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The Correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman Metrics When the Cosmological Constant Is Greater than Zero

Carlo Maria Pace

ICRANet, Piazza della Repubblica, Pescara, Italy
Email: pace@icra.it

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Abstract

In a recent article we have corrected the traditional derivation of the Schwarzschild metric when the cosmological constant is equal to zero, thus obtaining the formulation of the correct Schwarzschild metric when the cosmological constant is equal to zero, which formulation is different from that of the traditional Schwarzschild metric when the cosmological constant is equal to zero. Then, in another article by starting from this correct Schwarzschild metric when the cosmological constant is equal to zero, we have corrected also the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is equal to zero. In this article, by starting from these correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is equal to zero, we obtain the formulations of the correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero. Moreover, we analyse these correct results and their consequences. Finally, we propose some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

Keywords

General Theory of Relativity, Schwarzschild, Reissner-Nordstrøm, Kerr, Kerr-Newman, Metric, Cosmological Constant, Event Horizon, Black Hole

1. Introduction

In a recent article [1], we have corrected the traditional derivation of the Schwarzschild metric when the cosmological constant is equal to zero, thus obtaining the formulation of the correct Schwarzschild metric when the cosmological constant is equal to zero, which formulation is different from that of the traditional Schwarzschild metric when the cosmological constant is equal to zero.

Then, in another article [2] by starting from this correct Schwarzschild metric when the cosmological constant is equal to zero, we have corrected also the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is equal to zero.

Now, in this article, by starting from these correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is equal to zero, we want to obtain the formulations of the correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero.

Moreover, we will analyse these results and their consequences.

Finally, we will see the experimental prospects, proposing in particular some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

2. The Correct Schwarzschild Solution When the Cosmological Constant Is Equal to Zero

In this article all the formulas are expressed with the velocity of light $c \equiv 1$, unless otherwise indicated.

As for the Schwarzschild solution, that is the solution that represents the curved space-time geometry surrounding a spherically symmetric mass distribution, we have that the commonly used expression when the cosmological constant is equal to zero is [3]:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 - \frac{1}{1 - \frac{2GM}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{1}$$

where M is (a constant, which is equal to) the total mass of the system.

In the (1) there is a singularity (event horizon) when $r = 2GM$.

But the expression (1) is erroneous: in fact, as we have shown in [1], when the cosmological constant is equal to zero the correct Schwarzschild solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\mathrm{d}s^2 = \left(1 + \frac{2GM}{r_g}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \frac{2GM}{r_g}}\mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{2}$$

where M is (a constant, which is equal to) the total mass of the system, and t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (2) is expressed in curved coordinates.

We can note that in the (2) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

On the other hand, we have also shown in [1] that, when the cosmological constant is equal to zero, the correct Schwarzschild solution expressed as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, is:

$$\mathrm{d}s_g^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{3}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, r (or r_g) is always greater than zero since the Schwarzschild solution when the cosmological constant is equal to zero is a solution for vacuum region surrounding a spherically symmetric mass distribution.

As for the relation between r_g and r, we have [1]:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{4}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{5}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g^2\mathrm{d}r_g}{r_g+2GM} \tag{6}$$

From which, we have [1]:

$$r^2 = r_g^2 - 4GMr_g + 8G^2M^2\ln\left(1+\frac{r_g}{2GM}\right) \tag{7}$$

According to the (7) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases as can be seen by differentiating the right side of the (7) with respect to r_g or directly from the (6). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as can be seen directly from the (5).

Furthermore, for $r_g \gg 2GM$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also

$r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that for $\frac{2GM}{r} \ll 1$ the presence of r_g instead of r in the (3) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \ll 2GM$ we have $r^2 \cong \frac{r_g^3}{3GM}$.

We can note that we have not any singularity in the correct expressions (2) and (3) for any value of $r > 0$ and of $r_g > 0$. Consequently, we can infer that there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, the coefficients of $\mathrm{d}t^2$ in (3) and of $\mathrm{d}t_g^2$ in (2) are always >0, while the coefficients of $\mathrm{d}r^2$ in (3) and of $\mathrm{d}r_g^2$ in (2) are always <0. Therefore since the coefficients of $\mathrm{d}t^2$ in (3) and of $\mathrm{d}t_g^2$ in (2) are always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since the coefficients of $\mathrm{d}r^2$ in (3) and of $\mathrm{d}r_g^2$ in (2) are always not positive, also here the spatial coordinate r (or r_g) does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

As we have already shown in [1], the formula of the correct ratio between the times when they are measured in two different positions in this case is:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GM}{r_{g1}}}{1+\frac{2GM}{r_{g2}}}} \tag{8}$$

Therefore in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

Analogously, as we have already shown in [1], the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ is:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\frac{2GM}{r_{g1}}}{1+\frac{2GM}{r_{g2}}}} \tag{9}$$

This is the correct formula for the gravitational redshift of light in the correct

Schwarzschild metric when the cosmological constant is equal to zero. Therefore, in this case the gravitational redshift is always not infinite for any value of $r > 0$ and of $r_g > 0$.

We can note that the two correct formulas (8) and (9) are different from those that can be obtained from the incorrect expression of the Schwarzschild solution when the cosmological constant is equal to zero by means of an incorrect procedure [1] [3], but are equal at the first order in $\frac{2GM}{r}$ (to the incorrect formulas). In fact, in this case the incorrect formulas analogous of the two correct formulas (8) and (9) are respectively [1] [3]:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1-\frac{2GM}{r_2}}{1-\frac{2GM}{r_1}}} \tag{10}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1-\frac{2GM}{r_2}}{1-\frac{2GM}{r_1}}} \tag{11}$$

As we have already noted in [1], the light formally moves in a straight line relative to the reference system curved for expressing the presence of the gravitational field. Therefore, we can impose the condition $\mathrm{d}s_g^2 = 0$ for the propagation of light. Consequently from the (3) if we take a radial motion of the light in the commonly used coordinates ($\mathrm{d}\theta = 0$ and $\mathrm{d}\varphi = 0$) we have:

$$\frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}\right)\mathrm{d}r^2 = 0 \tag{12}$$

From which we have that the correct radial velocity of light in the commonly used coordinates is equal to [1] [3]:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{1+\frac{2GM}{r_g}} \tag{13}$$

We can note that the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field. Moreover this formula is in agreement with the available experimental data [3].

On the other hand, by using the incorrect formula (1), we obtain that the incorrect radial velocity of light in the commonly used coordinates is equal to [3]:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = 1-\frac{2GM}{r} \tag{14}$$

3. The Correct Schwarzschild Solution When the Cosmological Constant Is Greater than Zero

When the cosmological constant Λ is greater than zero (instead of being equal to zero), the Einstein's field equation in vacuum, as H. C. Ohanian, R. Ruffini and S. Weinberg state [3] [5], is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} \tag{15}$$

instead of:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \tag{16}$$

Consequently, when the cosmological constant Λ is greater than zero, the commonly used Schwarzschild solution is [3]:

$$\begin{aligned} \mathrm{d}s^2 = {} & \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right)\mathrm{d}t^2 - \frac{1}{1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}}\mathrm{d}r^2 \\ & - r^2 \mathrm{d}\theta^2 - r^2 \sin^2\theta \mathrm{d}\varphi^2 \end{aligned} \tag{17}$$

where M is (a constant, which is equal to) the total mass of the system.

