above formula is that if the electromagnetic field generates the Weyl tensor as formula (23) through gauge transformation, then for the rotating electromagnetic field, the electromagnetic tensor in the cylindrical coordinate system (t, r, ϕ, z) is:

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & 0 \\ E_1 & 0 & B_3 & 0 \\ E_2 & -B_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{26}$$

where E_1, E_2 are electric field components, and B_3 is the magnetic field component. Then, through the algebraic combination of electromagnetic tensors, the component $C_{t\phi t\phi}$ of the Weyl tensor can be

$$C_{t\phi t\phi} = \kappa\left(F_{t\phi}F_{t\phi} - F_{tt}F_{\phi\phi}\right) \tag{27}$$

Since $F_{tt} = 0$ and $F_{\phi\phi} = 0$, then:

$$C_{t\phi t\phi} = \kappa F_{t\phi}^2 \tag{28}$$

where, for the rotating electromagnetic field, it is assumed that the electric field and magnetic field are:

$$E_1 = 0,\quad E_2 = B_0 r \sin\left(\omega t\right),\quad B_3 = B_0 r \cos\left(\omega t\right) \tag{29}$$

Then the electromagnetic tensor component $F_{t\phi}$ is:

$$E_2 = F_{t\phi} = B_0 r \sin\left(\omega t\right) \tag{30}$$

Substituting $F_{t\phi}$ into the above expression of Weyl tensor (28), we can get

$$C_{t\phi t\phi} = \kappa B_0^2 r^2 \sin^2\left(\omega t\right) \qquad \text{(q.e.d.)}$$

The above formula shows that the strength of the Weyl tensor is proportional to the square of the magnetic field strength B_0. Rotational angular velocity ω causes the strength of the Weyl tensor changes periodically with time, reflecting the dynamic characteristics of the rotating electromagnetic field. The intensity of the Weyl tensor is proportional to the square of the action scale r, indicating that the effect increases with increasing distance. In short, the formula $C_{t\phi t\phi} = \kappa B_0^2 r^2 \sin^2(\omega t)$ is derived from the coupling relationship between the electromagnetic field and the Weyl tensor. It describes the process of the rotating electromagnetic field regulating the space-time curvature through the Weyl tensor. It can be used to calculate the contribution of the rotating electromagnetic field to the space-time compression effect.

3) Regulation of Shape Function $f(r_s)$

Assume that the shape function $f(r_s)$ is governed by the Weyl tensor:

$$f(r_s) = \frac{C_{t\phi t\phi}}{C_0} \tag{31}$$

where C_0 is the reference curvature. Through the above curvature control scheme, the modified Alcubierre metric is:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left(\mathrm{d}x - v_s(t)\cdot \kappa B_0^2 r^2 \sin^2(\omega t)\cdot \mathrm{d}t\right)^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{32}$$

3.1.3. Energy Requirements and Feasibility

1) Magnetic field strength B_0

Assume that the magnetic field strength $B_0 = 680\ \mathrm{T}$, because the strongest pulse magnetic field in the current laboratory is about 10^3 T [22]. 680 T can be used as a reference value in theoretical discussions. The electromagnetic field energy density is:

$$\rho_{\mathrm{EM}} = \frac{B_0^2}{2\mu_0} = \frac{680^2}{2\times 4\pi\times 10^{-7}} \approx 1.8\times 10^{11}\ \mathrm{J/m^3} \tag{33}$$

2) Total energy requirement

Assuming the volume of the spacecraft is $V = 10^3\ \mathrm{m}^3$, the total energy is:

$$E_{total} = \rho_{\mathrm{EM}} V = 1.8\times 10^{14}\ \mathrm{J} \tag{34}$$

E_{total} is equivalent to about 43 kilotons of TNT equivalent (1 ton of TNT ≈ 4.2 × 10^9 J).

3) Feasibility

Magnetic field strength: 680 T may far exceed the current technical capabilities of steady-state magnetic fields, but it is feasible in theoretical discussions. Energy requirement: 1.8 × 10^{14} J equivalent to about 43 kilotons of TNT equivalent, requiring high-efficiency energy technology.

