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The Gravitational Potential and the Gravitational Force According to the Correct Reissner-Nordstrøm, Kerr and Kerr-Newman Metrics

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Abstract

In a recent article, we have corrected the traditional derivation of the Schwarzschild metric, thus obtaining the formulation of the correct Schwarzschild metric, which is different from the traditional Schwarzschild metric. Then, in another article by starting from this correct Schwarzschild metric, we have corrected also the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics. On the other hand, in a third article, always by starting from this correct Schwarzschild metric, we have obtained the formulas of the correct gravitational potential and of the correct gravitational force in the case described by this metric. Now, in this article, by starting from these correct Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and proceeding in a manner analogous to this third article, we obtain the formulas of the correct gravitational potential and of the correct gravitational force in the cases described by these metrics. Moreover, we analyze these correct results and their consequences. Finally, we propose some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

Keywords

General Theory of Relativity, Schwarzschild, Reissner-Nordstrøm, Kerr, Kerr-Newman, Metric, Gravitational Potential, Gravitational Force, Orbital Motion

1. Introduction

In a recent article [1], we have corrected the traditional derivation of the Schwarzschild metric, thus obtaining the formulation of the correct Schwarzschild metric, which is different from the traditional Schwarzschild metric.

Then, in another article [2] by starting from this correct Schwarzschild metric we have corrected also the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics.

On the other hand, in a third article [3], always by starting from this correct Schwarzschild metric, we have obtained the formulas of the correct gravitational potential and of the correct gravitational force in the case described by this metric.

Now, in this article, by starting from these correct Reissner-Nordstrøm, Kerr and Kerr-Newman metrics, we want to obtain the formulas of the correct gravitational potential and of the correct gravitational force in the cases described by these metrics.

To do this, we will start from the analogous results obtained for the correct Schwarzschild metric [3] to extend by analogy these results to the correct Reissner-Nordstrøm, Kerr and Kerr-Newman metrics.

In particular, as in the article [3], we will obtain both the correct formulas of the gravitational potential and of the gravitational force related to the correct Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and the analogous incorrect formulas related to the incorrect Reissner-Nordstrøm, Kerr and Kerr-Newman metrics.

Moreover, we will analyze these results and their consequences.

Finally, we will see the experimental prospects, proposing in particular some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

2. The Gravitational Potential According to the Correct Schwarzschild Metric

In this article all the formulas are expressed with the velocity of light $c \equiv 1$, unless otherwise indicated.

We have shown in [1] that the correct Schwarzschild solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\mathrm{d}s^2 = \left(1+\frac{2GM}{r_g}\right)\mathrm{d}t_g^2 - \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{1}$$

where M is (a constant, which is equal to) the total mass of the system, and t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, the Equation (1) is expressed in curved coordinates.

On the other hand, we have also shown in [1] that the correct Schwarzschild solution expressed as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, is:

$$\mathrm{d}s_g^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{2}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, r (or r_g) is always greater than zero since the Schwarzschild solution is a solution for vacuum region surrounding a spherically symmetric mass distribution.

As for the relation between r_g and r, we have [1]:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{3}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{4}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g^2\mathrm{d}r_g}{r_g+2GM} \tag{5}$$

From which, we have [1]:

$$r^2 = r_g^2 - 4GMr_g + 8G^2M^2\ln\left(1+\frac{r_g}{2GM}\right) \tag{6}$$

According to the (6) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases as can be seen by differentiating the right side of the (6) with respect to r_g or directly from the (5). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as can be seen directly from the (4).

Furthermore, for $r_g \gg 2GM$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that for $\frac{2GM}{r} \ll 1$ the presence of r_g instead of r in the (2) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \ll 2GM$ we have $r^2 \cong \frac{r_g^3}{3GM}$.

Moreover, from the (3) we have:

$$2r\mathrm{d}r = \frac{2r_g\mathrm{d}r_g}{1+\frac{2GM}{r_g}} \tag{7}$$

From which, we have:

$$\frac{\mathrm{d}r_g}{\mathrm{d}r} = \frac{r}{r_g}\left(1+\frac{2GM}{r_g}\right) \tag{8}$$

Instead, the common erroneous expression for the Schwarzschild solution is [4]:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{r}\right)\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{9}$$

As we have already shown in [3], we can start from the equation (7.99) of the book of H. C. Ohanian and R. Ruffini [4] for the case of the Schwarzschild metric, which equation is:

$$\mathrm{e}^N\dot{t}^2 - \mathrm{e}^L\dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2 = 1 \tag{10}$$

with:

$$\dot{t} = \mathcal{E}\mathrm{e}^{-N} \tag{11}$$

$$\dot{\phi} = \frac{\ell}{r^2} \tag{12}$$

where $\mathcal{E}$ and ℓ are two constants, that are equal, respectively, to the energy per unit mass and to the angular momentum per unit mass.

By using the (11) and the (12) we can rewrite the (10) as [3]:

$$\mathcal{E}^2\mathrm{e}^{-N} - \mathrm{e}^L\dot{r}^2 - r^2\dot{\theta}^2 - \sin^2\theta\frac{\ell^2}{r^2} = 1 \tag{13}$$

In particular, we consider, as H. C. Ohanian and R. Ruffini [4], the case in which $\theta = \pi/2$. In this case the (13) becomes [3]:

$$\mathcal{E}^2\mathrm{e}^{-N} - \mathrm{e}^L\dot{r}^2 - \frac{\ell^2}{r^2} = 1 \tag{14}$$

Now, according to the common incorrect treatment of the Schwarzschild metric we have that $\mathrm{e}^{-L} = \mathrm{e}^N = 1-2GM/r$ [4], for which the (14) becomes [3] [4]:

$$\dot{r}^2 + \left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right) = \mathcal{E}^2 \tag{15}$$

In this equation, the second term on the left side plays the role of an effective gravitational potential [3] [4]. For which the expression of the effective gravitational potential according to the incorrect Schwarzschild solution is this expression [3] [4]:

$$\mathcal{V}_{incorr}(r) = \sqrt{\left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)} \tag{16}$$

This formula for the effective gravitational potential, which formula is present in the book of H. C. Ohanian and R. Ruffini [4], is perfectly equivalent to the analogous formulas that are present in the book of C. W. Misner, K. S. Thorne and J. A. Wheeler [5] and in the book of L. D. Landau and E. M. Lifšits [6].

But we know, as we have seen in [1], that the correct treatment of the Schwarzschild metric, when we use an expression analogue to that in which the linear theory is commonly expressed, provides instead that $\mathrm{e}^{-L} = \mathrm{e}^{N} = 1\Big/\left(1+\dfrac{2GM}{r_g}\right)$, since in this case we have the (2) instead of the (9), for which we have from the (14) not the Equation (15) but this other equation [3]:

$$\dot{r}^2 + \frac{1}{1+\dfrac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right) = \mathcal{E}^2 \tag{17}$$

We can note that the (17) is equal to the (15) at the first order in $\dfrac{2GM}{r}$.

Moreover, from the correct Equation (17) we can obtain, in a manner similar to that used by H. C. Ohanian and R. Ruffini [4] for obtaining the (16) from the incorrect Equation (15), that the correct expression of the effective gravitational potential is equal to [3]:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right)} \tag{18}$$

We can note that while the correct Expression (18) assumes positive real values for any value of r and r_g greater than zero, the incorrect Expression (16) instead assumes non-real values for the values of r less than $2GM$ and a value equal to zero for $r = 2GM$.

At large distances, taking into account that in this case $r_g \cong r$, we have [4]:

$$\begin{aligned}\mathcal{V}_{corr}(r) &= \sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right)} \\ &\cong \sqrt{\left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)} \cong 1-\frac{GM}{r}+\frac{\ell^2}{2r^2}\end{aligned} \tag{19}$$

Therefore in this case (that is, in the Newtonian limit) we have that the (18) and the (16) are approximately equal between them and are approximately equal to the Newtonian effective gravitational potential plus one [4].

From this we can infer that the (18) and the (16) are the analogue of the Newtonian effective gravitational potential plus one [3].

Now we consider the case in which $\ell = 0$. In this case the correct Expression (18) becomes:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}} \tag{20}$$

We can note that, according to the (20), $\mathcal{V}_{corr}(r)$ is always positive for any value of r and r_g greater than zero, and tends to zero for r (or r_g) that tends to zero.

While the incorrect Expression (16) becomes:

$$\mathcal{V}_{incorr}\left(r\right)=\sqrt{1-\frac{2GM}{r}} \tag{21}$$

We can note that, according to the (21), $\mathcal{V}_{incorr}\left(r\right)$ assumes non-real values for the values of r less than $2GM$ and a value equal to zero for $r=2GM$.

The correct Expression (20) in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), taking into account that in this case $r_g \cong r$, becomes:

$$\mathcal{V}_{corr}\left(r\right)=\sqrt{\frac{1}{1+\frac{2GM}{r_g}}}\cong\sqrt{1-\frac{2GM}{r}}\cong 1-\frac{GM}{r} \tag{22}$$

Therefore we have that in this case the correct Expression (20) becomes approximately equal to the incorrect Expression (21), and both the (20) and the (21) are approximatively equal to the Newtonian gravitational potential plus one.

From this we can infer that the (20) and the (21) are the analogue of the Newtonian gravitational potential plus one [3].

Consequently, from the (20) we can infer that the correct expression of the gravitational potential $V_{corr}\left(r\right)$ is [3]:

$$V_{corr}\left(r\right)=\sqrt{\frac{1}{1+\frac{2GM}{r_g}}}-1 \tag{23}$$

Obviously, according to the (23), $V_{corr}\left(r\right)$, in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (23), for $r_g \to +\infty$ (or $r \to +\infty$) $V_{corr}\left(r\right)\to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $V_{corr}\left(r\right)\to -1$.

Instead, from the (21) we can infer that the incorrect expression of the gravitational potential $V_{incorr}\left(r\right)$ is [3]:

$$V_{incorr}\left(r\right)=\sqrt{1-\frac{2GM}{r}}-1 \tag{24}$$

Obviously, also here, according to the (24), $V_{incorr}\left(r\right)$, in the case of $r \gg 2GM$, becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (24), $V_{incorr}\left(r\right)$ assumes non-real values for the values of r less than $2GM$ and a value equal to -1 for $r=2GM$. Furthermore, according to the (24), for $r \to +\infty$ $V_{incorr}\left(r\right)\to 0$.