Because Λ is very small there is a range of radii (in cgs units, $\frac{GM}{c^2} \ll r \ll \frac{c}{\sqrt{\Lambda}}$) in which the space-time geometry is nearly flat. At the lower end of this range, the effect of the mass M dominates; at the upper end of this range, the effect of the cosmological term dominates. Moreover, in the Newtonian approximation the Newtonian potential becomes [3]:

$$\Phi = -\frac{GM}{r} - \frac{\Lambda}{6} r^2 \tag{18}$$

In the (17) there are singularities (event horizons) when:

$$1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} = 0 \tag{19}$$

On the other hand, from the (17), by analogy with the derivation of the correct Schwarzschild solution when $\Lambda = 0$ [1], we can infer that the correct Schwarzschild solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, when $\Lambda > 0$ is equal to:

$$\begin{aligned} \mathrm{d}s^2 = {} & \left(1 + \frac{2GM}{r_g} + \frac{\Lambda r_g^2}{3}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \frac{2GM}{r_g} + \frac{\Lambda r_g^2}{3}}\mathrm{d}r_g^2 \\ & - r_g^2 \mathrm{d}\theta_g^2 - r_g^2 \sin^2\theta_g \mathrm{d}\varphi_g^2 \end{aligned} \tag{20}$$

where M is (a constant, which is equal to) the total mass of the system, and t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this Equation (20) is expressed in curved coordinates.

Therefore, when $\Lambda > 0$ we have $\frac{2GM}{r_g} + \frac{\Lambda r_g^2}{3}$ instead of $\frac{2GM}{r_g}$, in the correct formulas.

Moreover, we can note that in the (20) the coefficient of dt_g^2 is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Analogously to our treatment of the correct Schwarzschild solution when the cosmological constant is equal to zero [1] we can express the correct Schwarzschild solution, when the cosmological constant is greater than zero, as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates. In this case we have:

$$\begin{aligned} \mathrm{d}s_g^2 = & \frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}\right)\mathrm{d}r^2 \\ & - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \end{aligned} \tag{21}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, also here, r (or r_g) is always greater than zero since the Schwarzschild solution when the cosmological constant is greater than zero is a solution for vacuum region surrounding a spherically symmetric mass distribution.

As for the relation between r_g and r, we have:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}}\mathrm{d}r_g^2 \tag{22}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}}\mathrm{d}r_g^2 \tag{23}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{6r_g^2\mathrm{d}r_g}{3r_g+6GM+\Lambda r_g^3} \tag{24}$$

According to the (24) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (24). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (23).

For r_g not too large we have that the (24) is practically equal to the analogous formula with $\Lambda = 0$ [1], also because we know that the cosmological constant is very small.

Instead for $r_g \to +\infty$ we have that the term with the cosmological constant dominates. In this case from the (24) we have:

$$r^2 \cong \int_1^{r_g} \frac{6r_g^2 \mathrm{d}r_g}{\Lambda r_g^3} = \frac{6}{\Lambda} \ln\left(r_g\right) \tag{25}$$

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expressions (20) and (21) for any value of $r > 0$ (or of $r_g > 0$). Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, also here in the (20) and in the (21) the coefficients of $\mathrm{d}t_g^2$ and of $\mathrm{d}t^2$ are always >0, therefore since the coefficients of $\mathrm{d}t_g^2$ and of $\mathrm{d}t^2$ are always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since in the (20) and in the (21) the coefficients of $\mathrm{d}r_g^2$ and of $\mathrm{d}r^2$ are always not positive, also here the spatial coordinate does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\dfrac{2GM}{r_{g1}}+\dfrac{\Lambda r_{g1}^2}{3}}{1+\dfrac{2GM}{r_{g2}}+\dfrac{\Lambda r_{g2}^2}{3}}} \tag{26}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\dfrac{2GM}{r_{g1}}+\dfrac{\Lambda r_{g1}^2}{3}}{1+\dfrac{2GM}{r_{g2}}+\dfrac{\Lambda r_{g2}^2}{3}}} \tag{27}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (26) and (27) are different from those that can be obtained from the incorrect expression of the Schwarzschild solution when the cosmological constant is greater than zero by means of an incorrect procedure [1] [3]. In particular, in this case the incorrect formulas analogous of the two correct formulas (26) and (27) are respectively:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1 - \frac{2GM}{r_2} - \frac{\Lambda r_2^2}{3}}{1 - \frac{2GM}{r_1} - \frac{\Lambda r_1^2}{3}}} \tag{28}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1 - \frac{2GM}{r_2} - \frac{\Lambda r_2^2}{3}}{1 - \frac{2GM}{r_1} - \frac{\Lambda r_1^2}{3}}} \tag{29}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1] [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{1 + \frac{2GM}{r_g} + \frac{\Lambda r_g^2}{3}} \tag{30}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

Instead in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = 1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} \tag{31}$$

4. The Correct Reissner-Nordstrøm Solution When the Cosmological Constant Is Equal to Zero

As for the Reissner-Nordstrøm solution [3] [4], that is the solution that represents the curved space-time geometry surrounding an electrically charged mass, we have that the commonly used expressions for the space-time interval and the electric field, when the cosmological constant is equal to zero, are respectively [3]:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)\mathrm{d}t^2 - \frac{1}{1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{32}$$

and:

$$E(r)=\frac{Q}{r^2} \tag{33}$$

where M and Q are two constants: M is the total mass of the system and Q is the total electric charge of the system.

In the (32) there are singularities (event horizons) when:

$$1-\frac{2GM}{r}+\frac{GQ^2}{r^2}=0 \tag{34}$$

that is, when:

$$r^2-2GMr+GQ^2=0 \tag{35}$$

or, better yet, when:

$$r=GM\pm\sqrt{G^2M^2-GQ^2} \tag{36}$$

Obviously there are real singularities (event horizons) only when $G^2M^2-GQ^2\geq 0$.

But the expression (32) is erroneous: in fact, by means of the fact that for $Q=0$ the correct Reissner-Nordstrøm solution when the cosmological constant is equal to zero must be equal to the correct Schwarzschild solution when the cosmological constant is equal to zero, we can infer, as we have already noted in [2], that, when the cosmological constant is equal to zero, the correct Reissner-Nordstrøm solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2\equiv\mathrm{d}t^2-\mathrm{d}r^2-r^2\mathrm{d}\theta^2-r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned}\mathrm{d}s^2=&\left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}t_g^2-\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2\\&-r_g^2\mathrm{d}\theta_g^2-r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2\end{aligned} \tag{37}$$

where M and Q are two constants: M is the total mass of the system and Q is the total electric charge of the system. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (37) is expressed in curved coordinates.

We can note that in the (37) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Analogously to the case of the Schwarzschild metric when the cosmological constant is equal to zero [1], we can write also in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates. In this case we have [2]:

$$\mathrm{d}s_g^2 = \frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{38}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, also here, r (or r_g) is always greater than zero since the Reissner-Nordstrøm solution when the cosmological constant is equal to zero is a solution for vacuum region surrounding an electrically charged mass.

As for the relation between r_g and r, we have [2]:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \tag{39}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \tag{40}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g^3\mathrm{d}r_g}{r_g^2+2GMr_g+GQ^2} \tag{41}$$

From which, we have [2]:

$$\begin{aligned} r^2 = {} & r_g^2 - 4GMr_g + G\left(4GM^2-Q^2\right)\ln\left(1+\frac{r_g^2+2GMr_g}{GQ^2}\right) \\ & + \frac{G^2M\left(3Q^2-4GM^2\right)}{\sqrt{G^2M^2-GQ^2}} \\ & \cdot\ln\left(\frac{GM+r_g-\sqrt{G^2M^2-GQ^2}}{GM+r_g+\sqrt{G^2M^2-GQ^2}}\cdot\frac{GM+\sqrt{G^2M^2-GQ^2}}{GM-\sqrt{G^2M^2-GQ^2}}\right) \end{aligned} \tag{42}$$

According to the (42) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (41). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (40).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (38) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \to 0^+$ we have $r \cong \frac{r_g^2}{\sqrt{2GQ^2}}$, as we can see easily from the (41).