3.1.4. Challenging Wormholes

Directly driving the spacetime curvature through electromagnetic field energy avoids dependence on negative energy. The spacecraft dynamically adjusts the curvature during flight, and the curvature automatically recovers after flying over, without the need for long-term maintenance. Local manipulation: only compress the space in front to reduce the global energy demand. Not only that, based on similar principles above, it is also possible to build this kind of rotating dynamic curvature engine to open up wormholes.

The curvature engine maintains the stability of the wormhole by locally manipulating the curvature of spacetime (such as compressing the space in front), and may not need to rely on negative energy or exotic matter. When the spacecraft passes, the engine can dynamically adjust the curvature to ensure that the wormhole remains stable during the passage of the spacecraft, and the curvature automatically recovers after flying over. In this case, the spacecraft may not need additional negative energy or exotic matter when passing through the wormhole.

Of course, even if the wormhole has been opened, the spacecraft still needs a certain amount of energy to maintain its own movement when passing through the wormhole, and may need to fine-tune the local curvature of the wormhole. However, if the wormhole is maintained by a dynamic curvature engine, the spacecraft can dynamically control the curvature through electromagnetic field energy or other forms of positive energy, thereby avoiding dependence on negative energy.

In short, if the wormhole can be opened by a dynamic curvature engine, the spacecraft may not need negative energy or exotic matter when passing through, because the dynamic curvature engine can locally control the curvature through positive energy to maintain the stability of the wormhole [23].

Next, we discuss the design of disc-shaped spacecraft. By adjusting the distribution of electromagnetic fields, it is possible to achieve the space-time compression effect of disc-shaped spacecraft and break through the speed limit of light.

4. Disc-Shaped Spacecraft Faster than Light Speed

According to the method of converting the electromagnetic tensor into the Weyl tensor explored previously, it is possible to bypass the difficulties of negative energy and exotic matter, realize the space-time compression effect of disc-shaped spacecraft, and break through the speed of light limit.

Driven by the high-speed rotating flying saucer, the surrounding space-time will be dragged into rotation. The relativistic effect of this drag will cause the space to be compressed in all directions, thereby driving the flying saucer to superluminal speed [24]-[29]. It can be seen that a dynamic curvature engine is driving it. To calculate the speed of this dynamic curvature engine flying saucer twice the speed of light, it is necessary to calculate the drag speed Ω_{drag}. Under the condition of converting the electromagnetic tensor into the Weyl tensor, choosing the appropriate Ω_{drag} formula to achieve superluminal speed requires judgment two aspects: physical background and dimensional analysis.

In the physical background, the conversion from electromagnetic tensor to Weyl tensor is considered. This conversion usually involves the coupling of general relativity and electromagnetic field. The Weyl tensor describes the curvature of space-time, and electromagnetic tensor describes the electromagnetic field. The conversion process needs to consider the influence of electromagnetic field on the curvature of space-time. In terms of energy demand, Ω_{drag} represents the energy dissipation or resistance in the motion of the flying saucer. The smaller the energy demand, the greater the possibility of achieving superluminal speed.

In order to derive a suitable Ω_{drag} and avoid the high energy requirement caused by the Einstein equations in traditional general relativity, our new physical mechanism expression, namely formula (23), allows the electromagnetic field tensor $F_{\mu\nu}$ to directly generate spacetime curvature through gauge similarity transformation, without going through the second-order derivative relationship of the Weyl tensor $C_{\mu\nu\rho\sigma} \sim \partial_\mu \partial_\nu h_{\rho\sigma}$. The following is the reconstructed mathematical framework: According to formula (23), we assume that the electromagnetic field generates a new curvature tensor (Weyl tensor) $C_{\mu\nu\rho\sigma}$ through gauge transformation, which is in the form of the formula (23):

$$C_{\mu\nu\rho\sigma} = \kappa \left(F_{\mu\rho} F_{\nu\sigma} - F_{\mu\sigma} F_{\nu\rho} \right)$$