On the other hand, the correct Expression (23) implies that the energy of a massive particle is equal to [3]:

$$E_{corr}\left(r\right)=m_0\gamma\left[V_{corr}\left(r\right)+1\right]=m_0\gamma\sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \tag{25}$$

Now the fact that $V_{corr}(r)+1$ is always greater than zero for any value of r and r_g greater than zero implies that a massive particle, for any value of r and r_g greater than zero, has always a correct escape velocity $v_{e_{corr}}$ less than the velocity of light c. In particular we have that the massive particle, in order to escape, must have a γ that multiplied by the value of $V_{corr}(r)+1$ be equal to one or to a number greater than one, in fact in this case the $E_{corr}(r)$ would be equal to or greater than the energy of rest mass of the same particle: a such value of γ is always possible if $V_{corr}(r)+1>0$ since γ can assume all the values ≥ 1.

More precisely we have:

$$\gamma\left(v_{e_{corr}}\right)\sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}}=1 \tag{26}$$

From which, we have:

$$\frac{1}{\sqrt{1-v_{e_{corr}}^{2}}}\sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}}=1 \tag{27}$$

From which, since the square of the velocity of a massive particle is always < 1, we have:

$$\frac{1}{1+\dfrac{2GM}{r_g}}=1-v_{e_{corr}}^{2} \tag{28}$$

From which, we have [3]:

$$v_{e_{corr}}=\sqrt{\frac{1}{1+\dfrac{r_g}{2GM}}} \tag{29}$$

We can note that, according to the (29), $v_{e_{corr}}$ is always less than 1 (that is, always less than the velocity of light c) and for $r_g \to +\infty$ (or $r \to +\infty$) $v_{e_{corr}} \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $v_{e_{corr}} \to 1$ (that is, tends to the velocity of light c).

On the other hand, a photon is always free to escape away by decreasing its frequency. In fact in this case the (25) becomes [3]:

$$E_{corr}(r)=h\nu\left[V_{corr}(r)+1\right]=h\nu\sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}} \tag{30}$$

And the photon can escape away with the new correct frequency $\nu_{0_{corr}}$ at the infinite equal to [3]:

$$\nu_{0_{corr}}=\nu\sqrt{\frac{1}{1+\dfrac{2GM}{r_g}}} \tag{31}$$

On the contrary, by using the incorrect Expression (24) instead of the correct Expression (23) we would have, in place of the (25), (30) and (31), the following incorrect formulas [3]:

$$E_{incorr}(r) = m_0\gamma\left[V_{incorr}(r)+1\right] = m_0\gamma\sqrt{1-\frac{2GM}{r}} \tag{32}$$

$$E_{incorr}(r) = h\nu\left[V_{incorr}(r)+1\right] = h\nu\sqrt{1-\frac{2GM}{r}} \tag{33}$$

$$\nu_{0_{incorr}} = \nu\sqrt{1-\frac{2GM}{r}} \tag{34}$$

We can note that these incorrect Formulas (32), (33) and (34) assume non-real values for the values of r less than $2GM$ and a value equal to zero for $r = 2GM$. This fact clearly implies that according to these incorrect formulas there is no possibility of escape for a massive particle or a photon for $r \le 2GM$.

As for the incorrect escape velocity $v_{e_{incorr}}$ of a massive particle for $r > 2GM$, according to the incorrect Formula (32), we have:

$$\gamma\left(v_{e_{incorr}}\right)\sqrt{1-\frac{2GM}{r}} = 1 \tag{35}$$

From which, we have:

$$\frac{1}{\sqrt{1-v_{e_{incorr}}^2}}\sqrt{1-\frac{2GM}{r}} = 1 \tag{36}$$

From which, since the square of the velocity of a massive particle is always < 1, we have:

$$1-\frac{2GM}{r} = 1-v_{e_{incorr}}^2 \tag{37}$$

From which, we have [3]:

$$v_{e_{incorr}} = \sqrt{\frac{2GM}{r}} \tag{38}$$

We can note that, according to the (38), taking into account that in this case $r > 2GM$, $v_{e_{corr}}$ is always less than 1 (that is, always less than the velocity of light c) and for $r \to +\infty$ $v_{e_{incorr}} \to 0$, and for $r \to 2GM^{+}$ $v_{e_{incorr}} \to 1$ (that is, tends to the velocity of light c).

Therefore, the correct formulas, contrary to the commonly accepted theory (that is, contrary to the incorrect formulas), do not entail any event horizon and, consequently, any black hole: in fact, as we have noted, according to the correct formulas (contrary to the incorrect formulas) the massive particles are always free to escape away with an escape velocity less than the velocity of light c and the photons are always free to escape away by decreasing their frequency.

On the other hand, the correct formulas, contrary to the commonly accepted theory, are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [3] [7]-[12], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [3].

3. The Gravitational Force According to the Correct Schwarzschild Metric

According to the traditional way of deriving the gravitational force from the gravitational potential, we can calculate the correct expression of the gravitational force on a unit mass $F_{corr}\left(r\right)$ directly from the Formula (23) of the correct gravitational potential, taking into account the (8) [3]:

$$\begin{aligned} F_{corr}\left(r\right) &= \frac{\mathrm{d}V_{corr}\left(r\right)}{\mathrm{d}r} \\ &= \frac{1}{2\sqrt{\frac{1}{1+\frac{2GM}{r_g}}}}\frac{\left(-1\right)}{\left(1+\frac{2GM}{r_g}\right)^2}2GM\left(-1\right)\frac{1}{r_g^2}\frac{r}{r_g}\left(1+\frac{2GM}{r_g}\right) \\ &= \frac{GMr}{r_g^3\sqrt{1+\frac{2GM}{r_g}}} \end{aligned} \tag{39}$$

Obviously, to obtain the correct expression $F_{corr}\left(r,m\right)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (39) by that generic mass [3]:

$$F_{corr}\left(r,m\right) = \frac{GMmr}{r_g^3\sqrt{1+\frac{2GM}{r_g}}} \tag{40}$$

Of course, in this Expression (40), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

According to the (40), $F_{corr}\left(r,m\right)$ assumes always real positive values for any value of r and r_g greater than zero.

Here we can note that, according to the (40), $F_{corr}\left(r,m\right)$ in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), taking into account that in this case $r_g \cong r$, becomes approximately equal to the Newtonian gravitational force and for $r_g \to +\infty$ (or $r \to +\infty$) tends to zero.

Moreover, according to the (40), in the case of $r_g \ll 2GM$ (or $r \ll 2GM$), taking into account that in this case $r^2 \cong \frac{r_g^3}{3GM}$, $F_{corr}\left(r,m\right)$ is approximately equal to $\frac{m}{3^{\frac{5}{6}}\sqrt{2}\left(GM\right)^{\frac{1}{3}}r^{\frac{2}{3}}}$ and therefore for $r_g \to 0^+$ (or $r \to 0^+$) tends to $+\infty$, *i.e.* the correct gravitational force tends to be infinitely attractive.

Now, in the case of a stable circular orbital motion, by considering the correct gravitational force expressed by the Formula (40) as the centripetal force of this motion we have [3]:

$$\frac{GMmr}{r_g^3\sqrt{1+\frac{2GM}{r_g}}} = m\frac{v^2}{r} \tag{41}$$

From which, we have:

$$\frac{GMr^2}{r_g^3\sqrt{1+\frac{2GM}{r_g}}}=v^2 \tag{42}$$

In the case of $r_g \ll 2GM$ (or $r \ll 2GM$), taking into account that in this case $r^2 \cong \frac{r_g^3}{3GM}$, the (42) becomes approximately:

$$\frac{1}{3}\sqrt{\frac{r_g}{2GM}} \cong v^2 \tag{43}$$

From the (43) we have that for $r_g \to 0^+$ (or $r \to 0^+$) $v \to 0$.

Instead in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), taking into account that in this case $r_g \cong r$, the (42) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{44}$$

From the (44) we have that for $r \to +\infty$ (or $r_g \to +\infty$) $v \to 0$.

On the other hand, by using the Formula (24) of the incorrect gravitational potential for calculating the incorrect expression of the gravitational force on a unit mass we have [3]:

$$F_{incorr}(r)=\frac{\mathrm{d}V_{incorr}(r)}{\mathrm{d}r}=\frac{1}{2\sqrt{1-\frac{2GM}{r}}}(-2GM)\left(-\frac{1}{r^2}\right)=\frac{GM}{r^2\sqrt{1-\frac{2GM}{r}}} \tag{45}$$

Obviously, to obtain the incorrect expression $F_{incorr}(r,m)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (45) by that generic mass [3]:

$$F_{incorr}(r,m)=\frac{GMm}{r^2\sqrt{1-\frac{2GM}{r}}} \tag{46}$$

Of course, also here, in this Expression (46), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that, according to the (46), $F_{incorr}(r,m)$ in the case of $r \gg 2GM$ becomes approximately equal to the Newtonian gravitational force and for $r \to +\infty$ tends to zero.

Moreover, according to the (46), $F_{incorr}(r,m)$ for $r \to 2GM^+$ tends to $+\infty$, *i.e.* tends to be infinitely attractive, and for the values of $r < 2GM$ assumes non-real values.

On the other hand, in the case of a stable circular orbital motion, by considering the incorrect gravitational force expressed by the Formula (46) as the centripetal force of this motion we have [3]:

$$\frac{GMm}{r^2\sqrt{1-\frac{2GM}{r}}}=m\frac{v^2}{r} \tag{47}$$

Now, we have that the left side of the (47) for the values of $r < 2GM$ assumes non-real values and for $r \to 2GM^+$ tends to $+\infty$, while the right side of the (47) always assumes finite real values for all the values of $r > 0$: for which there are not solutions of the (47) for the values of $r \le 2GM$.

Moreover, for the values of $r > 2GM$, from the (47) we have:

$$\frac{GM}{r\sqrt{1-\frac{2GM}{r}}} = v^2 \tag{48}$$

In the case of $r \gg 2GM$ the (48) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{49}$$

From the (49) we have that for $r \to +\infty$ $v \to 0$.

4. The Gravitational Potential According to the Correct Reissner-Nordstrøm Metric

As for the Reissner-Nordstrøm solution [4] [5], that is the solution that represents the curved space-time geometry surrounding an electrically charged mass, we have that the commonly used expressions for the space-time interval and the electric field are respectively [4]:

$$\begin{aligned} \mathrm{d}s^2 = \left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}\mathrm{d}r^2 \\ -r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \end{aligned} \tag{50}$$

and:

$$E(r) = \frac{Q}{r^2} \tag{51}$$

where M and Q are two constants: M is the total mass of the system and Q is the total electric charge of the system.

