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The Gravitational Potential and the Gravitational Force According to the Correct Schwarzschild Metric

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Abstract

In a recent article, we have corrected the traditional derivation of the Schwarzschild metric, thus obtaining the formulation of the correct Schwarzschild metric, which is different from the traditional Schwarzschild metric. In this article, by starting from this correct Schwarzschild metric, we obtain the formulas of the correct gravitational potential and of the correct gravitational force in the case described by this metric. Moreover, we analyse these correct results and their consequences. Finally, we propose some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

Keywords

General Theory of Relativity, Schwarzschild, Metric, Gravitational Potential, Gravitational Force, Stable Circular Orbital Motion

1. Introduction

In a recent article [1], we have corrected the traditional derivation of the Schwarzschild metric, thus obtaining the formulation of the correct Schwarzschild metric, which is different from the traditional Schwarzschild metric.

In this article, by starting from this correct Schwarzschild metric, we want to obtain the formulas of the correct gravitational potential and of the correct gravitational force in the case described by this metric.

Therefore, in this article, we will use this correct Schwarzschild metric to correct the traditional derivation of the effective gravitational potential.

To do this we will follow such traditional derivation, replacing however what comes from the traditional Schwarzschild metric with what comes analogously from the correct Schwarzschild metric.

Then, by using the Newtonian limit, we will also obtain the expression of the correct gravitational potential, which is different from the analogue incorrect expression obtained from the traditional Schwarzschild metric.

Moreover, we will analyse these results and their consequences.

Furthermore, we will use these expressions of the gravitational potential to derive the expressions of the correct gravitational force and of the incorrect gravitational force, respectively.

Here too we will analyse these results and their consequences.

Finally, we will see the experimental prospects, proposing in particular some possible crucial experiments between the commonly accepted theory and the same theory corrected according to this article.

2. The Correct Schwarzschild Solution

In this article all the formulas are expressed with the velocity of light $c \equiv 1$, unless otherwise indicated.

Now, to begin with, we will recall in this section what of [1] will be useful for this article.

We have shown in [1] that the correct Schwarzschild solution expressed as the formally flat metric in the commonly used coordinates ($\mathrm{d}s^2 \equiv \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2$), which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time, is:

$$\mathrm{d}s^2 = \left(1+\frac{2GM}{r_g}\right)\mathrm{d}t_g^2 - \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{1}$$

where M is (a constant, which is equal to) the total mass of the system, and t_g, r_g, θ_g, φ_g are the space-time coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time. In other words, the equation (1) is expressed in curved coordinates.

We can note that in the (1), the coefficient of $\mathrm{d}t_g^2$ is always greater than one (except in the case where there is no gravitational field, in which case this coefficient is equal to one), and the more intense the gravitational field is, the greater this coefficient is. We have already shown in [1] that this is a sign that the coordinates are curved and is closely related to the fact that the clocks in a gravitational field run more slowly.

On the other hand, we have also shown in [1] that the correct Schwarzschild solution expressed as the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, is:

$$\mathrm{d}s_g^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{2}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates (that is, $\mathrm{d}s_g^2 \equiv \mathrm{d}t_g^2 - \mathrm{d}r_g^2 - r_g^2 \mathrm{d}\theta_g^2 - r_g^2 \sin^2\theta_g \mathrm{d}\varphi_g^2$).

Obviously, r (or r_g) is always greater than zero since the Schwarzschild solution is a solution for vacuum region surrounding a spherically symmetric mass distribution.

As for the relation between r_g and r, we have:

$$\mathrm{d}r^2 = \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{3}$$

From which, we have:

$$\int_0^{r^2} \mathrm{d}r^2 = \int_0^{r_g^2} \frac{1}{1+\frac{2GM}{r_g}}\mathrm{d}r_g^2 \tag{4}$$

From which, we have:

$$r^2 = \int_0^{r_g} \frac{2r_g^2 \mathrm{d}r_g}{r_g + 2GM} \tag{5}$$

From which, we have:

$$r^2 = \left[r_g^2 - 4GMr_g + 8G^2M^2 \ln\left(2GM + r_g\right)\right]_0^{r_g} \tag{6}$$

From which, we have:

$$r^2 = r_g^2 - 4GMr_g + 8G^2M^2 \ln\left(1+\frac{r_g}{2GM}\right) \tag{7}$$

According to the (7) for $r_g \to 0^+$ also $r \to 0^+$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases as can be seen by differentiating the right side of the (7) with respect to r_g or directly from the (5). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as can be seen directly from the (4).