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expressions (37) and (38) for any value of $r > 0$ and of $r_g > 0$. Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, also here the coefficients of $\mathrm{d}t^2$ in (38) and of $\mathrm{d}t_g^2$ in (37) are always >0, while the coefficients of $\mathrm{d}r^2$ in (38) and of $\mathrm{d}r_g^2$ in (37) are always <0. Therefore, since the coefficients of $\mathrm{d}t^2$ in (38) and of $\mathrm{d}t_g^2$ in (37) are always not negative, the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since the coefficients of $\mathrm{d}r^2$ in (38) and of $\mathrm{d}r_g^2$ in (37) are always not positive, also here the spatial coordinate r (or r_g) does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GM}{r_{g1}}+\frac{GQ^2}{r_{g1}^2}}{1+\frac{2GM}{r_{g2}}+\frac{GQ^2}{r_{g2}^2}}} \tag{43}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\frac{2GM}{r_{g1}}+\frac{GQ^2}{r_{g1}^2}}{1+\frac{2GM}{r_{g2}}+\frac{GQ^2}{r_{g2}^2}}} \tag{44}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (43) and (44) are different from those that can be obtained from the incorrect expression of the Reissner-Nordstrøm solution when the cosmological constant is equal to zero by means of an incorrect procedure [1]-[3]. In fact, in this case the incorrect formulas analogous of the two correct formulas (43) and (44) are respectively [1]-[3]:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1-\frac{2GM}{r_2}+\frac{GQ^2}{r_2^2}}{1-\frac{2GM}{r_1}+\frac{GQ^2}{r_1^2}}} \tag{45}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1-\frac{2GM}{r_2}+\frac{GQ^2}{r_2^2}}{1-\frac{2GM}{r_1}+\frac{GQ^2}{r_1^2}}} \tag{46}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}} \tag{47}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = 1-\frac{2GM}{r}+\frac{GQ^2}{r^2} \tag{48}$$

5. The Correct Reissner-Nordstrøm Solution When the Cosmological Constant Is Greater than Zero

By analogy with the (17) and the (32), we can infer that, when the cosmological constant Λ is greater than zero, the commonly accepted Reissner-Nordstrøm solution is:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{r}-\frac{\Lambda r^2}{3}+\frac{GQ^2}{r^2}\right)\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{r}-\frac{\Lambda r^2}{3}+\frac{GQ^2}{r^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{49}$$

where M and Q are two constants: M is the total mass of the system and Q is the total electric charge of the system.

In the (49) there are singularities (event horizons) when:

$$1-\frac{2GM}{r}-\frac{\Lambda r^2}{3}+\frac{GQ^2}{r^2}=0 \tag{50}$$

On the other hand, by analogy with the (20) and the (37), we can infer that, when the cosmological constant Λ is greater than zero, the correct Reissner-Nordstrøm solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\mathrm{d}s^2=\left(1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}t_g^2-\frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 -r_g^2\mathrm{d}\theta_g^2-r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{51}$$

where M and Q are two constants: M is the total mass of the system and Q is the total electric charge of the system. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (51) is expressed in curved coordinates.

We can note that in the (51) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Analogously to our treatment of the correct Reissner-Nordstrøm solution when the cosmological constant Λ is equal to zero [2] we can express the correct Reissner-Nordstrøm solution, when the cosmological constant Λ is greater than zero, as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates. In this case we have:

$$\mathrm{d}s_g^2=\frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g^2}}\mathrm{d}t^2-\left(1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}r^2 -r^2\mathrm{d}\theta^2-r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{52}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, also here, r (or r_g) is always greater than zero since the Reissner-Nordstrøm solution when the cosmological constant is greater than zero is a solution for vacuum region surrounding an electrically charged mass.

As for the relation between r_g and r, we have:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \tag{53}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GM}{r_g}+\frac{\Lambda r_g^2}{3}+\frac{GQ^2}{r_g{}^2}}\mathrm{d}r_g^2 \tag{54}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{6r_g^3\mathrm{d}r_g}{3r_g^2+6GMr_g+\Lambda r_g^4+3GQ^2} \tag{55}$$

According to the (55) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (55). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (54).

For r_g not too large we have that the (55) is practically equal to the analogous formula with $\Lambda = 0$ [2], also because we know that the cosmological constant is very small. In particular, for $r_g \to 0^+$ we have $r \cong \frac{r_g^2}{\sqrt{2GQ^2}}$, as we can see easily from the (55).

Instead for $r_g \to +\infty$ we have that the term with the cosmological constant dominates. In this case from the (55) we have:

$$r^2 \cong \int_1^{r_g}\frac{6r_g^3\mathrm{d}r_g}{\Lambda r_g^4} = \frac{6}{\Lambda}\ln\left(r_g\right) \tag{56}$$

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expressions (51) and (52) for any value of $r > 0$ (or of $r_g > 0$). Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, also here in the (51) and in the (52) the coefficients of $\mathrm{d}t_g^2$ and of $\mathrm{d}t^2$ are always >0, therefore since the coefficients of $\mathrm{d}t_g^2$ and of $\mathrm{d}t^2$ are always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since in the (51) and in the (52) the coefficients of $\mathrm{d}r_g^2$ and of $\mathrm{d}r^2$ are always not positive, also here the spatial coordinate does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{dt_{g2}}{dt_{g1}} = \sqrt{\frac{1 + \frac{2GM}{r_{g1}} + \frac{\Lambda r_{g1}^2}{3} + \frac{GQ^2}{r_{g1}^2}}{1 + \frac{2GM}{r_{g2}} + \frac{\Lambda r_{g2}^2}{3} + \frac{GQ^2}{r_{g2}^2}}} \quad (57)$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/dt_{g1}$ and $\nu_2 = 1/dt_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1 + \frac{2GM}{r_{g1}} + \frac{\Lambda r_{g1}^2}{3} + \frac{GQ^2}{r_{g1}^2}}{1 + \frac{2GM}{r_{g2}} + \frac{\Lambda r_{g2}^2}{3} + \frac{GQ^2}{r_{g2}^2}}} \quad (58)$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (57) and (58) are different from those that can be obtained from the incorrect expression of the Reissner-Nordstrøm solution when the cosmological constant is greater than zero by means of an incorrect procedure [1]-[3]. In particular, in this case the incorrect formulas analogous of the two correct formulas (57) and (58) are respectively:

$$\frac{dt_2}{dt_1} = \sqrt{\frac{1 - \frac{2GM}{r_2} - \frac{\Lambda r_2^2}{3} + \frac{GQ^2}{r_2^2}}{1 - \frac{2GM}{r_1} - \frac{\Lambda r_1^2}{3} + \frac{GQ^2}{r_1^2}}} \quad (59)$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1 - \frac{2GM}{r_2} - \frac{\Lambda r_2^2}{3} + \frac{GQ^2}{r_2^2}}{1 - \frac{2GM}{r_1} - \frac{\Lambda r_1^2}{3} + \frac{GQ^2}{r_1^2}}} \quad (60)$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{dr}{dt} = \frac{1}{1 + \frac{2GM}{r_g} + \frac{\Lambda r_g^2}{3} + \frac{GQ^2}{r_g^2}} \quad (61)$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = 1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} + \frac{GQ^2}{r^2} \tag{62}$$

6. The Correct Kerr Solution When the Cosmological Constant Is Equal to Zero

As for the Kerr solution, that is the solution that represents the curved space-time geometry surrounding a rotating mass, we have that the commonly used expression when the cosmological constant is equal to zero is [3]:

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2 \mathrm{d}\theta^2 - \left(r^2 + a^2\right)\sin^2\theta \mathrm{d}\varphi^2 \\ - \frac{2GMr}{\rho^2}\left(\mathrm{d}t - a\sin^2\theta \mathrm{d}\varphi\right)^2 \end{aligned} \tag{63}$$

where ρ^2 and Δ are functions of r and θ:

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{64}$$

$$\Delta \equiv r^2 - 2GMr + a^2 \tag{65}$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass.

In the (63) there are singularities (event horizons) when:

$$r^2 - 2GMr + a^2 = 0 \tag{66}$$

that is, when:

$$r = GM \pm \sqrt{G^2M^2 - a^2} \tag{67}$$

Obviously there are real singularities (event horizons) only when $G^2M^2 - a^2 \geq 0$.