But the Expression (50) is erroneous: in fact, by means of the fact that the correct Reissner-Nordstrøm solution must be equal to the correct Schwarzschild solution for $Q = 0$, we can infer, as we have already noted in [2], that the correct Reissner-Nordstrøm solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned} \mathrm{d}s^2 = \left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}t_g^2 - \frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \\ -r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \end{aligned} \tag{52}$$

where M and Q are two constants: M is the total mass of the system and

Q is the total electric charge of the system. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this Equation (52) is expressed in curved coordinates.

Analogously to the case of the Schwarzschild metric, we can write also in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates. In this case we have [2]:

$$\begin{aligned}\mathrm{d}s_g^2 = {} & \frac{1}{1+\dfrac{2GM}{r_g}+\dfrac{GQ^2}{r_g^2}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right)\mathrm{d}r^2 \\ & -r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2\end{aligned} \tag{53}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$).

Obviously, r (or r_g) is always greater than zero since the Reissner-Nordstrøm solution is a solution for vacuum region surrounding an electrically charged mass.

As for the relation between r_g and r, we have [2]:

$$\mathrm{d}r^2 = \frac{1}{1+\dfrac{2GM}{r_g}+\dfrac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \tag{54}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1+\dfrac{2GM}{r_g}+\dfrac{GQ^2}{r_g^2}}\mathrm{d}r_g^2 \tag{55}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g^3\mathrm{d}r_g}{r_g^2+2GMr_g+GQ^2} \tag{56}$$

From which, we have [2]:

$$\begin{aligned}r^2 = {} & r_g^2 - 4GMr_g + G\left(4GM^2-Q^2\right)\ln\left(1+\frac{r_g^2+2GMr_g}{GQ^2}\right) \\ & +\frac{G^2M\left(3Q^2-4GM^2\right)}{\sqrt{G^2M^2-GQ^2}} \\ & \cdot\ln\left(\frac{GM+r_g-\sqrt{G^2M^2-GQ^2}}{GM+r_g+\sqrt{G^2M^2-GQ^2}}\cdot\frac{GM+\sqrt{G^2M^2-GQ^2}}{GM-\sqrt{G^2M^2-GQ^2}}\right)\end{aligned} \tag{57}$$

According to the (57) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (56). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (55).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (53) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \to 0^+$ we have $r \cong \frac{r_g^2}{\sqrt{2GQ^2}}$, as we can see easily from the (56).

Moreover, from the (54) we have:

$$2r\mathrm{d}r = \frac{2r_g\mathrm{d}r_g}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}} \tag{58}$$

From which, we have:

$$\frac{\mathrm{d}r_g}{\mathrm{d}r} = \frac{r}{r_g}\left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right) \tag{59}$$

Now, by analogy with the case of the correct Schwarzschild metric [3] we can infer that the correct expression of the gravitational potential in this case is:

$$V_{corr}\left(r\right) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}}-1 \tag{60}$$

We can see that the (60) for $Q=0$ becomes equal to the (23).

On the other hand, we can note that, according to the (60), $-1 < V_{corr}\left(r\right) < 0$ for any value of $r > 0$ and of $r_g > 0$.

Obviously, according to the (60), $V_{corr}\left(r\right)$, in the case of $Q=0$ and $r_g \gg 2GM$ (or $r \gg 2GM$), taking into account that in this case $r_g \cong r$, becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (60), for $r_g \to +\infty$ (or $r \to +\infty$) $V_{corr}\left(r\right) \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $V_{corr}\left(r\right) \to -1$.

Instead, always by analogy with the case of the incorrect Schwarzschild metric [3] we can infer that the incorrect expression of the gravitational potential in this case is:

$$V_{incorr}\left(r\right) = \sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}-1 \tag{61}$$

Also here, we can see that the (61) for $Q=0$ becomes equal to the (24).

On the other hand, we can note that, according to the (61), $V_{incorr}\left(r\right)$ assumes non-real values when $1-\frac{2GM}{r}+\frac{GQ^2}{r^2} < 0$ (that is, when $GM^2 > Q^2$ and $GM - \sqrt{G^2M^2 - GQ^2} < r < GM + \sqrt{G^2M^2 - GQ^2}$) and a value equal to −1 when

$1-\frac{2GM}{r}+\frac{GQ^2}{r^2}=0$ (that is, when $GM^2 \geq Q^2$ and $r = GM \pm \sqrt{G^2M^2 - GQ^2}$).

Obviously, also here, according to the (61), $V_{incorr}(r)$, in the case of $Q=0$ and $r \gg 2GM$, becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (61), for $r \to +\infty$ $V_{incorr}(r) \to 0$ and, if $Q \neq 0$, for $r \to 0^+$ $V_{incorr}(r) \to +\infty$.

Furthermore the (60) implies that the correct energy of a massive particle has the form of:

$$E_{corr}(r) = m_0\gamma\left[V_{corr}(r)+1\right] = m_0\gamma\sqrt{\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \tag{62}$$

Now the fact that $V_{corr}(r)+1$ is always greater than zero for any value of r and r_g greater than zero implies that a massive particle, for any value of r and r_g greater than zero, has always a correct escape velocity $v_{e_{corr}}$ less than the velocity of light c. In particular we have that the massive particle, in order to escape, must have a γ that multiplied by the value of $V_{corr}(r)+1$ be equal to one or to a number greater than one, in fact in this case the $E_{corr}(r)$ would be equal to or greater than the energy of rest mass of the same particle: a such value of γ is always possible if $V_{corr}(r)+1>0$ since γ can assume all the values ≥ 1.

More precisely, in this case, by analogy with the case of the correct Schwarzschild metric [3], we have:

$$\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}} = 1 - v_{e_{corr}}^2 \tag{63}$$

From which, we have:

$$v_{e_{corr}} = \sqrt{\frac{\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \tag{64}$$

We can note that, according to the (64), $v_{e_{corr}}$ is always less than 1 (that is, always less than the velocity of light c) and for $r_g \to +\infty$ (or $r \to +\infty$) $v_{e_{corr}} \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $v_{e_{corr}} \to 1$ (that is, tends to the velocity of light c).

On the other hand a photon is always free to escape away by decreasing its frequency. In fact in this case the (62) becomes:

$$E_{corr}(r) = h\nu\left[V_{corr}(r)+1\right] = h\nu\sqrt{\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \tag{65}$$

And the photon can escape away with the new frequency ν_0 at the infinite equal to:

$$\nu_{0_{corr}} = \nu\sqrt{\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \tag{66}$$

On the contrary, by using the incorrect Expression (61) instead of the correct Expression (60) we would have, in place of the (62), (65) and (66), the following incorrect formulas:

$$E_{incorr}(r) = m_0\gamma\left[V_{incorr}(r)+1\right] = m_0\gamma\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}} \tag{67}$$

$$E_{incorr}(r) = h\nu\left[V_{incorr}(r)+1\right] = h\nu\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}} \tag{68}$$

$$\nu_{0_{incorr}} = \nu\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}} \tag{69}$$

We can note that these incorrect Formulas (67), (68) and (69) assume non-real values when $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}<0$ (that is, when $GM^2>Q^2$ and $GM-\sqrt{G^2M^2-GQ^2}<r<GM+\sqrt{G^2M^2-GQ^2}$) and a value equal to zero when $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}=0$ (that is, when $GM^2\geq Q^2$ and $r=GM\pm\sqrt{G^2M^2-GQ^2}$). This fact clearly implies that according to these incorrect formulas there is no possibility of escape for a massive particle or a photon when $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\leq 0$ and also for all values of r less than any value of r for which $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\leq 0$ since to escape a particle would have to pass through all the larger radius values without getting trapped at any radius (that is, when $GM^2\geq Q^2$ and $r\leq GM+\sqrt{G^2M^2-GQ^2}$).

As for the incorrect escape velocity $v_{e_{incorr}}$ of an untrapped massive particle when $1>1-\frac{2GM}{r}+\frac{GQ^2}{r^2}>0$, according to the incorrect Formula (67), by analogy with the case of the incorrect Schwarzschild metric [3], we have:

$$1-\frac{2GM}{r}+\frac{GQ^2}{r^2}=1-v_{e_{incorr}}^2 \tag{70}$$

From which, we have:

$$v_{e_{incorr}} = \sqrt{\frac{2GM}{r}-\frac{GQ^2}{r^2}} \tag{71}$$

We can note that, in the (71), because of the condition $1>1-\frac{2GM}{r}+\frac{GQ^2}{r^2}>0$, $0<\frac{2GM}{r}-\frac{GQ^2}{r^2}<1$, for which $v_{e_{incorr}}$ is always less than one (that is, always less than the velocity of light c).

Moreover, according to the (71), for $r \to +\infty$ $v_{e_{incorr}} \to 0$.

On the other hand, an untrapped massive particle when $1-\frac{2GM}{r}+\frac{GQ^2}{r^2} \ge 1$ does not need to lose kinetic energy to free itself.

Therefore, the correct formulas, contrary to the commonly accepted theory (that is, contrary to the incorrect formulas), do not entail any event horizon and, consequently, any black hole: in fact, as we have noted, according to the correct formulas (contrary to the incorrect formulas) the massive particles are always free to escape away with an escape velocity less than the velocity of light c and the photons are always free to escape away by decreasing their frequency.

On the other hand, the correct formulas, contrary to the commonly accepted theory, are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [3] [7]-[12], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [3].

5. The Gravitational Force According to the Correct Reissner-Nordstrøm Metric

According to the traditional way of deriving the gravitational force from the gravitational potential, we can calculate the correct expression of the gravitational force on a unit mass $F_{corr}(r)$ directly from the Formula (60) of the correct gravitational potential, taking into account the (59):

$$\begin{aligned} F_{corr}(r) &= \frac{\mathrm{d}V_{corr}(r)}{\mathrm{d}r} \\ &= \frac{1}{2\sqrt{\frac{1}{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}}} \frac{(-1)}{\left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right)^2} \\ &\quad \cdot \left[2GM\frac{(-1)}{r_g^2}+GQ^2\frac{(-2)}{r_g^3}\right]\frac{r}{r_g}\left(1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}\right) \\ &= \frac{\frac{GMr}{r_g^3}+\frac{GQ^2 r}{r_g^4}}{\sqrt{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \end{aligned} \tag{72}$$

Obviously, to obtain the correct expression $F_{corr}(r,m)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (72) by that generic mass:

$$F_{corr}(r,m) = \frac{\frac{GMmr}{r_g^3}+\frac{GQ^2 mr}{r_g^4}}{\sqrt{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}} \tag{73}$$

Of course, in this Expression (73), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

We can note that the (73) for $Q=0$ becomes equal to the (40).

