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On the Implementation of the Method of Derivation of Lorentz Transformations Used by Einstein

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Abstract

The Lorentz transformations are the mathematical basis of Einstein's theory of special relativity. We conduct a thorough examination of the method of derivation of the Lorentz transformations used by Einstein and identify the cause of the incorrect implementation of the method. The cause is related to the incorrect proof of the equality $\varphi(v)=1$ for the unknown function $\varphi(v)$ arising in the process of derivation of the Lorentz transformations. We develop proof of the equality $\varphi(v)=1$ and eliminate the cause of the incorrect implementation of the method of derivation of the Lorentz transformations used by Einstein.

Keywords

Einstein, Lorentz Transformations, Mass, Energy, Lagrangian

1. Introduction

This paper examines the method of derivation of the Lorentz transformations used by Einstein and identifies the cause of the incorrect implementation of the method. The cause is related to the incorrect proof of the equality $\varphi(v)=1$ for the unknown function $\varphi(v)$ arising in the process of derivation of the Lorentz transformations. As it will be shown in this paper, we develop proof of the equality $\varphi(v)=1$ and eliminate the cause of the incorrect implementation of the method of derivation of the Lorentz transformations used by Einstein.

2. Kinematic Part

In [1] [2] Einstein proposed a method of derivation of the Lorentz transformations based on obtaining transformations

$$x' = \varphi(v)\gamma(v)(x - vt), \quad y' = \varphi(v)y, \quad z' = \varphi(v)z,$$
$$t' = \varphi(v)\gamma(v)\left(t - \frac{vx}{c^2}\right), \tag{1}$$

where a point event is determined by the variables x, y, z, t with respect to system S, and by the variables x', y', z', t' with respect to system S'. System S' moves parallel to the *x*-axis with a constant speed v relative to system S. c is the speed of light in vacuum and $\varphi(v)$ is a function to be determined. $\gamma(v) = 1\Big/\sqrt{1 - \frac{v^2}{c^2}}$.

Transformations (1) ensure the transfer of the spherical surface

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

to a spherical surface

$$x^2 + y^2 + z^2 = c^2 t^2.$$

Thus, equation $c^2 t'^2 - x'^2 - y'^2 - z'^2 = 0$ is invariant under transformations of Equation (1).

This is the first stage of derivation of the Lorentz transformations used by Einstein. Note that the transformations of Equation (1) are linear due to homogeneity, which Einstein attributes to space and time.

It is important to note that to obtain transformations of Equation (1), Einstein used only two postulates: the principle of constancy of the speed of light and the principle of relativity.

It should be noted that there is a third postulate or assumption implicit in [1]: "If a material point is at rest relatively to this system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the method of Euclidean geometry, and can be expressed in Cartesian co-ordinates."

The Lorentz transformations have the form

$$x' = \gamma(v)(x - vt), \quad y' = y, \quad z' = z,$$
$$t' = \gamma(v)\left(t - \frac{vx}{c^2}\right), \tag{2}$$

therefore, the second stage of the derivation is associated with the proof of the equality

$$\varphi(v) = 1, \tag{3}$$

which ensures that the Lorentz transformations are obtained.

Einstein [2] [3] proves the equality in the following way: If we introduce a third system S'', which is equivalent to S and S', is moving with the velocity $-v$ relative to S', and is oriented relative to S' in the same way S' is oriented to relative to S, we obtain, by twofold application of Equation (1) we have just found,

$$\begin{aligned}
t'' &= \varphi(-v)\gamma(-v)\left(t' + \frac{vx'}{c^2}\right) = \varphi(-v)\varphi(v)t, \\
x'' &= \varphi(-v)\gamma(-v)(x' + vt') = \varphi(-v)\varphi(v)x, \\
y'' &= \varphi(-v)y' = \varphi(-v)\varphi(v)y, \\
z'' &= \varphi(-v)z' = \varphi(-v)\varphi(v)z.
\end{aligned} \tag{4}$$

Since the coordinate origins of S and S' coincide permanently, the axes have identical directions and the systems are "equivalent", this substitution is the identity, so that

$$\varphi(-v)\varphi(v) = 1. \tag{5}$$

Further, since the relation between y and y' cannot depend on the sign of v, we have

$$\varphi(v) = \varphi(-v). \tag{6}$$

Thus, $\varphi(v) = 1$, and the transformations Equation (1) read (2).

Let us show that the method proposed by Einstein for proving Equation (3) is not proof, and within the framework of the method of derivation of the Lorentz transformations used by Einstein, it is impossible to prove Equation (3).

