Journal of Modern Physics, 2024, 15, 2274-2315
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Mach's Principle Revised: Is the Inertia, and also Gravitational Interaction of Bodies, Determined by Their Long-Range Gravitational Interaction with Distant Matter in the Universe?

Stilian Lukov

Institute for Space Research at the Bulgarian Academy of Sciences, Sofia, Bulgaria
Email: slukov@space.bas.bg, Lukovstilian50@gmail.com

How to cite this paper: Lukov, S. (2024) Mach's Principle Revised: Is the Inertia, and also Gravitational Interaction of Bodies, Determined by Their Long-Range Gravitational Interaction with Distant Matter in the Universe? *Journal of Modern Physics*, **15**, 2274-2315.
https://doi.org/10.4236/jmp.2024.1512093

Received: April 2, 2024
Accepted: November 25, 2024
Published: November 28, 2024

Abstract

The work analyzes the basic assumption in Mach's principle, namely that the inertia of material bodies is determined by their gravitational interaction with distant masses in the universe. However, while Mach's principle is based on the so-called "long-range gravitational interaction" characterized by an infinitely large propagation velocity, our study is based on a "modified" long-range principle, assuming a very large but finite propagation velocity of the gravitational interaction between local material objects and distant matter. Thus, it is postulated that there are two types of gravitational interaction—short-range gravitational interaction between local objects and long-range gravitational interaction between local objects and distant matter in the universe, which are characterized by different propagation speeds, but with the same gravitational constant. On the basis of the modified long-range principle, a model of distant matter is built in the form of a hollow spherical layer with negligible thickness. The phenomenological assumption is made that the movement with acceleration of the local reference frame (RF) is related to a change in the spherically symmetric distribution of the lines of gravitational interaction of this RF with distant matter, which is expressed in a corresponding asymmetric distribution of the effective mass density on the hollow sphere. A simplified (idealized) model of the effective change of the hollow sphere of distant matter by cutting off separate segments of the sphere is proposed. On the basis of the model, the possibility of representing the inertial effects in three simplest types of reference frames through a corresponding gravitational interaction is considered: 1) inertial RF; 2) RF moving in a straight line with constant acceleration; 3) RF rotating with constant angular velocity. Expressions were obtained for the gravitational accelerations acting on the test body located inside the hollow sphere with a corresponding change ("cutting"). It is concluded that these accelerations can in a first approximation represent the inertial accelerations of the main types noted above. It is shown that in order to obtain reasonable values of the truncation parameters of the hollow sphere, it is necessary to assume that the gravitational interaction inside this sphere is not of the Newtonian type, *i.e.* the same depends on the distance not according to the law $1/r^2$, but according to modified law with a non-integer (fractional) exponent. This law corresponds to a fractal structure of the source of attraction inside the truncated sphere of distant matter. The issue of the possibility of the supposed modified long-range interaction is briefly discussed on the basis of a comparison of the finding a connection with the lines of force of the same with the "cosmic strings" assumed by a number of researchers, along which corresponding excitations (waves, particles) moving at super-light speed. The work advances the idea of the presence of unity and at the same time oppositeness of the inertia of material objects and the known gravitational interaction between them, which are generated by the properties of symmetry of the long-range gravitational interaction. Moreover, while the inertia of the bodies is due to the violation of this symmetry caused by their movement with acceleration, the gravitational interaction between the bodies is due to the aspiration to restore the symmetry of a far-reaching gravitational interaction, which is disturbed by the presence of local material bodies. In the conclusion of the work, the important physico-philosophical significance of Mach's principle is emphasized, expressed in the understanding that not only the world of microscopic objects ("micro-world"), but also the world of huge cosmic objects ("mega-world") can have a corresponding impact on our "macroscopic" world.

Keywords

Inertia, Gravitatrion, Mach's Principle, Equivalence Principle

1. Introduction

As is well known, in his works Newton gives an explanation of the emergence of inertial forces during the accelerated movement of bodies, which basically boils down to the following: there are two types of body movements—relative and absolute. Relative motion is motion relative to other material bodies and does not involve inertial forces, while absolute motion is relative to the so-called "absolute space" and is accompanied by the emergence of inertial forces [1]. Of course, this difference made an impression right away and caused a justified criticism—as we know, initially by Berkeley [2] and later—in a much clearer form by Mach [3] [4]. At the same time, both of them (much more precisely Mach) make an alternative assumption—there is only relative motion, which has two forms: motion relative to nearby bodies, which is not associated with interaction and corresponding inertial forces, and motion relative to the distant matter in the universe (the "distant fixed" stars), where there is interaction and inertial effects [3]. Noting the much more precise formulation of this assumption than Mach, Einstein christened it "Mach's Principle" [5].

But, in addition to the revolutionary view that all movements, including those with acceleration (including rotational movements), are relative, Mach also gave an unequivocal answer to the question of the nature of inertial phenomena—inertia arises from the gravitational interaction of an accelerated moving body with distant matter. Therefore, according to Mach, inertia is reducible to gravity. Of course, this is a very general statement, clearly in need of a corresponding quantitative analysis, which unfortunately Mach did not do.

It should be noted that both earlier and now, the question of a more or less strict explanation of inertial phenomena, including in the context of Mach's principle, has not escaped the attention of researchers, as previously (for example, [6]-[11]), and currently (for example, [12]-[16]).

In our research, we will also try to answer the question—is it possible to quantitatively explain the main types of inertial effects through the corresponding gravitational interaction of material bodies with distant matter in the universe, adhering to and at the same time modifying the Newtonian theory of gravity (NTG). We must fade away, that at the time, Mach had in mind that the gravitational interaction of bodies with distant matter can be described precisely within the framework of Newtonian gravity.

2. Preliminary Remarks: Main Points in Mach's Principle

Perhaps it would not be far from the truth to say that there is hardly any other stated idea in physics which has given rise to so many different interpretations and evaluations as Mach's idea concerning the source of the inertia of bodies. On this occasion, the famous American physicist Robert Dicke called him the "Many-faced Mach". Probably, the reason for this is only the purely verbal formulation of the idea, without corresponding mathematical and quantitative analysis. Proceeding to our attempt at a quantitative formulation of Mach's idea, we should briefly note the main points of its verbal formulation, which more or less become clear from his works.

1) Kinematic and dynamic aspects of Mach's principle

In the formulation of Mach's principle, two aspects clearly stand out—kinematic and dynamic. The kinematic aspect is expressed in Mach's opinion that all movements are relative, including movements with acceleration (for example, rotation). Moreover, the most general motion of a given body is the motion relative to the rest of the matter as a whole ("the system of distant fixed stars"). From here follows the statement that the movement of the body relative to the distant matter is equivalent to the movement of the same in the opposite direction relative to the stationary body, which means that in the kinematic aspect there is complete symmetry.

The dynamic aspect concerns the interaction between the body and distant matter. According to Mach, all bodies in the universe are interconnected, *i.e.* interact with each other. But, at the same time, the system of distant interconnected objects considered as a whole (the "distant fixed stars") also affects the individual bodies. At first glance, it can be concluded that Mach's idea actually replaces one "metaphysical" assumption with another, also metaphysical assumption: the inertial phenomena observed in the accelerated RF are due not to the movement of the same relative to "absolute space", and of their movement relative to the "distant matter in the universe". Although the totality of distant stars is conceivable as something material, it is actually inaccessible to direct or indirect measurement by appropriate physical apparatus.

But, Mach adds something essential in his idea, namely that the mediator between the local RF and the distant matter is the gravitational interaction between them. Then, if we can measure certain specific characteristics of this interaction, we can judge the truth of the assumption. Obviously, this is possible if there is a certain (quantitative!) model of this interaction that reflects its specific characteristics. The question arises—is there reciprocity in this interaction? It is reasonable to assume that the insignificant in size and mass body could not cause any change in the structure (shape, mass density distribution) of the huge in size and mass distant matter. Here, however, in contrast to this conception, we assume that there is a certain symmetry also in the dynamic aspect. In other words, if the acceleration of the body is equal to the acceleration of the distant matter, then this causes a corresponding change in the structure (shape) of the distant matter, for example, a flattening towards the equator of the hollow sphere of the distant matter. This change precisely gives rise to a reverse gravitational action of the distant matter on the body, understood as the action of "inertial" forces and causes a change in the shape of the same. This is conventionally illustrated in **Figure 1** for the case of rotational motion.

It should be noted that the change in the structure of the distant matter in question is effective, *i.e.* the same only formally represents the real interaction of the accelerated RF with distant matter. In fact, the very gravitational interaction

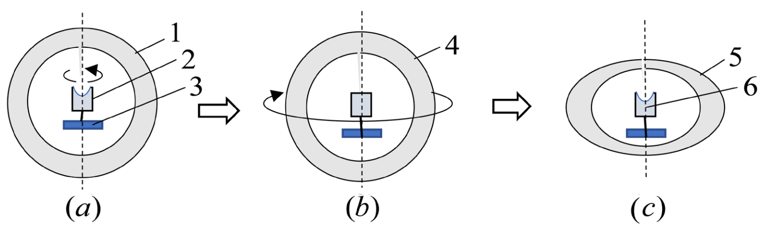

Figure 1. Representation of the dynamic interaction between the accelerated moving body and the distant matter in the universe: (a) Rotational motion of the local RF relative to the local "immobile" RF: 1—Hollow sphere of the distant matter (DM); 2—Local RF ("Newton's bucket of water"); 3—Bulk RF (the earth's surface); (b) Rotation of distant matter in the opposite direction to LRF: 4—rotating Sphere of DM; (c) Change in the shape (flattening) of the DM sphere as a result of the rotation: 5—Sphere of DM with the flattened shape; 6—change of shape of water in Newton's bucket.

of the local RF with the distant matter changes, which is presented as a change in the shape, the distribution of the mass density, and the form of the DM.

2) Action of near and far masses on the inertia of bodies

According to Mach, if the rotating bucket in Newton's experiment is massive enough, then its rotation will also cause the water to rotate and rise up the walls. In practice, real bodies have no such mass and cannot produce such effects. This, however, can be done by distant matter, which has an enormous total mass, and regardless of vast distances, will act on water in the same way. In our opinion, this is about another effect—the so-called "gravitational induction", where when one body is moving with acceleration, it causes another nearby body to also move with acceleration. A number of specialists, including Einstein, at one time considered this possible effect (e.g. [17]). The effect, however, has a local character and should be distinguished from the global effect of the interaction of bodies with distant matter.

According to Mach the difference between the two types of gravitational interaction is only quantitative. Our view here differs from that of Mach—in our opinion, the gravitational interaction of individual bodies with distant matter considered as a single material object has specific features that distinguish it from the gravitational interaction between two separately taken "isolated" bodies. Determining these features is exactly the goal we will try to achieve in the present study.

3) The principle of long-range gravitational interaction—fundamental in Mach's principle and its modification proposed here

As we have already noted, in his view on the indicated "most general" gravitational interaction—between macroscopic bodies and the universe, Mach completely relies on Newtonian mechanics and the Newtonian theory of gravitation (NTG). At the same time, the main principle in Newtonian gravity—the "Principle of long-range interaction" makes it possible to ignore the problem of the huge distance to the distant matter—this huge distance can always be "overcome" by a corresponding huge mass of the distant matter or the universe as a whole [3] [4]. Moreover, this principle makes it possible to ignore the problem of time—the infinite speed of propagation of the gravitational interaction according to NTG means that there is no "reaction time" for inertial effects—they occur immediately after the application of the accelerating force on the body. A third important aspect of the principle of long-range action is that it implies the action of one material object on another "through empty space", *i.e.* without material support.

It is interesting to note that Newton himself protested against this notion, but nevertheless he formulated his theory of gravitation on the basis of this conception, and Mach, who criticized Newton's view of the nature of inertia, was a firm supporter of this principle. According to him, the principle of long-range action "has only temporarily yielded to the opposite principle of short-range action, and it is not excluded that it will return to physics in the future" [4].

But, currently, as we know, the opposite concept prevails in modern physics—the "principle of short-range action", which is valid for all types of interactions including gravity, and which is often identified with the "Field Theory", according to which the interaction between material objects is carried out directly by means of a material carrier (the corresponding type of matter—*i.e.* n. "field"). But, bearing in mind the ultimate transmission speed of the interaction in this theory—the speed of light in a vacuum, the problem immediately arises—how then could very distant objects (stars, galaxies, etc.) affect the material bodies around us, as this implies Mach's principle? (At the time, especially after the creation of General Relativity (GR), Einstein was cool with Mach's principle, saying that it did not "connect" with the principle of field theory [18]).

Then, the thought naturally arises—isn't it possible to bring the two principles together—long-range and short-range? It should be noted that such an idea has been expressed more than once (for example, [19]). In our study, we will also adopt, in a certain sense, an intermediate position, representing a kind of "Modified principle of the long-range gravitational action":

The modified long-range gravitational interaction (MLRGI) of bodies with distant matter is characterized by the same Newtonian gravitational constant G as the known short-range gravitational interaction (SHRGI) between them, but with a finitely large-valued propagation speed w that many times exceeds the speed of the SHRGI c

$$G_{LRGA} = G_{SHRGA} = G; \quad v_{LRGA} = w \succ\succ v_{SHRGA} = c; \quad w/c \succ\succ 1\,. \tag{1}$$

The question can be raised—what exactly can this so-called "intermediate" approach change in the given case? In short, according to the modified principle of long-range gravitational interaction there will be a very small but still finite reaction time Δt_{react} for bodies against their acceleration under the action of an external force

$$\Delta t_{ireac} = t_{InertEffAp} - t_{ExtForAct}\,; \quad w = \frac{2R_{DM}}{\Delta t_{ireac}}\,, \tag{2}$$

where $t_{ExtForAct}, t_{InertEffAp}$ are, respectively, the time instant at which the external accelerating force is turned on and the instant at which the inertial reaction occurs, R_{DM} is the distance between the accelerated body and to DM.

At the same time, it follows from principle of Modified LRGI that there will be a vast, yet finite in size, region of the much larger bulk Universe that can exert an influence on the bodies within it. Thirdly, this principle provides for the existence of a corresponding material carrier, related to the gravitational interaction between the accelerating body and distant matter.

4) The concept of "reference frame" (RF) in the Mach's principle

In his writings, Mach criticized Newton's understanding of the motion and inertia of bodies, but nevertheless relied on Newtonian mechanics and theory of gravitation. At the same time, as is well known, the principle of relativity of location and movement is firmly adopted in Newtonian mechanics: the position and movement of a given body can only be determined with respect to other bodies. In other words, in order to determine the location and movement of bodies, it is necessary to set a corresponding reference frame—another body or a system of connected (fixed) bodies. But, speaking of the motion of the rotating bucket relative to "absolute space", Newton actually deviated from this concept. It is here that Mach directs his criticism—according to him, one cannot speak of position and motion relative to something imaginary that cannot serve as a reference frame. Instead, he suggests talking about determining the motion (and position) of bodies relative to distant matter in the universe or, relative to distant fixed stars. Although this notion is not as well formed as the notion of immediate (local) RF, in which distances can be measured in a given coordinate system, it can be said that this is how Mach raises the concept of RF to the highest rank—the level of the universe at hole. When Mach speaks of motion relative to the "distant fixed stars", he effectively assumes that the aggregate of distant stars is something fixed and unchanging, even though these stars are actually moving (some of them at enormous speeds) relative to each other or to another system of space objects. But, this resembles our idealized view of the reference system as a rigid body that is unchanging, even though it is composed of connected particles that are constantly moving!