According to the (73), $F_{corr}(r,m)$ assumes always real positive values for any value of r and r_g greater than zero.

Moreover, according to the (73), $F_{corr}(r,m)$ at large distances, taking into account that in this case $r_g \cong r$, becomes approximately equal to the Newtonian gravitational force and for $r_g \to +\infty$ (or $r \to +\infty$) tends to zero.

On the other hand, according to the (73), for $r_g \to 0^+$ (or $r \to 0^+$), taking into account that in this case $r \cong \frac{r_g^2}{\sqrt{2GQ^2}}$, $F_{corr}(r,m)$ is approximately equal to $\frac{m}{\sqrt{2}\left(2GQ^2\right)^{\frac{1}{4}}}\frac{1}{\sqrt{r}}$ and tends to $+\infty$, *i.e.* the correct gravitational force tends to be infinitely attractive.

Now, in the case of a stable circular orbital motion, by considering the gravitational force expressed by the correct Formula (73) as the centripetal force of this motion we have:

$$\frac{\frac{GMmr}{r_g^3}+\frac{GQ^2mr}{r_g^4}}{\sqrt{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}}=m\frac{v^2}{r} \tag{74}$$

From which, we have:

$$\frac{\frac{GMr^2}{r_g^3}+\frac{GQ^2r^2}{r_g^4}}{\sqrt{1+\frac{2GM}{r_g}+\frac{GQ^2}{r_g^2}}}=v^2 \tag{75}$$

Moreover for $r_g \to 0^+$ (or $r \to 0^+$), taking into account that in this case $r \cong \frac{r_g^2}{\sqrt{2GQ^2}}$, the (75) becomes approximately:

$$\frac{r_g}{2\sqrt{GQ^2}} \cong v^2 \tag{76}$$

From the (76) we have that for $r_g \to 0^+$ (or $r \to 0^+$) also $v \to 0$.

Instead for $r_g \to +\infty$ (or $r \to +\infty$), taking into account that in this case $r_g \cong r$, the (75) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{77}$$

From the (77) we have that for $r \to +\infty$ (or $r_g \to +\infty$) $v \to 0$.

On the other hand, by using the Formula (61) of the incorrect gravitational potential for calculating the incorrect expression of the gravitational force on a unit mass we have:

$$\begin{aligned}F_{incorr}\left(r\right)&=\frac{\mathrm{d}V_{incorr}\left(r\right)}{\mathrm{d}r}\\&=\frac{1}{2\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}}\left[\left(-2GM\right)\left(-\frac{1}{r^2}\right)+GQ^2\frac{\left(-2\right)}{r^3}\right]\\&=\frac{\frac{GM}{r^2}-\frac{GQ^2}{r^3}}{\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}}\end{aligned}\tag{78}$$

Obviously, to obtain the incorrect expression $F_{incorr}\left(r,m\right)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (78) by that generic mass:

$$F_{incorr}\left(r,m\right)=\frac{\frac{GMm}{r^2}-\frac{GQ^2m}{r^3}}{\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}}\tag{79}$$

Of course, also here, in this Expression (79), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that the (79) for $Q=0$ becomes equal to the (46).

According to the Expression (79), $F_{incorr}\left(r,m\right)$ is equal to zero when $Q^2=Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\neq 0$, assumes non-real values when $Q^2\neq Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}<0$, is negative when $Q^2>Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}>0$, and is positive when $Q^2<Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}>0$.

Moreover, according to the (79), $F_{incorr}\left(r,m\right)$ for $r\to+\infty$ becomes approximately equal to the Newtonian gravitational force and tends to zero.

On the other hand, according to the (79), for $r\to0^+$ $F_{incorr}\left(r,m\right)$ is approximately equal to $-\frac{\sqrt{GQ^2}m}{r^2}$ and tends to $-\infty$, *i.e.* the incorrect gravitational force tends to be infinitely repulsive.

Furthermore, in the case of a stable circular orbital motion, by considering the incorrect gravitational force expressed by the Formula (79) as the centripetal force of this motion we have:

$$\frac{\frac{GMm}{r^2}-\frac{GQ^2m}{r^3}}{\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}}=m\frac{v^2}{r}\tag{80}$$

From which, we have:

$$\frac{\frac{GM}{r}-\frac{GQ^2}{r^2}}{\sqrt{1-\frac{2GM}{r}+\frac{GQ^2}{r^2}}}=v^2\tag{81}$$

Now, we have that the left side of the (81) assumes non-real values when $Q^2 \neq Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}<0$, and is negative when $Q^2 > Mr$ and $1-\frac{2GM}{r}+\frac{GQ^2}{r^2}>0$: in these cases there are not solutions of the (81) since $0 \leq v^2 \leq 1$.

Moreover for $r \to 0^+$ the (81) becomes approximately:

$$-\frac{\sqrt{GQ^2}}{r} \cong v^2 \tag{82}$$

From the (82) we have that for $r \to 0^+$ there is not solution since the left side of the (82) tends to $-\infty$ and $0 \leq v^2 \leq 1$.

On the other hand, for $r \to +\infty$ the (81) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{83}$$

From the (83) we have that for $r \to +\infty$ $v \to 0$.

6. The Gravitational Potential According to the Correct Kerr Metric

As for the Kerr solution, that is the solution that represents the curved space-time geometry surrounding a rotating mass, we have that the commonly used expression is [4]:

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2\mathrm{d}\theta^2 - \left(r^2+a^2\right)\sin^2\theta\mathrm{d}\varphi^2 \\ -\frac{2GMr}{\rho^2}\left(\mathrm{d}t - a\sin^2\theta\mathrm{d}\varphi\right)^2 \end{aligned} \tag{84}$$

where ρ^2 and Δ are functions of r and θ:

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{85}$$

$$\Delta \equiv r^2 - 2GMr + a^2 \tag{86}$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass.

But the Expression (84) is erroneous: in fact, by means of the fact that the correct Kerr solution must be equal to the correct Schwarzschild solution for $a = 0$, we can infer, as we have already noted in [2], that the correct Kerr solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 - \rho^2\mathrm{d}\theta_g^2 - \left(r_g^2+a^2\right)\sin^2\theta_g\mathrm{d}\varphi_g^2 \\ +\frac{2GMr_g}{\rho^2}\left(\mathrm{d}t_g - a\sin^2\theta_g\mathrm{d}\varphi_g\right)^2 \end{aligned} \tag{87}$$

where ρ^2 and Δ are functions of r_g and θ_g:

$$\rho^2 \equiv r_g^2 + a^2 \cos^2\theta_g \tag{88}$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 \tag{89}$$

and M and a are constants: M is the total mass of the system and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this Equation (87) is expressed in curved coordinates.

Now, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$. In fact the (87) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes [2]:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g}{\rho^2}\right)\mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 \tag{90}$$

where ρ^2 and Δ are functions of r_g:

$$\rho^2 \equiv r_g^2 + a^2 \cos^2\theta_g \tag{91}$$

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 \tag{92}$$

Therefore the (90) can be written as [2]:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \dfrac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{93}$$

Now by analogy with the previous cases we can write in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as [2]:

$$\mathrm{d}s_g^2 = \frac{1}{1 + \dfrac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}t^2 - \left(1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}r^2 \tag{94}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$) and θ_g is a constant.

Obviously, r (or r_g) is always greater than zero since the Kerr solution is a solution for vacuum region surrounding a rotating mass.

As for the relation between r_g and r, we have [2]:

$$\mathrm{d}r^2 = \frac{1}{1 + \dfrac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{95}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1 + \dfrac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{96}$$

From which, we have:

$$r^2 = \int_0^{r_g} \frac{2r_g\left(r_g^2 + a^2\cos^2\theta_g\right)\mathrm{d}r_g}{r_g^2 + 2GMr_g + a^2} \tag{97}$$

From which, we have [2]:

$$\begin{aligned} r^2 = {} & r_g^2 - 4GMr_g + \left(-a^2\sin^2\theta_g + 4G^2M^2\right)\ln\left(1 + \frac{r_g^2 + 2GMr_g}{a^2}\right) \\ & + \frac{GM\left(2a^2 + a^2\sin^2\theta_g - 4G^2M^2\right)}{\sqrt{G^2M^2 - a^2}} \\ & \cdot \ln\left(\frac{GM + r_g - \sqrt{G^2M^2 - a^2}}{GM + r_g + \sqrt{G^2M^2 - a^2}} \cdot \frac{GM + \sqrt{G^2M^2 - a^2}}{GM - \sqrt{G^2M^2 - a^2}}\right) \end{aligned} \tag{98}$$

According to the (98) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (97). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (96).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g\left(1 - \frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1 + \frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (94) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \to 0^+$ in the case of $\cos^2\theta_g \neq 0$ we have $r \cong r_g\left|\cos\theta_g\right|$, while in the case of $\cos^2\theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2a^2}}$, as we can see easily from the (97).

Moreover, from the (95) we have:

$$2r\mathrm{d}r = \frac{2r_g\mathrm{d}r_g}{1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}} \tag{99}$$

From which, we have:

$$\frac{\mathrm{d}r_g}{\mathrm{d}r} = \frac{r}{r_g}\left(1 + \frac{2GMr_g + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right) \tag{100}$$

Now, also here, by analogy with the case of the correct Schwarzschild metric [3] we can infer that the correct expression of the gravitational potential $V_{corr}\left(r\right)$ in this case is equal to:

$$V_{corr}\left(r\right) = \sqrt{\frac{1}{1 + \frac{2GMr_g}{r_g^2 + a^2\cos^2\theta_g}}} - 1 \tag{101}$$

We can see that the (101) for $a = 0$ becomes equal to the (23).

On the other hand, we can note that, according to the (101), $-1<V_{corr}\left(r\right)<0$ for any value of $r>0$ and of $r_g>0$.

Obviously, according to the (101), in the case of $r_g \gg 2GM$ (or $r \gg 2GM$) and $a=0$, taking into account that in this case $r \cong r_g$, $V_{corr}\left(r\right)$ becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (101), $V_{corr}\left(r\right)\to 0$ both for $r_g \to +\infty$ (or $r\to+\infty$) and, in the case of $\cos^2\theta_g \neq 0$, for $r_g \to 0^+$ (or $r\to 0^+$). On the other hand, in the case of $\cos^2\theta_g=0$, according to the (101), $V_{corr}\left(r\right)\to -1$ for $r_g\to 0^+$ (or $r\to 0^+$).

Instead, always by analogy with the case of the incorrect Schwarzschild metric [3], we can infer that the incorrect expression of the gravitational potential $V_{incorr}\left(r\right)$ is equal to:

$$V_{incorr}\left(r\right)=\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}}-1 \tag{102}$$

We can see that the (102) for $a=0$ becomes equal to the (24).