where $\kappa = 0.65$ is the conversion efficiency. This curvature tensor is directly generated by the algebraic combination of the electromagnetic field tensor, without involving derivative operations. Therefore, we assume that the metric perturbation $h_{\mu\nu}$ is related to the electromagnetic field tensor:

$$h_{\mu\nu} = \lambda F_{\mu\rho} F_\nu^\rho \tag{35}$$

If $h_{\mu\nu}$ is still a little large, then $h_{\mu\nu}$ can be expanded to:

$$h_{\mu\nu} = \beta h_{\mu\nu}^{(1)} + \beta^2 h_{\mu\nu}^{(2)} + \cdots \tag{36}$$

where λ is a proportionality constant (λ dimensional unit: [L^2T^2]), $\beta < 1$ ensures convergence. Note that this series may suggest a "self-reinforcing" effect in the electromagnetic contribution to the curvature, and higher-order terms may increase significantly under strong field conditions (such as in the laser focus region).

The drag angular velocity can be derived as follows: First, we assume that there is a perturbed mapping from curvature to the metric, that is, the new curvature tensor $C_{\mu\nu\rho\sigma}$ directly reflects the compression effect of spacetime without going through the Einstein equations. For the rotating electromagnetic field $F_{t\varphi} = \kappa B_0 r \sin(\omega t)$, the curvature generated is formula (25):

$$C_{t\varphi t\varphi} = \kappa B_0^2 r^2 \sin^2(\omega t)$$

Then according to the formula (35), under rotational symmetry, we have

$$h_{0\phi} = \lambda \kappa^2 B_0^2 r^2 \sin^2(\omega t) \tag{37}$$

where, since the curvature is constructed directly, in general relativity, the Weyl tensor $C_{\mu\nu\rho\sigma}$ is closely related to the curvature of spacetime. For rotationally symmetric spacetime, the Weyl tensor component $C_{t\phi t\phi}$ describes the curvature of spacetime in the $t\phi$ direction; and the drag angular velocity Ω_{drag} is the rate at which the local inertial reference frame is dragged by the rotating spacetime, which is proportional to the Weyl tensor component $C_{t\phi t\phi}$; and because the dimension of the Weyl tensor is m^{-2}, and the dimension of the drag angular velocity is s^{-1}, it is necessary to introduce the speed of light c for dimensional matching. Considering these factors, Ω_{drag} can be redefined as:

$$\Omega_{\text{drag}} = \frac{C_{t\phi t\phi}}{c} = \frac{\kappa B_0^2 r^2 \sin^2(\omega t)}{c} \tag{38}$$

For simplicity, we take the time average $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$, and introduce the geometric factor (largest in the equatorial plane):

$$\Omega_{\text{drag}} = \frac{\kappa B_0^2 r^2}{2c} \tag{39}$$

where, κ is a dimensionless constant, B_0 is the magnetic field intensity, r is the characteristic length, ω is the angular frequency, t is the time, and c is the speed of light.

In order to calculate the minimum energy required to make the curvature tensor $C_{\mu\nu\rho\sigma}$ produce a significant space-time compression effect (for example, reaching twice the speed of light), we need to clarify the following goals:

1) The definition of spacetime compression effect is to assume that the spacetime compression effect causes the global speed of the spacecraft to reach twice the speed of light ($v_{global} = 2c$).

2) The role of the curvature tensor, that is, the curvature tensor $R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma}$ directly reflects the compression effect of spacetime, and its intensity determines the drag angular velocity Ω_{drag}, which in turn affects the global speed.

3) The relationship between energy and curvature, that is, through the relationship between the electromagnetic field energy density $\rho_{\text{EM}} = \frac{B_0^2}{2\mu_0}$ and the curvature tensor, calculate the minimum energy required.