Furthermore, for $r_g \gg 2GM$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that for $\frac{2GM}{r} \ll 1$ the presence of r_g instead of r in the (2) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand, for $r_g \ll 2GM$ we have $r^2 \cong \frac{r_g^3}{3GM}$.

In particular for $r_g = 2GM$, $r^2 = 4G^2M^2\left(2\ln 2 - 1\right)$, and so in this case we have $r = 2GM\sqrt{2\ln 2 - 1} \cong 1.24GM$.

Moreover, for $r_g = 4GM$, $r^2 = 8G^2M^2 \ln 3$, and so in this case we have $r = 2GM\sqrt{2\ln 3} \cong 2.96GM$.

Instead for $r_g = 8GM$, $r^2 = 8G^2M^2\left(4 + \ln 5\right)$, and so in this case we have

$r = 2GM\sqrt{2(4+\ln 5)} \cong 6.70GM$.

Furthermore, from the (3) we have:

$$2r\mathrm{d}r = \frac{2r_g \mathrm{d}r_g}{1+\frac{2GM}{r_g}} \tag{8}$$

From which, we have:

$$\frac{\mathrm{d}r_g}{\mathrm{d}r} = \frac{r}{r_g}\left(1+\frac{2GM}{r_g}\right) \tag{9}$$

Instead, the common erroneous expression for the Schwarzschild solution is [2]:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{r}\right)\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{10}$$

We can note that we have not any singularity in the coefficients of $\mathrm{d}t^2$ (or of $\mathrm{d}t_g^2$) and of $\mathrm{d}r^2$ (or of $\mathrm{d}r_g^2$) in the correct expressions (1) and (2) for any value of $r > 0$ and of $r_g > 0$.

In particular, the coefficients of $\mathrm{d}t^2$ in (2) and of $\mathrm{d}t_g^2$ in (1) are always >0, while the coefficients of $\mathrm{d}r^2$ in (2) and of $\mathrm{d}r_g^2$ in (1) are always <0. Therefore since the coefficients of $\mathrm{d}t^2$ in (2) and of $\mathrm{d}t_g^2$ in (1) are always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [2] [3]. Moreover, since the coefficients of $\mathrm{d}r^2$ in (2) and of $\mathrm{d}r_g^2$ in (1) are always not positive, also here the spatial coordinate r (or r_g) does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [2] [3]. Therefore we can say that, according to the correct expressions (1) and (2), there is not any event horizon and therefore that there is not any black hole [1]-[3].

Moreover, according to the correct expressions (1) and (2), the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [2] [3].

For further information on the correct Schwarzschild solution, in particular how it is obtained, please refer to [1].

3. The Gravitational Potential According to the Correct Schwarzschild Metric

We can start from the equation (7.99) of the book of H. C. Ohanian and R. Ruffini [2] for the case of the Schwarzschild metric, which equation is:

$$\mathrm{e}^N\dot{t}^2 - \mathrm{e}^L\dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2 = 1 \tag{11}$$

with:

$$\dot{t} = \mathcal{E}\mathrm{e}^{-N} \tag{12}$$

$$\dot{\phi} = \frac{\ell}{r^2} \tag{13}$$

where $\mathcal{E}$ and ℓ are two constants, that are equal, respectively, to the energy per unit mass and to the angular momentum per unit mass.

