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Neutrinos Described as Vacuum Energy Excitations Predict Observed Neutrino Mass Sum

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Abstract

Reliable observations find only three neutrino mass eigenstates, oscillating between each other as neutrinos travel through space, and limit the sum of the three neutrino masses. At a minimum, any reliable description of neutrinos must allow only three neutrino mass eigenstates and predict a neutrino mass sum consistent with observations. This paper describes neutrinos as spheres, with radius one quarter of their Compton wavelength and thickness of the Planck length, surrounding a central core along their rotation axis, with diameter of the Planck length. This description of neutrinos as excitations of the vacuum energy allows only three neutrino mass eigenstates and predicts a neutrino mass sum consistent with observations.

Keywords

Neutrino Mass Sum Prediction, Electron Neutrino Mass from Vacuum Energy Density

1. Introduction

This description of neutrinos as spheres, with radius one quarter of their Compton wavelength and thickness of the Planck length, surrounding a central core along their rotation axis, with diameter of the Planck length, allows only three neutrino mass eigenstates. Describing neutrinos as excitations of cosmic dark energy responsible for observed accelerating expansion of the universe and equating electron neutrino energy density to vacuum energy density (*i.e.*, cosmic dark energy density) then predicts a neutrino mass sum consistent with observations.

2. A Reliable Description of Neutrinos

Neutrinos (and charged Standard Model fermions) can be reliably described as spheres with radius r, mass m, and density $\rho = m\Big/\left(\frac{4}{3}\pi r^3\right)$ consisting of spherical shells (with mass m_S, thickness of the Planck length $l_P = \sqrt{\frac{\hbar G}{c^3}}$, and matter density $\rho_S l_P$ per unit area) enclosing cylinders along their rotation axis (with mass m_A, diameter l_P, and matter density $\rho_A l_P^2$ per unit length). The resulting equation for neutrino radius

$$\begin{aligned}\frac{4}{3}\pi\rho r^3 &= m_S + m_A \\ &= \rho_S l_P 4\pi r^2 + \rho_A \pi l_P^2 (2r)\end{aligned}$$

can be rewritten as

$$\rho r^3 - 3\rho_S l_P r^2 - \frac{3}{2}\rho_A l_P^2 r = ar^3 + br^2 + cr = 0$$

with $a = \rho$, $b = -3\rho_S l_P$ and $c = -\frac{3}{2}\rho_A l_P^2$. The discriminant $b^2c^2 - 4ac^3$ of the cubic equation is positive and the three real roots of the equation correspond to the radii of three neutrino states.

3. Neutrino Masses

Characteristic lengths of neutrinos are their Compton wavelengths $\lambda = \frac{\hbar}{mc}$. If electron neutrinos are spheres with radius $r = \frac{1}{4}\frac{\hbar}{mc}$ and the lowest energy density in the universe (cosmic vacuum energy density $\rho_v = 5.83 \times 10^{-30}\ \text{g/cm}^3$), they have mass $m_1 = \left[\frac{\pi}{6}\rho_v\left(\frac{\hbar}{c}\right)^3\right]^{\frac{1}{4}} = 2.02 \times 10^{-36}\ \text{g} = 0.0013\ \text{eV}$. Neutrino oscillation data [1] [2] predict $m_2 = \sqrt{m_1^2 + 7.42 \times 10^{-5}\,(\text{eV})^2} = 0.00871\ \text{eV}$ and $m_3 = \sqrt{0.5(m_1 + m_2)^2 + 2.517 \times 10^{-3}\,(\text{eV})^2} = 0.0507\ \text{eV}$, resulting in neutrino mass sum = 0.0607 eV consistent with the minimum neutrino mass sum presented in Reference 3.

4. Conclusion

The description of neutrinos presented above allows only three neutrino mass eigenstates and predicts a neutrino mass sum consistent with observations [3]. As background, the Appendix reviews holographic analysis of Standard Model fermions described as spheres that:

- accounts for mass and charge of the nine charged Standard Model fermions;
- specifies electron mass, to six significant figures, in terms of fundamental constants $\alpha, \hbar, G, \Lambda$ and Ω_Λ; and
- explains matter dominance in the universe.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Background on Holographic Analysis of Charged Standard Model Fermions and Matter Dominance

Introduction to holographic analysis

Holographic analysis: 1) explains why three Standard Model fermions are in each charge state $e, \frac{2}{3}e$ and $-\frac{1}{3}e$; 2) relates electron mass to up and down quarks masses; and 3) specifies electron mass in terms of fine structure constant α, Planck's constant $\hbar$, gravitational constant G, cosmological constant Λ, and vacuum energy fraction Ω_Λ.