Obviously, also here, according to the (102), $V_{incorr}\left(r\right)$, in the case of $r\gg 2GM$ and $a=0$, becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (102), $V_{incorr}\left(r\right)$ assumes non-real values when $1-\frac{2GMr}{r^2+a^2\cos^2\theta}<0$ (that is, when $G^2M^2>a^2\cos^2\theta$ and $GM-\sqrt{G^2M^2-a^2\cos^2\theta}<r<GM+\sqrt{G^2M^2-a^2\cos^2\theta}$) and a value equal to −1 when $1-\frac{2GMr}{r^2+a^2\cos^2\theta}=0$ (that is, when $G^2M^2\geq a^2\cos^2\theta$ and $r=GM\pm\sqrt{G^2M^2-a^2\cos^2\theta}$).

Furthermore, according to the (102), $V_{incorr}\left(r\right)\to 0$ both for $r\to+\infty$ and, in the case of $\cos^2\theta\neq 0$, for $r\to 0^+$. On the other hand, in the case of $\cos^2\theta=0$, according to the (102), $V_{incorr}\left(r\right)$ assumes non-real values for $r<2GM$.

On the other hand, the (101) implies that the correct energy of a massive particle has the form of:

$$E_{corr}\left(r\right)=m_0\gamma\left[V_{corr}\left(r\right)+1\right]=m_0\gamma\sqrt{\frac{1}{1+\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}} \tag{103}$$

Now the fact that $V_{corr}\left(r\right)+1$ is always greater than zero for any value of r and r_g greater than zero implies that a massive particle, for any value of r and r_g greater than zero, has always a correct escape velocity $v_{e_{corr}}$ less than the velocity of light c. In particular we have that the massive particle, in order to escape, must have a γ that multiplied by the value of $V_{corr}\left(r\right)+1$ be equal to one or to a number greater than one, in fact in this case the $E_{corr}\left(r\right)$ would be equal to or greater than the energy of rest mass of the same particle: a such value of γ is always possible if $V_{corr}\left(r\right)+1>0$ since γ can assume all the values ≥1.

More precisely, in this case, by analogy with the case of the correct Schwarzschild metric [3], we have:

$$\frac{1}{1+\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}=1-v_{e_{corr}}^2 \tag{104}$$

From which, we have:

$$v_{e_{corr}}=\sqrt{\frac{\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}{1+\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}} \tag{105}$$

We can note that, according to the (105), $v_{e_{corr}}$ is always less than 1 (that is, always less than the velocity of light c) and tends to zero both for $r_g\to+\infty$ (or $r\to+\infty$) and, in the case of $\cos^2\theta_g\neq0$, for $r_g\to0^+$ (or $r\to0^+$). Instead, in the case of $\cos^2\theta_g=0$, according to the (105), $v_{e_{corr}}\to1$ (that is, tends to the velocity of light c) for $r_g\to0^+$ (or $r\to0^+$).

On the other hand, a photon is always free to escape away by decreasing its frequency. In fact in this case the (103) becomes:

$$E_{corr}(r)=h\nu\left[V_{corr}(r)+1\right]=h\nu\sqrt{\frac{1}{1+\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}} \tag{106}$$

And the photon can escape away with the new frequency ν_0 at the infinite equal to:

$$\nu_{0_{corr}}=\nu\sqrt{\frac{1}{1+\frac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}} \tag{107}$$

On the contrary, by using the incorrect Expression (102) instead of the correct Expression (101) we would have, in place of the (103), (106) and (107), the following incorrect formulas:

$$E_{incorr}(r)=m_0\gamma\left[V_{incorr}(r)+1\right]=m_0\gamma\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}} \tag{108}$$

$$E_{incorr}(r)=h\nu\left[V_{incorr}(r)+1\right]=h\nu\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}} \tag{109}$$

$$\nu_{0_{incorr}}=\nu\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}} \tag{110}$$

We can note that these incorrect Formulas (108), (109) and (110) assume non-real values when $1-\frac{2GMr}{r^2+a^2\cos^2\theta}<0$ (that is, when $G^2M^2>a^2\cos^2\theta$ and $GM-\sqrt{G^2M^2-a^2\cos^2\theta}<r<GM+\sqrt{G^2M^2-a^2\cos^2\theta}$) and a value equal

to zero when $1-\frac{2GMr}{r^2+a^2\cos^2\theta}=0$ (that is, when $G^2M^2 \ge a^2\cos^2\theta$ and $r = GM \pm \sqrt{G^2M^2 - a^2\cos^2\theta}$). This fact clearly implies that according to these incorrect formulas there is no possibility of escape for a massive particle or a photon when $1-\frac{2GMr}{r^2+a^2\cos^2\theta} \le 0$ and also for all values of r less than any value of r for which $1-\frac{2GMr}{r^2+a^2\cos^2\theta} \le 0$ since to escape a particle would have to pass through all the larger radius values without getting trapped at any radius (that is, when $G^2M^2 \ge a^2\cos^2\theta$ and $r \le GM + \sqrt{G^2M^2 - a^2\cos^2\theta}$).

As for the incorrect escape velocity $v_{e_{incorr}}$ of an untrapped massive particle when $1-\frac{2GMr}{r^2+a^2\cos^2\theta} > 0$, according to the incorrect Formula (108), by analogy with the case of the incorrect Schwarzschild metric [3], we have:

$$1-\frac{2GMr}{r^2+a^2\cos^2\theta} = 1 - v_{e_{incorr}}^2 \tag{111}$$

From which, we have:

$$v_{e_{incorr}} = \sqrt{\frac{2GMr}{r^2+a^2\cos^2\theta}} \tag{112}$$

We can note that in the (112), because of the condition $1-\frac{2GMr}{r^2+a^2\cos^2\theta} > 0$, $\frac{2GMr}{r^2+a^2\cos^2\theta} < 1$, for which $v_{e_{incorr}}$ is always less than 1 (that is, always less than the velocity of light c).

Moreover, according to the (112), $v_{e_{incorr}} \to 0$ both for $r \to +\infty$ and, in the case of $\cos^2\theta \ne 0$, for $r \to 0^+$. Instead, in the case of $\cos^2\theta = 0$, the (112) is valid only for $r > 2GM$.

Therefore, the correct formulas, contrary to the commonly accepted theory (that is, contrary to the incorrect formulas), do not entail any event horizon and, consequently, any black hole: in fact, as we have noted, according to the correct formulas (contrary to the incorrect formulas) the massive particles are always free to escape away with an escape velocity less than the velocity of light c and the photons are always free to escape away by decreasing their frequency.

On the other hand, the correct formulas, contrary to the commonly accepted theory, are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [3] [7]-[12], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [3].

7. The Gravitational Force According to the Correct Kerr Metric

According to the traditional way of deriving the gravitational force from the gravitational potential, we can calculate the correct expression of the gravitational force on a unit mass $F_{corr}\left(r\right)$ directly from the Formula (101) of the correct gravitational potential, taking into account the (100):

$$\begin{aligned}
F_{corr}\left(r\right) &= \frac{\mathrm{d}V_{corr}\left(r\right)}{\mathrm{d}r} \\
&= \frac{1}{2\sqrt{\dfrac{1}{1+\dfrac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}}}}\frac{\left(-1\right)}{\left(1+\dfrac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}\right)^2} \\
&\quad\cdot\left[\frac{2GM}{r_g^2+a^2\cos^2\theta_g}+2GMr_g\frac{\left(-1\right)2r_g}{\left(r_g^2+a^2\cos^2\theta_g\right)^2}\right] \\
&\quad\cdot\frac{r}{r_g}\left(1+\frac{2GMr_g+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right) \\
&= \frac{GM\left(r_g^2-a^2\cos^2\theta_g\right)\dfrac{r}{r_g}\left(1+\dfrac{2GMr_g+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}{\left(r_g^2+a^2\cos^2\theta_g\right)^2\left(1+\dfrac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}\right)^{\frac{3}{2}}}
\end{aligned} \tag{113}$$

Obviously, to obtain the correct expression $F_{corr}\left(r,m\right)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (113) by that generic mass:

$$F_{corr}\left(r,m\right) = \frac{GMm\left(r_g^2-a^2\cos^2\theta_g\right)\dfrac{r}{r_g}\left(1+\dfrac{2GMr_g+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}{\left(r_g^2+a^2\cos^2\theta_g\right)^2\left(1+\dfrac{2GMr_g}{r_g^2+a^2\cos^2\theta_g}\right)^{\frac{3}{2}}} \tag{114}$$

Of course, in this Expression (114), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

We can note that the (114) for $a=0$ becomes equal to the (40).

Moreover, we can see that, according to the (114), the sign of the correct gravitational force is negative when ${r_g}^2 < a^2\cos^2\theta_g$: in this case the correct gravitational force is repulsive.

In particular, according to the (114), for $r_g\to+\infty$ (or $r\to+\infty$), taking into account that in this case $r_g\cong r$, $F_{corr}\left(r,m\right)$ becomes approximately equal to the Newtonian gravitational force and tends to zero.

Furthermore, according to the (114), for $r_g\to0^+$ (or $r\to0^+$), in the case of $\cos^2\theta_g\neq0$, taking into account that in this case $r\cong r_g\left|\cos\theta_g\right|$, $F_{corr}\left(r,m\right)$ tends to $-\dfrac{GMm}{a^2\left|\cos^3\theta_g\right|}$ and therefore the correct gravitational force tends to be repulsive, while in the case of $\cos^2\theta_g=0$, taking into account that in this case

$r \cong \frac{r_g^2}{\sqrt{2a^2}}$, $F_{corr}(r,m)$ is approximately equal to $\frac{m|a|^{\frac{1}{4}}}{2^{\frac{3}{8}}4\sqrt{GM}\,r^{\frac{3}{4}}}$ and therefore tends to $+\infty$, *i.e.* the correct gravitational force tends to be infinitely attractive.

On the other hand, by using the Formula (102) of the incorrect gravitational potential for calculating the incorrect expression of the gravitational force on a unit mass we have:

$$\begin{aligned} F_{incorr}(r) &= \frac{\mathrm{d}V_{incorr}(r)}{\mathrm{d}r} \\ &= \frac{1}{2\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}}} \\ &\quad \cdot\left[\frac{-2GM}{r^2+a^2\cos^2\theta}-2GMr\frac{(-1)}{\left(r^2+a^2\cos^2\theta\right)^2}2r\right] \\ &= \frac{GM\left(r^2-a^2\cos^2\theta\right)}{\left(r^2+a^2\cos^2\theta\right)^2\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}}} \end{aligned} \tag{115}$$

Obviously, to obtain the incorrect expression $F_{incorr}(r,m)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (115) by that generic mass:

$$F_{incorr}(r,m) = \frac{GMm\left(r^2-a^2\cos^2\theta\right)}{\left(r^2+a^2\cos^2\theta\right)^2\sqrt{1-\frac{2GMr}{r^2+a^2\cos^2\theta}}} \tag{116}$$

Of course, also here, in this Expression (116), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that the (116) for $a=0$ becomes equal to the (46).