By using the (12) and the (13) we can rewrite the (11) as:

$$\mathcal{E}^2 e^{-N} - e^L \dot{r}^2 - r^2\dot{\theta}^2 - \sin^2\theta \frac{\ell^2}{r^2} = 1 \tag{14}$$

In particular, we consider, as H. C. Ohanian and R. Ruffini [2], the case in which $\theta = \pi/2$. In this case the (14) becomes:

$$\mathcal{E}^2 e^{-N} - e^L \dot{r}^2 - \frac{\ell^2}{r^2} = 1 \tag{15}$$

Now, according to the common incorrect treatment of the Schwarzschild metric we have that $e^{-L} = e^N = 1 - 2GM/r$ [2], for which the (15) becomes:

$$\frac{\mathcal{E}^2}{1-\frac{2GM}{r}} - \frac{\dot{r}^2}{1-\frac{2GM}{r}} - \frac{\ell^2}{r^2} = 1 \tag{16}$$

From which, we have [2]:

$$\dot{r}^2 + \left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right) = \mathcal{E}^2 \tag{17}$$

In this equation, the second term on the left side plays the role of an effective gravitational potential [2]. For which the expression of the effective gravitational potential according to the incorrect Schwarzschild solution is this expression [2]:

$$\mathcal{V}_{incorr}(r) = \sqrt{\left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)} \tag{18}$$

This formula for the effective gravitational potential, which formula is present in the book of H. C. Ohanian and R. Ruffini [2], is perfectly equivalent to the analogous formulas that are present in the book of C. W. Misner, K. S. Thorne and J. A. Wheeler [3] and in the book of L. D. Landau and E. M. Lifšits [4].

But we know, as we have seen in [1], that the correct treatment of the Schwarzschild metric, when we use an expression analogue to that in which the linear theory is commonly expressed, provides instead that $e^{-L} = e^N = 1/\left(1+\frac{2GM}{r_g}\right)$, since in this case we have the (2) instead of the (10), for which we have from the (15) not the equation (16) but this other equation:

$$\left(1+\frac{2GM}{r_g}\right)\mathcal{E}^2 - \left(1+\frac{2GM}{r_g}\right)\dot{r}^2 - \frac{\ell^2}{r^2} = 1 \tag{19}$$

We can note that the (19) is equal to the (16) at the first order in $\frac{2GM}{r}$.

Now we can rewrite the (19) as:

$$\dot{r}^2 + \frac{1}{1+\frac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right) = \mathcal{E}^2 \tag{20}$$

From this correct equation we can obtain, in a manner similar to that used by H. C. Ohanian and R. Ruffini [2] for obtaining the (18) from the incorrect equation (17), that the correct expression of the effective gravitational potential is equal to:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right)} \tag{21}$$

We can note that while the correct expression (21) assumes positive real values for any value of r and r_g greater than zero, the incorrect expression (18) instead assumes non-real values for the values of r less than $2GM$ and a value equal to zero for $r = 2GM$.

At large distances, we have [2]:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}}\left(1+\frac{\ell^2}{r^2}\right)} \cong \sqrt{\left(1-\frac{2GM}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)} \cong 1-\frac{GM}{r}+\frac{\ell^2}{2r^2} \tag{22}$$

Therefore in this case (that is, in the Newtonian limit) we have that the (21) and the (18) are approximately equal between them and are approximately equal to the Newtonian effective gravitational potential plus one [2].

From this we can infer that the (21) and the (18) are the analogue of the Newtonian effective gravitational potential plus one.

Now we consider the case in which $\ell = 0$. In this case the correct expression (21) becomes:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \tag{23}$$

We can note that, according to the (23), $\mathcal{V}_{corr}(r)$ is always positive for any value of r and r_g greater than zero, and tends to zero for r (or r_g) that tends to zero.

While the incorrect expression (18) becomes:

$$\mathcal{V}_{incorr}(r) = \sqrt{1-\frac{2GM}{r}} \tag{24}$$

We can note that, according to the (24), $\mathcal{V}_{incorr}(r)$ assumes non-real values for the values of r less than $2GM$ and a value equal to zero for $r = 2GM$.

The correct expression (23) in the case of $r_g \gg 2GM$ (or $r \gg 2GM$) becomes:

$$\mathcal{V}_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \cong \sqrt{1-\frac{2GM}{r}} \cong 1-\frac{GM}{r} \tag{25}$$

Therefore we have that in this case the correct expression (23) becomes approximately equal to the incorrect expression (24), and both the (23) and the (24) are approximatively equal to the Newtonian gravitational potential plus one.

From this we can infer that the (23) and the (24) are the analogue of the Newtonian gravitational potential plus one.