3. Main Types of Inertial Effects and Corresponding Reference Frames to Be Analyzed in the Present Study

It should be noted that in his works Mach actually treated only one type of inertial effects—the centrifugal ones, which Newton also sought to explain. But, as is known, other types of inertial effects can be specified, for example, those that occur in rectilinear motion with constant acceleration, Coriolis accelerations, etc. It is close to the mind to assume that the different types of inertial effects should correspond to a different type of gravitational interaction of the given body with distant matter.

We will not dwell here on a detailed analysis of the different types of inertial effects, and in our study we will consider only three types of inertial effects and the corresponding local RF in which these effects exist:

1) Absence of inertial accelerations in the so-called "inertial" RF (IRF) moving in a straight line at a constant speed relative to another bulk RF that is stationary or also moving in a straight line at a constant speed;

2) Rectilinear inertial accelerations occurring in non-inertial RF, which move in a straight line with constant acceleration relative to a stationary (inertial) RF;

3) Centrifugal inertial accelerations occurring in non-inertial RF that rotate with a constant angular velocity relative to a stationary (inertial) RF.

4. Towards a Model of Distant Matter in the Universe: Our Model

4.1. Basic Assumptions

Proceeding to the analysis of the essence of Mach's assumption, we will focus first of all on the selection of an adequate model of the distant matter in the universe, allowing to carry out approximate but realistic calculations. It should be noted that in known studies, the model of the universe in the form of a hollow sphere (spherical layer) with a small thickness compared to the radius of the sphere is widely used (for example, [20]). The reason for accepting such a model is given by the weakness of the gravitational interaction. As a consequence, nearby material objects have almost no influence on a given material object, while the large enough distant matter (spherical layer) contains enough total mass to be able to exert such an influence. It should be noted that such a model assumes that the DM sphere (Sphere DM) coincides with the Universe itself, *i.e.* outside this sphere there is no matter. In the case where it is assumed that the universe has infinite dimensions and the principle of long-range action is accepted, this model will be refuted by the Neumann-Zeeliger paradox, since more and more distant layers of matter will have an influence and, as a result, the total the action of matter in Sphere DM will grow to infinity.

In the present study, we suggest that the Sphere DM does not coincide with the Universe itself, which may have far larger dimensions. But, then, perceiving the proposed modification of the long-range principle with a very large but finite interaction velocity, we can justify the formation of the DM sphere with a finite layer thickness as a result of the action of the two factors—the weakness of the gravitational interaction, which requires the inclusion of a sufficiently large size (radius) part of the bulk Universe and the finite speed of this interaction, which “cuts off” the influence of very distant regions.

This is qualitatively represented in **Figure 2(a)**, where the formation of the resulting dependence of the effective mass $M_{ef} = M_{ef}(R)$ of the DM from the radial distance R is illustrated as a result of the intersection of the two dependences:

a) Growth of the effective mass proportional to the third power of the size (radius) $M_{ef1} \propto \rho_0 R^3$ where ρ_0 is the average density of matter in the universe, and

b) A rapid decrease in the effective mass after a certain distance related to the supreme velocity of propagation of the LRGI according to an exponential law analogous to Yukawa’s potential [21] $M_{ef2} \propto \rho_0 e^{-kR}$, where k depend (are proportional) to the rate of long-range gravitational interaction w.

Then, the resulting region in which distant matter can effectively act will be determined by the intersection of the two dependences $M_{efres}(R) = M_{ef1}(R) \otimes M_{ef2}(R)$, where the sign $\otimes$ stands for functional multiplication (**Figure 2(a)**).

As stated above, according to Mach, all bodies in the universe are connected, both to each other and to distant matter through the modified long-range gravitational interaction. Figuratively speaking, according to this concept, between the separate body and the distant matter, there are a kind of “gravitational connections” understood as lines of force of the Modified LRGI. This is tentatively depicted in **Figure 2(b)** by the double-sided arrows between the point-like sample body and the continuous spherical layer of distant matter.

In the present study, we will strengthen the idealization of the known Sphere DM model by assuming that the thickness of the hollow sphere (spherical layer) is infinitesimally small, *i.e.* we have a model with a massive hollow infinitely thin sphere (**Figure 2(c)**). This means that we go from the real resultant function of the effective action of distant matter to an idealized "Dirac-delta" function $M_{efresDir} = M_{efres}\delta(R_0)$; $\delta(R_0) = \begin{cases} 1 & if \quad R = R_0 \\ 0 & if \quad R \neq R_0 \end{cases}$, which has an infinite-valued derivative but a finite eigenvalue (**Figure 2(a)**).

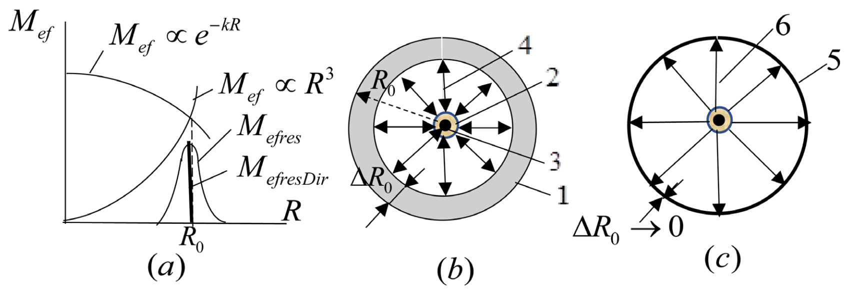

Figure 2. To the explanation of the DM model: (a) Illustration of the formation of a spherical region (layer) of finite thickness as a result of the intersection of two opposite dependences of the effective mass on the radius of the sphere; (b) Idealized model of DM in the form of a hollow sphere with finite layer thickness: 1—sphere; 2—local RF; 3—test body in LRF; 4—lines of the long-range gravitational interaction; (c) Super idealization of the model of the distant matter in the form of a hollow sphere with an infinitesimal thickness: 5—an infinitely thin hollow sphere; 6—one-sided arrows on lines of the Modified LRGI.

Here again, the lines of Modified LRGI are presented in the form of one-sided arrows pointing from the Local RF to any point on the inner surface of the sphere. It should be noted that in the figure the arrows are discretely located, but in reality they form a continuous set ("continuum"), since the points of the Local RF are connected to each point of the inner surface of the sphere DM.

It should be noted that the proposed model makes it possible to explain the above-noted (item 2) difference in the action of near and far masses on inertial effects, as a consequence of the fact that each individual body forms its "own" (individual) Sphere DM, which is independent of other bodies. In addition, the model makes it possible to imagine a "movement" of the Sphere DM in accordance with the movement of the test body relative to the local RF, which actually reflects the fact that in each subsequent position the given body forms a new Sphere DM, which is a "copy" of the previous one (**Figure 3(a)**). In fact, the set of lines of the gravitational interaction of the test body with distant matter is actually moved (**Figure 3(b)**).

Adopting such an idealized model of DM makes it possible to represent the assumed change of the parameters of the DM sphere (sphere shape, thickness of the spherical layer, mass distribution in it, etc.) only by removing parts of the sphere (spherical segments), which we will refer to as "effective truncation of the DM sphere". Here we remind again that the term "effectively" means that it is not

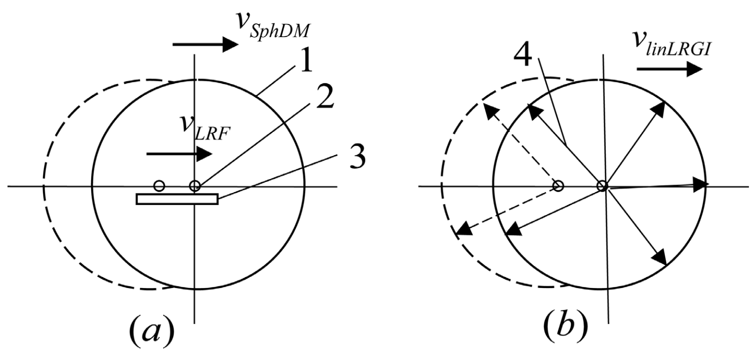

Figure 3. (a) Imaginary displacement of the Sphere DM following the displacement of the test body relative to the local RF: 1—Individual Sphere DM; 2—test body; 3—local RF; (b) Actual displacement of the gravitational interaction lines following test body relocation: 4—gravitational interaction lines.

the DM sphere itself that actually changes, but the parameters of the long-range gravitational interaction inside it (for example, a change in the uniform distribution of the density of force lines of this interaction). This will be shown further in the analysis of the specific three types of RF we noted above.

Moreover, our model enables a simplified representation of the gravitational interaction between the Sphere DM and the sample body inside it based on a corresponding modification of the Newtonian theory of gravity. In our further investigation based on the proposed target model, we will adopt the following basic assumptions:

a) A given individual segment of the infinitely thin Sphere DM attracts a point-like massive sample body as a complete massive object located in the center of the segment;

b) The mass of the object is equal to the total mass of the separated segment, which is proportional to the area of the same with a uniform distribution of the surface mass density;

c) The dependence on the distance to the object is determined by a power law $\left(\propto 1/R^n\right)$, whose power indicator can be different from the power indicator in NTG $(n=2)$.

To prove assumption (a), we consider the equatorial section of the DM sphere with finite layer thickness and with a separate spherical segment depicted in Figure 4(a).

According to the figure, we can represent the spherical segment of finite thickness 1 as the result of subtracting from each other two filled concentric spherical sectors 2, 3 with radii $R_{s1}=R_0$ respectively $R_{c2}=R_0-\Delta R_0$, where $R_0, \Delta R_0$ are the radius of the sphere and the thickness of the spherical layer, respectively.

Using the well-known expression for the position of the center of gravity of a spherical sector [22]

$$z_{ci}=\frac{3}{4}\left(R_i-\frac{1}{2}h_{si}\right);(i=1,2), \tag{3}$$

for the position of the center of gravity of the spherical segment 1 we can write

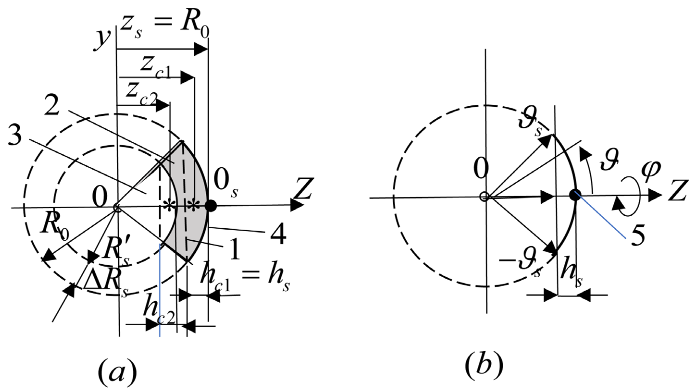

Figure 4. a) Towards the proof of the center of attraction of the spherical segment: 1—spherical segment with finite thickness; 2—outer spherical sector with a radius equal to the radius of the sphere; 3—internal sector with a smaller radius; b) 4—spherical segment with infinitesimal thickness: 5—effective massive object located in the center of attraction of the segment.

$$z_s = z_{c1} - z_{c2} = R - \frac{3}{4}\left[\left(R_{c1} - R_{c2}\right) - \frac{1}{2}\left(h_{c1} - h_{c2}\right)\right], \tag{4}$$

where $h_{c1}; h_{c2}$ are the heights of the sectors that represent the outer surfaces of the spherical sectors, according to **Figure 4(a)**.

Then, when the radius of the inner sector tends to the radius of the outer sector, *i.e.* to the radius of the sphere $R_{c2} \to R_{c1} = R_0$, we have $h_{c2} \to h_{c1} = h_s$ and

$$z_s = R\ , \tag{5}$$

i.e. the center of gravity of the residue the massive spherical segment of infinitesimal thickness, which is actually also the center of attraction, will coincide with its geometrical center (**Figure 4(b)**).

Assumption (b) follows from the vector summation of the forces of the modified long range gravitational interaction acting on the test body, which can be represented as

$$\boldsymbol{F}_{res} = F_1 \boldsymbol{e}_Z \int_0^{2\pi} \mathrm{d}\varphi \int_{-\vartheta_s}^{\vartheta_s} \frac{1}{2} \cos\vartheta \mathrm{d}\vartheta = \boldsymbol{e}_Z 2\pi F_1 \sin\vartheta\Big|_{-\vartheta_s}^{\vartheta_s}, \tag{6}$$

where $\boldsymbol{F}_{res} = F_{res}\boldsymbol{e}_Z$ is the resultant force, which is directed along the axis of symmetry $0Z$, is the $\boldsymbol{F}_1 = F_1 \boldsymbol{e}_F(\varphi, \vartheta)$ is the "unit" force, the direction of which is determined by the angles (φ, ϑ), being $\pm\vartheta_s$ is the boundaries of the spherical segment at elevation angle, $\boldsymbol{e}_Z$ is the unit vector along the axis $0Z$ (**Figure 4(a)**).

But, in an analogous way, the area of the segment of the sphere with radius R_0, if we pass from integration along the angle to integration along the radius, bearing in mind that $\cos\vartheta = (R_0 - r)/R_0' = 1 - r/R_0$

$$S_s = R_0 \int_0^{2\pi} \mathrm{d}\varphi \int_{-\vartheta_s}^{\vartheta_s} \frac{1}{2} \cos\vartheta \mathrm{d}\vartheta = 2\pi R_0 \int_{R_0 - h_s}^{R_0} r \mathrm{d}r = 2\pi R_0 r\Big|_{R_0 - h_s}^{R_0} = 2\pi R_0 h_s. \tag{7}$$

Therefore, in the expression (5) we can replace the result of the integration over an angle in place by a simple multiplication over the area of the spherical segment

$$\boldsymbol{F}_{res} = F_1 S_s \boldsymbol{e}_Z = 2\pi F_1 R_0 h_s \boldsymbol{e}_Z . \quad (8)$$

Since the force $\boldsymbol{F}_{res}$ is a force of gravitational attraction to the center of the segment, for the attraction force per unit mass, *i.e.* for the acceleration of the test body to the segment, we can write

$$\boldsymbol{a}_{sg} = -G\frac{M_s}{R^n}\boldsymbol{e}_Z = -G\frac{\rho'_{s0} S_s}{R^n}\boldsymbol{e}_Z = -G\frac{2\pi R_0 \rho'_{s0} h_s}{R^n}\boldsymbol{e}_Z , \quad (9)$$

where ρ'_{s0} is the "effective" surface density of the mass.

This density is related to the real bulk density (provided that it is uniformly distributed in the sphere bounded by the sphere) by the simple relation

$$\rho'_{S0} = \frac{\frac{4}{3}\pi R_0^3}{4\pi R_0^2}\rho'_0 = \frac{1}{3}\rho_0 R_0 . \quad (10)$$

Since we assume that when changing the shape of the Sphere DM after removing a separate segment from it, the total amount of lines of Modified LRGI is preserved, which are only redistributed on the remaining surface, then the above ratio should be specified, as instead of the full area of sphere $S_0 = 4\pi R_0^2$, put the reduced area resulting from the truncation of the sphere $S'_0 = 4\pi R_0^2 - S_{ccut}$ where S_{ccut} is the area of the cut segment or segments.