First, the relations between systems S'' and S' and systems S' and S belong to different types of system relations, and we have no reason to use function $\varphi(-v)$ in Equation (4). We argue that Einstein, when proving Equation (3), used, in the general case, an illegal equating of the unknown function $\varphi(-v)$, which in essence represents the motion of system S'' relative to the system S' (which we will re-designate below as unknown function $\psi(-v)$), and the unknown function $\varphi(-v)$, designated by the same symbol, which in essence represents the motion of system S' relative to system S. Thus, Einstein illegitimately applied Equation (6), which applies to the motion of system S' relative to system S, to the solution of Equation (5), which now has the form $\psi(-v)\varphi(v) = 1$.

Thus, Einstein proved not the equality $\varphi^2(v) = 1$, but the equality $\psi(-v)\varphi(-v) = 1$.

This is sufficient to consider that Einstein's proof of Equation (3) using the three systems S, S', S'' is not proof.

Second, Einstein's proof of Equation (3) was based on the construction of inverse transformations for system S', but by using an additional system S''. Let us exclude the additional system S'' from consideration and find the inverse transformations for system S' by transferring it from a state of motion to a state of rest relative to system S. Inverse transformations can be found from direct transformations (1). We have

$$x = \frac{1}{\varphi(v)}\gamma(v)(x' + vt'), \quad y = \frac{1}{\varphi(v)}y', \quad z = \frac{1}{\varphi(v)}z', \quad t = \frac{1}{\varphi(v)}\gamma(v)\left(t' + \frac{vx'}{c^2}\right),$$

where $\varphi(v) \neq 0$, therefore, taking into account Equation (6), we have $\frac{1}{\varphi(v)}\varphi(-v) = 1$. This means there is no reason to believe that the function $\varphi(v)$

should satisfy only the condition $\varphi(v)=1$.

Everything indicates that Einstein, in his derivation of the Lorentz transformations, made a volitional decision to assign a value equal to one to this function in order to remove the uncertainty caused by the presence of an unknown function $\varphi(v)$.

It is known that the transformations of Equation (1) were deduced respectively by Lorentz [4] and Poincaré [5]. They also equated $\varphi(v)=1$ in transformations of Equation (1) without proper grounds. In particular, Poincaré [5] implicitly assumed reciprocity, *i.e.* $\frac{1}{\varphi(v)}=\varphi(-v)$ without careful justification.

Poincaré did not see any reason that $\varphi(v)$ must equal one from a physical standpoint, although he recognized that transformations of Equation (1) form a group (in mathematical sense) only if $\varphi(v)=1$.

Thus, all three founders of relativistic physics, H. Lorentz, A. Poincaré, and A. Einstein, put the function $\varphi(v)$ equal to one, without the correct proof, that is, the function $\varphi(v)$ actually disappears from the formulas of Equation (1). In such truncated form, the Lorentz transformations entered into thousands of papers and textbooks. Nowadays few people remember their initially general form with the function $\varphi(v)$.

By the way, Einstein [6], while discussing in 1921 the theory of H. Weil, wrote that it would be worth trying to change the theory of relativity by assuming that the geometric quality called the four-dimensional interval s ($s^2=x^2+y^2+z^2-c^2t^2$) will stretch or shrink by $\varphi(v)$ times (along with the length and time scales); only the condition $s=0$ is invariant. In view of that, it seems that the details of the correct proof of the equality $\varphi(v)=1$ are relevant at this time and the attempt to prove Equation (3) is justified.

To my knowledge, as yet, no proper proof of the equality $\varphi(v)=1$ has been made. However, this does not mean that it is impossible to prove Equation (3).

The assignment by a volitional decision of the fulfillment of Equation (3) gives us the reason to consider the Lorentz transformations as a result of processes associated not only with kinematics but also with dynamics and to prove Equation (3).