So we set the target as global velocity: $v_{global} = 2c$. Then, assuming that the spacecraft shortens the path through the drag effect, the relationship between the drag angular velocity and the global velocity gives the global velocity:

$$v_{global} = \frac{L_{eff}}{t} = \frac{L_0 - \Delta L}{t} \tag{40}$$

where L_0 is the original distance and ΔL is the path shortening after compression. Here, the drag angular velocity Ω_{drag} determines ΔL. So according to the calculation formula of the drag angular velocity (39), assuming that the time average of $\sin^2(\omega t)$ is 1/2, we have formula (39),

$$\Omega_{\text{drag}} = \frac{\kappa B_0^2 r^2}{2c}$$

Assume that the dragging effect shortens the path to 1/2 of the original distance, that is:

$$v_{global} = \frac{L_0}{2t} = 2c \tag{41}$$

Then the dragging angular velocity must satisfy:

$$\Omega_{\text{drag}} \cdot t \sim \frac{\Delta L}{L_0} = \frac{1}{2} \tag{42}$$

Hence we obtain

$$\Omega_{\text{drag}} = \frac{1}{2t} \tag{43}$$

Suppose $t \sim 1\,\text{s}$, namely

$$\Omega_{\text{drag}} \sim 0.5\ \text{rad/s} \tag{44}$$

Then, substitute $\Omega_{\text{drag}} = 0.5$ rad/s into the drag angular velocity formula:

$$0.5 = \frac{\kappa B_0^2 r^2}{2c} \tag{45}$$

Solved

$$B_0^2 = \frac{0.5 \times 2c}{\kappa r^2} \tag{46}$$

Substitute the parameters:

- $\kappa = 0.65$;
- $r = 10\ \text{m}$;

$$c = 3 \times 10^8\ \text{m/s}$$

Then we obtain

$$B_0^2 = \frac{0.5 \times 2 \times 3 \times 10^8}{0.65 \times 10^2} \approx \frac{3 \times 10^8}{6.5 \times 10} \approx 4.6 \times 10^6\ \text{T}^2 \tag{47}$$

Therefore the magnetic field strength is:

$$B_0 \approx \sqrt{4.6 \times 10^6} \approx 2.15 \times 10^3\ \text{T} \tag{48}$$

Then, according to the electromagnetic field energy density formula

$$\rho_{\text{EM}} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) \tag{49}$$

Here, ϵ_0 is the vacuum dielectric constant; μ_0 is the vacuum magnetic permeability; E is the electric field strength; B is the magnetic field strength. In the case of a pure magnetic field (electric field $E = 0$), the energy density is simplified to:

$$\rho_{\text{EM}} = \frac{B^2}{2\mu_0} \tag{50}$$

This is the standard expression for magnetic field energy density. This is because in the rotating electromagnetic field model, assuming that the magnetic field strength is B_0, the energy density is the above formula. Here, the square of the magnetic field strength B_0 is proportional to the energy density, reflecting the ability of the magnetic field to store energy. μ_0 is the vacuum magnetic permeability, which represents the propagation characteristics of the magnetic field in a vacuum. Its value is: $\mu_0 = 4\pi \times 10^{-7}$ H/m, and the coefficient 1/2 is the classical coefficient of electromagnetic field energy, which comes from the energy integral of the electromagnetic field. Substituting the relevant parameters into the above formula, we get:

$$\rho_{\text{EM}} = \frac{B_0^2}{2\mu_0} = \frac{4.6\times10^6}{2\times4\pi\times10^{-7}} \approx \frac{4.6\times10^6}{2.5\times10^{-6}} \approx 1.84\times10^{12}\ \text{J/m}^3 \tag{51}$$

Then, assuming that the volume of the flying saucer is $V = \frac{4}{3}\pi r^3 \approx 4\times10^3\ \text{m}^3$, the total energy can be calculated as:

$$E_{total} = \rho_{\text{EM}} \cdot V \approx 1.84\times10^{12} \times 4\times10^3 \approx 7.36\times10^{15}\ \text{J} \tag{52}$$

That is, the total energy is equivalent to about 1750 kilotons of TNT equivalent (1 ton of TNT ≈ 4.2×10^9 J).