On the other hand, the assumption (c) of a possible distinction of the value of the degree indicator from the value $n = 2$ of the same in NTG and what this value should be, we will consider when applying the present model for the analysis of the first three ranks of RF, which were indicated above.

4.2. Basic Parameters of the Proposed Distant Matter Model

It is important to note that our concept of "distant matter" is based on the well-known idealized view of the currently observed region of the Universe (the so-called "Metagalaxy"), in the form of a sphere (ball) with a finite value of radius and uniformly distributed average density of matter. We are inclined to accept this idealized view, since it allows for the purposes of quantitative estimation of the model to use the currently accepted parameters of the observable part of the universe—radius R_U and average volume density of matter ρ_U.

Then, equating the sphere of distant matter in our model with the currently observable part of the Universe, we will further use the currently accepted parameters [23], *i.e.* we will assume that

$$\rho_0 = \rho_U \approx 10^{-26}\ \text{kg/m}^3 ; \quad R_0 = R_U \approx 10^{26}\ \text{m} . \quad (11)$$

This assumption actually means that the point P of intersection of the two dependences $M_{ef1}(R) \propto \rho_0 R^3$ and $M_{ef2}(R) \propto \rho_0 e^{-kR}$ in **Figure 2(a)** corresponds precisely to this distance. Of course, the current value of the size of the observable part of the universe is tentatively determined on the basis of many different observational data and cannot be considered final. But, for the quantitative calculations based on our model, we will accept these values, which can subsequently be further refined.

On the basis of these parameters, we can roughly estimate the speed of Modified LRGI *w*, if the delay time of the DM reaction Δt_{reacDM} on the test body after application of the external force is extremely small, but not equal to zero.

Obviously, the determination of this time should be based on a corresponding experimental measurement. But, still, for a rough estimate of the value of this delay, let us make a completely arbitrary analogy between the DM sphere and the microscopic atomic structure consisting of a central nucleus and an electron shell, which has a spherical shape.

As is known, the main interaction between the nucleus and the shell of an atom is electromagnetic interaction, which is transmitted at the speed of light *c*. It follows from this that if any disturbance occurs in the impact (electrostatic attraction) of the nucleus on the electronic shell of the atom, then the reaction on the shell should be transmitted with a delay of the order of $\Delta t_{cereacA} \approx \frac{2R_a}{c} = \frac{2\times10^{-8}}{3\times10^{8}} = 0.77\times10^{-16} \approx 10^{-16}\ \text{s}$, where $R_a \approx 10^{-8}\ \text{m}$ is the typical size (radius) of the atom [24].

Let us assume that there is a kind of recurrence in the hierarchical structure of matter, so that the "megastructure" under consideration (the DM sphere) resembles its "microstructure" (the atom), so that the response time of the spherical DM is close to the reaction time of the electron shell of the atom, then for the value of the velocity of the Modified LRGI we get $\Delta t_{reacDM} = \Delta t_{reacA} \approx 10^{-16}\ \text{s}$.

We will note that such a concept in a certain sense approaches the well-known concept of the existence of peculiar "atom-like" gravitational structures—so-called "Gravitational atoms" (for example, [25]). Hence, for the value of the transmission rate of this interaction we get

$$w = \frac{2R_0}{\Delta t_{DMreac}} \approx \frac{2\times10^{26}}{10^{-16}} = 2\times10^{42}\ \text{m/s}. \tag{12}$$

Of course, this assumption is quite conditional, and the real inrtial response delay time of real material objects and the corresponding inertial transfer rate can only be determined on the basis of relevant experiments.

5. The Model in Action: Representation of the Main Types of Inertial Effects by Corresponding Gravitational Accelerations inside the Distant Matter Sphere

5.1. Absence of Inertial Accelerations in Inertial RF as the Absence of Gravitational Accelerations inside the DM Sphere

Consider a local RF that is stationary or moving in a straight line with constant velocity relative to a bulk stationary RF (the "Galilean ship" relative to the Earth). As we know, no inertial forces arise in such an RF, which is reflected in the Galileo-Newton Mechanical Principle of Relativity: Mechanical processes in an RF that moves in a straight line at a constant speed proceed in the same way as when this RF is stationary. Here, our aim is to elucidate whether such type of RFs can be mapped to corresponding RFs located inside the DM sphere.

As noted above, in our model we assume that when the local RF is located at the center of the Sphere DM, then between each point of the RF and the inner surface of the Sphere DM there exist radially directed lines of modified long-range gravitational interaction, which are uniformly distributed given the uniformity of the mass distribution in the spherical layer (**Figure 2(a)**), respectively, in our idealized model—on the surface of the sphere (**Figure 2(c)**).

Then, when the local RF is located at the center of the hollow sphere of the DM, there will be a complete compensation of the sum of the forces of Modified LRGI and, therefore, no gravitational forces will act on the Local RF (and the test bodies inside it). Here two questions arise: does the indicated compensation of the forces of Local RF remain in force a) when the position of the Local RF is shifted from the center of the Sphere DM? and b) in rectilinear motion of Local RF with constant speed relative to bulk stationary RF?

It should be noted that the question of whether a body located inside a hollow sphere with a uniform distribution of mass density will be attracted to the inner surface of the sphere was already dealt with by Newton. As is well known, he proves that at any point inside the hollow sphere attraction will be absent given the mutual compensation of the forces of attraction to arbitrarily taken sufficiently small opposite regions (spherical segments) of the hollow sphere [26]. Moreover, this conclusion follows not only from the point of view of the Newtonian theory of gravitation, but also from the point of view of the far more perfect Einsteinian theory of gravitation, *i.e.* the General relativity. However, a number of specialists disagree with this opinion and consider that inside the hollow sphere there will be a "residual" gravity that will act on the test body, increasing as it approaches the surface of the sphere (for example, [27]).

Let us discuss this question from the perspective of our assumptions laid out in the previous point. For this purpose, we will consider our model of the distant matter in the form of a hollow sphere of infinitesimal thickness, with the condition that the test body is located at an arbitrary point that does not coincide with the center of the sphere (**Figure 5**).

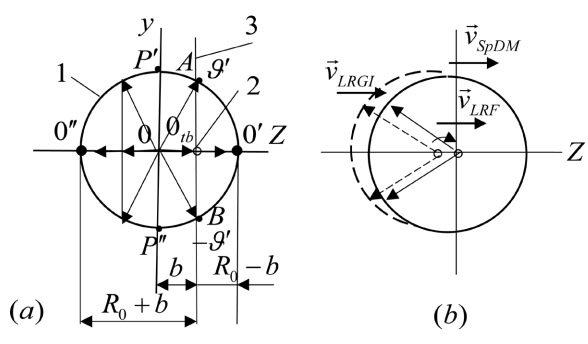

Figure 5. (a) Towards the determination of the resultant force of attraction of the test body to the inner surface of the infinitely thin hollow sphere: 1—hollow sphere; 2—test body; 3—secant plane perpendicular to the radius passing through the point of placement of the test body; (b) Moving the lines of the Modified LRGI and the Sphere DM when the position of the Local RF is shifted.

According to the figure, if the local RF is located not at the center of the DM sphere, but at an arbitrary point inside it, then we can mentally construct a secant plane (3) that passes through the test point body and is perpendicular to the radius passing through this point. Obviously, this plane will divide the sphere into two complementary segments: small $(A0'B)$ and large $(A0''B)$. Then, according to our assumption in point 4, the two sectors will attract the test body in opposite directions, *i.e.* the resultant action will be determined by the difference of these opposing forces.

Bearing in mind what was stated in the previous point and expression (8), for the resulting gravitational acceleration of the test body, which is directed along the axis of symmetry $0'00'' \equiv 0Z$ we can write

$$\boldsymbol{a}_{gsres} = a_{gsres}\boldsymbol{e}_Z;$$
$$a_{gsres} = -\kappa_g\left(\frac{S_{s1}+\Delta S}{R_0^n} - \frac{S_{s2}-\Delta S}{R_0^n}\right) = -k_g\frac{(1/2)(S_{sph}-S_{sph})}{R_0^n} \equiv 0, \tag{13}$$

where S_{s1}, S_{s2} are the areas of the small and large segments, respectively, and ΔS is the area of the part of the sphere (girdle) $(P'P''AB)$, k_g is the coefficient of proportionality.

This gridle is added to the small segment, respectively, is subtracted from the large segment, since the direction of the force lines of the Modified LRGI is reversed according to the figure. Therefore, the resulting areas of the two segments become equal to half the area of the sphere. As can be seen from the above expression, the resultant gravitational acceleration is equal to zero at any displacement of the test body from the center of the sphere and at any value of the exponent, *i.e.* under an arbitrary law of gravitational interaction. This result shows that inside the hollow sphere gravity will be absent due to its "self-compensation".

Then, the absence of gravitational accelerations in the hollow sphere gives us reason to assume that it imitates the local RF, which is stationary relative to the bulk (also stationary) RF, in which inertial forces and accelerations are absent (Galileo's ship, when it is stationary).

But, we have to account for the fact that when the RF moves at a constant speed, the mechanical processes proceed in the same way when this axis is stationary ("Galileo's ship, when it moves at a constant speed "relative to water, *i.e.* Earth). In other words, we should explain what the taken-for-granted Galileo-Newton mechanical principle of relativity is based on?

Our model makes it possible to explain the existence of the indicated principle on the basis of the understanding that the constant-velocity relative motion of the local RF with respect to a bulky stationary RF does not change the structure of the lines of Modified LRGI of this Local RF with the distant matter—the same as if "move" along with the movement of Local RF (**Figure 5(b)**). Hence, it follows that the specified full compensation of the gravitational forces acting on the side of the hollow sphere on the LRF and the other bodies in it, in the stationary state of the LRF relative to the bulky RF, continued to be preserved.

It is important to note that the realization of an inertial LRF moving in a straight line with a constant velocity relative to another bulk RF that is also inertial is an idealization and in practice is possible only approximately, since any more general RFs actually moves with non-constant velocity as a result of the (local) gravitational interaction with other objects. But, then the "imitation" of the inertial RF as RF, which is in the ideal-shaped DM sphere, is also an idealization, given that the presence of other material objects in the sphere will distort the ideal shape of the lines of Modified LRGI and full compensation of gravitational forces inside the hollow sphere will not be realized.

5.2. Rectilinear Inertial Acceleration in the Accelerated RF as a Static Gravitational Acceleration Occurring in the Unilaterally Truncated Sphere of DM

The simplest type of non-inertial RF are those that move in a straight line with constant acceleration (for example, a bus that moves on a level road with a uniformly increasing or decreasing speed). In this case, as is known, significant changes occur in the state of the bodies located in the accelerated RF. So, for example, we can imagine a "rectilinear" version of Newton's experiment with the bucket of water by replacing the bucket with a parallelepiped-shaped container partially filled with water. The vessel is placed on a trolley which is stationary or can move in a straight line at a constant speed or with a constant acceleration. Obviously, when the cart is stationary or moving at a constant speed, the water in it remains stationary and has an unchanged flat surface. But, when the trolley moves with constant acceleration, the water changes its shape—it falls on the front wall of the vessel in the direction of movement and rises on the back wall. It is clear that the explanation of this changed state of water according to Newton and according to Mach is the same as in the experiment with the rotating bucket of water.

But, how do we usually explain this difference in the state of the test bodies (in this case, the water in the vessel) in rectilinear motion with constant velocity and in rectilinear motion with acceleration? It should be noted that the use of a fluid body as an indicator of inertial phenomena is associated with a number of side assumptions (for example, an assumption of ideal incompressibility of the fluid) that complicate the analysis, so here we will use a model of a test body in the form of a massive hard ball whose dimensions are small enough to be neglected.

Then, in the smoothed type of non-inertial RF, the speed of movement changes only in absolute value, while its direction remains unchanged

$$\boldsymbol{v}_{LRF} = v_{LRF}\boldsymbol{e}_L; \quad v_{LRF} = v_{LRF}(t) \neq const; \quad \boldsymbol{e}_L = const, \tag{14}$$

where $\boldsymbol{e}_L$ is the unit vector in the direction of the line of motion of the Local RF.

In Figure 6(a), the local RF is depicted in the form of a wagon, which moves under the action of a constant external force $\boldsymbol{F}_{ext}$ relative to the stationary RF.

From the point of view of the observer located in the bulky RF (3) (The Eart), the wagon (1) moves with a constant rectilinear acceleration $\boldsymbol{a}_{LRF} = \frac{1}{m_{LRF}}\boldsymbol{F}_{ext}$,

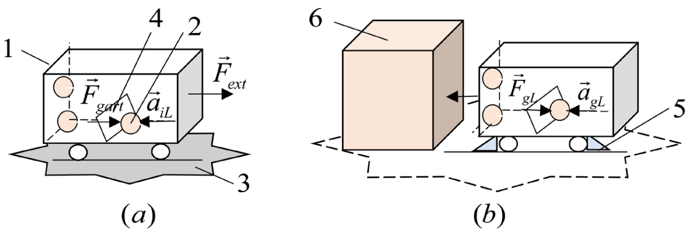

Figure 6. (a) A wagon moving under the action of an external force in a straight line with constant acceleration relative to the stationary RF: 1—wagon; 2—massive ball; 3—stationary RF; 4—stoping barrier; (b) Stationary (locked) wagon located in an external uniform gravitational field created by a massive body: 5—locking devices, 6—external massive body

where m_{LRF} is the mass of the wagon, while the massive ball (2) located in the wagon is stationary. When the barrier (4) reaches the ball, it will exert pressure on it, trying to dislodge it. But, the presence of a specific property—"inertial mass" of the ball, is the reason why it resists the acceleration. This will cause a back reaction on the ball, which will act on the barrier with a force $\boldsymbol{F}_{TB} = -m_{TB}\boldsymbol{a}_{LRF}$. This force is sometimes called the "drag force" $\boldsymbol{F}_{Ldrag}$ because it will reduce the acceleration of the LRF due to the addition of the (inert) mass of the test body (ball) to the mass of the RF.

But, from the point of view of the observer, who is located in the LRF (wagon), the ball will move in a straight line relative to the stationary wagon with constant acceleration $\boldsymbol{a}_{tb} = -\boldsymbol{a}_{LRF}$. Reaching the barrier, the ball will impact it with force $\boldsymbol{F}_{iL} = -\boldsymbol{F}_{tb} = m_{tb}\boldsymbol{a}_{LRF}$. The back reaction of the barrier to this force represents the "rectilinear artificial gravity force" $\boldsymbol{F}_{Lart} = -\boldsymbol{F}_{tb} = \boldsymbol{F}_{il}$.

The force per unit mass represents the "Field of rectilinear inertial acceleration" (FRLIA)

$$\boldsymbol{a}_{iL} = \boldsymbol{a}_{iL}\left(x_{\parallel}, x_{\perp i}, t\right), \tag{16}$$

where $x_{\parallel}, x_{\perp}$ are the longitudinal and the transverse coordinates, respectively, t is a current time.