Moreover we have that the coefficient g_{00} is equal to zero when:

$$r^2 - 2GMr + a^2\cos^2\theta = 0 \tag{68}$$

that is, when:

$$r = GM \pm \sqrt{G^2M^2 - a^2\cos^2\theta} \tag{69}$$

Therefore there are two distinct surfaces where g_{00} is equal to zero. These surfaces are infinite-redshift surfaces. Obviously such surfaces really exist only where $G^2M^2 - a^2\cos^2\theta \geq 0$.

But the expression (63) is erroneous: in fact, by means of the fact that for $a = 0$ the correct Kerr solution when the cosmological constant is equal to zero must be equal to the correct Schwarzschild solution when the cosmological constant is equal to zero, we can infer, as we have already noted in [2], that, when the cosmological constant is equal to zero, the correct Kerr solution expressed as the formally flat metric in the commonly used coordinates ($ds^2 \equiv dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned} ds^2 = dt_g^2 - \frac{\rho^2}{\Delta} dr_g^2 - \rho^2 d\theta_g^2 - \left(r_g^2 + a^2\right)\sin^2\theta_g d\varphi_g^2 \\ + \frac{2GMr_g}{\rho^2}\left(dt_g - a\sin^2\theta_g d\varphi_g\right)^2 \end{aligned} \quad (70)$$

where ρ^2 and Δ are functions of r_g and θ_g :

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \quad (71)$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 \quad (72)$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (70) is expressed in curved coordinates.

We can note that in the (70) the coefficient of dt_g^2 is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expression (70) for any value of $r_g > 0$. Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

Furthermore, the coefficient of dt_g^2 in the (70) is always ≥1, therefore, since the coefficient of dt_g^2 is always not zero, there is not any infinite-redshift surface and, since the coefficient of dt_g^2 is always not negative, the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since the coefficient of dr_g^2 in the (70) is always not positive, also here the spatial coordinate r_g does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Now, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $d\theta_g = 0$

and $\mathrm{d}\varphi_g = 0$. In fact the (70) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g}{\rho^2}\right)\mathrm{d}t_g^2 - \frac{\rho^2}{\Delta} dr_g^2 \tag{73}$$

where ρ^2 and Δ are functions of r_g:

$$\rho^2 \equiv r_g^2 + a^2 \cos^2\theta_g \tag{74}$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 \tag{75}$$

Therefore the (73) can be written as:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{76}$$

Now by analogy with the previous cases [1] [2] we can write in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as:

$$\mathrm{d}s_g^2 = \frac{1}{1 + \frac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}t^2 - \left(1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}r^2 \tag{77}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$) and θ_g is a constant.

Obviously, also here, r (or r_g) is always greater than zero since the Kerr solution when the cosmological constant is equal to zero is a solution for vacuum region surrounding a rotating mass.

As for the relation between r_g and r, we have [2]:

$$\mathrm{d}r^2 = \frac{1}{1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{78}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{79}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g\left(r_g^2 + a^2\cos^2\theta_g\right)\mathrm{d}r_g}{r_g^2 + 2GMr_g + a^2} \tag{80}$$

From which, we have [2]:

$$\begin{aligned} r^2 = {} & r_g^2 - 4GMr_g + \left(-a^2\sin^2\theta_g + 4G^2M^2\right)\ln\left(1 + \frac{r_g^2 + 2GMr_g}{a^2}\right) \\ & + \frac{GM\left(2a^2 + a^2\sin^2\theta_g - 4G^2M^2\right)}{\sqrt{G^2M^2 - a^2}} \\ & \cdot\ln\left(\frac{GM + r_g - \sqrt{G^2M^2 - a^2}}{GM + r_g + \sqrt{G^2M^2 - a^2}}\cdot\frac{GM + \sqrt{G^2M^2 - a^2}}{GM - \sqrt{G^2M^2 - a^2}}\right) \end{aligned} \tag{81}$$

According to the (81) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (80). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (79).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (77) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \to 0^+$ in the case of $\cos^2\theta_g \neq 0$ we have $r \cong r_g\left|\cos\theta_g\right|$, while in the case of $\cos^2\theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2a^2}}$, as we can see easily from the (80).

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GMr_{g1}}{r_{g1}^2 + a^2\cos^2\theta_g}}{1+\frac{2GMr_{g2}}{r_{g2}^2 + a^2\cos^2\theta_g}}} \tag{82}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\frac{2GMr_{g1}}{r_{g1}^2 + a^2\cos^2\theta_g}}{1+\frac{2GMr_{g2}}{r_{g2}^2 + a^2\cos^2\theta_g}}} \tag{83}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (82) and (83) are different from those that can be obtained from the incorrect expression of the Kerr solution when the cosmological constant is equal to zero by means of an incorrect procedure [1]-[3]. In fact, in this case the incorrect formulas analogous of the two correct formulas (82) and (83) are respectively [1]-[3]:

$$\frac{dt_2}{dt_1} = \sqrt{\frac{1 - \frac{2GMr_2}{r_2^2 + a^2\cos^2\theta}}{1 - \frac{2GMr_1}{r_1^2 + a^2\cos^2\theta}}} \tag{84}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1 - \frac{2GMr_2}{r_2^2 + a^2\cos^2\theta}}{1 - \frac{2GMr_1}{r_1^2 + a^2\cos^2\theta}}} \tag{85}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{dr}{dt} = \sqrt{\frac{1}{\left(1 + \frac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}\right)\left(1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)}} \tag{86}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{dr}{dt} = \sqrt{\left(1 - \frac{2GMr}{r^2 + a^2\cos^2\theta}\right)\left(1 + \frac{-2GMr + a^2\sin^2\theta}{r^2 + a^2\cos^2\theta}\right)} \tag{87}$$

7. The Correct Kerr Solution When the Cosmological Constant Is Greater than Zero

By analogy with the (17) and the (63), we can infer that, when the cosmological constant Λ is greater than zero, the commonly accepted Kerr solution is:

$$\begin{aligned} ds^2 = {} & dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \left(r^2 + a^2\right)\sin^2\theta d\varphi^2 \\ & - \frac{2GMr + \frac{\Lambda}{3}r^4}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 \end{aligned} \tag{88}$$

where ρ^2 and Δ are functions of r and θ:

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{89}$$

$$\Delta \equiv r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2 \tag{90}$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass.

In the (88) there are singularities (event horizons) when:

$$r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2 = 0 \quad (91)$$

Moreover, in the (88) the coefficient g_{00} is equal to zero (and therefore we have infinite redshift) when:

$$r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2\cos^2\theta = 0 \quad (92)$$

On the other hand, by analogy with the (20) and the (70), we can infer that, when the cosmological constant Λ is greater than zero, the correct Kerr solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned}\mathrm{d}s^2 = {} & \mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 - \rho^2\mathrm{d}\theta_g^2 - \left(r_g^2 + a^2\right)\sin^2\theta_g\mathrm{d}\varphi_g^2 \\ & + \frac{2GMr_g + \frac{\Lambda}{3}r_g^4}{\rho^2}\left(\mathrm{d}t_g - a\sin^2\theta_g\mathrm{d}\varphi_g\right)^2\end{aligned} \quad (93)$$

where ρ^2 and Δ are functions of r_g and θ_g:

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \quad (94)$$

$$\Delta \equiv r_g^2 + 2GMr_g + \frac{\Lambda}{3}r_g^4 + a^2 \quad (95)$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (93) is expressed in curved coordinates.