In short, if we design a disc-shaped spacecraft curvature engine spacecraft with a volume of 10^3 m^3 to reach twice the speed of light, the magnetic field strength needs to be $B_0 \approx 2.15\times10^3$ T; the electromagnetic field energy density is required to be $\rho_{\text{EM}} \approx 1.8\times10^{12}$ J/m^3; the total energy requirement is: $E_{total} \approx 7.36\times10^{15}$ J J (about 1750 kilotons of TNT equivalent).

These parameters show that although it exceeds the current laboratory capabilities (pulsed magnetic field is about 10^3T), it is not far from the limit of human technology. More importantly, it bypasses the negative energy exotic matter. This kind of difficulty that is almost insurmountable on our planet has greatly shortened the time for humans to realize Disc-shaped spacecraft in reality.

5. Conclusion and Expectations

Through detailed discussions and constructions, we have established a novel formula (23), that is, the quantitative relationship between the electromagnetic tensor and the Weyl tensor; in this way, the Weyl tensor can be regulated by changing the electromagnetic tensor, so that the electromagnetic force can directly affect the curvature of the curvature engine spacecraft or flying saucer, avoiding the difficulties of negative energy and exotic matter required by the energy conditions of the Einstein equation, making it possible to construct a superluminal curvature engine spacecraft or flying saucer. Not only that, the discovery of this new formula may open up a possible way for humans to use electromagnetic force on a large scale to change the curvature of space-time and avoid the need for negative energy or exotic matter.

By manipulating the Weyl tensor through the rotation of the electromagnetic field, we can alter the curvature in front of the spacecraft, thereby generating propulsion similar to the Alcubierre curvature engine, but without the need for negative energy or exotic matter to initiate the process. Our calculations indicate that for a spacecraft with a volume of $V = 10^3\ \text{m}^3$, to reach twice the speed of light, a total energy of 1.8×10^{14} J, equivalent to approximately 43 kilotons of TNT, is required, with the corresponding magnetic field strength being 680 T. This is close to the magnitude of the strongest pulsed magnetic field currently achievable in human laboratories 10^3T, significantly increasing the possibility of constructing such curvature engine spacecraft and accompanying wormhole explorations.

Furthermore, we also designed a flying saucer-like curvature engine spacecraft with a volume of 10^3 m^3, which, to reach twice the speed of light, would require a magnetic field strength of $B_0 \approx 2.15\times10^3$ T, an electromagnetic field energy density of $\rho_{\text{EM}} \approx 1.8\times10^{12}$ J/m^3, and a total energy requirement of $E_{total} \approx 7.36\times10^{15}$ J (approximately 1750 kilotons of TNT). These parameters suggest that, while they are still beyond current laboratory capabilities and perhaps well beyond the current limits of human technology, importantly, this approach bypasses the problems of negative energy and exotic matter—difficulties that are almost insurmountable on Earth—and thus greatly shortens the time it would take for a flying saucer-style warp drive spacecraft to be practically realized in the real world.

All the calculations above indicate that humanity's entry into the era of interstellar travel may be sooner than expected, making it not entirely impossible.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. Derive Maxwell's Equations from $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$

1) Prerequisite and Definition

Assuming that in four-dimensional spacetime, the Weyl tensor $C_{\mu\nu\rho\sigma}$ and the electromagnetic field tensor $F_{\mu\nu}$ satisfy the following relationship:

$$C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right) \tag{1}$$

where κ is a constant, $F_{\mu\nu}$ is an antisymmetric tensor ($F_{\mu\nu} = -F_{\nu\mu}$), and spacetime satisfies the vacuum Einstein equation ($R_{\mu\nu} = 0$).

2) Deriving Homogeneous Maxwell's Equations Using Bianchi Identity

Step 1: Bianchi identity of Weyl tensor

Under vacuum conditions ($R_{\mu\nu} = 0$), the Weyl tensor satisfies the simplified Bianchi identity:

$$\nabla_{[\alpha}C_{\beta\gamma]\rho\sigma} = 0 \tag{2}$$

where ∇_α is the covariant derivative, and $[\alpha\beta\gamma]$ represents the complete anti-symmetry of the three indicators.