Holographic analysis is based on quantum mechanics, general relativity, thermodynamics, and Shannon information theory. Bousso's [4] review of holographic analysis indicates only about 5×10^{122} of bits of information on the event horizon will ever be available to describe physics in our universe with cosmological constant [5] $\Lambda = 1.088 \times 10^{-56}\ \text{cm}^2$.

The radius of the event horizon $R_H = \sqrt{3/\Lambda} = 1.66 \times 10^{28}\ \text{cm}$. With Hubble constant [5] $H_0 = 67.4\ \text{km}/(\text{sec} \cdot \text{Mpc})$, critical energy density $\rho_{crit} = \frac{3H_0^2}{8\pi G}$, gravitational constant [5] $G = 6.67430 \times 10^{-8}\ \text{cm}^3/(\text{g} \cdot \text{sec}^2)$, and vacuum energy fraction [5] $\Omega_\Lambda = 0.685$, mass of the observable universe within the event horizon is $M_H = \frac{4}{3}\pi(1-\Omega_\Lambda)\rho_{crit}R_H^3 = (0.187\ \text{g/cm}^2)R_H^2$. So M_H is the total mass of the bits of information necessary to describe all physics within the event horizon, indicating the bits of information describing a particle with definite mass m within the universe are available on a spherical surface around the particle with radius $r = \sqrt{\frac{m}{M_H}}R_H$.

Charged Standard Model fermions

Charged Standard Model fermions described as spheres with holographic radius r, mass $m = (0.187\ \text{g/cm}^2)r^2$, and matter density $\rho = m\Big/\left(\frac{4}{3}\pi r^3\right)$ have a surface mass component and an axial mass component along their rotation axis. Holographic analysis is based in part on general relativity, and general relativity is not valid at distances less than the Planck length $l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.61625 \times 10^{-33}\ \text{cm} = 1.61625 \times 10^{-20}\ \text{fm}$, so fermion surface mass components are treated as spherical shells with mass m_S, thickness l_P, and matter density $\rho_S l_P$ per unit area, while axial mass components with mass m_A are treated as cylinders with diameter l_P and matter density $\rho_A l_P^2$ per unit length. A cubic equation for charged fermion holographic radius in each charge state is

$$\frac{4}{3}\pi\rho r^3 = m_S + m_A = \rho_S l_P 4\pi r^2 + \rho_A \pi l_P^2 (2r)$$

or

$$\rho r^3 - 3\rho_S l_P r^2 - \frac{3}{2}\rho_A l_P^2 r = ar^3 + br^2 + cr = 0$$

with $a=\rho$, $b=-3\rho_S l_P$ and $c=-\frac{3}{2}\rho_A l_P^2$. The discriminant $b^2c^2-4ac^3$ of the cubic equation is positive and the three real roots of the equation correspond to holographic radii of three fermions per charge state. Parameters ρ_S and ρ_A are determined by the holographic radii r_1, r_2, and r_3 using Nickalls' solutions [6] to cubic equations involving

$$r_N=\frac{r_1+r_2+r_3}{3} \text{ and } \delta^2=\frac{\left(r_1-r_N\right)^2}{4}+\frac{\left(r_2-r_3\right)^2}{12}$$

so $\rho_S l_P = r_N\rho$, from $r_N=-b/3$, and $\rho_A l_P^2 = 2\rho\left(r_N^2-\delta^2\right)$, from $\delta^2=\frac{b^2-3ac}{9a^2}$.

Tangential velocity v_T of points on charged fermion surfaces are found from $I\omega=\hbar/2$, where charged fermion moment of inertia $I=I_S+I_A$, with shell moment of inertia $I_S=\frac{2}{3}m_S r^2$, axial moment of inertia $I_A=\frac{m_A}{2}\left(\frac{l_P}{2}\right)^2$, and $I_S \gg I_A$. The electron is the only Standard Model fermion with $v_T=\frac{\hbar}{4mr_N}>c$, but points on the electron surface are not particles and cannot send signals, so no particle or signal travels with speed $>c$.

All persistent structures in the universe are composed of electrons, protons, and neutrons. Protons and neutrons are composed of up and down quarks. The lowest mass charged Standard Model fermion in each charge state (electron, up quark, and down quark) are constituents of all persistent structures in the universe. Holographic analysis then provides succinct explanations relating lowest mass Standard Model fermions in each charge state.