We can note that in the (93) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expression (93) for any value of $r_g > 0$. Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, also here the coefficient of $\mathrm{d}t_g^2$ is always ≥1, therefore, since the coefficient of $\mathrm{d}t_g^2$ is always not zero, there is not any infinite-redshift surface and, since the coefficient of $\mathrm{d}t_g^2$ is always not negative, the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since the coefficient of $\mathrm{d}r_g^2$ is always not positive, also here the spatial coordinate r_g does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Now, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$. In fact the (93) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4}{\rho^2}\right)\mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 \tag{96}$$

where ρ^2 and Δ are functions of r_g:

$$\rho^2 \equiv r_g^2 + a^2 \cos^2\theta_g \tag{97}$$

$$\Delta \equiv r_g^2 + 2GMr_g + \frac{\Lambda}{3} r_g^4 + a^2 \tag{98}$$

Therefore the (96) can be written as:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{99}$$

Now by analogy with the previous cases [1] [2] we can write this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as:

$$\mathrm{d}s_g^2 = \frac{1}{1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}t^2 - \left(1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}r^2 \tag{100}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$) and θ_g is a constant.

Obviously, also here, r (or r_g) is always greater than zero since the Kerr solution when the cosmological constant is greater than zero is a solution for vacuum region surrounding a rotating mass.

As for the relation between r_g and r, we have:

$$\mathrm{d}r^2 = \frac{1}{1 + \frac{2GMr_g + \frac{\Lambda}{3} r_g^4 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{101}$$

From which, we have:

$$\int_0^{r^2} \mathrm{d}r^2 = \int_0^{r_g^2} \frac{1}{1+\frac{2GMr_g + \frac{\Lambda}{3} r_g^4 + a^2 \sin^2 \theta_g}{r_g^2 + a^2 \cos^2 \theta_g}} \mathrm{d}r_g^2 \tag{102}$$

From which, we have:

$$r^2 = \int_0^{r_g} \frac{6r_g \left(r_g^2 + a^2 \cos^2 \theta_g\right) \mathrm{d}r_g}{3r_g^2 + 3a^2 + 6GMr_g + \Lambda r_g^4} \tag{103}$$

According to the (103) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (103). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (102).

For r_g not too large we have that the (103) is practically equal to the analogous formula with $\Lambda = 0$ [2], also because we know that the cosmological constant is very small. In particular, for $r_g \to 0^+$ in the case of $\cos^2 \theta_g \neq 0$ we have $r \cong r_g \left|\cos \theta_g\right|$, while in the case of $\cos^2 \theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2a^2}}$, as we can see easily from the (103).

Instead for $r_g \to +\infty$ we have that the term with the cosmological constant dominates. In this case from the (103) we have:

$$r^2 \cong \int_1^{r_g} \frac{6r_g^3 \mathrm{d}r_g}{\Lambda r_g^4} = \frac{6}{\Lambda} \ln\left(r_g\right) \tag{104}$$

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GMr_{g1} + \frac{\Lambda}{3} r_{g1}^4}{r_{g1}^2 + a^2 \cos^2 \theta_g}}{1+\frac{2GMr_{g2} + \frac{\Lambda}{3} r_{g2}^4}{r_{g2}^2 + a^2 \cos^2 \theta_g}}} \tag{105}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\dfrac{2GMr_{g1}+\dfrac{\Lambda}{3}r_{g1}^{\ 4}}{r_{g1}^{\ 2}+a^2\cos^2\theta_g}}{1+\dfrac{2GMr_{g2}+\dfrac{\Lambda}{3}r_{g2}^{\ 4}}{r_{g2}^2+a^2\cos^2\theta_g}}} \tag{106}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (105) and (106) are different from those that can be obtained from the incorrect expression of the Kerr solution when the cosmological constant is greater than zero by means of an incorrect procedure [1]-[3]. In particular, in this case the incorrect formulas analogous of the two correct formulas (105) and (106) are respectively:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1-\dfrac{2GMr_2+\dfrac{\Lambda}{3}r_2^4}{r_2^2+a^2\cos^2\theta}}{1-\dfrac{2GMr_1+\dfrac{\Lambda}{3}r_1^4}{r_1^2+a^2\cos^2\theta}}} \tag{107}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1-\dfrac{2GMr_2+\dfrac{\Lambda}{3}r_2^4}{r_2^2+a^2\cos^2\theta}}{1-\dfrac{2GMr_1+\dfrac{\Lambda}{3}r_1^4}{r_1^2+a^2\cos^2\theta}}} \tag{108}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{\frac{1}{\left(1+\dfrac{2GMr_g+\dfrac{\Lambda}{3}r_g^4}{r_g^2+a^2\cos^2\theta_g}\right)\left(1+\dfrac{2GMr_g+\dfrac{\Lambda}{3}r_g^4+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}} \tag{109}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{\left(1-\frac{2GMr+\dfrac{\Lambda}{3}r^4}{r^2+a^2\cos^2\theta}\right)\left(1+\frac{-2GMr-\dfrac{\Lambda}{3}r^4+a^2\sin^2\theta}{r^2+a^2\cos^2\theta}\right)} \tag{110}$$

8. The Correct Kerr-Newman Solution When the Cosmological Constant Is Equal to Zero

As for the Kerr-Newman solution [3] [4], that is the solution that represents the curved space-time geometry surrounding an electrically charged rotating mass, we have that the commonly used expression when the cosmological constant is equal to zero is [4]:

$$\begin{aligned} ds^2 = {} & \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)d\varphi - a dt\right]^2 \\ & - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \end{aligned} \tag{111}$$

where ρ^2 and Δ are functions of r and θ:

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{112}$$

$$\Delta \equiv r^2 - 2GMr + a^2 + GQ^2 \tag{113}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass.

In the (111) there are singularities (event horizons) when:

$$r^2 - 2GMr + a^2 + GQ^2 = 0 \tag{114}$$

that is, when:

$$r = GM \pm \sqrt{G^2M^2 - a^2 - GQ^2} \tag{115}$$

Obviously there are real singularities (event horizons) only when $G^2M^2 - a^2 - GQ^2 \geq 0$.

Moreover, in the (111) we have that the coefficient g_{00} is equal to zero when:

$$r^2 - 2GMr + a^2\cos^2\theta + GQ^2 = 0 \tag{116}$$

that is, when:

$$r = GM \pm \sqrt{G^2M^2 - a^2\cos^2\theta - GQ^2} \tag{117}$$

Therefore there are two distinct surfaces where g_{00} is equal to zero. These surfaces are infinite-redshift surfaces. Obviously such surfaces really exist only where $G^2M^2 - a^2\cos^2\theta - GQ^2 \geq 0$.

But also here the expression (111) is erroneous: in fact, by means of the fact that the correct Kerr-Newman solution when the cosmological constant is equal to zero must be equal for $Q = 0$ to the correct Kerr solution when the cosmological constant is equal to zero, and must be equal for $a = 0$ to the correct Reissner-Nordstrøm solution when the cosmological constant is equal to zero, and must be equal for $Q = 0$ and $a = 0$ to the correct Schwarzschild solution when the cosmological constant is equal to zero, we can infer, as we have already noted in [2], that, when the cosmological constant is equal to zero, the correct Kerr-Newman solution expressed as the formally flat metric in the commonly used coordinates ($ds^2 \equiv dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\mathrm{d}s^2 = \frac{\Delta}{\rho^2}\left(\mathrm{d}t_g - a\sin^2\theta_g \mathrm{d}\varphi_g\right)^2 - \frac{\sin^2\theta_g}{\rho^2}\left[\left(r_g^2 + a^2\right)\mathrm{d}\varphi_g - a\mathrm{d}t_g\right]^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 - \rho^2\mathrm{d}\theta_g^2 \tag{118}$$

where ρ^2 and Δ are functions of r_g and θ_g:

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \tag{119}$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 + GQ^2 \tag{120}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (118) is expressed in curved coordinates.