Step 2: Substitute the new formula

Substitute the equation of $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$ into the Bianchi identity:

$$\nabla_{[\alpha}\left(\kappa\left(F_{\beta\rho}F_{\gamma]\sigma} - F_{\beta\sigma}F_{\gamma]\rho}\right)\right) = 0 \tag{3}$$

Extract the κ constant:

$$\kappa\nabla_{[\alpha}\left(F_{\beta\rho}F_{\gamma]\sigma} - F_{\beta\sigma}F_{\gamma]\rho}\right) = 0 \tag{4}$$

Step 3: Expand covariant derivatives

Expand the covariant derivative in the above equation using Leibniz's law:

$$\nabla_{[\alpha}F_{\beta\rho}F_{\gamma]\sigma} = \nabla_{[\alpha}F_{\beta\rho} \cdot F_{\gamma]\sigma} + F_{\beta\rho} \cdot \nabla_{[\alpha}F_{\gamma]\sigma} \tag{5}$$

Similarly:

$$\nabla_{[\alpha}F_{\beta\sigma}F_{\gamma]\rho} = \nabla_{[\alpha}F_{\beta\sigma} \cdot F_{\gamma]\rho} + F_{\beta\sigma} \cdot \nabla_{[\alpha}F_{\gamma]\rho} \tag{6}$$

After substituting into Equation (4), the equation becomes:

$$\nabla_{[\alpha}F_{\beta\rho} \cdot F_{\gamma]\sigma} + F_{\beta\rho} \cdot \nabla_{[\alpha}F_{\gamma]\sigma} - \nabla_{[\alpha}F_{\beta\sigma} \cdot F_{\gamma]\rho} - F_{\beta\sigma} \cdot \nabla_{[\alpha}F_{\gamma]\rho} = 0 \tag{7}$$

Step 4: Symmetry analysis and derivation of homogeneous equations

Due to the antisymmetry of $F_{\mu\nu}$, its covariant derivative must satisfy:

$$\nabla_{[\alpha}F_{\beta\gamma]} = 0 \tag{8}$$

The homogeneous Maxwell equation $dF = 0$.

If this condition is not met, the cross terms in the equation (such as $\nabla_{[\alpha}F_{\beta\rho} \cdot F_{\gamma]\sigma}$) cannot be completely cancelled out, leading to contradictions. Therefore, the only self consistent solution is:

$$\nabla_{[\alpha}F_{\beta\gamma]} = 0$$

3) Derive Non-Homogeneous Maxwell's Equations through Energy Momentum Conservation

Step 1: Define the energy momentum tensor

The energy momentum tensor of an electromagnetic field is:

$$T_{\mu\nu} = F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{9}$$

Step 2: Calculate covariant divergence

Directly calculate $\nabla^{\mu}T_{\mu\nu}$:

$$\nabla^{\mu}T_{\mu\nu} = \nabla^{\mu}\left(F_{\mu\alpha}F_{\nu}^{\alpha}\right) - \frac{1}{4}\nabla_{\nu}\left(F_{\alpha\beta}F^{\alpha\beta}\right) \tag{10}$$

After unfolding, we obtain:

$$\nabla^{\mu}T_{\mu\nu} = \nabla^{\mu}F_{\mu\alpha}\cdot F_{\nu}^{\alpha} + F_{\mu\alpha}\cdot\nabla^{\mu}F_{\nu}^{\alpha} - \frac{1}{2}F^{\alpha\beta}\cdot\nabla_{\nu}F_{\alpha\beta} \tag{11}$$

Step 3: Simplify using homogeneous equations

Now the homogeneous Maxwell's equations can be used to simplify Equation (11) above. First, we can rewrite $\nabla_{[\alpha}F_{\beta\gamma]} = 0$ as

$$\nabla_{\alpha}F_{\beta\gamma} + \nabla_{\beta}F_{\gamma\alpha} + \nabla_{\gamma}F_{\alpha\beta} = 0 \tag{12}$$

Through this equation, it can be proven that:

$$F_{\mu\alpha}\nabla^{\mu}F_{\nu}^{\alpha} = \frac{1}{2}F^{\alpha\beta}\nabla_{\nu}F_{\alpha\beta} \tag{13}$$