3. Dynamic Part

Suppose a material particle with a rest mass m_0 is displaced by force F through displacement $\mathrm{d}x$. The relation between energy and work done, $\mathrm{d}E=F\cdot\mathrm{d}x=F\cdot V\mathrm{d}t$, is fundamental, where $F=\mathrm{d}\left[m(V)V\right]/\mathrm{d}t$, and m(V) is the mass of the moving particle, $m(0)=m_0$. If we additionally assume that the energy of the material moving particle is $E=m(V)c^2>0$, then

$$\begin{aligned}&\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{\mathrm{d}\left[m(V)c^2\right]}{\mathrm{d}t}=\frac{\mathrm{d}\left[m(V)V\right]}{\mathrm{d}t}V=\left[V\frac{\mathrm{d}m}{\mathrm{d}t}+m\frac{\mathrm{d}V}{\mathrm{d}t}\right]V\\&\to c^2\frac{\mathrm{d}m}{\mathrm{d}t}=V^2\frac{\mathrm{d}m}{\mathrm{d}t}+mV\frac{\mathrm{d}V}{\mathrm{d}t}\to c^2\mathrm{d}m=V^2\mathrm{d}m+mV\mathrm{d}V\\&\to\int_{m_0}^{m}\frac{\mathrm{d}m}{m}=\int_0^V\frac{V\mathrm{d}V}{c^2-V^2}\to\ln\left(\frac{m}{m_0}\right)=-\frac{1}{2}\ln\left(1-\frac{V^2}{c^2}\right).\end{aligned}\tag{7}$$

From Equation (7) we have

$$m(V)=\frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}}. \tag{8}$$

Therefore from Equation (8) we obtain

$$E=m(V)c^2=\frac{m_0c^2}{\sqrt{1-\frac{V^2}{c^2}}} \tag{9}$$

It is known [7] that Equation (9) is consistent with available experimental physics data.

Henceforth, when studying motion of a material particle, we proceed from the Principle of Least Action.

The action between the initial time t_1 and the final time t_2 is

$$\Phi=\int_{t_1}^{t_2} L\mathrm{d}t, \tag{10}$$

where the coefficient (integrand) L at $\mathrm{d}t$ is called the Lagrangian for a given material particle.

Let us consider one directional motion along the *x*-axis of the free material particle of rest mass m_0 with velocity $V=\frac{\mathrm{d}x}{\mathrm{d}t}=\dot{x}(t)$.

Momentum can be used to reverse engineer the Lagrangian.

Momentum $p=m(V)V=\frac{\partial L}{\partial V}$, therefore from Equation (8) we obtain

$$L=\int\frac{m_0V\mathrm{d}V}{\sqrt{1-\frac{V^2}{c^2}}}=-m_0c^2\sqrt{1-\frac{\dot{x}^2(t)}{c^2}}+\text{const.} \tag{11}$$

As the constant in the Lagrangian is not reflected in the equation of motion, it can be omitted.

From Equations (10) and (11) we have

$$\Phi=-m_0c^2\sqrt{1-\frac{v^2}{c^2}}\int_{t_1}^{t_2}\mathrm{d}t=-m_0c^2\sqrt{1-\frac{v^2}{c^2}}\left(t_2-t_1\right),$$

where $\dot{x}(t)=v=\text{const}$.

In the S_0 frame associated with the material particle, we have

$$\Phi=-m_0c^2\left(t_{02}-t_{01}\right)=-m_0c^2T_0,$$

where T_0 is the time interval in the S_0 frame.

Action Φ should not depend on the choice of one or another inertial system because the Laws of Physics are frame independent (Einstein's second postulate).

The action Φ is frame independent, if

$$T_0=\sqrt{1-\frac{v^2}{c^2}}\left(t_2-t_1\right)=\sqrt{1-\frac{v^2}{c^2}}T' \tag{12}$$

where T' is the time interval in the moving frame.

However, from Equation (1) we have

$$T' = \varphi(-v)\frac{t_{02} + \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \varphi(-v)\frac{t_{01} + \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \varphi(-v)\frac{t_{02} - t_{01}}{\sqrt{1-\frac{v^2}{c^2}}} = \varphi(-v)\frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (13)$$

Therefore, from Equations (12) and (13), we obtain $\varphi(-v) = 1$, and taking into account Equation (6), we prove Equation (3).

Thus, we have proven Equation (3) if the additional assumption $E = m(V)c^2 > 0$ is satisfied.

4. Conclusions

Einstein's theory [1] derives a relativistic geometry for spacetime in the absence of matter, and then applies it in the presence of matter for deriving rules for how that matter behaves at high relative speeds.

My paper raises the question: Is there a correct way to complete the method of derivation of the Lorentz transformations used by Einstein without a necessary additional assumption being in disagreement with known principles or available experimental physics data?

In the present paper, the appropriate conclusion is reached that to turn Einstein's two-postulate relativistic theory of empty space into a correct mathematical theory then applied in the presence of matter, the additional assumption $E = m(V)c^2 > 0$, which is associated with matter, must be satisfied.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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