We will briefly note the main properties of this field.

a) Field of RLIA is proportional to the external accelerating (stopping) force, assuming that the mass of the accelerated body (Local RF) is constant, *i.e.* the field of rectilinear inertial accelerations is linear

$$\boldsymbol{a}_{iL} = -k_{iL}\boldsymbol{F}_{ext}; \quad k_{iL} = \frac{1}{m_{LRF}} = const. \tag{17}$$

b) Field of RLIA is strictly uniform and does not depend either on the longitudinal $x_{\parallel}$ or on the transverse coordinates $x_{\perp i}$ of the point where the test body is located in the Local RF, *i.e.*

$$\boldsymbol{a}_{iL} = \boldsymbol{a}_{iL}\left(x_{\parallel}, x_{\perp i}\right) = const. \tag{18}$$

c) Field of RLIA does not depend on time *t*, *i.e.* the field of linear inertial accelerations is a static field

$$\boldsymbol{a}_{iL} = \boldsymbol{a}_{iL}\left(t\right) = const. \tag{19}$$

d) When changing the direction of accelerated movement of the Local RF, the FRLIA changes its sign to the opposite

$$\boldsymbol{a}_{LRF} \to -\boldsymbol{a}_{LRF} \quad \to \quad \boldsymbol{a}_{iL} \to -\boldsymbol{a}_{il} \ . \tag{20}$$

e) When the uniformly accelerating motion of Local RF change to a uniformly decelerating motion, the sign of the rectlinear acceleration field also changes its sign to the opposite

$$\boldsymbol{a}_{LRFacc} \to -\boldsymbol{a}_{LRFdrag} \quad \to \quad \boldsymbol{a}_{iL} \to -\boldsymbol{a}_{il} \ . \tag{21}$$

Then, the question—what is the origin of the considered inertial effects during the movement of the RF with acceleration?—can be answered—the origin is precisely the presence of inertial mass of the bodies (in this case—the massive ball).

But, as we know, according to the principle of equality of inertial and gravitational masses of bodies, we can assume that there is only one common characteristic—"mass" of bodies, which is the source, as of inertial, as well as the gravitational phenomena (in particular, the mutual attraction between them).

As is well known, the experimentally confirmed equality of inertial and gravitational masses allowed Einstein to formulate his famous "Equivalence principle", which he illustrated through his thought experiment with lift ("Einstein's lift"). At the same time, his goal was to explain a relatively unknown phenomenon—gravity, with another relatively much more familiar phenomenon—the inertia of bodies. In the given case, we have the opposite goal—to explain the inertia of the bodies through the action of a corresponding gravitational field arising in the RF, which is moving accelerated.

Here, for the purposes of our analysis, we will make use of a modification of this thought experiment illustrated in **Figure 6(b)**. In it, instead of the wagon moving with constant acceleration shown in **Figure 6(a)**, here the same wagon is "stuck", *i.e.* the same is stationary relative to the bulky RF. Near it is located a massive object (6) with sufficient mass and large enough dimensions, so that it creates a static uniform gravitational field of the same magnitude as the inertial field arising in the accelerated moving wagon. Then, if the action of the gravitational field of the external source on the test bodies (balls) located inside it is equal to the action of the field of inertial accelerations in the case when the wagon moves with constant acceleration, the logical question follows—what is this source and where is it located? It is to this question that Mach answers with his intuitive guess—it is the totality of the "distant fixed stars" understood as a united material object.

Then, developing Mach's idea, we should specify in this case what should be the structure of the distant material object so that it can create a gravitational field, the action of which on the test bodies is equivalent to the action of rectilinear motion with constant acceleration of the local RF on the test bodies in it.

Based on the model of interaction of the local material object with the DM represented by lines Modified LRGI connecting the body to the individual points on the inner side of the hollow sphere of the DM (**Figure 2(b)**), we should assume that during movement with acceleration the uniform distribution of these lines violates. These lines seem to recede from the front side of the Sphere DM (in the direction of movement of the LRF, *i.e.* in the opposite direction of movement of the sphere) and thicken towards the back side of the sphere (**Figure 7(a)**). This change can be represented by an equivalent change in the shape of the spherical layer, which thins in the direction of the accelerated movement and thickens in the opposite direction (**Figure 7(b)**). In our idealized model, the variation of the thickness of the spherical layer is replaced by an equivalent one-sided truncation of the infinitely thin DM sphere, *i.e.* with the removal of a part (segment) of the hollow sphere (**Figure 7(c)**).

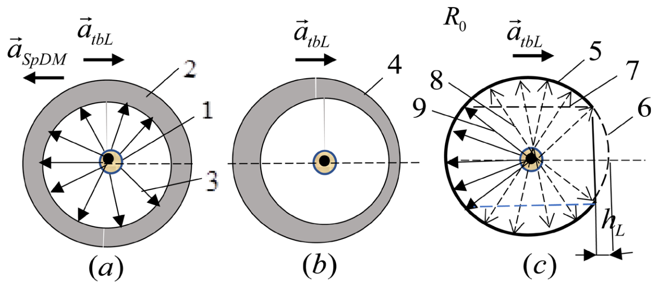

Figure 7. (a) Change in the distribution of the lines of Modified LRGI during straight-line movement of Sphere DM with constant acceleration opposite to the acceleration of LRF: 1—LRF; 2—Sphere DM; 3—MLRGI lines; b) Equivalent effective variation of the thickness of the Sphere DM layer: 4—spherical layer with uneven thickness; c) Equivalent effective one-sided cutting of the infinitely thin Sphere DM according to the proposed model: 5—infinitely thin Sphere DM; 6—cut spherical segment; 7—projection of the cut segment on the opposite wall of the sphere; 8—mutually compensating ("passive") lines of MLRGI; 9—"active" MLRGI lines.

Obviously, in contrast to the complete compensation of gravitational forces in the hollow uncut sphere with a homogeneous mass distribution, discussed in the previous point, in this case this compensation will be incomplete. Then, if we mentally consider the equivalents of all the long-range accelerations directed oppositely, it is seen that they will mutually compensate each other in all parts of the sphere, except for the segment obtained by projecting the cut segment (6) on the opposite wall of the sphere, *i.e.* the projected segment (7). The equivalent of the uncompensated accelerations from this segment will be directed along the line passing through the point of location of the RF and the center point of the segment (8) (**Figure 7(c)**).

Hence, according to the assumptions mentioned in item 4.1, the gravitational acceleration will be proportional to the area of the cut spherical segment, *i.e.* of the projection of the same on the opposite side of the Sphere DM and assuming that the attraction to this segment occurs according to the power law $\left(\propto 1/R_0^n\right)$, we have

$$\boldsymbol{a}_{gL} = -G\frac{\rho'_{s0}S_s}{R_0^n}\boldsymbol{e}_Z = -G\frac{2\pi\rho'_{s0}R_0h_L}{R_0^n}\boldsymbol{e}_Z = -G\frac{2\pi\rho'_{s0}R_0^2\overline{h}_L}{R_0^n}\boldsymbol{e}_Z, \tag{22}$$

where $h_L; \overline{h}_L = h_L/R_0$ is the absolute and relative value of the one-sided truncation (parameter of the truncation) of the sphere, is the unit vector in the direction of the axis of the segment (**Figure 8(b)**).

Considering that

$S'_{sph} = S_{sph} - S_s = 4\pi R_0^2 - 2\pi R_0 h_L = 4\pi R_0^2\left(1 - \frac{h_L}{2R_0}\right) = 4\pi R_0^2\left(1 - \frac{1}{2}\bar{h}_L\right)$, for the resultant surface mass density of the segment we obtain

$$\rho'_{s0} = \frac{V_{sph}}{S'_{sph}}\rho_0 = \frac{4\pi R_0^3 \rho_0}{3.4\pi R_0^2\left(1 - \frac{1}{2}\bar{h}_L\right)} = \frac{\rho_0 R_0}{3\left(1 - \frac{1}{2}\bar{h}_L\right)}, \tag{23}$$

Then, for (22) we get

$$\boldsymbol{a}_{gL} = -\frac{2\pi}{3} G\rho_0 \frac{R_0^3}{R_0^n} \frac{\bar{h}_L}{1 - \frac{1}{2}\bar{h}_L} \boldsymbol{e}_Z. \tag{24}$$

For the case when the intercept of Sphere DM is small compared to its radius $\bar{h}_L \prec\prec 1$, from which it follows,

$$\boldsymbol{a}'_{gL} = -\frac{2\pi}{3} G\rho_0 \frac{R_0^3}{R_0^n} \bar{h}_L \boldsymbol{e}_Z. \tag{25}$$

Then, for Newton's law of attraction $(n = 2)$ for (24) and (25) we have

$$\boldsymbol{a}_{gL(1)} = -\frac{2\pi}{3} G\rho_0 R_0 \frac{\bar{h}_L}{1 - \frac{1}{2}\bar{h}_L} \boldsymbol{e}_Z; \quad \boldsymbol{a}'_{gL(1)} = -\frac{2\pi}{3} G\rho_0 R_0 \bar{h}_L \boldsymbol{e}_Z. \tag{26}$$

From here we can determine the value of the parameter of one-sided truncation of the SphDM necessary to obtain a set value of the gravitational acceleration

$$\bar{h}_{L(2)} = \frac{3\left|\boldsymbol{a}_{gL(2)}\right|}{2\pi G\rho_0 R_0}. \tag{27}$$

Here, as in what follows, we will use as standard the so-called "unit rectilinear gravitational acceleration", which should be equal to the corresponding "unit rectilinear inertial acceleration"

$$\left|\boldsymbol{a}_{gL1}\right| = \left|\boldsymbol{a}_{iL1}\right| = \frac{F_{i1}}{m_1} = \frac{1\ \text{N}}{1\ \text{kg}} = 1\ \text{m/s}^2. \tag{28}$$

Then, setting the values of the main parameters of Sphere DM according to item 4.3 and the value of G, we get

$$\bar{h}_{L1(2)} = \frac{3.1}{2 \times 3.14 \times 6.67 \times 10^{-11} \times 10^{-26} \times 10^{26}} \approx 7 \times 10^9, \tag{29}$$

which is absurd (the part cannot be greater than the whole!).

Obviously, this result leads to the reasonable question—what is the reason for this contradiction—the inaccuracy of our model or the inapplicability of Newton's law in the given case?

Then, let us suppose that the mass of the entire observable part of the universe is concentrated at a point which is located at a distance from the test body equal to the supposed radius of that part $R_U \approx 10^{26}$ m. It follows that in order to obtain a unit acceleration at the point of location of the test body, according to Newton's law of attraction, the required mass of the material point should be

$$M'_U = \frac{|\boldsymbol{a}_{g1}| R_U^2}{G} = \frac{1\times(10^{26})^2}{6.67\times10^{-11}} \approx 1.5\times10^{62}\ \text{kg}. \quad (30)$$

while the estimated mass of the observed part of the universe is approximately ≈ 10^{52} kg, *i.e.* 10 orders of magnitude smaller! Therefore, we have reason to assume that Newton's law in the considered case (inside the unilaterally truncated sphere of DM) is inapplicable.

Then, let's assume that more realistic in this case is the "weakened power law" when $n \prec 2$. So, let's initially take the nearest integer $(n=1)$. Then, instead of (24) and (25), we will have

$$\boldsymbol{a}_{gL(1)} = -\frac{2\pi}{3}\tilde{G}'\rho_0 R_0^2 \frac{\bar{h}_L}{1-\frac{1}{2}\bar{h}_L}\boldsymbol{e}_Z; \quad \boldsymbol{a}'_{gL(1)} = -\frac{2\pi}{3}\tilde{G}'\rho_0 R_0^2 \bar{h}_L \boldsymbol{e}_Z. \quad (31)$$

In the above expressions, instead of the standard Newtonian gravitational constant *G*, we have substituted the "modified" gravitational constant $\tilde{G}'$ having dimension $\left[\frac{\text{m}^2}{\text{kg}\cdot\text{s}^2}\right]$, so that the dimensions on both sides of the ratios equalize. Then the question naturally arises, does the numerical value of the reduced constant also change? This question can obviously be answered by a corresponding experimental study. Here, for the purpose of our rough calculation, we will assume that the numerical value of the reduced gravitational constant remains the same as that of the standard Newtonian constant. Some reason for this assumption is given by the fact that the change in the dimensionality of the reduced gravitational constant is precisely related to the transition from voluminous to the surface mass density in our DM model in the form of a hollow infinitely thin sphere, in which the numerical value of the gravitational constant does not change.

Then, from (31) we determine the parameter of one-sided truncation of Sphere DM

$$\bar{h}_{L(1)} = \frac{3|\boldsymbol{a}'_{gL}|}{2\pi G\rho_0 R_0^2}. \quad (32)$$

Hence, for the required truncation to obtain the unit acceleration, we have

$$\bar{h}_{L1(1)} = \frac{3.1}{2\times3.14\times6.67\times10^{-11}\times10^{-26}\times(10^{26})^2} \approx 7\times10^{-17}, \textit{ i.e.}$$

$$h_{L(1)} = \bar{h}_{L(1)} R_0 = 7\times10^9\ m = 7\times10^6\ \text{km}, \quad (33)$$

which is already an acceptable value.

But another problem arises here. So, as we know the field which is characterized by potential and tension as follows

$$\varphi_g(R) = GM \ln R \quad \to \quad \boldsymbol{a}_g(R) = \frac{\mathrm{d}\varphi_g}{\mathrm{d}t} = -\frac{GM}{R}\boldsymbol{e}_R, \tag{34}$$

corresponds to a source in the form of an infinitely thin and infinitely long massive fiber ($\boldsymbol{e}_R$ is a unit vector in the direction of the radius perpendicular to the fiber).

With this type of field, in addition to the dependence of the magnitude of the acceleration on the distance to the source, there will also be a dependence of this magnitude on the polar angle χ $(a_g \propto \cos\chi)$, which is determined by the orientation of the fiber in a plane perpendicular to the line of motion. This means that if the test body has a drawn shape (filament, cylinder, downloaded ellipsoid, etc.), on the same, in addition to attraction to the source (the massive fiber), a torque will arise, tending to orient the test body parallel to the fiber (Figure 8(a)).

But, as experience shows, such a dependence of the field of accelerations in the rectilinearly moving RF with constant acceleration is not detected. From this follows the conclusion that the field with a structure corresponding to the field of an infinite massive thread is not suitable to imitate the field of linear accelerations in the indicated type of RF.

Of course, we can avoid this difficulty by running multiple infinite massive fibers across the center of the segment. Thus, in the end, we will go from an infinite massive fibre to an infinite massive plane (5 in Figure 8(a)).