According to the Expression (116), $F_{incorr}(r,m)$ is equal to zero when $r^2=a^2\cos^2\theta$ and $1-\frac{2GMr}{r^2+a^2\cos^2\theta}\neq 0$, assumes non-real values when $r^2\neq a^2\cos^2\theta$ and $1-\frac{2GMr}{r^2+a^2\cos^2\theta}<0$, is negative when $r^2<a^2\cos^2\theta$ and $1-\frac{2GMr}{r^2+a^2\cos^2\theta}>0$, and is positive when $r^2>a^2\cos^2\theta$ and $1-\frac{2GMr}{r^2+a^2\cos^2\theta}>0$.

Moreover, according to the (116), $F_{incorr}(r,m)$ for $r\to+\infty$ becomes approximately equal to the Newtonian gravitational force and tends to zero.

On the other hand, according to the (116), for $r\to 0^+$ $F_{incorr}(r,m)\to\frac{-GMm}{a^2\cos^2\theta}$ (that is, tends to be repulsive) in the case of $\cos^2\theta\neq 0$, while in the case of $\cos^2\theta=0$ the right side of the (116) assumes non-real values for the values of $r<2GM$.

8. The Gravitational Potential According to the Correct Kerr-Newman Metric

As for the Kerr-Newman solution [4] [5], that is the solution that represents the curved space-time geometry surrounding an electrically charged rotating mass, we have that the commonly used expression is [5]:

$$\begin{aligned} \mathrm{d}s^2 = {} & \frac{\Delta}{\rho^2}\left(\mathrm{d}t - a\sin^2\theta \mathrm{d}\varphi\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)\mathrm{d}\varphi - a\mathrm{d}t\right]^2 \\ & -\frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2\mathrm{d}\theta^2 \end{aligned} \tag{117}$$

where ρ^2 and Δ are functions of r and θ:

$$\Delta \equiv r^2 - 2GMr + a^2 + GQ^2 \tag{118}$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta \tag{119}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass.

But also here the Expression (117) is erroneous: in fact, by means of the fact that the correct Kerr-Newman solution must be equal to the correct Kerr solution for $Q=0$, and must be equal to the correct Reissner-Nordstrøm solution for $a=0$, and must be equal to the correct Schwarzschild solution for $Q=0$ and $a=0$, we can infer, as we have already noted in [2], that the correct Kerr-Newman solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\begin{aligned} \mathrm{d}s^2 = {} & \frac{\Delta}{\rho^2}\left(\mathrm{d}t_g - a\sin^2\theta_g \mathrm{d}\varphi_g\right)^2 - \frac{\sin^2\theta_g}{\rho^2}\left[\left(r_g^2+a^2\right)\mathrm{d}\varphi_g - a\mathrm{d}t_g\right]^2 \\ & -\frac{\rho^2}{\Delta}\mathrm{d}r_g^2 - \rho^2\mathrm{d}\theta_g^2 \end{aligned} \tag{120}$$

where ρ^2 and Δ are functions of r_g and θ_g:

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 + GQ^2 \tag{121}$$

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \tag{122}$$

and M, Q and a are constants: M is the total mass of the system, Q is the total electric charge of the system, and a is the spin angular momentum of the system per unit mass. Moreover t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, this Equation (120) is expressed in curved coordinates.

Now, also in this case, it is very complicated to express this metric as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, but however we can obtain easily a such expression for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$. In fact the (120) for $\mathrm{d}\theta_g = 0$ and $\mathrm{d}\varphi_g = 0$ becomes [2]:

$$\mathrm{d}s^2 = \frac{\Delta - a^2 \sin^2\theta_g}{\rho^2}\mathrm{d}t_g^2 - \frac{\rho^2}{\Delta}\mathrm{d}r_g^2 \tag{123}$$

where ρ^2 and Δ are functions of r_g:

$$\Delta \equiv r_g^2 + 2GMr_g + a^2 + GQ^2 \tag{124}$$

$$\rho^2 \equiv r_g^2 + a^2\cos^2\theta_g \tag{125}$$

Therefore the (123) can be written as [2]:

$$\mathrm{d}s^2 = \left(1 + \frac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}t_g^2 - \frac{1}{1 + \dfrac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{126}$$

Now by analogy with the previous cases we can write in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, as [2]:

$$\mathrm{d}s_g^2 = \frac{1}{1 + \dfrac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}t^2 - \left(1 + \frac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}\right)\mathrm{d}r^2 \tag{127}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2$) and θ_g is a constant.

Obviously, r (or r_g) is always greater than zero since the Kerr-Newman solution is a solution for vacuum region surrounding an electrically charged rotating mass.

As for the relation between r_g and r, we have [2]:

$$\mathrm{d}r^2 = \frac{1}{1 + \dfrac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{128}$$

From which, we have:

$$\int_0^{r^2}\mathrm{d}r^2 = \int_0^{r_g^2}\frac{1}{1 + \dfrac{2GMr_g + GQ^2 + a^2\sin^2\theta_g}{r_g^2 + a^2\cos^2\theta_g}}\mathrm{d}r_g^2 \tag{129}$$

From which, we have:

$$r^2 = \int_0^{r_g}\frac{2r_g\left(r_g^2 + a^2\cos^2\theta_g\right)\mathrm{d}r_g}{r_g^2 + 2GMr_g + GQ^2 + a^2} \tag{130}$$

From which, we have [2]:

$$\begin{aligned} r^2 = {} & r_g^2 - 4GMr_g + \left(4G^2M^2 - a^2\sin^2\theta_g - GQ^2\right) \\ & \cdot \ln\left(1 + \frac{r_g^2 + 2GMr_g}{GQ^2 + a^2}\right) + \frac{GM\left(2a^2 + a^2\sin^2\theta_g + 3GQ^2 - 4G^2M^2\right)}{\sqrt{G^2M^2 - GQ^2 - a^2}} \\ & \cdot \ln\left(\frac{GM + r_g - \sqrt{G^2M^2 - GQ^2 - a^2}}{GM + r_g + \sqrt{G^2M^2 - GQ^2 - a^2}} \cdot \frac{GM + \sqrt{G^2M^2 - GQ^2 - a^2}}{GM - \sqrt{G^2M^2 - GQ^2 - a^2}}\right) \end{aligned} \tag{131}$$

According to the (131) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases, as we can see directly from the (130). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as we can see directly from the (129).

Furthermore, for $r_g \to +\infty$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that, in this case, the presence of r_g instead of r in the (127) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand for $r_g \to 0^+$ in the case of $\cos^2\theta_g \neq 0$ we have $r \cong r_g\sqrt{\frac{a^2\cos^2\theta_g}{GQ^2+a^2}}$, while in the case of $\cos^2\theta_g = 0$ we have $r \cong \frac{r_g^2}{\sqrt{2\left(GQ^2+a^2\right)}}$, as we can see easily from the (130).

Moreover, from the (128) we have:

$$2r\mathrm{d}r = \frac{2r_g\mathrm{d}r_g}{1+\frac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}} \tag{132}$$

From which, we have:

$$\frac{\mathrm{d}r_g}{\mathrm{d}r} = \frac{r}{r_g}\left(1+\frac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right) \tag{133}$$

Now, also here, by analogy with the case of the correct Schwarzschild metric [3] we can infer that the correct expression of the gravitational potential $V_{corr}(r)$ in this case is equal to:

$$V_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}}}-1 \tag{134}$$

We can see that the (134) for $a=0$ becomes equal to the (60), for $Q=0$ becomes equal to the (101) and for $a=0$ and $Q=0$ becomes equal to the (23).

On the other hand, we can note that, according to the (134), $-1 < V_{corr}(r) < 0$ for any value of $r > 0$ and of $r_g > 0$.

Obviously, according to the (134), in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), $a=0$ and $Q=0$, taking into account that in this case $r \cong r_g$, $V_{corr}(r)$ becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (134), for $r_g \to +\infty$ (or $r \to +\infty$) $V_{corr}(r) \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $V_{corr}(r) \to -1$ in the case of $\cos^2\theta_g = 0$ and $V_{corr}(r) \to \sqrt{\frac{1}{1+\frac{GQ^2}{a^2\cos^2\theta_g}}}-1$ in the case of $\cos^2\theta_g \neq 0$.

Instead, always by analogy with the case of the incorrect Schwarzschild metric [3], we can infer that the incorrect expression of the gravitational potential $V_{incorr}(r)$ is equal to:

$$V_{incorr}(r)=\sqrt{1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}}-1 \tag{135}$$

We can see that the (135) for $a=0$ becomes equal to the (61), for $Q=0$ becomes equal to the (102) and for $a=0$ and $Q=0$ becomes equal to the (24).

Obviously, also here, according to the (135), in the case of $r \gg 2GM$, $a=0$ and $Q=0$, $V_{incorr}(r)$ becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (135), $V_{incorr}(r)$ assumes non-real values when $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}<0$ (that is, when $G^2M^2>GQ^2+a^2\cos^2\theta$ and $GM-\sqrt{G^2M^2-GQ^2-a^2\cos^2\theta}<r<GM+\sqrt{G^2M^2-GQ^2-a^2\cos^2\theta}$) and a value equal to −1 when $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}=0$ (that is, when $G^2M^2\geq GQ^2+a^2\cos^2\theta$ and $r=GM\pm\sqrt{G^2M^2-GQ^2-a^2\cos^2\theta}$).

Furthermore, according to the (135), for $r\to+\infty$ $V_{incorr}(r)\to 0$, and for $r\to 0^+$ $V_{incorr}(r)\to+\infty$ in the case of $\cos^2\theta=0$ and $V_{incorr}(r)\to\sqrt{1+\frac{GQ^2}{a^2\cos^2\theta}}-1$ in the case of $\cos^2\theta\neq 0$.