Electrons, the only charged Standard Model fermions persisting in isolation in the universe, have the smallest mass and holographic radius of the nine charged Standard Model fermions. Up quarks, with twice the electron holographic radius, have four times the electron mass. Down quarks, with three times the electron holographic radius, have nine times the electron holographic mass.

Protons are composed of two up quarks and one down quark, and neutrons are composed of two down quarks and one up quark. Isolated neutrons decay to protons, so up quarks must have lower mass than down quarks, consistent with experimental data. Each lepton has a corresponding neutrino, but neutrinos oscillate between mass states when propagating through space and are not persistent structures in the universe. Neutrinos are not consistently related to holographic radii and characteristic lengths of neutrinos are Compton wavelengths $\lambda=\frac{\hbar}{mc}$.

Electron mass

Using electron holographic radius $r_e=\sqrt{\frac{m_e}{M_H}}R_H$, holographic analysis specifies electron mass to six significant figures in terms of fundamental constants

$\alpha, \hbar, G, \Lambda$ and Ω_Λ. Our universe is so large it is almost flat, and Friedmann's equation $H_0^2 = \frac{8\pi G}{3}\rho_{crit} + \frac{\Lambda c^2}{3}$ identifies $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$. Since $M_H = \frac{4}{3}\pi(1-\Omega_\Lambda)\rho_{crit}R_H^3$, $M_H = \frac{(1-\Omega_\Lambda)c^2}{2G\Omega_\Lambda}\sqrt{\frac{3}{\Lambda}}$ is constant in time.

Electrostatic potential energy of electron charge e and positron charge $-e$ separated by $2r_e$ is $V = -\frac{e^2}{2r_e} = -\frac{\alpha\hbar c}{2r_e}$, with Planck's constant [5] $\hbar = 1.05457\times10^{-27}\ \text{g}\cdot\text{cm}^2/\text{sec}$. Two adjacent spheres with holographic radii r_e, a precursor for electron-positron pair production, have total energy $E = 2m_ec^2 - \frac{\alpha\hbar c}{2r_e} = 0$ when $r_e = \frac{\alpha\hbar c}{4m_ec^2}$.

The two equations for r_e give $\frac{\alpha\hbar c}{4m_ec^2} = \sqrt{\frac{m_e}{M_H}}R_H$ and electron mass

$$m_e = \left[\left(\frac{\alpha\hbar^2}{32}\right)\left(\frac{1-\Omega_\Lambda}{G\Omega_\Lambda}\sqrt{\frac{\Lambda}{3}}\right)\right]^{1/3}.$$

If $\Lambda = 1.08800\times10^{-56}\ \text{cm}^2$ and $\Omega_\Lambda = 0.6853855$ (within PDG [5] 2023 error bars) electron mass is specified to six significant figures, since gravitational constant G is only known to six significant figures.

Matter dominance

Charged bits on Standard Model fermion surfaces must be at the rotation axis, to avoid radiation from accelerated charge. Bits of information on the horizon can indicate presence of a charged Standard Model fermion somewhere along the axis between diametrically opposed bits of information on opposite hemispheres of the horizon, so charge $\pm e/6$ must be associated with each bit of information. One, two, or three bit pairs on opposite surfaces of spherical Standard Model fermions at their rotation axis specify charge $-e/3$ or $2e/3$ quarks, or charge e leptons.

A closed universe beginning by a quantum fluctuation from nothing [7] must be charge neutral, with equal numbers of $e/6$ and $-e/6$ bits. Regardless of details of how bits of information specify protons or anti-protons, configurations specifying protons differ in 6 bits from configurations specifying anti-protons. In any physical system, energy is transferred to change bits from one state to another, and $e/6$ bits with lower energy than $-e/6$ bits result in more matter than antimatter in a closed universe, as discussed below.