As is known, a source in the form of an infinite massive plane creates a uniform gravitational field, the acceleration of which does not depend on the distance to the source and is characterized by a potential and tension, which are described by the expressions

$$\varphi_g(R) = -GMR; \quad \to \quad \boldsymbol{a}_g(R) = \frac{\mathrm{d}\varphi_g}{\mathrm{d}R}\boldsymbol{e}_R = -GM\boldsymbol{e}_R = const. \tag{35}$$

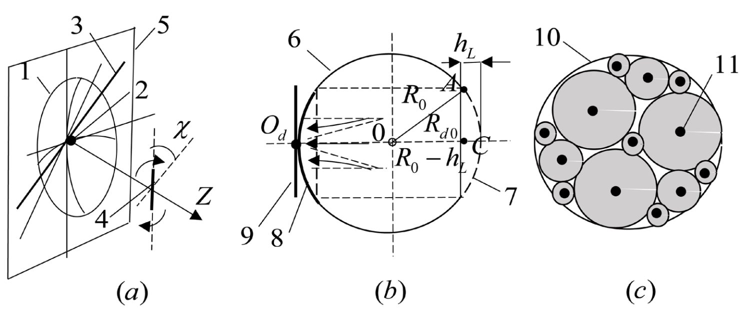

Figure 8. (a) Spherical segment (1), which is obtained as a result of projecting the cut segment on the back side of the sphere: 1—spherical segment; 2—center of the segment; 3—infinite in length massive fiber; 4—test body with an elongated shape ("pin"); 5—infinite massive plane; b) Meridional section of the unilaterally truncated sphere of DM: 6—sphere; 7—cut segment; 8—projection of the cut segment on the opposite side of the sphere; 9—section of the massive disk creating an attraction field in the truncated sphere; c) Structure of the attracting object (massive disk) in the form of a fractal spherical circle: 10—frontal view of the fractal circle in the form of a special fractal (Apollonius gasket); 11—local centers of attraction.

(In this case the unit vector $\boldsymbol{e}_R$ is directed along the normal to the massive plane).

Then, in order to consider this case, we should pass in the generalized law (25) a power law with a zero power exponent $(n = 0)$

$$\boldsymbol{a}_{gL(1)} = -\frac{2\pi}{3} G\rho_0 R_0^3 \frac{\overline{h}_L}{1-\frac{1}{2}\overline{h}_L}\boldsymbol{e}_Z; \quad \boldsymbol{a}'_{gL(1)} = -\frac{2\pi}{3} G\rho_0 R_0^3 \overline{h}_L \boldsymbol{e}_Z. \tag{36}$$

Hence, for the case when $\overline{h}_R \prec\prec 1$ we get

$$\overline{h}_{L(0)} = \frac{3\left|\boldsymbol{a}_{gL}\right|}{2\pi G\rho_0 R_0^3}. \tag{37}$$

Obviously, in this case also another modification of the gravitational constant $\tilde{G}''$ with dimension $\left[\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right]$. But again assuming that the numerical value of this constant is preserved, for the required value of the truncation parameter to obtain unit acceleration we get

$$\overline{h}_{L(0)} \approx 7\times10^{-43}; \quad h_{L1(0)} = \overline{h}_{L1(0)} R_0 \approx 7\times10^{-17}\ \text{m}. \tag{38}$$

As can be seen, the obtained value of the truncation parameter of the DM sphere is extremely small (microscopic), which casts doubt on its credibility given the huge ('megascopic') scales of the DM sphere. Thus, we again face the dilemma, this time in the opposite direction—towards small values of the truncation parameter. Then, the question again follows—what is the reason for this implausible value—the imperfection of our model or the properties of the gravitational interaction inside the unilaterally truncated Sphere DM?

Undoubtedly, our model of distant matter as a hollow infinitely thin sphere is highly idealized. But, let us assume that, apart from the imperfection of the infinitely thin sphere model, there is also a factor related to the specific properties of the long-range gravitational interaction inside the hollow Sphere DM.

Thus, as we have already noted, a uniform gravitational field is theoretically created by a source in the form of an infinite in size and infinitely thin plane. This is, of course, an idealization, since real sources of infinitely large dimensions (massive fibers of infinite length and massive planes of infinite size) do not exist. In the given case, we will also try to find an imaginary source of the gravitational field inside the truncated sphere of the DM, which to a large enough extent corresponds to the features of the gravitational interaction between the material objects inside the truncated sphere and the inner surface of the same. What exactly are the requirements for such a hypothetical source? This, in the case of a DM one-sided truncated sphere, is precisely the requirement that such a source produces a field sufficiently close to the uniform field, while not leading to unrealistic (microscopic) sizes of the sphere truncation parameter.

Our hypothesis in this case is based on the assumption that the exponent in the law of attraction can take non-integer (*i.e.* fractional) values. But, as we know, there are objects with so-called "fractal" spatial and temporal structure that can be described with fractional dependencies (fractional dimensionality, etc.) [28] [29].

In the given case, it is assumed that if the object creating a field of attraction to the inner surface of the Sphere DM has such a fractal structure, then the same should create a gravitational field that is quasi-uniform, while the same does not have unrealistically small (microscopic and submicroscopic) dimensions, as required by the zero-valued power-law case of attraction discussed above. We can imagine this object as a massive disk with a diameter equal to the diameter of the base of the cut spherical segment, which touches the sphere at the center point O_d of the projected spherical segment (**Figure 8(b)**). This disk will actually act as a "starting" element to obtain the actual fractal structure.

According to the figure, from the triangle 0AC in **Figure 8(b)**, for the radius of the touching massive disk we have

$$R_{d0} = \sqrt{R_0^2 - \left(R_0 - h_L\right)^2} \approx \sqrt{2R_0 h_L} = R_0\sqrt{2\overline{h}_L} . \tag{39}$$

Then, for the value of this radius at a power exponent $n = 1$ and for $h_R \prec\prec R_0$, according to (33), we obtain

$$R_{d0L1} = \sqrt{2\times10^{26}\times7\times10^{9}} \approx 1.18\times10^{17}\ \text{m} . \tag{40}$$

Let us now consider as an illustration the case of a source in the form of a massive disk with a preselected fractal structure. A well-known such structure, etc. "Apollonius gasket" type fractal, which has different variants [30] [31].

Figure 8(c) shows the "standard" Apollonius gasket, which is constructed by initially inscribing three identical touching circles in an initial "base" circle, then in the remainder between them the number $3\times2+1=7$ of even smaller also touching circles is inscribed etc. [31]. This type of fractal can have two variants—lightly filled (inner circles are removed) and heavily filled (inner circles remain, their surroundings are removed).

Here we will consider the case of a heavily filled fractal. We can introduce a parameter "fractal fill factor"

$$k_{FA} = \frac{S_\Sigma}{S_0} = \frac{S_\Sigma}{4\pi R_{sd0L1}^2} = 3r_1^2 + 3r_2^2 + 7r_3^2 + \cdots + pr_n^2 , \tag{41}$$

where R_{d0}, S_{a0} are the radius and area of the initial filled massive disk, respectively, S_Σ is the total area of the filled circles inscribed in it, $r_i = \dfrac{R_i}{R_{d0}}, i = 1,2,\cdots,n$ are their relative radii, i, p are the iteration number and number of circles with the smallest radius for the final iteration (n—final number).

For our illustrative example, without claiming accuracy and limiting ourselves to the first three iterations with an approximate value of the relative radii of the inscribed circles, we can assume

$$k_{FA} \approx 3\times0.46^2 + 3\times0.21^2 + 7\times0.04^2 \approx 0.7783 . \tag{42}$$

As can be seen, this value is a fractional number in the interval $0 \prec k_{FA} \prec 1$.

Then, in accordance with our hypothesis stated above, we can assume that the above value in a rough approximation can be taken as the value of the exponent in the general expression for the magnitude of the gravitational acceleration (22), *i.e.*

$$n = k_{FA} \approx 0.7783\,. \tag{43}$$

Hence, for the magnitude of the gravitational acceleration and for the relative value of the truncation parameter, we have

$$\boldsymbol{a}'_{gL(0.77)} = -\frac{2\pi}{3} G\rho_0 \frac{R_0^3}{R_0^{0.7783}} \bar{h}_L \boldsymbol{e}_Z\,; \quad \bar{h}_{L(0.77)} = \frac{3\left|\boldsymbol{a}_{gL}\right| R_0^{0.7783}}{2\pi\rho_0 R_0^3}\,. \tag{44}$$

Substituting the values of the corresponding quantities, for the required value of the truncation parameter to obtain unit acceleration, we obtain

$$\begin{aligned} &\bar{h}_{L1(0.77)} = \frac{3\times1\times1.72\times10^{20}}{2\times3.14\times6.67\times10^{-11}\times10^{-26}\times\left(10^{26}\right)^{0.7783}} \approx 1.25\times10^{-22}\,; \\ &h_{L1(0.77)} \approx 1.25\times10^{4}\ \text{m} = 12.5\ \text{km}. \end{aligned} \tag{45}$$

As can be seen, in contrast to the case with a zero value of the exponent, where the truncation parameter assumes microscopic values, in the given case with the assumed fractional value of the exponent, the truncation parameter has already far larger (macroscopic) values.

Then, for the radius of the fractal circle according to the expression (37) we have

$$R_{FC} \approx \sqrt{2\times10^{26}\times1.25\times10^{4}} \approx 1.58\times10^{15}\ \text{m} = 1.58\times10^{12}\ \text{km}\,. \tag{46}$$

The obtained value even has a super-macroscopic (“megascopic”) scale.

Of course, taking into account more iterations in the procedure for calculating the fractal filling factor in (30), and also adopting another, more appropriate version of the “Apollonius gasket” fractal [32], could to provide a more accurate value of this parameter.

But, on the other hand, can we consider that the attraction field within the limits of the cylindrical region with a radius equal to the radius of the fractal circle (**Figure 8(b)**) is close to the uniform field, *i.e.* much closer than the field that would create a uniform massive disk of the same dimensions? In our opinion, the grounds for such an assumption are given by the properties of the fractal structure of the disc—it does not have a “single” center of attraction, like a uniform disc, but many distributed centers (**Figure 8(c)**), which brings the attraction field closer to a strictly uniform field.

Here again we should note that the representations introduced here of a “fractal structure” of the gravitational field source in the DM truncated sphere and a “non-integer-valued power exponent” in the law of attraction are in fact “effective”. In fact, they only represent the real properties of the gravitational interaction inside the hollow one-sided truncated sphere of the DM.

What has been stated in this paragraph gives reason to conclude that, by its properties and structure, the static gravitational interaction inside the unilaterally truncated sphere of the DM largely represents the inertial linear effects observed in the Local RF, which moves in a straight line with constant acceleration which are "static". But, at the same time, there are certain differences.

Thus, unlike the field of inertial forces in rectilinearly moving with constant acceleration Local RF, which is linear ((expression (17)), the field of static gravitational forces inside the unilaterally truncated sphere of the DM is nonlinear, since the same depends nonlinearly on the truncation parameter $\overline{h}_L$. It can be approximately assumed to be linear for small truncation parameters (expressions (26)). Second, while the field of rectilinear inertial accelerations is strictly uniform, the field of rectilinear gravitational accelerations according to the model can only be assumed to be "quasi-uniform", in accordance with the additional assumptions in the fractional power-law model in Sphere DM and fractal structure of the effective attraction source. This field can be assumed to be uniform only for sufficiently small values of the parameter of one-sided truncation of the Sphere DM,

5.3. The Inertial Centrifugal Acceleration in the Rotating RF as a Gravitational Centrifugal-Like Acceleration in the Bilaterally Truncated Sphere of DM

This is, in fact, the case of rotational motion that Newton considered in his thought experiment and to which Mach directed his criticism.

Unlike motion with rectilinear constant acceleration, where the linear velocity of a point on the local RF remains constant in direction but changes in magnitude, in the rotation of a this RF with constant angular velocity, the linear velocity of a point remains constant in magnitude but changes at any moment by direction, i.e

$$\boldsymbol{v}_{LRF} = v_{LRF}\boldsymbol{e}_L; \quad v_{LRF} = v_{LRF}(t) = const; \quad \boldsymbol{e}_L = \boldsymbol{e}_L(t) \neq const. \tag{47}$$

Figure 9(a) shows a Local RF in the form of a hard disk (1), which rotates around an axis (2) passing through its center 3, relative to a bulky stationary RF (3). A test body in form a massive ball (4) is fixed to the disk.

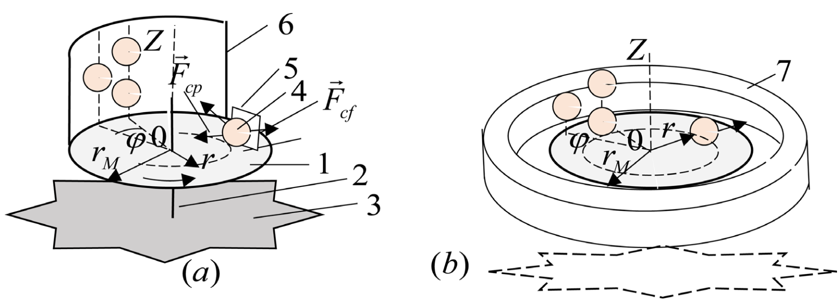

Figure 9. (a) Local RF in the form of a hard disk, which rotates around an axis passing through its center with a constant angular velocity relative to a bulky stationary RF: 1—disk; 2—axis of rotation; 3—bulk stationary RF; 4—test body (massive ball) attached to the disc; 5—stopping barrier; 6—wall of the cylindrical cabin; (b) The same hard disk, but stationary and located in the center of a symmetrically encompassing massive body creating a radial gravitational field directed towards it: 7—a massive body (toroid) creating the gravitational field with cylindrical symmetry.

When the disk rotates at a constant angular velocity, a "centripetal" force $\boldsymbol{F}_{cp}$ acts on the fixed ball, which causes the direction of the instantaneous rectilinear motion of the ball $\boldsymbol{F}_{cp}$ to deviate toward the center of the disk. This force is balanced by an equal and oppositely directed "centrifugal" force $\boldsymbol{F}_{cf} = m_{TB}\omega^2 r\boldsymbol{e}_r = -\boldsymbol{F}_{cp}$, where m_{TB}, ω, r are, respectively, the mass of the ball, the angular velocity of rotation, and the radial distance from the axis of rotation.

If the ball is released from the disc, then from the observer's located on bulk fixed RF point of view, the ball continues to move along a line perpendicular to the radial direction with a constant linear velocity $\boldsymbol{v}_t = \omega r\boldsymbol{e}_\varphi$, where $\boldsymbol{e}_\varphi$ is the unit vector in the direction of the circle of rotation. But, from the point of view of the observer, who is located in the rotating RF (disc), the released ball will move in a straight line directed along the radius from the center of rotation (**Figure 9(a)**).

When the ball reaches an obstacle (for example, the inner wall of a cylindrical cabin with a base coinciding with the disk), it will, under the action of centrifugal force, press against the wall of the cabin, tending to move it. The response of the cabin wall is known as "artificial radial gravity force" $\boldsymbol{F}_{rart} = F_{rart}\boldsymbol{e}_r = -\boldsymbol{F}_{cf}$.

$$\boldsymbol{a}_{iR} = \omega^2 r\boldsymbol{e}_r . \tag{48}$$

Analogously to the previous case, we can consider this centrifugal acceleration as a function of the cylindrical coordinates (r, φ, Z) of the point in the Local RF and the time t and speak of a "field of centrifugal inertial accelerations" (Field of CfIA) $\boldsymbol{a}_{iR} = \boldsymbol{a}_{iR}(r, \varphi, Z, t)$, where r, φ, Z is radial coordinate, azimuth angle and position angle ((**Figure 9(a)**).