We can note that in the (118) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expression (118) for any value of $r_g > 0$. Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

Moreover, also here the coefficient of $\mathrm{d}t_g^2$ in the (118) is always ≥1, therefore, since the coefficient of $\mathrm{d}t_g^2$ is always not zero, there is not any infinite-redshift surface and, since the coefficient of $\mathrm{d}t_g^2$ is always not negative, the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Furthermore, since the coefficient of $\mathrm{d}r_g^2$ in the (118) is always not positive, also here the spatial coordinate r_g does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Now, also in this case, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$. In fact the (118) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes:

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta_g}{\rho^2} dt_g^2 - \frac{\rho^2}{\Delta} dr_g^2 \tag{121}$$

where ρ^2 and Δ are functions of r_g:

$$\rho^2 \equiv r_g^2 + a^2 \cos^2\theta_g \tag{122}$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 + GQ^2 \tag{123}$$

Therefore the (121) can be written as:

$$ds^2 = \left(1 + \frac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}\right) dt_g^2 - \frac{1}{1 + \frac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}} dr_g^2 \tag{124}$$

Now by analogy with the previous cases [1] [2] we can write in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as:

$$ds_g^2 = \frac{1}{1 + \frac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}} dt^2 - \left(1 + \frac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right) dr^2 \tag{125}$$

where ds_g^2 is a metric formally flat in the curved coordinates (that is, $ds_g^2 \equiv dt_g^2 - dr_g^2 - r_g^2 d\theta_g^2 - r_g^2 \sin^2\theta_g d\varphi_g^2$) and θ_g is a constant.

Obviously, also here, r (or r_g) is always greater than zero since the Kerr-Newman solution when the cosmological constant is equal to zero is a solution for vacuum region surrounding an electrically charged rotating mass.

As for the relation between r_g and r, we have [2]:

$$dr^2 = \frac{1}{1 + \frac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}} dr_g^2 \tag{126}$$

From which, we have:

$$\int_0^{r^2} dr^2 = \int_0^{r_g^2} \frac{1}{1 + \frac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}} dr_g^2 \tag{127}$$

From which, we have:

$$r^2 = \int_0^{r_g} \frac{2r_g\left(r_g^2 + a^2\cos^2\theta_g\right) dr_g}{r_g^2 + 2GMr_g + GQ^2 + a^2} \tag{128}$$

From which, we have [2]:

$$\begin{aligned} r^2 &= r_g^2 - 4GMr_g + \left(4G^2M^2 - a^2\sin^2\theta_g - GQ^2\right) \cdot \ln\left(1 + \frac{r_g^2 + 2GMr_g}{GQ^2 + a^2}\right) \\ &+ \frac{GM\left(2a^2 + a^2\sin^2\theta_g + 3GQ^2 - 4G^2M^2\right)}{\sqrt{G^2M^2 - GQ^2 - a^2}} \\ &\cdot \ln\left(\frac{GM + r_g - \sqrt{G^2M^2 - GQ^2 - a^2}}{GM + r_g + \sqrt{G^2M^2 - GQ^2 - a^2}} \cdot \frac{GM + \sqrt{G^2M^2 - GQ^2 - a^2}}{GM - \sqrt{G^2M^2 - GQ^2 - a^2}}\right) \end{aligned} \tag{129}$$

According to the (129) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (128). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (127).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g \left(1 - \frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1 + \frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (125) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand for $r_g \to 0^+$ in the case of $\cos^2\theta_g \neq 0$ we have $r \cong r_g \sqrt{\frac{a^2\cos^2\theta_g}{GQ^2 + a^2}}$, while in the case of $\cos^2\theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2\left(GQ^2 + a^2\right)}}$, as we can see easily from the (128).

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1 + \dfrac{2GMr_{g1} + GQ^2}{r_{g1}^2 + a^2\cos^2\theta_g}}{1 + \dfrac{2GMr_{g2} + GQ^2}{r_{g2}^2 + a^2\cos^2\theta_g}}} \tag{130}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1 + \dfrac{2GMr_{g1} + GQ^2}{r_{g1}^2 + a^2\cos^2\theta_g}}{1 + \dfrac{2GMr_{g2} + GQ^2}{r_{g2}^2 + a^2\cos^2\theta_g}}} \tag{131}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (130) and (131) are different from those that can be obtained from the incorrect expression of the Kerr-Newman solution when the cosmological constant is equal to zero by means of an incorrect procedure [1]-[3]. In fact, in this case the incorrect formulas analogous of the two correct formulas (130) and (131) are respectively [1]-[3]:

$$\frac{dt_2}{dt_1} = \sqrt{\frac{1+\frac{-2GMr_2+GQ^2}{r_2^2+a^2\cos^2\theta}}{1+\frac{-2GMr_1+GQ^2}{r_1^2+a^2\cos^2\theta}}} \tag{132}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{1+\frac{-2GMr_2+GQ^2}{r_2^2+a^2\cos^2\theta}}{1+\frac{-2GMr_1+GQ^2}{r_1^2+a^2\cos^2\theta}}} \tag{133}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{dr}{dt} = \sqrt{\frac{1}{\left(1+\frac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}\right)\left(1+\frac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}} \tag{134}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{dr}{dt} = \sqrt{\left(1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}\right)\left(1+\frac{-2GMr+GQ^2+a^2\sin^2\theta}{r^2+a^2\cos^2\theta}\right)} \tag{135}$$

9. The Correct Kerr-Newman Solution When the Cosmological Constant Is Greater than Zero

By analogy with the (17) and the (111), we can infer that, when the cosmological constant Λ is greater than zero, the commonly accepted Kerr-Newman solution is:

$$\begin{aligned} ds^2 = {} & \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)d\varphi - a dt\right]^2 \\ & -\frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \end{aligned} \tag{136}$$

where ρ^2 and Δ are functions of r and θ:

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{137}$$

$$\Delta \equiv r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2 + GQ^2 \tag{138}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass.

In the (136) there are singularities (event horizons) when:

$$r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2 + GQ^2 = 0 \tag{139}$$

Moreover, in the (136) the coefficient g_{00} is equal to zero (and therefore we have infinite redshift) when:

$$r^2 - 2GMr - \frac{\Lambda}{3}r^4 + a^2\cos^2\theta + GQ^2 = 0 \tag{140}$$

On the other hand, by analogy with the (20) and the (118), we can infer that, when the cosmological constant Λ is greater than zero, the correct Kerr-Newman solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned}\mathrm{d}s^2 = {} & \frac{\Delta}{\rho^2}\left(\mathrm{d}t_g - a\sin^2\theta_g\mathrm{d}\varphi_g\right)^2 \\ & -\frac{\sin^2\theta_g}{\rho^2}\left[\left(r_g{}^2 + a^2\right)\mathrm{d}\varphi_g - a\mathrm{d}t_g\right]^2 \\ & -\frac{\rho^2}{\Delta}\mathrm{d}r_g^2 - \rho^2\mathrm{d}\theta_g^2\end{aligned} \tag{141}$$

where ρ^2 and Δ are functions of r_g and θ_g:

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \tag{142}$$

$$\Delta \equiv r_g^2 + 2GMr_g + \frac{\Lambda}{3}r_g^4 + a^2 + GQ^2 \tag{143}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this equation (141) is expressed in curved coordinates.

We can note that in the (141) the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

Also in this case, as in that of the correct Schwarzschild solution when the cosmological constant is equal to zero [1], we have not any singularity in the correct expression (141) for any value of $r_g > 0$. Consequently we can infer that also here there is not any event horizon and therefore there is not any black hole [3] [4].

In particular, also here the coefficient of $\mathrm{d}t_g^2$ is always ≥1, therefore, since the coefficient of $\mathrm{d}t_g^2$ is always not zero, there is not any infinite-redshift surface and, since the coefficient of $\mathrm{d}t_g^2$ is always not negative, the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Moreover, since the coefficient of $\mathrm{d}r_g^2$ is always not positive, also here the spatial coordinate r_g does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Therefore, also here the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Now, also in this case, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$. In fact the (141) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes:

$$\mathrm{d}s^2 = \frac{\Delta - a^2 \sin^2\theta_g}{\rho^2}\mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 \tag{144}$$

where ρ^2 and Δ are functions of r_g:

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \tag{145}$$

$$\Delta \equiv r_g^2 + 2GMr_g + \frac{\Lambda}{3}r_g^4 + a^2 + GQ^2 \tag{146}$$

Therefore the (144) can be written as:

$$\begin{aligned}\mathrm{d}s^2 = &\left(1 + \frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}t_g^2 \\ &- \frac{1}{1 + \frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2\end{aligned} \tag{147}$$

Now by analogy with the previous cases [1] [2] we can write this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as:

$$\begin{aligned}\mathrm{d}s_g^2 = &\frac{1}{1 + \frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}t^2 \\ &- \left(1 + \frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}r^2\end{aligned} \tag{148}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$) and θ_g is a constant.