Therefore, from the (23) we can infer that the correct expression of the gravitational potential $V_{corr}(r)$ is:

$$V_{corr}(r) = \sqrt{\frac{1}{1+\frac{2GM}{r_g}}} - 1 \quad (26)$$

Obviously, according to the (26), $V_{corr}(r)$, in the case of $r_g \gg 2GM$ (or $r \gg 2GM$), becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (26), for $r_g \to +\infty$ (or $r \to +\infty$) $V_{corr}(r) \to 0$, and for $r_g \to 0^+$ (or $r \to 0^+$) $V_{corr}(r) \to -1$.

Instead, from the (24) we can infer that the incorrect expression of the gravitational potential $V_{incorr}(r)$ is:

$$V_{incorr}(r) = \sqrt{1-\frac{2GM}{r}} - 1 \quad (27)$$

Obviously, also here, according to the (27), $V_{incorr}(r)$, in the case of $r \gg 2GM$, becomes approximately equal to the Newtonian gravitational potential.

Moreover, we can note that, according to the (27), $V_{incorr}(r)$ assumes non-real values for the values of r less than $2GM$ and a value equal to −1 for $r = 2GM$. Furthermore, according to the (27), for $r \to +\infty$ $V_{incorr}(r) \to 0$.

On the other hand, the correct expression (26) implies that the energy of a massive particle is equal to:

$$E_{corr}(r) = m_0\gamma\left[V_{corr}(r)+1\right] = m_0\gamma\sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \quad (28)$$

Now the fact that $V_{corr}(r)+1$ is always greater than zero for any value of r and r_g greater than zero implies that a massive particle, for any value of r and r_g greater than zero, has always a correct escape velocity $v_{e_{corr}}$ less than the velocity of light c. In particular we have that the massive particle, in order to escape, must have a γ that multiplied for the value of $V_{corr}(r)+1$ be equal to one or to a number greater than one, in fact in this case the $E_{corr}(r)$ would be equal to or greater than the energy of rest mass of the same particle: a such value of γ is always possible if $V_{corr}(r)+1>0$ since γ can assume all the values ≥ 1.

More precisely we have:

$$\gamma\left(v_{e_{corr}}\right)\sqrt{\frac{1}{1+\frac{2GM}{r_g}}} = 1 \quad (29)$$

From which, we have:

$$\frac{1}{\sqrt{1-v_{e_{corr}}^{2}}}\sqrt{\frac{1}{1+\frac{2GM}{r_g}}}=1 \tag{30}$$

From which, since the square of the velocity of a massive particle is always <1, we have:

$$\frac{1}{1+\frac{2GM}{r_g}}=1-v_{e_{corr}}^{2} \tag{31}$$

From which, we have:

$$v_{e_{corr}}^{2}=\frac{\frac{2GM}{r_g}}{1+\frac{2GM}{r_g}}=\frac{1}{1+\frac{r_g}{2GM}} \tag{32}$$

From which, we have:

$$v_{e_{corr}}=\sqrt{\frac{1}{1+\frac{r_g}{2GM}}} \tag{33}$$

We can note that, according to the (33), for $r_g \to +\infty$ (or $r \to +\infty$) $v_{e_{corr}} \to 0$, and for $r_g \to 0^{+}$ (or $r \to 0^{+}$) $v_{e_{corr}} \to 1$ (that is, tends to the velocity of light *c*).

On the other hand, a photon is always free to escape away by decreasing its frequency. In fact in this case the (28) becomes:

$$E_{corr}(r)=h\nu\left[V_{corr}(r)+1\right]=h\nu\sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \tag{34}$$

And the photon can escape away with the new correct frequency $\nu_{0_{corr}}$ at the infinite equal to:

$$\nu_{0_{corr}}=\nu\sqrt{\frac{1}{1+\frac{2GM}{r_g}}} \tag{35}$$

On the contrary, by using the incorrect expression (27) instead of the correct expression (26) we would have, in place of the (28), (34) and (35), the following incorrect formulas:

$$E_{incorr}(r)=m_0\gamma\left[V_{incorr}(r)+1\right]=m_0\gamma\sqrt{1-\frac{2GM}{r}} \tag{36}$$

$$E_{incorr}(r)=h\nu\left[V_{incorr}(r)+1\right]=h\nu\sqrt{1-\frac{2GM}{r}} \tag{37}$$

$$v_{0_{incorr}} = v\sqrt{1-\frac{2GM}{r}} \tag{38}$$

We can note that these incorrect formulas (36), (37) and (38) assume non-real values for the values of r less than $2GM$ and a value equal to zero for $r = 2GM$. This fact clearly implies that according to these incorrect formulas there is no possibility of escape for a massive particle or a photon for $r \leq 2GM$.