We will note the main properties of this field:

a) At constant angular velocity $(\omega = const)$, the field of CfIA intensity is proportional to the displacement from the center *r*.

b) Field of centrifugal forces, unlike of the field of rectilinear inertial forces, is not a completely uniform field, since it depends (linearly) on the radial coordinate and does not depend on the azimuthal and axial angles

$$\boldsymbol{a}_{iR} \propto r \neq const; \quad \boldsymbol{a}_{iR} = \boldsymbol{a}_{iR}(\varphi, Z) = const . \tag{49}$$

It should be noted that unlike the field of an infinitely long massive filament, which weakens proportionally to the distance from the filament and which can be characterized as a field with "cylindrical symmetry", the field of centrifugal forces, on the contrary, increases proportionally to the distance from the axis of rotation and can be characterized as a field with "anti-cylindrical symmetry".

c) Field of centrifugal accelerations does not depend on time, *i.e.* so is a "static" field

$$\boldsymbol{a}_{iR} = \boldsymbol{a}_{iR}(t) = const . \tag{50}$$

d) Unlike the case when the RF moves in a straight line with constant acceleration, in the RF rotating with a constant angular velocity, when the direction of rotation is reversed, the centrifugal inertial acceleration in the RF does not change its sign

$$\boldsymbol{\omega}_{LRF} \to -\boldsymbol{\omega}_{LRF} \quad \to \quad \boldsymbol{a}_{iR} \to \boldsymbol{a}_{iR} \,. \tag{51}$$

Then, similarly to the previous case, let us assume that the rotating disk is stationary (relative to the bulk RF), but near it there is a source of gravitational field, the action of which is similar to the centrifugal inertial field in the case when the disk rotates. Again the question arises, what should this source be?

Based on the above-mentioned cylindrical (anti-cylindrical) symmetry of fields of centrifugal accelerations, we can conclude that the supposed source should be similar to a body with a toroidal shape encompassing the Local RF (disk) (**Figure 9(b)**).

Hence, similar to the general considerations on the basis of which the model of the modified (one-sidedly truncated) hollow sphere of DM is built in the previous case, we can assume what should be the corresponding modification (truncation) of the hollow sphere so that it can represent the inertial effects in the rotating LRF by equivalent gravitational effects in this modified hollow sphere. Thus, let us mentally imagine a real hollow sphere with a finite thickness of the layer and a uniform distribution of the mass density in it, in the center of which there is a local axis rotating with a constant angular velocity around a fixed (in this case—vertically oriented) axis. According to Mach's notion, this is equivalent to rotating the Sphere DM in the opposite direction about the same axis relative to the stationary RF (**Figure 10(a)**). This change in the set of lines, considered as a complete material object, can be represented by an equivalent change in the thickness of the spherical layer of the Sphere DM—this layer thins around the poles and thickens around the equator of the sphere (**Figure 10(b)**).

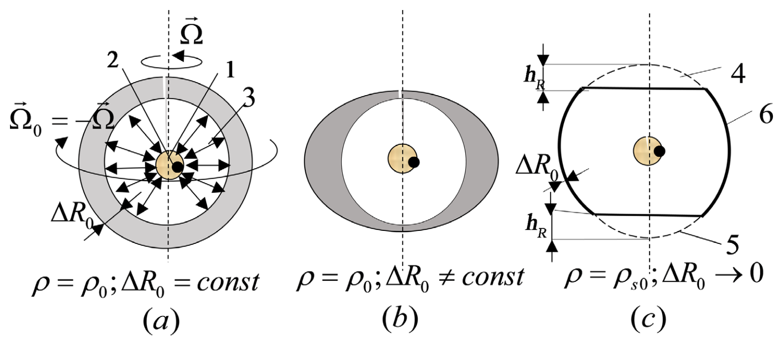

Figure 10. (a) Change of the density distribution of the Modified LRGI lines upon rotation of the LRF, *i.e.* with equivalent rotation of Sphere DM in the opposite direction 1—LRF; 2—Modified LRGI lines; (b) Equivalent effective variation of the thickness of the spherical layer: 3—variation of the thickness of the spherical layer (thinning around the poles and thickening towards the equator of Sphere DM); (c) Equivalent bilaterally symmetric cutting of the infinitely thin Sphere DM according to the proposed model: 4, 5—cut circumpolar segments, 6—meridional section of the central spherical belt

Then, analogously to the previous case, we can assume that in fact this rotation causes a change in the distribution of the lines of force of the Modified LRGI—they withdraw from the poles of the sphere and condense towards the equator of the same (**Figure 10(a)**). Again by analogy with the previous case, in our idealized model this change is represented by an equivalent bilaterally symmetric cutting of the hollow infinitely thin Sphere DM (two diametrically located spherical segments of the same height are removed from this sphere (**Figure 10(c)**).

Then, for the effective surface mass density of the remaining part of the sphere instead of (23) we will have

$$\rho_{S0} = \rho'_{S0} \frac{4\pi R_0^2}{4\pi R_0^2 - 4\pi R_0 h_R} = \frac{\rho_0 R_0}{3} \frac{1}{1-\bar{h}_R}, \quad (52)$$

where $\bar{h}_R = h_R / R_0$, and $h_R, \bar{h}_R$ are the absolute and relative depth (parameter) of the bilateral cutting of the Sphere DM.

To determine the gravitational acceleration in this case, similarly to item 5.2, we will again proceed from the assumption that this acceleration is proportional to the area of the cut parts of the sphere, *i.e.* on the two circumpolar segments

$$\left|\boldsymbol{a}_{gR}\right| \propto 2S_s = 4\pi R_0 h_R = 4\pi R_0^2 \bar{h}_R. \quad (53)$$

Here the question may arise—since the lines of MLRGI are concentrated in the central spherical belt, which remains after the bilateral truncation of the sphere, we assume that is determined not by the area of the belt, but by the area of the two truncated segments?

The basis for this assumption is given by the comparison with the picture of the lines of Modified LRGI in the unilaterally cut Sphere DM shown in **Figure 7(c)** (paragraph 5.2). From here, by analogy, it can be concluded that in the area of the central spherical belt, in addition to mutually compensating ("passive") lines, there are also "active" lines, the number of which is proportional to the area of the cut segments (**Figure 11(a)**). Thus, in fact, we have a kind of "projection" of the cut segments on the central belt of the sphere (**Figure 11(b)**).

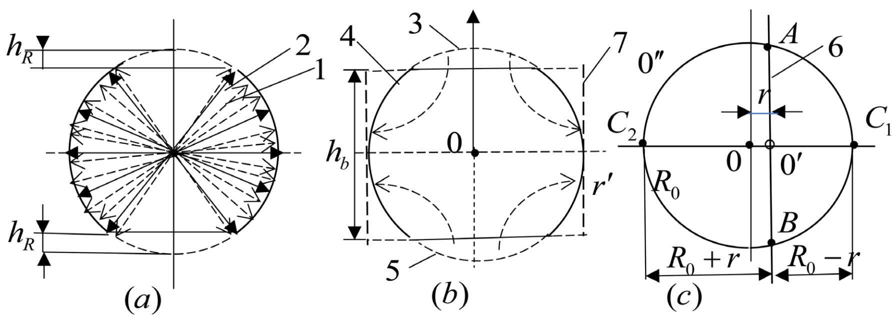

Figure 11. (a) "Passive" (mutually compensating) (1) and "active" (2) lines of Modified LRGI existing in the area of the central spherical belt; (b) Equivalent "projection" of the cut circumpolar spherical segments (3, 4) onto the central spherical belt (5); (c) Equatorial section of the Sphere DM: 6—a plane perpendicular to the equatorial plane crossing the Sphere DM at a distance r from the center.

Then, we can record

$$\left|\boldsymbol{a}_{gR}\right| \propto \rho_{s0} \Delta S_b = \rho_{s0} \left(S_{b2} - S_{b1}\right), \quad (54)$$

where $\Delta S_{b1,2}$ is the difference in the areas of the two parts of the central belt, which are obtained from the crossing of the same by a plane perpendicular to the equatorial plane and displaced from the center of Sphere DM at a distance (**Figure 11(c)**).

According to **Figure 11(a)**, **Figure 11(b)**, for the areas of the two parts of the central belt we have

$$S_{b1,2} = \pi R_0 R_{1,2} = \pi R_0 \left(R_0 \mp r \right). \tag{55}$$

Then, for the gravitational acceleration inside the bilaterally truncated Sphere DM we can write

$$\boldsymbol{a}_{gR} = -\frac{1}{3} G \rho_0 R_0 \frac{2\bar{h}_R}{1-\bar{h}_R} \pi R_0^2 \left[\frac{R_0 + r}{\left(R_0 + r\right)^n} - \frac{R_0 - r}{\left(R_0 - r\right)^n} \right] \boldsymbol{e}_r , \tag{56}$$

where $r, \boldsymbol{e}_r$ is displacement from the center of the sphere and a unit vector along its direction (**Figure 11(c)**).

Given the small value of the displacement compared to the radius of the Sphere DM $\left(r \prec\prec R_0 \right)$ the above expression can be simplified

$$\boldsymbol{a}_{gR} = -\frac{2\pi}{3} G \rho_0 \bar{h}_R \frac{R_0^3}{R_0^n} \left(-2r\right) \boldsymbol{e}_r = \frac{4\pi}{3} G \rho_0 \bar{h}_R \frac{R_0^3}{R_0^n} r \boldsymbol{e}_r . \tag{57}$$

As seen from the above expression, the radial gravitational acceleration is proportional to the displacement from the center r and has the opposite sign, meaning that it is directed from the center to the periphery of the disk, similar to the radial inertial centrifugal acceleration.

Hence for the exact and approximate (at $\bar{h}_R \prec\prec 1$) expression we have

$$\boldsymbol{a}_{gR} = \frac{4\pi}{3} G \rho_0 \frac{R_0^3}{R_0^n} \frac{\bar{h}_R}{1-\bar{h}_R} r \boldsymbol{e}_r ; \quad \boldsymbol{a}_{gR} = \frac{4\pi}{3} G \rho_0 \frac{R_0^3}{R_0^n} \bar{h}_R \boldsymbol{e}_r . \tag{58}$$

It should be noted that the presence of the multiplier will violate the equality of the dimensions of the quantities on the left and right sides of the ratio (58). To restore this equality, it is necessary to reduce the gravitational constant into a new constant $\tilde{G}'''$ with a correspondingly changed dimension. Here, analogously to the previous paragraph, we assume that this modified constant keeps its numerical value equal to the value of the standard gravitational constant.

Let us now, as in the previous point, test the validity of the obtained expression (58) at a value of the power exponent $n = 2$, *i.e.* for Newton's law of attraction.

Thus, instead of the general expressions (58), we have

$$\boldsymbol{a}_{gR} = \frac{4\pi}{3} G \rho_0 R_0 \frac{\bar{h}_R}{1-\bar{h}_R} r \boldsymbol{e}_r ; \quad \boldsymbol{a}_{gR} = \frac{4\pi}{3} G \rho_0 R_0 \bar{h}_R r \boldsymbol{e}_r . \tag{59}$$

Using only the approximate expression, for the relative and for the absolute depth of bilateral cutting of Sphere DM we get

$$\bar{h}_{R(2)} = \frac{3\left|\boldsymbol{a}_{\partial gR(2)}\right|}{8\pi G \rho_0 R_0 r} ; \quad h_{R(2)} = \bar{h}_{R(2)} R_0 . \tag{60}$$

Then, similar to the previous paragraph, we can introduce a parameter "unit centrifugal-like gravitational acceleration" equal in value to "unit centrifugal inertial acceleration"

$$\left|\boldsymbol{a}_{gR1}\right| = \left|\boldsymbol{a}_{icf}\right| = \left|\omega^2 r\right|_1 = 1\frac{1}{\text{s}^2}\times 1\,\text{m} = 1\frac{\text{m}}{\text{s}^2}. \tag{61}$$

Putting in (60) $r_1 = 1\,\text{m}$, we obtain the same absurd values for the truncation depths as in the previous paragraph

$$\bar{h}_{R1(2)} \approx 3.5\times 10^{9};\quad h_{R1(2)} \approx 3.5\times 10^{35}\,\text{m}. \tag{62}$$

This means that even in the case considered here, the law of attraction with a power exponent $n = 2$ is unacceptable.

Then, let's check the fulfillment of the exponent condition $n = 1$.

Thus, instead of (58) we get

$$\boldsymbol{a}_{gR} = \frac{4\pi}{3}G\rho_0 R_0^2\frac{\bar{h}_R}{1-\bar{h}_R}r\boldsymbol{e}_r;\quad \boldsymbol{a}_{gR} = \frac{4\pi}{3}G\rho_0 R_0^2\,\bar{h}_R\boldsymbol{e}_r. \tag{63}$$

Again using the approximate expression, we have

$$\boldsymbol{a}_{gR(1)} = \frac{4\pi}{3}G\rho_0 R_0^2\bar{h}_R r\boldsymbol{e}_r;\quad \bar{h}_{R(1)} = \frac{3\left|\boldsymbol{a}_{gR(1)}\right|}{4\pi G R_0^2\rho_0 r}, \tag{64}$$

and instead of (62) we get

$$\bar{h}_{R1(1)} \approx 3.5\times 10^{-17};\quad h_{R1(1)} \approx 3.5\times 10^{9}\,\text{m}. \tag{65}$$

We also obtained this value in the case of representing the rectilinear inertial acceleration as an equivalent rectilinear gravitational acceleration in the unilaterally truncated Sphere DM discussed in the previous paragraph, but there we abandoned the value of the exponent $(n = 1)$ due to the inappropriate structure of the field of the equivalent source of attraction (infinite rectilinear fiber) which possesses polarization properties absent in the rectilinear inertial acceleration field.

In the given case, however, the source field in the form of an infinitely long massive filament turns out to be appropriate given the cylindrical symmetry of the imaginary source in the form of a cylinder described around the Sphere DM (Figure 9(b)).

But, then, as in the previous paragraph, the assumption of an infinite length of the source in the form of a massive fiber remains unacceptable. Therefore, by analogy with the previous paragraph, we should look for a way out of this difficulty by switching to a non-integer (fractional) value of the exponent, but this time this value should be in the interval $1 \prec n \prec 2$.

Then, analogously to the previous point, we will assume that the fractional value of the indicator can be associated with a corresponding fractal structure of the source of attraction.

In the given case, given the axial symmetry of the rotating LRF, the initial appearance of the imaginary source of attraction to the inner wall of the bilaterally truncated Sphere DM should also have axial symmetry.

Hence, starting from the massive toroid depicted in **Figure 9(b)**, we can imagine this source as an infinitely thin massive cylinder with a radius equal to the radius of the Sphere DM and touching at the equator of the sphere. The height of this cylinder should be equal to the height of the spherical belt remaining between the two-sided cutting of the sphere (**Figure 12(a)**).