Obviously, also here, r (or r_g) is always greater than zero since the Kerr-

Newman solution when the cosmological constant is greater than zero is a solution for vacuum region surrounding an electrically charged rotating mass.

As for the relation between r_g and r, we have:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{149}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GMr_g + \frac{\Lambda}{3}r_g^4 + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{150}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{6r_g\left(r_g^2 + a^2\cos^2\theta_g\right)\mathrm{d}r_g}{3r_g^2 + 3a^2 + 6GMr_g + \Lambda r_g^4 + 3GQ^2} \tag{151}$$

According to the (151) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (151). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (150).

For r_g not too large we have that the (151) is practically equal to the analogous formula with $\Lambda = 0$ [2], also because we know that the cosmological constant is very small. In particular, for $r_g \to 0^+$ in the case of $\cos^2\theta_g \neq 0$ we have $r \cong r_g\sqrt{\frac{a^2\cos^2\theta_g}{GQ^2 + a^2}}$, while in the case of $\cos^2\theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2\left(GQ^2 + a^2\right)}}$, as we can see easily from the (151).

Instead for $r_g \to +\infty$ we have that the term with the cosmological constant dominates. In this case from the (151) we have:

$$r^2 \cong \int_1^{r_g}\frac{6r_g^3\mathrm{d}r_g}{\Lambda r_g^4} = \frac{6}{\Lambda}\ln\left(r_g\right) \tag{152}$$

In this case the formula (8) of the correct ratio between the times when they are measured in two different positions becomes:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GMr_{g1} + \frac{\Lambda}{3}r_{g1}^4 + GQ^2}{r_{g1}^2 + a^2\cos^2\theta_g}}{1+\frac{2GMr_{g2} + \frac{\Lambda}{3}r_{g2}^4 + GQ^2}{r_{g2}^2 + a^2\cos^2\theta_g}}} \tag{153}$$

Therefore also in this case we have that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly). And the more intense the gravitational field is, the more slowly the clocks go (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

While (in this case) the formula (9) of the correct ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ becomes:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\dfrac{2GMr_{g1}+\dfrac{\Lambda}{3}r_{g1}^4+GQ^2}{r_{g1}^2+a^2\cos^2\theta_g}}{1+\dfrac{2GMr_{g2}+\dfrac{\Lambda}{3}r_{g2}^4+GQ^2}{r_{g2}^2+a^2\cos^2\theta_g}}} \tag{154}$$

This is the correct formula for the gravitational redshift of light in this case. Therefore, also here the gravitational redshift is always not infinite for any value of $r_g > 0$ (or of $r > 0$), as in the correct Schwarzschild solution when the cosmological constant is equal to zero [1].

We can note that the two correct formulas (153) and (154) are different from those that can be obtained from the incorrect expression of the Kerr-Newman solution when the cosmological constant is greater than zero by means of an incorrect procedure [1]-[3]. In particular, in this case the incorrect formulas analogous of the two correct formulas (153) and (154) are respectively:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1+\dfrac{-2GMr_2-\dfrac{\Lambda}{3}r_2^4+GQ^2}{r_2^2+a^2\cos^2\theta}}{1+\dfrac{-2GMr_1-\dfrac{\Lambda}{3}r_1^4+GQ^2}{r_1^2+a^2\cos^2\theta}}} \tag{155}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\dfrac{-2GMr_2-\dfrac{\Lambda}{3}r_2^4+GQ^2}{r_2^2+a^2\cos^2\theta}}{1+\dfrac{-2GMr_1-\dfrac{\Lambda}{3}r_1^4+GQ^2}{r_1^2+a^2\cos^2\theta}}} \tag{156}$$

In this case the formula (13) of the correct radial velocity of light in the commonly used coordinates [1]-[3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{\frac{1}{1+\dfrac{2GMr_g+\dfrac{\Lambda}{3}r_g^4+GQ^2}{r_g^2+a^2\cos^2\theta_g}}} \cdot \sqrt{\frac{1}{1+\dfrac{2GMr_g+\dfrac{\Lambda}{3}r_g^4+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}}} \tag{157}$$

We can note that, also here, the correct radial velocity of light in the commonly used coordinates is always ≤1, and is equal to 1 only when there is not a gravitational field.

On the other hand, in this case the formula (14) of the incorrect radial velocity of light in the commonly used coordinates [3] becomes:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{1+\frac{-2GMr-\frac{\Lambda}{3}r^4+GQ^2}{r^2+a^2\cos^2\theta}} \cdot\sqrt{1+\frac{-2GMr-\frac{\Lambda}{3}r^4+GQ^2+a^2\sin^2\theta}{r^2+a^2\cos^2\theta}} \tag{158}$$

10. The Consequences of These Correct Metrics

We have seen that these correct metrics of Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman when the cosmological constant is greater than zero do not imply any event horizon and, consequently, these metrics do not imply any black hole.

In fact, in these metrics there is not any singularity in general at any value of $r>0$ or of $r_g>0$.

Moreover, according to these metrics, for any value of $r>0$ or of $r_g>0$ the coefficient of $\mathrm{d}t^2$ and the coefficient of $\mathrm{d}t_g^2$ always remain positive and the coefficient of $\mathrm{d}r^2$ and the coefficient of $\mathrm{d}r_g^2$ always remain negative. Therefore, since the coefficient of $\mathrm{d}t^2$ and the coefficient of $\mathrm{d}t_g^2$ are always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4]. Furthermore, since the coefficient of $\mathrm{d}t^2$ and the coefficient of $\mathrm{d}t_g^2$ are always not equal to zero the gravitational redshift is always not infinite. On the other hand, since the coefficient of $\mathrm{d}r^2$ and the coefficient of $\mathrm{d}r_g^2$ are always not positive the spatial coordinate r or r_g does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [4].

Consequently, the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [4].

11. Experimental Prospects

11.1. The Available Experimental Data

As we have already noted in [1] [2], as for the experimental data obtained with the help of x-ray astronomy the proof that we have found black holes, and therefore event horizons, is based only on the fact that we have found invisible objects which have masses that are too great, according to the commonly accepted theory, for not being black holes [3] [6] [7]. But according to the correct theory of this article, whatever the masses and the dimensions of these invisible objects are, we never have black holes, and therefore we never have event horizons. Consequently such experimental data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

On the other hand, as we have already noted in [1] [2], with regard to the experimental data of the so-called gravitational waves (obtained by the LIGO collaboration) of a collision between two black holes, such gravitational waves were detected only below measurement errors, *i.e.* the signals detected were lower than the background noise (cf. chapter 6 of [6]. See also [8]). Furthermore the models expected from the theory were used for selecting the signals from the background noise (cf. chapter 6 of [6]) with the help of supercomputers: obviously, this is an incorrect practice which cannot produce any significant data. The awareness of the non-significance of the LIGO collaboration data is now widespread [8]-[12]. Obviously, also such data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

As for the alleged photos of black holes, as we have already noted in [1] [2], they were formed with the help of special algorithms from something compatible with the white noise. In other words, these photos were extracted from something compatible with the white noise only on the basis of the images that were expected by the researchers, with the help of appropriate algorithms loaded onto supercomputers (cf. the Section "Imaging a Black Hole" of [13]. See also [14]). Therefore, also in this case the researchers wanted to measure something that is below measurement errors, and so these photos are completely unreliable. On the other hand, serious doubts have now spread about the reliability of these photos [15]-[17]. Consequently such photos cannot prove anything and in particular cannot discriminate in any way between the commonly accepted theory and the same theory corrected according to this article.