As for the incorrect escape velocity $v_{e_{incorr}}$ of a massive particle for $r > 2GM$, according to the incorrect formula (36), we have:

$$\gamma\left(v_{e_{incorr}}\right)\sqrt{1-\frac{2GM}{r}} = 1 \tag{39}$$

From which, we have:

$$\frac{1}{\sqrt{1-v_{e_{incorr}}^2}}\sqrt{1-\frac{2GM}{r}} = 1 \tag{40}$$

From which, since the square of the velocity of a massive particle is always < 1, we have:

$$1-\frac{2GM}{r} = 1-v_{e_{incorr}}^2 \tag{41}$$

From which, we have:

$$v_{e_{incorr}} = \sqrt{\frac{2GM}{r}} \tag{42}$$

We can note that, according to the (42), for $r \to +\infty$ $v_{e_{incorr}} \to 0$, and for $r \to 2GM^+$ $v_{e_{incorr}} \to 1$ (that is, tends to the velocity of light c).

Therefore, the correct formulas, contrary to the commonly accepted theory (that is, contrary to the incorrect formulas), do not entail any event horizon and, consequently, any black hole: in fact, as we have noted, according to the correct formulas (contrary to the incorrect formulas) the massive particles are always free to escape away with an escape velocity less than the velocity of light c and the photons are always free to escape away by decreasing their frequency.

On the other hand, the correct formulas, contrary to the commonly accepted theory, are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [5]-[10], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1].

4. The Gravitational Force According to the Correct Schwarzschild Metric

According to the traditional way of deriving the gravitational force from the gravitational potential, we can calculate the correct expression of the gravitational force on a unit mass $F_{corr}(r)$ directly from the formula (26) of the correct gravitational potential, taking into account the (9):

$$\begin{aligned} F_{corr}\left(r\right) &= \frac{\mathrm{d}V_{corr}\left(r\right)}{\mathrm{d}r} \\ &= \frac{1}{2\sqrt{\frac{1}{1+\frac{2GM}{r_g}}}} \frac{-1}{\left(1+\frac{2GM}{r_g}\right)^2} 2GM\left(-1\right)\frac{1}{r_g^2}\frac{\mathrm{d}r_g}{\mathrm{d}r} \\ &= \frac{GM}{r_g^2}\frac{1}{\left(1+\frac{2GM}{r_g}\right)^{\frac{3}{2}}}\frac{r}{r_g}\left(1+\frac{2GM}{r_g}\right) \\ &= \frac{GMr}{r_g^3}\frac{1}{\sqrt{1+\frac{2GM}{r_g}}} \end{aligned} \tag{43}$$

Obviously, to obtain the correct expression $F_{corr}\left(r,m\right)$ of the gravitational force on a generic mass *m*, it is sufficient to multiply the (43) by that generic mass:

$$F_{corr}\left(r,m\right) = \frac{GMmr}{r_g^3}\frac{1}{\sqrt{1+\frac{2GM}{r_g}}} \tag{44}$$

Of course, in this expression (44), *m* is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Here we can note that, according to the (44), $F_{corr}\left(r,m\right)$ in the case of $r_g \gg 2GM$ (or $r \gg 2GM$) becomes approximately equal to the Newtonian gravitational force and for $r_g \to +\infty$ (or $r \to +\infty$) tends to zero.