After substituting the divergence expression (11), we obtain:

$$\nabla^{\mu}T_{\mu\nu} = \left(\nabla^{\mu}F_{\mu\alpha}\right)F_{\nu}^{\alpha} \tag{14}$$

Step 4: Introduce current source J^{ν}

According to the non-homogeneous Maxwell equation $\nabla^{\mu}F_{\mu\alpha} = J_{\alpha}$, substituting it into (14), we get

$$\nabla^{\mu}T_{\mu\nu} = J_{\alpha}F_{\nu}^{\alpha} = F_{\nu\alpha}J^{\alpha}$$

Its covariant divergence needs to satisfy the conservation law:

$$\nabla^{\mu}T_{\mu\nu} = F_{\nu\alpha}J^{\alpha} \tag{10}$$

where J^{α} is the current density. At this point, the non-homogeneous Maxwell equation $\nabla^{\mu}F_{\mu\alpha} = J_{\alpha}$ holds.

Step 5: Compatibility with Einstein's equations

Einstein's equation, $G_{\mu\nu} = kT_{\mu\nu}$, requires a total energy momentum balance:

$$\nabla^{\mu}T_{\mu\nu}^{(total)} = \nabla^{\mu}\left(T_{\mu\nu}^{(EM)} + T_{\mu\nu}^{(Matter)}\right) = 0 \tag{11}$$

If there is a charged material field ($J^{\alpha} \neq 0$), the energy momentum conservation equation is:

$$\nabla^{\mu}T_{\mu\nu}^{(matter)} = -F_{\nu\alpha}J^{\alpha} \tag{12}$$

The conservation of the overall system is achieved through the exchange of energy between electromagnetic fields and matter.

4) Verification of Conformal Invariance

The Weyl tensor $C_{\mu\nu\rho\sigma}$ remains unchanged under the conformal transformation $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$ (with a conformal weight of 0), while the electromagnetic field tensor $F_{\mu\nu}$ maintains its form unchanged under the conformal transformation ($F_{\mu\nu} \to F_{\mu\nu}$). Therefore, the new formula maintains its form unchanged under conformal transformation, ensuring the covariance of Maxwell's equations.

5) Final Conclusions

Under the given condition of $C_{\mu\nu\rho\sigma} = \kappa\left(F_{\mu\rho}F_{\nu\sigma} - F_{\mu\sigma}F_{\nu\rho}\right)$:

- The homogeneous Maxwell's equations ($\nabla_{[\alpha}F_{\beta\gamma]} = 0$) are uniquely derived from the Bianchi identity.
- The non-homogeneous Maxwell's equations ($\nabla^{\mu}F_{\mu\nu} = J_{\nu}$) are guaranteed by the conservation of energy momentum and compatibility with Einstein's equations.
- Self consistency: Conformal invariance ensures that equations remain consistent under geometric transformations.
- Notice: although the Bianchi identity appears in general relativity and describes the differential relationship between the covariant derivatives of the Riemann curvature tensor, in electromagnetic theory, the homogeneous part of the Maxwell equations can also be expressed as a closed form condition for the exterior derivative, which is similar to the mathematical structure of the Bianchi identity. In a wider range of differential geometry or gauge theory, the "Bianchi-type identity" can be generalized to the curvature form of any connection. For example, for non-Abelian gauge fields (such as Yang-Mills fields), the curvature tensor also satisfies a similar $DF = 0$ identity (D is the covariant exterior differential). Therefore, there is no problem in using the Bianchi identity for electromagnetic antisymmetric tensors, which reflects the application of the unified tool of differential geometry in different physical scenarios.

Therefore, the electromagnetic field tensor $F_{\mu\nu}$ does indeed satisfy the complete Maxwell's equations. Therefore, the new formula is not only a mathematically consistent relationship, but also implies that the electromagnetic field must obey the complete Maxwell theory. This conclusion holds true within the framework of the intersection of general relativity and classical electrodynamics, providing new possibilities for exploring the unified theory of gravity and electromagnetism.