On the other hand, the (134) implies that the correct energy of a massive particle has the form of:

$$E_{corr}(r)=m_0\gamma\left[V_{corr}(r)+1\right]=m_0\gamma\sqrt{\frac{1}{1+\frac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}}} \tag{136}$$

Now the fact that $V_{corr}(r)+1$ is always greater than zero for any value of r and r_g greater than zero implies that a massive particle, for any value of r and r_g greater than zero, has always a correct escape velocity $v_{e_{corr}}$ less than the velocity of light c. In particular we have that the massive particle, in order to escape, must have a γ that multiplied by the value of $V_{corr}(r)+1$ be equal to one or to a number greater than one, in fact in this case the $E_{corr}(r)$ would be equal to or greater than the energy of rest mass of the same particle: a such value of γ is always possible if $V_{corr}(r)+1>0$ since γ can assume all the values ≥1.

More precisely, in this case, by analogy with the case of the correct Schwarzschild metric [3], we have:

$$\frac{1}{1+\frac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}}=1-v_{e_{corr}}^2 \tag{137}$$

From which, we have:

$$v_{e_{corr}} = \sqrt{\frac{\dfrac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}{1 + \dfrac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}} \tag{138}$$

We can note that, according to the (138), $v_{e_{corr}}$ is always less than 1 (that is, always less than the velocity of light c) and for $r_g \to +\infty$ (or $r \to +\infty$) $v_{e_{corr}} \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $v_{e_{corr}} \to \sqrt{\frac{\dfrac{GQ^2}{a^2\cos^2\theta_g}}{1 + \dfrac{GQ^2}{a^2\cos^2\theta_g}}}$ in the case of $\cos^2\theta_g \neq 0$ and $v_{e_{corr}} \to 1$ (that is, tends to the velocity of light c) in the case of $\cos^2\theta_g = 0$.

On the other hand, a photon is always free to escape away by decreasing its frequency. In fact in this case the (136) becomes:

$$E_{corr}(r) = h\nu\left[V_{corr}(r) + 1\right] = h\nu\sqrt{\frac{1}{1 + \dfrac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}} \tag{139}$$

And the photon can escape away with the new frequency ν_0 at the infinite equal to:

$$\nu_{0_{corr}} = \nu\sqrt{\frac{1}{1 + \dfrac{2GMr_g + GQ^2}{r_g^2 + a^2\cos^2\theta_g}}} \tag{140}$$

On the contrary, by using the incorrect Expression (135) instead of the correct Expression (134) we would have, in place of the (136), (139) and (140), the following incorrect formulas:

$$E_{incorr}(r) = m_0\gamma\left[V_{incorr}(r) + 1\right] = m_0\gamma\sqrt{1 + \frac{-2GMr + GQ^2}{r^2 + a^2\cos^2\theta}} \tag{141}$$

$$E_{incorr}(r) = h\nu\left[V_{incorr}(r) + 1\right] = h\nu\sqrt{1 + \frac{-2GMr + GQ^2}{r^2 + a^2\cos^2\theta}} \tag{142}$$

$$\nu_{0_{incorr}} = \nu\sqrt{1 + \frac{-2GMr + GQ^2}{r^2 + a^2\cos^2\theta}} \tag{143}$$

We can note that these incorrect Formulas (141), (142) and (143) assume non-real values when $1 + \frac{-2GMr + GQ^2}{r^2 + a^2\cos^2\theta} < 0$ (that is, when $G^2M^2 > GQ^2 + a^2\cos^2\theta$ and $GM - \sqrt{G^2M^2 - GQ^2 - a^2\cos^2\theta} < r < GM + \sqrt{G^2M^2 - GQ^2 - a^2\cos^2\theta}$) and a value equal to zero when $1 + \frac{-2GMr + GQ^2}{r^2 + a^2\cos^2\theta} = 0$ (that is, when $G^2M^2 \geq GQ^2 + a^2\cos^2\theta$ and $r = GM \pm \sqrt{G^2M^2 - GQ^2 - a^2\cos^2\theta}$). This fact clearly implies that according to these incorrect formulas there is no possibility of escape for a massive particle or a photon when $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}\leq 0$ and also for all values of r less than any value of r for which $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}\leq 0$ since to escape a particle would have to pass through all the larger radius values without getting trapped at any radius (that is, when $G^2M^2\geq GQ^2+a^2\cos^2\theta$ and $r\leq GM+\sqrt{G^2M^2-GQ^2-a^2\cos^2\theta}$).

As for the incorrect escape velocity $v_{e_{incorr}}$ of an untrapped massive particle when $1>1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}>0$, according to the incorrect Formula (141), by analogy with the case of the incorrect Schwarzschild metric [3], we have:

$$1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}=1-v_{e_{incorr}}^2 \tag{144}$$

From which, we have:

$$v_{e_{incorr}}=\sqrt{\frac{2GMr-GQ^2}{r^2+a^2\cos^2\theta}} \tag{145}$$

We can note that in the (145), because of the condition $1>1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}>0$, $0<\frac{2GMr-GQ^2}{r^2+a^2\cos^2\theta}<1$, for which $v_{e_{incorr}}$ is always less than 1 (that is, always less than the velocity of light c).

Moreover, according to the (145), for $r\to+\infty$ $v_{e_{incorr}}\to 0$.

On the other hand, an untrapped massive particle when $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}\geq 1$ does not need to lose kinetic energy to free itself.

Therefore, the correct formulas, contrary to the commonly accepted theory (that is, contrary to the incorrect formulas), do not entail any event horizon and, consequently, any black hole: in fact, as we have noted, according to the correct formulas (contrary to the incorrect formulas) the massive particles are always free to escape away with an escape velocity less than the velocity of light c and the photons are always free to escape away by decreasing their frequency.

On the other hand, the correct formulas, contrary to the commonly accepted theory, are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [3] [7]-[12], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [3].

9. The Gravitational Force According to the Correct Kerr-Newman Metric

According to the traditional way of deriving the gravitational force from the gravitational potential, we can calculate the correct expression of the gravitational force on a unit mass $F_{corr}(r)$ directly from the Formula (134) of the correct gravitational potential, taking into account the (133):

$$\begin{aligned}
F_{corr}\left(r\right) &= \frac{\mathrm{d}V_{corr}\left(r\right)}{\mathrm{d}r} \\
&= \frac{1}{2\sqrt{\dfrac{1}{1+\dfrac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}}}}\frac{\left(-1\right)}{\left(1+\dfrac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}\right)^2} \\
&\quad\cdot\left[\frac{2GM}{r_g^2+a^2\cos^2\theta_g}+\left(2GMr_g+GQ^2\right)\frac{\left(-1\right)2r_g}{\left(r_g^2+a^2\cos^2\theta_g\right)^2}\right] \\
&\quad\cdot\frac{r}{r_g}\left(1+\frac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right) \\
&= \frac{GM\left(r_g^2-a^2\cos^2\theta_g\right)+GQ^2r_g}{\left(r_g^2+a^2\cos^2\theta_g\right)^2} \\
&\quad\cdot\frac{\dfrac{r}{r_g}\left(1+\dfrac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}{\left(1+\dfrac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}\right)^{\frac{3}{2}}}
\end{aligned} \tag{146}$$

Obviously, to obtain the correct expression $F_{corr}\left(r,m\right)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (146) by that generic mass:

$$\begin{aligned}
F_{corr}\left(r,m\right) &= \frac{GMm\left(r_g^2-a^2\cos^2\theta_g\right)+GQ^2mr_g}{\left(r_g^2+a^2\cos^2\theta_g\right)^2} \\
&\quad\cdot\frac{\dfrac{r}{r_g}\left(1+\dfrac{2GMr_g+GQ^2+a^2\sin^2\theta_g}{r_g^2+a^2\cos^2\theta_g}\right)}{\left(1+\dfrac{2GMr_g+GQ^2}{r_g^2+a^2\cos^2\theta_g}\right)^{\frac{3}{2}}}
\end{aligned} \tag{147}$$

Of course, in this Expression (147), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that the (147) for $a=0$ becomes equal to the (73), for $Q=0$ becomes equal to the (114) and for $a=0$ and $Q=0$ becomes equal to the (40).

According to the (147), for $r_g\to+\infty$ (or $r\to+\infty$), taking into account that in this case $r_g\cong r$, $F_{corr}\left(r,m\right)$ becomes approximately equal to the Newtonian gravitational force and tends to zero.

Moreover, according to the (147), for $r_g\to0^+$ (or $r\to0^+$), in the case of $\cos^2\theta_g\neq0$, taking into account that in this case $r\cong r_g\sqrt{\dfrac{a^2\cos^2\theta_g}{GQ^2+a^2}}$, $F_{corr}\left(r,m\right)$

tends to $-\frac{GMm\sqrt{GQ^2+a^2}}{\left(GQ^2+a^2\cos^2\theta_g\right)^{\frac{3}{2}}}$ and therefore the correct gravitational force tends to be repulsive, while in the case of $\cos^2\theta_g=0$, taking into account that in this case $r\cong\frac{r_g^2}{\sqrt{2\left(GQ^2+a^2\right)}}$, $F_{corr}\left(r,m\right)$ is approximately equal to $\frac{m\left(GQ^2+a^2\right)^{\frac{1}{4}}}{2^{\frac{3}{4}}\sqrt{GQ^2}\sqrt{r}}$ and therefore tends to $+\infty$, *i.e.* the correct gravitational force tends to be infinitely attractive.

On the other hand, by using the Formula (135) of the incorrect gravitational potential for calculating the incorrect expression of the gravitational force on a unit mass we have:

$$\begin{aligned}
F_{incorr}\left(r\right) &= \frac{\mathrm{d}V_{incorr}\left(r\right)}{\mathrm{d}r} \\
&= \frac{1}{2\sqrt{1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}}} \\
&\quad\cdot\left[\frac{-2GM}{r^2+a^2\cos^2\theta}+\left(-2GMr+GQ^2\right)\frac{\left(-1\right)}{\left(r^2+a^2\cos^2\theta\right)^2}2r\right] \\
&= \frac{GM\left(r^2-a^2\cos^2\theta\right)-GQ^2r}{\left(r^2+a^2\cos^2\theta\right)^2\sqrt{1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}}}
\end{aligned} \tag{148}$$

Obviously, to obtain the incorrect expression $F_{incorr}\left(r,m\right)$ of the gravitational force on a generic mass m, it is sufficient to multiply the (148) by that generic mass:

$$F_{incorr}\left(r,m\right)=\frac{GMm\left(r^2-a^2\cos^2\theta\right)-GQ^2mr}{\left(r^2+a^2\cos^2\theta\right)^2\sqrt{1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}}} \tag{149}$$

Of course, also here, in this Expression (149), m is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that the (149) for $a=0$ becomes equal to the (79), for $Q=0$ becomes equal to the (116) and for $a=0$ and $Q=0$ becomes equal to the (46).