Moreover, according to the (44), in the case of $r_g \ll 2GM$ (or $r \ll 2GM$), taking into account that in this case (as we have already noted) $r^2 \cong \frac{r_g^3}{3GM}$, we have:

$$F_{corr}\left(r,m\right) \cong \frac{GMmr}{3GMr^2}\frac{r^{\frac{1}{3}}\left(3GM\right)^{\frac{1}{6}}}{\sqrt{2GM}} = \frac{m}{3^{\frac{5}{6}}\sqrt{2}\left(GM\right)^{\frac{1}{3}}r^{\frac{2}{3}}} \tag{45}$$

We can note that, according to the (45), for $r \to 0^+$ the correct gravitational force tends to $+\infty$, *i.e.* tends to be infinitely attractive.

Now, in the case of a stable circular orbital motion, by considering the gravitational force expressed by the correct formula (44) as the centripetal force of this motion we have:

$$\frac{\frac{GMmr}{r_g^3}}{\sqrt{1+\frac{2GM}{r_g}}} = m\frac{v^2}{r} \tag{46}$$

From which, we have:

$$\frac{\frac{GMr^2}{r_g^3}}{\sqrt{1+\frac{2GM}{r_g}}}=v^2 \tag{47}$$

In the case of $r_g \ll 2GM$ (or $r \ll 2GM$), taking into account that in this case (as we have already noted) $r^2 \cong \frac{r_g^3}{3GM}$, the (47) becomes approximately:

$$\frac{1}{3}\sqrt{\frac{r_g}{2GM}} \cong v^2 \tag{48}$$

From the (48) we have that for $r_g \to 0^+$ (or $r \to 0^+$) $v \to 0$.

Instead in the case of $r_g \gg 2GM$ (or $r \gg 2GM$) the (47) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{49}$$

From the (49) we have that for $r \to +\infty$ (or $r_g \to +\infty$) $v \to 0$.

On the other hand, by using the formula (27) of the incorrect gravitational potential for calculating the incorrect expression of the gravitational force on a unit mass we have:

$$F_{incorr}(r)=\frac{\mathrm{d}V_{incorr}(r)}{\mathrm{d}r}=\frac{1}{2\sqrt{1-\frac{2GM}{r}}}(-2GM)\left(-\frac{1}{r^2}\right)=\frac{GM}{r^2}\frac{1}{\sqrt{1-\frac{2GM}{r}}} \tag{50}$$

Obviously, to obtain the incorrect expression $F_{incorr}(r,m)$ of the gravitational force on a generic mass *m*, it is sufficient to multiply the (50) by that generic mass:

$$F_{incorr}(r,m)=\frac{GMm}{r^2}\frac{1}{\sqrt{1-\frac{2GM}{r}}} \tag{51}$$

Of course, also here, in this expression (51), *m* is equal to $m_0\gamma$ for a massive particle and is equal to $h\nu$ for a photon.

Also here we can note that, according to the (51), $F_{incorr}(r,m)$ in the case of $r \gg 2GM$ becomes approximately equal to the Newtonian gravitational force and for $r \to +\infty$ tends to zero.

Moreover, according to the (51), $F_{incorr}(r,m)$ for $r \to 2GM^+$ tends to $+\infty$, *i.e.* tends to be infinitely attractive, and for the values of $r < 2GM$ assumes non-real values.

On the other hand, in the case of a stable circular orbital motion, by considering the incorrect gravitational force expressed by the formula (51) as the centripetal force of this motion we have:

$$\frac{\frac{GMm}{r^2}}{\sqrt{1-\frac{2GM}{r}}}=m\frac{v^2}{r} \tag{52}$$

Now, we have that the left side of the (52) for the values of $r < 2GM$ assumes non-real values and for $r \to 2GM^{+}$ tends to $+\infty$, while the right side of the (52) always assumes finite real values for all the values of $r > 0$: for which there are not solutions of the (52) for the values of $r \le 2GM$.

Moreover, for the values of $r > 2GM$, from the (52) we have:

$$\frac{\frac{GM}{r}}{\sqrt{1-\frac{2GM}{r}}} = v^2 \tag{53}$$

In the case of $r \gg 2GM$ the (53) becomes approximately:

$$\frac{GM}{r} \cong v^2 \tag{54}$$

From the (54) we have that for $r \to +\infty$ $v \to 0$.