Moreover, the corrections, that we have proposed in this article, to the commonly accepted theory are very small in the normal experimental situations (for example in the solar system), so the fact that, in these situations, so far no difference has been noted between the commonly accepted theory and the experimental results is not strange. In fact, in the usual case of $\frac{2GM}{r} \ll 1$, $\frac{GQ^2}{r^2}$ negligible compared to $\frac{2GM}{r}$, a^2 negligible compared to r^2 and the cosmological constant negligible, we have that the difference between the previsions of the commonly accepted theory and the previsions of the same theory corrected according to this article is, in the experiments commonly performed to test the General Theory of Relativity, only at the second order in $\frac{2GM}{r}$ [1] [2]. And all the experiments conducted so far in the solar system have not had errors so small as to test differences at the second order in $\frac{2GM}{r}$ [3].

Furthermore in this article we have taken into consideration the contribution to the metrics by the cosmological constant, but we know well that no cosmological constant has so far been directly measured, so the fact that there is so far no difference measured, with respect to the part due to the cosmological constant, between the previsions of the commonly accepted theory and the previsions of the same theory corrected according to this article should not be considered strange.

Therefore, in conclusion, there is no available experimental data that can discriminate between the commonly accepted theory and the same theory corrected according to this article.

11.2. Some Proposals for a Crucial Experiment

On the other hand, a crucial experiment could be done, which discriminates between the commonly accepted theory and the same theory corrected according to this article, by taking advantage of the high precision and sensitivity of the latest atomic clocks.

In fact, as we have shown in [1], when the cosmological constant is equal to zero the ratio of the passage of time in the gravitational field according to the correct Schwarzschild metric to that according to the commonly accepted Schwarzschild metric, in the case of $\frac{2GM}{r} \ll 1$, is approximately equal to $1+\frac{1}{2}\left(\frac{2GM}{r}\right)^2$.

Now throughout the solar system we have effectively that $\frac{2GM}{r} \ll 1$ and the cosmological constant is negligible.

The term $\frac{1}{2}\left(\frac{2GM}{r}\right)^2$ is obviously expressed in the case with the velocity of light $c \equiv 1$. Instead in the case more general (in which c is not defined equal to 1) this term becomes $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$. Now, the term $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$ due to the solar mass on the surface of the Sun is approximately equal to 8.99×10^{-12}, while at the average distance of the Earth from the Sun this value becomes approximately equal to 1.95×10^{-16}. Obviously, externally to the Sun, such term decreases with the square of the distance from the centre of the Sun according to the formula.

On the other hand, the same term due to the mass of the Earth on the surface of the Earth is approximately equal to 9.69×10^{-19} and obviously also here, externally to the Earth, decreases with the square of the distance from the centre of the Earth according to the formula.

Moreover, now we have atomic clocks that have an error of 7.6×10^{-21} [18] [19] and therefore, as we have already noted in [1], we can measure such differences between the predictions of the commonly accepted Schwarzschild metric when the cosmological constant is equal to zero and those of the same theory corrected according to this article with appropriate temporal measurements made in the solar system.

In particular, as we have already noted in [1], we could do a crucial experiment, which discriminates between the commonly accepted Schwarzschild metric when the cosmological constant is equal to zero and the same metric corrected according to this article, by taking one such atomic clock to diverse convenient locations in the solar system for comparing its time measurements made at those various locations with the corresponding time measurements made by another similar clock here on Earth.

Now, since the corrections of this article to the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero strictly depend on the corrections of this article to the Schwarzschild metric when the cosmological constant is equal to zero [1] [2], we can say that this crucial experiment would also discriminate between the commonly accepted theory about the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero and the same theory corrected according to this article.

On the other hand, in the usual experimental tests of the General Theory of Relativity, no direct gravitational force measurements have been used so far because they are not very precise.

However, as we have already noted in [20] [21], according to the Schwarzschild metric when the cosmological constant is equal to zero, in the usual case of $\frac{2GM}{r} \ll 1$, taking into account that in this case (as we have already noted) $r_g \cong r\left(1+\frac{2GM}{r}\right)$, the correct expression of the gravitational force would be approximately equal to [20] [21]:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{r}\right) \tag{159}$$

and the incorrect expression of the gravitational force would be approximately equal to [20] [21]:

$$F_{incorr}(r,m) \cong \frac{GMm}{r^2}\left(1+\frac{GM}{r}\right) \tag{160}$$

The formulas (159) and (160) are expressed in the case with the velocity of light $c \equiv 1$. Of course, in the case more general (in which c is not defined equal to 1) we have that the (159) and the (160) become respectively:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{rc^2}\right) \tag{161}$$

$$F_{incorr}(r,m) \cong \frac{GMm}{r^2}\left(1+\frac{GM}{rc^2}\right) \tag{162}$$

As we have already noted in [20] [21], although no gravitational force measurements have been used to test the General Theory of Relativity so far, one might consider doing so by sending probes close to the Sun where the difference between the (161) and the (162) and the differences of the (161) and of the (162) with respect to the Newtonian gravitational force are larger, and the GM product of the Sun's mass is known with a smaller relative uncertainty than the GM product of the Earth's mass [22]. In fact, the relative error on GM product of the Sun's mass is approximately equal to 1 on 1.33×10^{10} [22], while the value of $\frac{GM}{rc^2}$ on the surface of the Sun is approximatively equal to 2.12×10^{-6}.

For which, according to the right sides of the formulas (161) and (162), it is possible to measure the contribution due to the corrections made to the Newtonian gravitational force both in the case of the commonly accepted Schwarzschild metric when the cosmological constant is equal to zero and in the case of the same metric corrected according to this article, if we measure the distance from the centre of the Sun with sufficient accuracy (and, of course, we know the mass m quite precisely). Obviously, in order to do a crucial experiment between the commonly accepted Schwarzschild metric when the cosmological constant is equal to zero and the same metric corrected according to this article we would also need to measure the gravitational forces with sufficient accuracy.

Moreover, the best way to carry out such a crucial experiment would be to perform a series of measurements of the gravitational force at different distances from the centre of the Sun, in order to also measure the trend of the gravitational force as a function of these distances.

Now, here too, since the corrections of this article to the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero strictly depend on the corrections of this article to the Schwarzschild metric when the cosmological constant is equal to zero [1] [2] [20] [21], we can say that this crucial experiment would also discriminate between the commonly accepted theory about the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero and the same theory corrected according to this article.

12. General Conclusions

In this article, by starting from our corrections to the traditional Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is equal to zero [1] [2], we have obtained the formulations of the correct Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero.

All these corrections have been made assuming that the General Theory of Relativity is valid, *i.e.* without changing the General Theory of Relativity [1] [2].

As we have seen, the correct solutions of all these metrics do not entail any event horizon and, consequently, any black hole, since there is not any black hole without an event horizon [1] [2] [20] [21]. Therefore, this article confutes all the physics that on the basis of all these metrics foresees the possibility of the existence of event horizons and black holes [1]-[4] [20] [21] [23] [24].

Moreover, we can note that all these correct metrics are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [20] [21] [24]-[29], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [20] [21].

On the other hand, we have seen that there is no available experimental data that can discriminate between the commonly accepted theory about the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero and the same theory corrected according to this article.

However, we have noted that, in theory, it is possible to perform in the solar system some crucial experiments, that discriminate between the commonly accepted theory about the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero and the same theory corrected according to this article. Therefore, it would be appropriate to try to make one of such crucial experiments.

Finally, according to this article, all the physics that is based on the incorrect Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero should be modified on the basis of the correct formulas that we have obtained.

Obviously, the introduction of such new correct formulas for the Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics when the cosmological constant is greater than zero can have many applications both in the gravitational physics and in the analysis of astronomical data.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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