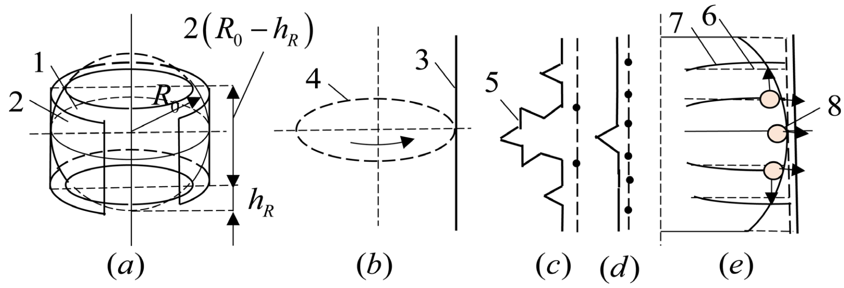

Figure 12. (a) Bilaterally cut Sphere DM (1) and a massive cylinder (2) circumscribed around it with a height equal to the height of the spherical belt; (b) Formation of the smooth massive circumscribed cylinder by straight line (3) rotating along the equatorial circle (4); (c) Formation of the non-smooth broken fractal cylinder by replacing the line with the "broken" Koch Curve (5); (d) An example of constructing a "smoothed" fractal of the Koch Curve type; (e) Lines of force of the strictly uniform (6) and the quasi-uniform (7) field in the meridional plane of the bilaterally cut Sphere DM, (8) test bodies (balls) located on the same meridional plane of the Sphere DM.

Unlike the case of linear acceleration, where a "sparse" fractal disc fractal structure of the "Apollonius gasket" was needed, the occupancy of which is always less than the dense disc, in the given case it turns out that a "extended" fractal cylinder (*i.e.* a cylinder whose area is always greater than the area of the original smooth cylinder). A cylinder with such a structure can be constructed if the forming line, which moves along a circle coinciding with the equatorial circle of the Sphere DM, is not a straight line, but a fractal broken line. A benchmark for such a line is the well-known fractal "Koch Curve" [32].

As is well known, the fractal "Koch Curve" representing a broken line has a greater length compared to a smooth straight line. From here we can conclude that the total area of the fractal broken cylinder, which is obtained when the broken section moves along the equatorial circumference of the sphere (**Figure 12(b)**), will also be larger than the starting area of the smooth cylinder.

Then, following the same procedure as in the previous point, we can enter a parameter—"coefficient of increase in the area of the cylinder" in the form of a relation

$$k_{FR} = \frac{S_\Sigma}{S_{smcyl}} = \frac{L_\Sigma}{L_{smcyl}}, \tag{66}$$

where $S_\Sigma, L_\Sigma; S_{smcyl}, L_{smcyl}$ are the area and length of the forming, respectively, the broken and the smooth cylinder.

We should immediately note that if here, as in the previous point, we assume that the coefficient of elongation is numerically equal to the fractional exponent of the modified law of attraction to the broken smooth cylinder, *i.e.*

$$k_{FR} = n, \tag{67}$$

then, in order to fulfill the condition $1 \prec n \prec 2$, it is obviously necessary that this coefficient does not exceed the value $n = 2$, which corresponds to Newton's law of attraction, which, as we have already established above, leads to a contradiction.

In the "standard" type of fractal "Koch Curve", the value of the elongation factor increases with increasing number of iterations of the law

$$k_{FRst} = \left(\frac{4}{3}\right)^p, \tag{68}$$

where $p = 1, 2, \cdots, p_{cr}$ is the number of iterations, is the "critical" p_{cr} number at which $k_{FR} \geq 2$.

In this case, k_{FR} it becomes greater than 2 on the second iteration $\left(k_{FRst}^{(2)} \simeq 3.16\right)$. This means that this type of fractal is "highly non-uniform".

But, a highly inhomogeneous fractal is not suitable in that the exponent will be too close to the value 2 corresponding to a highly inhomogeneous attraction field created by a point source.

Then, a "smoothed" variety of the "Koch Curve" fractal, defined by the dependence

$$k_{FRir(pcr)} = \left(\frac{q+1}{q}\right)^p, \tag{69}$$

where $q = 3, 5, 7, \cdots, (2u+1); (u = 1, 2, 3, \cdots)$ is the odd number of segments into which the entire segment is initially divided.

Based on the above formula and on successive samples, we obtain that for $q = 29$ after the fourth iteration $(p = 4)$, we get $k_{Fr} = n \approx 1.72$.

Then, instead of (62), we will have

$$\boldsymbol{a}_{gR(1.72)} = \frac{4\pi}{3} G\rho_0 R_0^{1.72} \overline{h}_R r \boldsymbol{e}_r; \quad \overline{h}_{R(1.72)} = \frac{3\left|\boldsymbol{a}_{gR(1.72)}\right|}{4\pi G\rho_0 R_0^{1.72} r}. \tag{70}$$

Substituting the values of the corresponding quantities, we get

$$\begin{aligned} &\overline{h}_{R(1.72)} = \frac{3\times 1}{4\times 3.14\times 6.67\times 10^{-11}\times 10^{26}\times 5.3\times 10^{44}\times 1} \approx 6.6\times 10^{-10}; \\ &h_{R(1.72)} \approx 6.6\times 10^{16}\ \text{m}. \end{aligned} \tag{71}$$

The obtained result is satisfactory from the point of view of the above-mentioned condition of "non-microscopicity" of the bilateral truncation parameter of Sphere DM. But, at the same time, we should note that by increasing the "smoothing" of the Koch fractal curve, we will also increase the degree of nonuniformity of the field in the meridional plane, since the attracting object (the broken cylinder) will also increasingly approach the smooth cylinder with final height.

Figure 12(d) shows the structure of the force lines of the strictly uniform (6) and the quasi-uniform (7) centrifugal-like gravitational field. As can be seen, in the case of the quasi-uniform field, in addition to radial centrifugal forces, transverse forces tending to move the test bodies (8) apart in a vertical plane will also act. These forces are actually related to the spherical shape of the residual spherical belt, which is obtained as a result of the bilateral cutting of the Sphere DM (**Figure 12(a)**).

As in the previously discussed case, as well as here, the question remains open—what is the law of attraction in the bilaterally truncated sphere of the DM, which can be approximated most accurately with a corresponding type of fractal structure of the source. Then, as well as the similar question in the previous point, the answer to this question should be sought in a corresponding experiment modeling to a large extent the theoretical model adopted here.

Based on the obtained results in this point, we can conclude that the gravitational centrifugal-like field created inside the hollow bilaterally truncated sphere of the DM basically mimics the field of inertial centrifugal forces arising in the local RF, which rotates with a constant angular velocity.

However, as in the case of the formation of rectilinear gravitational acceleration inside the unilaterally truncated Sphere DM, there are characteristic differences between the centrifugal-like gravitational acceleration field created in the bilaterally truncated Sphere DM and the field of inertial centrifugal accelerations existing in the rotating RFs. The main differences here are also reduced, as in the case of the field of rectilinear gravitational accelerations, to the presence of a non-linear dependence of the field on the magnitude of bilateral cutting due to the presence of the multiplier $\bar{h}'_R = \bar{h}_R / 1 - \bar{h}_R$, and also the presence of non-uniformity of the field in the meridional plane, which does not is found in the field of centrifugal inertial accelerations

6. Regarding a Possible Connection of the Modified Long Range Gravitational Interaction with Other Cosmic Phenomena

In item 4, we set forth the main assumption in the model, namely, that between the individual body (the local RF) and the DM sphere there are "connections" in the form of lines of modified long-range gravitational interaction (MLRGI). (Recall that Mach believed that all bodies in Nature are interconnected). Moreover, although individually such a line makes a negligible contribution to the interaction of LRFs with DM, the combined effect of all the lines, the number of which is enormous considering the huge surface area of Sphere DM, can be substantial. (As Mach pointed out, the weakening of the action of distant stars due to the huge distances to them can be overcome by the huge total mass of the same!) But, as already indicated, when the LRF is stationary or moving at a constant speed (relative to another LRF), the total action of the lines of the MLRGI is canceled due to the spherically symmetric distribution of the lines, leading to a mutual compensation of their action. However, when the LRF moves with acceleration, the distribution of the lines changes—in certain parts of the sphere, their density increases at the expense of the decrease of the same in other parts of the Sphere DM. As a result, full compensation is broken and a residual (uncompensated) gravitational interaction occurs inside the sphere.

But, here the logical questions immediately follow—what is the essence of this material intermediary and how does it carry out the interaction between the local RF and the distant matter—instantaneously, at a finite high superluminal speed, so there may be a certain delay of the inertial reaction relative to the instant of application of the accelerating force?

According to our model, it follows that there are two types of gravitational interaction: "local"—between the local bodies (or local RF), which is transmitted at the speed of light, and, "global"—between the local bodies and the DM sphere considered as connected (unified) object that is transmitted at a speed many times exceeding the speed of light and realizing the inertial properties of objects. Moreover, as we know, for the essence of the local gravitational interaction there are more or less assumed material "carriers"—for example, for the simplest Newtonian gravity, according to Lesage's hypothesis, these are supposed hard microscopic particles moving chaotically and causing convergence of two neighboring bodies that mutually "shadow" each other, in relativistic Einsteinian gravity the mediator is distorted space-time seen as a continuous material substance, in modern quantum field theories these are the supposed quanta of the gravitational field ("gravitons"). But, here immediately follows the reasonable question about the nature of the material mediator in the global gravitational interaction and how does it effect the interaction between the local RFs and the distant matter at superluminal but still finite speed?

Obviously, these questions can be given more or less definite answers, a corresponding new theory could give. However, perhaps it is appropriate, within the framework of the proposed modern hypotheses and theories in physics, to make some assumption about the nature of this material mediator.

Thus, the intuitive idea of the connections between the accelerated LRF and the Sphere DM as "infinitely thin" lines of enormous length in the proposed model suggests a certain similarity of this idea with the known hypothesis of the so-called "cosmic strings". The idea of the possibility of the existence of such objects in outer space stems from the general theory of strings—objects that, unlike material points, which have no dimensions, have only one dimension—length. Numerous works devoted to the theoretical justification of the idea of the existence of cosmic strings are known (for example, [33]). Then, we can assume that the interaction of the local object (the local RF) with the near end of the string is transmitted along the same to the other object (the distant matter considered as a connected whole) with an enormous speed that is commensurate with the cosmic scales and many times exceeds the speed of light. Here the hypothesis can be continued further, suggesting a similarity of the waves or particles transmitting the interaction along the strings, with the so-called "tachyons"—excitations always moving at superluminal speeds [34]. In other words, we can assume that the gravitational interaction between the local RFs and the distant matter in the Universe, considered as a total object, is different from the gravitational interaction between the local RFs located inside the Sphere DM. This interaction is transmitted at a very high, but still finite speed, and is carried out by a corresponding type of particle (we can tentatively call them "inertons", as opposed to "gravitons", which are supposed to carry out the ordinary gravitational interaction).

7. Gravity and Inertia—Unity and Opposite: Symmetry of the Long Range Gravitational Interaction Violation and Recovery

As we noted at the beginning, Mach's principle is irrevocably related to the principle of long-range action, according to which one material object can act on another material object at an arbitrarily large distance, instantaneously and without a material intermediary. This suggests that the gravitational interaction of bodies with distant matter in the universe is characterized by an infinitely large propagation velocity. On the contrary, in the modification of Mach's principle proposed here, it is assumed that the rate of gravitational interaction of material objects with distant masses in the universe takes place at a huge but finite-valued propagation speed. This means that the action of the distant masses is formed by a vast but finite spatial region (the "distant matter sphere") and acts on the material objects within this region with an extremely small, finite time delay. Moreover, instead of assuming in Mach's principle that the action of distant masses takes place without a material intermediary, in his modification it is assumed that such an intermediary exists (the set of lines of "modified long-range gravitational interaction" connecting each body inside the sphere with the sphere itself considered as a single material object).

The idea of symmetry as the first basis of physical interactions is emphasized by a number of researchers and, in particular, for gravity (for example, [35]). In the present study, we also express a similar idea, namely that the global symmetry of the Modified LRGI can be the source of both the inertial phenomena in the accelerating RF and the effects of "local gravity" between local material objects.

According to Mach's principle revised here, the absence of inertial effects in inertial RFs moving at constant velocity is due to the strict "self-compensation" of the long-range gravitational interaction as a consequence of exact spherical symmetry. It can be assumed that this ultimately underlies the famous "Principle of Relativity".

In non-inertial RFs that move with acceleration, the exact spherical symmetry is broken, "decompensation" of Modified LRGI occurs, which is the reason for the occurrence of inertial effects in these RFs (**Figure 13(a)**).

Here we make an additional assumption, according to which the "local" gravitational effects are also due to a corresponding violation of the global spherical symmetry of the Modified long-range interaction, which occurs in the presence of material objects inside the Sphere DM.

Qualitatively, the idea of the existence of this local gravitational effect arising as a result of the broken spherical symmetry of the Modified LRGI is illustrated by **Figure 13(b)**.

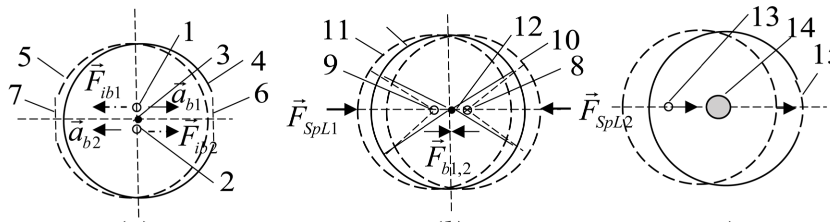

Figure 13. (a) Violation of the symmetry of the Sphere DM in the presence of bodies moving with acceleration relative to each other and occurrence of decompensation of the Modified LRGI, which is the source of the linear stopping inertial forces: 1, 2—two point-like massive bodies moving with acceleration relative to each other something else; 3—center of mass of the bodies; 4, 5—"individual" unilaterally cut Sphere DM of each of the bodies; 6, 7—cut segments of spheres; (b) Violation of the global symmetry of the Sphere DM in the presence of massive bodies inside it and striving to restore the symmetry as a source of the local mutual attraction of the bodies: 8, 9—two point-like massive bodies located at a certain distance from each other; 10, 11—individual Sphere DM for each of the bodies; 12—center of mass of the two bodies; (c) Representation of the (incomplete) compensation of gravitational and inertial forces in the local RFs, which "freely falls" in the gravitational field of a massive enough large body, as a partial local restoration of the broken symmetry of Modified LRGI: 13—local freely falling RF; 14—massive large body; 15—partial local restoration of symmetry of Modified LRGI.

According to this idea, in the presence of (at least two) bodies in the sphere of the DM, there is a violation of the spherical symmetry of the Modified LRGI expressed in the appearance of two non-coincident spheres of the DM. At the same time, there is a tendency to restore symmetry and the emergence of a "global" force of reconciliation of the two spheres, which is "transformed" into a "local" force of mutual attraction $\boldsymbol{F}_{SpL1,2}$ between the two bodies towards the common center of mass $\boldsymbol{F}_{b1,2}$ (**Figure 13(b)**). This generate a non-uniform local gravitational field related to the well-known Newton's theory of gravity, or considering that the speed of this interaction is equal to the speed of light, this leads to a relativistic theory of gravity, eg Einsteinian gravity (General Relativity) or another similar theories.