5. Experimental Prospects

5.1. The Available Experimental Data

As we have already noted in [1], as for the experimental data obtained with the help of x-ray astronomy the proof that we have found black holes, and therefore event horizons, is based only on the fact that we have found invisible objects which have masses that are too great, according to the commonly accepted theory, for not being black holes [2] [11] [12]. But according to the correct theory of this article, whatever the masses and the dimensions of these invisible objects are, we never have black holes, and therefore we never have event horizons. Therefore such experimental data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

On the other hand, as we have already noted in [1], with regard to the experimental data of the so-called gravitational waves (obtained by the LIGO collaboration) of a collision between two black holes, such gravitational waves were detected only below measurement errors, *i.e.* the signals detected were lower than the background noise (cf. chapter 6 of [11]). Furthermore the models expected from the theory were used for selecting the signals from the background noise (cf. chapter 6 of [11]) with the help of supercomputers: obviously, this is an incorrect practice which cannot produce any significant data. The awareness of the non-significance of the LIGO collaboration data is now widespread [13]-[16]. Obviously, also such data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

As for the alleged photos of black holes, as we have already noted in [1], they were formed with the help of special algorithms from something compatible with the white noise. In other words, these photos were extracted from something compatible with the white noise only on the basis of the images that were expected by the researchers, with the help of appropriate algorithms loaded onto supercomputers (cf. the Section "Imaging a Black Hole" of [17]. See also [18]). Therefore, also in this case the researchers wanted to measure something that is below measurement errors, and so these photos are completely unreliable. On the other hand, serious doubts have now spread about the reliability of these photos [19] [20]. Consequently such photos cannot prove anything and in particular cannot discriminate in any way between the commonly accepted theory and the same theory corrected according to this article.

Moreover, the corrections, that we have proposed in this article, to the commonly accepted theory are very small in the normal experimental situations (for example in the solar system), so the fact that, in these situations, so far no difference has been noted between the commonly accepted theory and the experimental results is not strange. In fact, as we have already noted in [1], in the usual case of $\frac{2GM}{r} \ll 1$ we have that the difference between the previsions of the erroneous Schwarzschild solution together with the erroneous classical limit and the previsions of the correct Schwarzschild solution together with the correct classical limit is, in the experiments commonly performed to test the General Theory of Relativity, only at the second order in $\frac{2GM}{r}$. And all the experiments conducted so far in the solar system have not had errors so small as to test differences at the second order in $\frac{2GM}{r}$ [2].

Therefore, in conclusion, there is no available experimental data that can discriminate between the commonly accepted theory and the same theory corrected according to this article.

5.2. Some Proposals for a Crucial Experiment

On the other hand, a crucial experiment could be done, which discriminates between the commonly accepted theory and the same theory corrected according to this article, by taking advantage of the high precision and sensitivity of the latest atomic clocks.

In fact, as we have shown in [1], the ratio of the passage of time in the gravitational field according to the correct Schwarzschild metric to that according to the commonly accepted Schwarzschild metric, in the case of $\frac{2GM}{r} \ll 1$, is approximately equal to $1+\frac{1}{2}\left(\frac{2GM}{r}\right)^2$.

Now throughout the solar system we have effectively $\frac{2GM}{r} \ll 1$.

The term $\frac{1}{2}\left(\frac{2GM}{r}\right)^2$ is obviously expressed in the case with the velocity of light $c \equiv 1$. Instead in the case more general (in which c is not defined equal to 1) this term becomes $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$. Now, the term $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$ due to the solar mass on the surface of the Sun is approximately equal to 8.99×10^{-12}, while at the average distance of the Earth from the Sun this value becomes approximately equal to 1.95×10^{-16}. Obviously, externally to the Sun, such term decreases with the square of the distance from the centre of the Sun according to the formula.

On the other hand, the same term due to the mass of the Earth on the surface of the Earth is approximately equal to 9.69×10^{-19} and obviously also here, externally to the Earth, decreases with the square of the distance from the centre of the Earth according to the formula. Of course, if we were to opt to use only the term due to the mass of the Earth we would have to do so between two points that have an approximatively equal contribution due to the mass of the Sun, so that the difference between these contributions due to the mass of the Sun be negligible compared to the difference between the correspondent contributions due to the mass of the Earth.