Temperature at the time of baryon formation was $T_B = \frac{2m_pc^2}{k} = 2.18\times10^3\ \text{K}$, with Boltzmann constant $k = 1.38\times10^{-16}$ (g cm^2/sec^2)/K and proton mass $m_p = 1.67\times10^{-24}\ \text{g}$. The radius of the universe at baryogenesis was [8] $R_B = \left(\frac{2.725}{2.18\times10^3}\right)R_0 \approx 10^{15}\ \text{cm}$, where 2.725 $^\circ$K is today's microwave background temperature and the radius of the universe today is $R_0 \approx 10^{28}$ cm. The time of baryogenesis t_B in seconds after the end of inflation is determined by Friedmann's equation $\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \left(\frac{8\pi G}{3}\right)\varepsilon(R)\left(\frac{R}{c}\right)^2 = -\kappa c^2$. After inflation, in a closed universe so large it is almost flat, the curvature parameter $\kappa \approx 0$. Energy density $\varepsilon(R) = \varepsilon_r\left(\frac{R_0}{R}\right)^4 + \varepsilon_m\left(\frac{R_0}{R}\right)^3 + \varepsilon_v$, where $\varepsilon_r, \varepsilon_m$ and ε_v are today's radiation, matter, and vacuum energy densities. Matter energy density $\varepsilon_m \approx 9 \times 10^{-9}$ (g cm^2 sec^{-2}/cm^3), and vacuum energy density were negligible in the early universe, so radiation dominated when $R \ll 10^{-5} R_0$ before radiation/matter equality. Integrating $\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \left(\frac{8\pi G}{3c^2}\right)\frac{\varepsilon_r R_0^4}{R^2} = \left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{A^2}{R^2} = 0$ where $A = \sqrt{\frac{8\pi G \varepsilon_r R_0^4}{3c^2}}$, from the end of inflation at $t = 0$ to t, determines $At = \frac{1}{2}\left(R(t)^2 - R_i^2\right)$, where R_i is the radius of the universe at the end of inflation and $R_B \gg R_i$. So $t_B = \frac{R_B^2 - R_i^2}{2A} \approx \frac{R_B^2}{2A} \approx 10^{-7}$ seconds. The distance from any point in the universe at baryogenesis to the horizon for that point [9] is

$$d_B = c\int_0^{t_B} \frac{\mathrm{d}t'}{R(t')} = \frac{cR_B}{A}\left[\sqrt{R_i^2 + 2At_B} - R_i\right] \approx c\frac{R_B^2}{A} \approx 10^4 \text{ cm}.$$

Surface gravity on the horizon at baryogenesis is $g_{HB} = G\frac{4\pi}{3}\frac{\varepsilon(R_B)}{c^2}d_B \approx \frac{4\pi G}{3c}\varepsilon_r\frac{R_0^4}{AR_B^2}$, and the associated horizon temperature [10] is $T_{HB} = \frac{\hbar}{2\pi ck}g_{HB} \approx 6 \times 10^{-7}$ K. Occupation probabilities of bits on the horizon at baryogenesis are proportional to their Boltzmann factors. If the energy of $e/6$ bits is $E - \Delta$ and the energy of $-e/6$ bits is $E + \Delta$, proton-antiproton ratio at baryogenesis is $\left(\mathrm{e}^{-\frac{E-\Delta}{kT_{HB}}} / \mathrm{e}^{-\frac{E+\Delta}{kT_{HB}}}\right)^6 = \mathrm{e}^{\frac{12\Delta}{kT_{HB}}} \approx 1 + \frac{12\Delta}{kT_{HB}}$ and the proton excess is $\frac{12\Delta}{kT_{HB}}$. The energy released when a $-e/6$ bit on the horizon changes to an $e/6$ bit raises another bit from $e/6$ to $-e/6$, ensuring charge conservation. The energy to change the state of bits on the horizon must be transferred by massless quanta with wavelengths related to the scale of the horizon, and the only macroscopic length characteristic of the horizon at baryogenesis is the circumference $2\pi R_B$. If the energy 2Δ to change the state of bits on the horizon (and corresponding bits within the universe) is the energy of massless quanta with wavelengths characteristic of a closed Friedmann universe with radius R_B at baryogenesis, $2\Delta = \frac{\hbar c}{R_B}$. Substituting from above, the proton excess at baryogenesis is $\frac{12\Delta}{kT_{HB}} = \left(\frac{24\pi c^2}{R_0}\right)\left(\frac{2.725}{T_B}\right)\sqrt{\frac{3}{8\pi G\varepsilon_r}} \approx 1.8 \times 10^{-9}$. WMAP [11] found (baryon density)/(microwave background photon density) = 6.1×10^{-10}. At baryogenesis, the number of protons, anti-protons, and photons were approximately equal. When almost all protons and anti-protons annihilated to two photons, the baryon to photon ratio became $\frac{1}{3}\left(1.8\times10^{-9}\right)=6\times10^{-10}$, in agreement with WMAP results.