From here it can be concluded that the basis of both the inertial effects and the local gravitational effects lies in the same cause—the change in the symmetry of the Modified LRGI.

But, while in the movement with acceleration of the local RF (material object) relative to the other RF (also a material object), we have a violation of the global symmetry of the Modified LRGI, which is actually the source of the corresponding inertial effects in this RFs, in the case of local gravitational effects, we have a tendency to restore the broken global symmetry due to the presence of material objects in Sphere of DM, which represents the source of the local gravitational effects.

On the basis of the above, we can still conditionally consider, on the basis of the above, we can still conditionally consider that the first mechanism "generates" the inertial mass of the bodies, and the second—their gravitational mass. But, the unity of the two mechanisms can be taken as the origin of the equality of the two masses, and also as the origin of the famous "Principle of Equivalence". At the same time, the opposition of the two mechanisms can be considered as the source of the partial mutual compensation of the gravitational and inertial effects in the so-called "Local Inertial RFs" (LRFs) that move freely "fall" in an external gravitational field. As is known, due to the difference in the symmetries of the local gravitational fields of real bodies and the symmetries of the inertial force fields, residual ("tidal") gravitational effects exist in LIRF.

The interpretation of the phenomenon of disappearance of gravity in the so-called "freely falling" or "locally inertial" reference systems as a process of "local partial (incomplete) restoration of the symmetry of the long-range gravitational interaction in these OS is illustrated qualitatively in **Figure 13(c)**.

The three aspects of the proposed idea are illustrated in the following block diagrame (**Figure 14**).

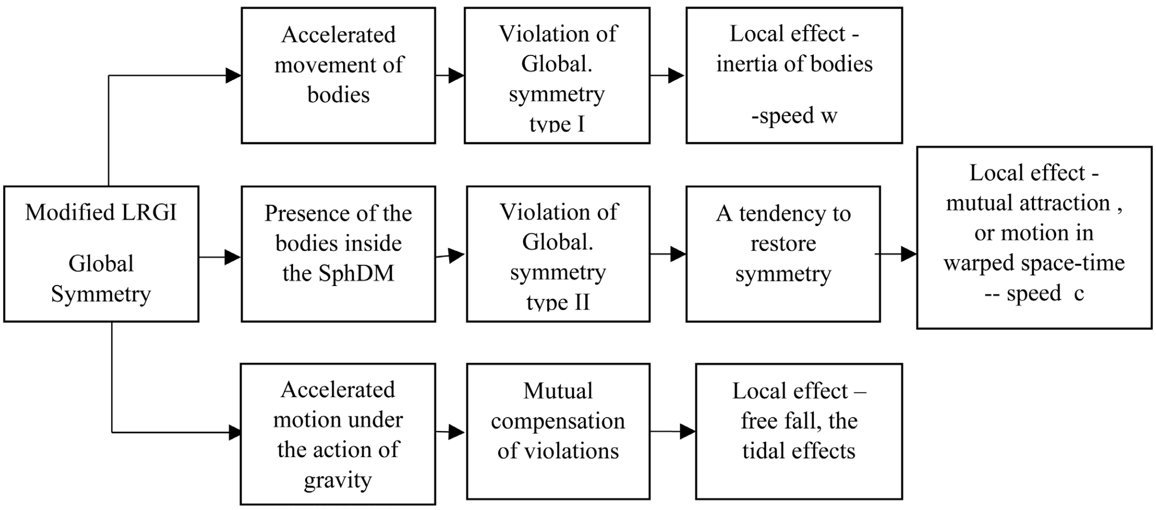

Figure 14. Block diagram illustrating the relationship between the symmetry breaking of the modified long-range gravitational interaction, the inertia and the local gravitational interaction of the bodies.

8. Main Results, Open Questions, Conclusion

At the end of our work, let us make a brief discussion of the main conclusions of our study.

First of all, we will note again that the main goal of our study, as stated at the beginning, is to present Mach's idea known as "Mach's Principle", in a more definite quantitative aspect allowing to represent the inertial accelerations occurring in an accelerated the moving local RF, as a consequence of the gravitational interaction of the same with the rest of the matter in the universe, which is assumed to be mainly concentrated at very large distances (the so-called "distant matter in the universe").

In the overview of the main points in Mach's principle made at the beginning, the leading role in it is emphasized by the so-called "long-range interaction principle" which completely ignores the interaction time between the local body and the distant matter (the system of distant and stationary stars). For the realization of this interaction, the only necessary and sufficient condition is the availability of a sufficiently large total mass to overcome the "monstrous" distances to distant matter. Such a view, however, leads to significant contradictions, for example, that according to it, the given accelerated moving body will act "instantaneously" on the entire mass of the remaining matter, which is impossible to quantitatively determine or estimate, even more so that if the universe has infinite size. In order to overcome this difficulty, in our study we adopt a kind of "modification of the long-range principle", assuming that the gravitational interaction between the accelerated body and the distant matter takes place with a huge but finite propagation speed. This situation, together with the fact of the extraordinary weakness of the gravitational interaction, makes it possible to limit the area (size) of the system of distant objects and to build an approximate, but still quantitative model of the distant matter.

In our research, on this basis, a model of this matter is built in the form of a hollow, infinitely thin sphere with a uniform distribution of surface mass density. Moreover, the work explicitly emphasizes that this model is "effective" and that it actually reflects the real properties of the "modified long-range gravitational interaction field". Thus, in fact, in the work it is assumed that there are two types of gravitational interaction: a short-range—between nearby bodies and a long-range—between the accelerated moving body and distant matter in the universe.

Thus, the simplified model in the form of a "hollow infinitely thin sphere of distant matter" actually effectively represents the real interaction of local material objects with distant matter, which is realized by a kind of material object as a collection of lines of the modified far-acting gravitational interaction. At the same time, the real change in the lines of this interaction as a result of the movement of the local RF with acceleration is represented by an effective change ("cutting") of the sphere of the DM. It is also important that, according to this model, the "individuality" property of inertial effects can be represented as a property of the formation of the DM sphere: each individual material object, or each local RF, forms "its" individual sphere, in the center of which it is located the object. The proposed model is further used to analyze the main (and simplest type) reference frames—the so-called "inertial" RF that move in a straight line at a constant velocity, axles that move in a straight line at a constant acceleration, and axles that rotate about a fixed axis at a constant angular velocity.

The approximate quantitative analysis made shows that the application of Newton's law of attraction characterized by inverse proportionality to the square of the distance in the model leads to contradictions with the data on the main parameters of the distant matter, if the data on the observed part of the universe (the "Metagalaxy" are accepted as the same"). This contradiction can be overcome if a non-Newtonian law of mutual attraction is adopted, namely, inverse proportionality with a fractional exponent of distance dependence. Such type of laws can conditionally be associated with a special ("fractal") structure of the source of attraction, which for the case of linearly accelerated motion is presented in the form of a disk with a fractal structure, and for the case of rotational motion is presented in the form of a fractal crimped cylinder. This approach allows, on the one hand, to avoid the absurd values of the DM sphere truncation parameter necessary to realize "single" gravitational accelerations inside the unilaterally or bilaterally truncated sphere, and on the other hand, to avoid the unrealistic microscopic values of this parameter, and also the infinite sizes of attraction sources in the truncated Sphere of DM.

The application of the proposed model to the main types of local RF mentioned above shows that the static gravitational accelerations created inside the hollow sphere of the DM represent in a first approximation the inertial accelerations in these RFs. At the same time, the work notes that the simulation of inertial acceleration fields as gravitational accelerations inside the truncated sphere according to the proposed model is not accurate. Moreover, according to the model, the fields of gravitational accelerations inside the truncated sphere of the DM are not strictly linear, unlike the fields of the corresponding inertial accelerations. Also, while the inertial acceleration fields are strictly uniform, the corresponding simulated gravity fields inside a unilaterally or bilaterally truncated DM sphere are "quasi-uniform". On the basis of the approximate analysis, the assumption is made that it is possible that the inertial acceleration fields themselves have such differences that exist in the gravitational acceleration fields inside the truncated Sphere of DM according to the proposed model. This gives reason to seek these distinctions on the basis of relevant experiments.

Proceeding from the conviction of the unity and, at the same time, the opposite of inertia and gravity, at the end of the work the assumption is made that the principle of symmetry of the modified long-range interaction lies at the basis of the same. From this point of view, during the accelerated movement of material objects, a violation of the global symmetry occurs, which is the source of the inertial mass of the same and of the corresponding inertial effects, while the presence of the material objects inside the DM sphere also leads to a violation of the global symmetry, but of another type. In this case, the tendency to restore symmetry becomes the source of the gravitational mass and the corresponding gravitational effects (the mutual attraction of bodies). Thus, the proposed concept can explain the experimentally confirmed equality of inertial and gravitational masses and the realization of the so-called "local" inertial systems that move ("freely fall") in an external gravitational field and in which a mutual compensation of gravitational and inertial accelerations occurs (the phenomenon of "weightlessness").

At the end of our study, we allow ourselves to conclude that within one of the main paradigms raised so far in physics—the principle of long-range action, it is possible to quantitatively substantiate Mach's idea that the inert properties of bodies are due to their gravitational interaction with distant matter in the universe. This was shown in the work for the main types of inertial forces (accelerations), to which a corresponding type of static gravitational forces (accelerations) is matched. But then the question can be asked—does this prove the truth of Mach's principle? That's hard to say. In our opinion, the importance of this principle and its new interpretation in the present work lies in the thought that our ideas about the properties of gravity in the region of ultra-large ("mega"-) scales are severely limited. The stated assumptions (and persistent searches) about the existence of the so-called "dark matter" and "dark energy" which can explain the peculiarities in the motion of such large objects as galaxies and which cannot explain the known theories of gravity. But then the assumption naturally arises that the "modified long-range gravitational interaction" assumed in the present work may be used in the explanation of the noted features in the movement of super-large space objects. This would probably require the development of a new theory of gravity applicable precisely on such scales, just as there is now more and more insistence on the development of a "Quantum Theory of Gravity" that can be applied to the realm of ultra-small ("micro"-) scales.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Newton, I. (1687) Philosophiae Naturalis Principia Mathematica. Joseph Streater. https://doi.org/10.5479/sil.52126.39088015628399

[2] Berkeley, G. (1726) The Principles of Human Knowledge. Aaron Rhames.

[3] Mach, E. (1911) History and Root of the Principle of the Conservation of Energy. Open Court.

[4] Mach, E. (1960) The Science of Mechanics: A Critical and Historical Account of Its Development. Open Court Pub. Co.

[5] Einstein, A. (1973) Letter to Ernst Mach, Zurich, 25 June 1913. In: Misner, C., Thorne, K.S. and Wheeler, J.A., Eds., *Gravitation*, W.H. Freeman.

[6] Sciama, D.W. (1953) On the Origin of Inertia. *Monthly Notices of the Royal Astronomical Society*, **113**, 34-42. https://doi.org/10.1093/mnras/113.1.34

[7] Dicke, R.H. (1961) Dirac's Cosmology and Mach's Principle. *Nature*, **192**, 440-441. https://doi.org/10.1038/192440a0

[8] Bondi, H. and Samuel, J. (1996) The Lense-Thirring Effect and Mach's Principle.

[9] Jannes, G. and Volovik, G.E. (2015) Emergent Physics on Mach's Principle and the Rotating Vacuum. *JETP Letters*, **102**, 73-79. https://doi.org/10.1134/s0021364015140052

[10] Telkamp, H. (2016) Machian Derivation of the Friedmann Equation. *Physical Review D*, **94**, Article ID: 043520. https://doi.org/10.1103/physrevd.94.043520

[11] Weinberg, S. (1972) Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. John Wiley & Sons, Inc.

[12] Putz, V. (2018) A Theory of Inertia Based on Mach's Principle.

[13] Schlatter, A. and Kastner, P.F. (2024) A Note on the Origin of Inertia.

[14] Das, S. (2023) Aspects of Machian Gravity (I): A Mathematical formulation for Mach's Principle.

[15] Chugreev, U.V. (2015) Mach's Principle in Relativistic Theory of Gravity. *Elementary Particles and Atomic Nuclei*, **12**, 281-298.

[16] Nikitin, A.P. (2020) Mach Principle and Principle of Relativity, Metaphisics, No. 2, Russia.

[17] Einstein, A. (1912) Gibt es eine Gravitationswirkung, die der *elektrodynamischen* Induc-tionswirkung analog ist? *Vierbeljahrshr*, **44**, 37-40.

[18] Einstein, A. (1965) Collection of Scientific Works. Vol. 1, "Science".

[19] Post, E.J. (2006) Mach's Principle in a Mixed Newton-Einstein Context. University of Houston.

[20] Ferraro, R. (2017) The Frame of Fixed Stars in Relational Mechanics.

[21] Benisty, D. (2022) Testing Modified Gravity via Yukawa Potential in Two Body Problem: Analytical Solution and Observational Constraints. *Physical Review D*, **106**, Article ID: 043001. https://doi.org/10.1103/physrevd.106.043001

[22] Afonin, A.M. (2006) Physical Foundations of Mechanics. MSTU. (In Russian)

[23] Roth, G.D. (2009) Handbook of Practical Astronomy. Springler.

[24] (2007) Handbook of Physical Measurements. 2nd Edition, Oxford University Press.

[25] Nielsen, N.G., Pallesandro, A. and Sloth, M.S. (2019) Gravitational Atoms.

[26] Newton, I. (1687) Philosophia Naturalis Principia Mathematica, London, p. 193, Theorem XXXI.

[27] Putekhin, P.V. (2021) Gravitational Forces inside a Hoop, a Sphere and between Two Points. Samizdat.

[28] Ghanbarian, B. and Hunt, A.G. (2017) Fractals, Concepts and Application in Geosciences Q.

[29] Lauwerier, H. and Lauwerier, H.A. (1991) Fractals: Endlessly Repeated Geometrical Figures. Princeton University Press.

[30] Boyd, D.W. (1973) The Residual Set Dimension of the Apollonian Packing. *Mathematika*, **20**, 170-174. https://doi.org/10.1112/s0025579300004745

[31] Bradford, A. (2022) Revisiting Apollonian Gaskets.

[32] Ungar, Š. (2007) The Koch Curve: A Geometric Proof. *The American Mathematical Monthly*, **114**, 61-66. https://doi.org/10.1080/00029890.2007.11920392

[33] Kibble, T.W.B. (1976) Topology of Cosmic Domains and Strings. *Journal of Physics A*: *Mathematical and General*, **9**, 1387-1398. https://doi.org/10.1088/0305-4470/9/8/029

[34] Bers, A., Fox, R., Kuper, C.G. and Lipson, S.G. (1971) The Impossibility of Free Tachyons. In: Kuper, C.G. and Peres, A., Eds., *Relativity and Gravitation*, Gordon and Breach Science Publishers, 41-46.

[35] Harlowa, D. and Shaghoulian, E. (2021) Global Symmetry, Euclidean Gravity, and the Black Hole Information Problem. arXiv 2010.10539v2.