According to the (149), $F_{incorr}\left(r,m\right)$ is equal to zero when $M\left(r^2-a^2\cos^2\theta\right)-Q^2r=0$ and $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}\neq0$, assumes non-real values when $M\left(r^2-a^2\cos^2\theta\right)-Q^2r\neq0$ and $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}<0$, is negative when $M\left(r^2-a^2\cos^2\theta\right)-Q^2r<0$ and $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}>0$, and is positive when $M\left(r^2-a^2\cos^2\theta\right)-Q^2r>0$ and $1+\frac{-2GMr+GQ^2}{r^2+a^2\cos^2\theta}>0$.

Moreover, according to the (149), $F_{incorr}\left(r,m\right)$ for $r\to+\infty$ becomes approximately equal to the Newtonian gravitational force and tends to zero.

On the other hand, according to the (149), for $r\to0^+$ $F_{incorr}\left(r,m\right)$ tends to $\frac{-GMm}{a^2\cos^2\theta\sqrt{1+\frac{GQ^2}{a^2\cos^2\theta}}}$ (that is, the incorrect gravitational force tends to be repulsive) in the case of $\cos^2\theta\neq0$, and is approximately equal to $-\frac{\sqrt{GQ^2}m}{r^2}$ and therefore tends to $-\infty$ (that is, the incorrect gravitational force tends to be infinitely repulsive) in the case of $\cos^2\theta=0$.

10. Experimental Prospects

10.1. The Available Experimental Data

As we have already noted in [1]-[3], as for the experimental data obtained with the help of x-ray astronomy the proof that we have found black holes, and therefore event horizons, is based only on the fact that we have found invisible objects which have masses that are too great, according to the commonly accepted theory, for not being black holes [4] [13] [14]. But according to the correct theory of this article, whatever the masses and the dimensions of these invisible objects are, we never have black holes, and therefore we never have event horizons. Consequently such experimental data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

On the other hand, as we have already noted in [1]-[3], with regard to the experimental data of the so-called gravitational waves (obtained by the LIGO collaboration) of a collision between two black holes, such gravitational waves were detected only below measurement errors, *i.e.* the signals detected were lower than the background noise (cf. chapter 6 of [13]). Furthermore the models expected from the theory were used for selecting the signals from the background noise (cf. chapter 6 of [13]) with the help of supercomputers: obviously, this is an incorrect practice which cannot produce any significant data. The awareness of the non-significance of the LIGO collaboration data is now widespread [15]-[18]. Obviously, also such data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

As for the alleged photos of black holes, as we have already noted in [1]-[3], they were formed with the help of special algorithms from something compatible with the white noise. In other words, these photos were extracted from something compatible with the white noise only on the basis of the images that were expected by the researchers, with the help of appropriate algorithms loaded onto supercomputers (cf. the Section "Imaging a Black Hole" of [19]. See also [20]). Therefore, also in this case the researchers wanted to measure something that is below measurement errors, and so these photos are completely unreliable. On the other hand, serious doubts have now spread about the reliability of these photos [21]-[23]. Consequently such photos cannot prove anything and in particular cannot discriminate in any way between the commonly accepted theory and the same theory corrected according to this article.

Moreover, the corrections, that we have proposed in this article, to the commonly accepted theory are very small in the normal experimental situations (for example in the solar system), so the fact that, in these situations, so far no difference has been noted between the commonly accepted theory and the experimental results is not strange. In fact, in the usual case of $\frac{2GM}{r} \ll 1$, $\frac{GQ^2}{r^2}$ negligible compared to $\frac{2GM}{r}$ and a^2 negligible compared to r^2, we have that the difference between the previsions of the commonly accepted theory and the previsions of the same theory corrected according to this article is, in the experiments commonly performed to test the General Theory of Relativity, only at the second order in $\frac{2GM}{r}$ [1]-[3]. And all the experiments conducted so far in the solar system have not had errors so small as to test differences at the second order in $\frac{2GM}{r}$ [4].

Therefore, in conclusion, there is no available experimental data that can discriminate between the commonly accepted theory and the same theory corrected according to this article.

10.2. Some Proposals for a Crucial Experiment

On the other hand, a crucial experiment could be done, which discriminates between the commonly accepted theory and the same theory corrected according to this article, by taking advantage of the high precision and sensitivity of the latest atomic clocks.

In fact, as we have shown in [1], the ratio of the passage of time in the gravitational field according to the correct Schwarzschild metric to that according to the commonly accepted Schwarzschild metric, in the case of $\frac{2GM}{r} \ll 1$, is approximately equal to $1+\frac{1}{2}\left(\frac{2GM}{r}\right)^2$.

Now throughout the solar system we have effectively $\frac{2GM}{r} \ll 1$.

The term $\frac{1}{2}\left(\frac{2GM}{r}\right)^2$ is obviously expressed in the case with the velocity of light $c \equiv 1$. Instead in the case more general (in which c is not defined equal to 1) this term becomes $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$. Now, the term $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$ due to the solar mass on the surface of the Sun is approximately equal to 8.99×10^{-12}, while at the average distance of the Earth from the Sun this value becomes approximately equal to 1.95×10^{-16}. Obviously, externally to the Sun, such term decreases with the square of the distance from the centre of the Sun according to the formula.

On the other hand, the same term due to the mass of the Earth on the surface of the Earth is approximately equal to 9.69×10^{-19} and obviously also here, externally to the Earth, decreases with the square of the distance from the centre of the Earth according to the formula.

Moreover, now we have atomic clocks that have an error of 7.6×10^{-21} [24] [25] and therefore, as we have already noted in [1], we can measure such differences between the predictions of the commonly accepted Schwarzschild metric and those of the same theory corrected according to this article with appropriate temporal measurements made in the solar system.

In particular, as we have already noted in [1], we could do a crucial experiment, which discriminates between the commonly accepted Schwarzschild metric and the same metric corrected according to this article, by taking one such atomic clock to diverse convenient locations in the solar system for comparing its time measurements made at those various locations with the corresponding time measurements made by another similar clock here on Earth.

Now, since the corrections of this article to the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics strictly depend on the corrections of this article to the Schwarzschild metric [1]-[3], we can say that this crucial experiment would also discriminate between the commonly accepted theory about the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and the same theory corrected according to this article.

On the other hand, in the usual experimental tests of the General Theory of Relativity, no direct gravitational force measurements have been used so far because they are not very precise.

However, in the usual case of $\frac{2GM}{r} \ll 1$, taking into account that in this case (as we have already noted) $r_g \cong r\left(1+\frac{2GM}{r}\right)$, the correct expression of the gravitational force (40) would be approximately equal to:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{r}\right) \tag{150}$$

And, always in the usual case of $\frac{2GM}{r} \ll 1$, the incorrect expression of the gravitational force (46) would be approximately equal to:

$$F_{incorr}(r,m) \cong \frac{GMm}{r^2}\left(1+\frac{GM}{r}\right) \tag{151}$$

The Formulas (150) and (151) are expressed in the case with the velocity of light $c \equiv 1$. Of course, in the case more general (in which c is not defined equal to 1) we have that the (150) and the (151) become respectively:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{rc^2}\right) \tag{152}$$

$$F_{incorr}\left(r,m\right)\cong\frac{GMm}{r^2}\left(1+\frac{GM}{rc^2}\right)\tag{153}$$

As we have already noted in [3], although no gravitational force measurements have been used to test the General Theory of Relativity so far, one might consider doing so by sending probes close to the Sun where the difference between the (152) and the (153) and the differences of the (152) and of the (153) with respect to the Newtonian gravitational force are larger, and the GM product of the Sun's mass is known with a smaller relative uncertainty than the GM product of the Earth's mass [26]. In fact, the relative error on GM product of the Sun's mass is approximately equal to 1 on 1.33×10^{10} [26], while the value of $\frac{GM}{rc^2}$ on the surface of the Sun is approximatively equal to 2.12×10^{-6}.

For which, according to the right sides of the Formulas (152) and (153), it is possible to measure the contribution due to the corrections made to the Newtonian gravitational force both in the case of the commonly accepted Schwarzschild metric and in the case of the same theory corrected according to this article, if we measure the distance from the centre of the Sun with sufficient accuracy (and, of course, we know the mass m quite precisely). Obviously, in order to do a crucial experiment between the commonly accepted Schwarzschild metric and the same metric corrected according to this article we would also need to measure the gravitational forces with sufficient accuracy.

Moreover, the best way to carry out such a crucial experiment would be to perform a series of measurements of the gravitational force at different distances from the centre of the Sun, in order to also measure the trend of the gravitational force as a function of these distances.

Now, here too, since the corrections of this article to the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics strictly depend on the corrections of this article to the Schwarzschild metric [1]-[3], we can say that this crucial experiment would also discriminate between the commonly accepted theory about the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and the same theory corrected according to this article.

11. General Conclusions

In this article, we started from our correction to the traditional Schwarzschild, Reissner-Nordstrøm, Kerr and Kerr-Newman metrics [1] [2] and to the gravitational potential and gravitational force according to the traditional Schwarzschild metric [3].

Such corrections have been performed by assuming that the General Theory of Relativity is valid, differently from other articles [27]-[31] that deal with the same topics of [3] and of this article and that instead start from proposals to change the General Theory of Relativity.

In this way, we have obtained new expressions for the gravitational potential and the gravitational force in the cases described by the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics.

As we have seen, these new expressions do not entail any event horizon and, consequently, any black hole.

Therefore, this article confutes all the physics that on the basis of the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics foresees the possibility of the existence of event horizons and black holes [1]-[5] [7] [32].

Moreover, we have noted that these new expressions are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [3] [7]-[12], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1] [3].

On the other hand, we have seen that there is no available experimental data that can discriminate between the commonly accepted theory about the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and the same theory corrected according to this article.

However, we have noted that, in theory, is possible to perform in the solar system some crucial experiments, that discriminate between the commonly accepted theory about the gravitational potential and the gravitational force according to the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics and the same theory corrected according to this article. Therefore, it would be appropriate to try to make one of such crucial experiments.

Finally, according to this article, all the physics that is based on the incorrect Reissner-Nordstrøm, Kerr and Kerr-Newman metrics, in particular on the incorrect expressions of the gravitational potential and of the gravitational force in the cases described by these metrics, should be modified on the basis of the correct formulas that we have calculated.

Obviously, the introduction of such new formulas for the gravitational potential and the gravitational force in the cases described by the Reissner-Nordstrøm, Kerr and Kerr-Newman metrics can have many applications both in the gravitational physics and in the analysis of astronomical data.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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