On the other hand, now we have atomic clocks that have an error of 7.6×10^{-21} [21] [22] and therefore we can measure such differences between the predictions of the commonly accepted theory and those of the same theory corrected according to this article with appropriate temporal measurements made in the solar system.

Now, in theory we could make a single measurement of time with one such atomic clock for being able to detect such differences, but in practice there can easily be errors due to the low precision in predicting the measurement results according to the commonly accepted theory (for example due to the low precision in knowing the mass of the Sun, the mass of the Earth and the universal gravitational constant G), so it is more convenient to make differential measurements, *i.e.* to measure the differences between the time measurements of two atomic clocks of such type placed respectively in two appropriate positions in the solar system. Indeed, it would be even better if the comparison between the time measurements of two atomic clocks of such type were made between several pairs of points of the solar system, thus revealing the difference between the two theories also on the basis of the trend of these differences as a function of the positions of at least one of these two clocks in the solar system.

Therefore we could do a crucial experiment, which discriminates between the commonly accepted Schwarzschild metric and the same metric corrected according to this article, by taking one such atomic clock to diverse convenient locations in the solar system for comparing its time measurements made at those various locations with the corresponding time measurements made by another similar clock here on Earth.

Now, since the corrections of this article to the gravitational potential and the gravitational force strictly depend on the corrections of this article to the Schwarzschild metric, we can say that this crucial experiment would also discriminate between the commonly accepted theory about the gravitational potential and the gravitational force and the same theory corrected according to this article.

On the other hand, in the usual experimental tests of the General Theory of Relativity, no direct gravitational force measurements have been used so far because they are not very precise.

However, the ratio between the correct expression of the gravitational force (44) and the incorrect expression of the gravitational force (51), in the usual case of $\frac{2GM}{r} \ll 1$, taking into account that in this case (as we have already noted) $r_g \cong r\left(1+\frac{2GM}{r}\right)$, is equal approximatively to:

$$\frac{F_{corr}(r,m)}{F_{incorr}(r,m)} = \frac{\dfrac{\dfrac{GMmr}{r_g^{\,3}}}{\sqrt{1+\dfrac{2GM}{r_g}}}}{\dfrac{\dfrac{GMm}{r^2}}{\sqrt{1-\dfrac{2GM}{r}}}} \cong \frac{\dfrac{\dfrac{GMmr}{r^3\left(1+\dfrac{2GM}{r}\right)^3}}{\sqrt{1+\dfrac{2GM}{r}}}}{\dfrac{\dfrac{GMm}{r^2}}{\sqrt{1-\dfrac{2GM}{r}}}} \cong \frac{1-\dfrac{GM}{r}}{\left(1+\dfrac{6GM}{r}\right)\left(1+\dfrac{GM}{r}\right)} \cong 1-\frac{8GM}{r} \quad (55)$$

In particular, in this case the correct expression of the gravitational force would be approximately equal to:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{r}\right) \quad (56)$$

And the incorrect expression of the gravitational force would be approximately equal to:

$$F_{incorr}(r,m) \cong \frac{GMm}{r^2}\left(1+\frac{GM}{r}\right) \quad (57)$$

The formulas (56) and (57) are expressed in the case with the velocity of light $c \equiv 1$. Of course, in the case more general (in which c is not defined equal to 1) we have that the (56) and the (57) become respectively:

$$F_{corr}(r,m) \cong \frac{GMm}{r^2}\left(1-\frac{7GM}{rc^2}\right) \quad (58)$$

$$F_{incorr}(r,m) \cong \frac{GMm}{r^2}\left(1+\frac{GM}{rc^2}\right) \quad (59)$$

Although no gravitational force measurements have been used to test the General Theory of Relativity so far, one might consider doing so by sending probes close to the Sun where the difference between the (58) and the (59) and the differences of the (58) and of the (59) with respect to the Newtonian gravitational force are larger, and the GM product of the Sun's mass is known with a smaller relative uncertainty than the GM product of the Earth's mass [23]. In fact, the relative error on GM product of the Sun's mass is approximately equal to 1 on 1.33×10^{10} [23], while the value of $\frac{GM}{rc^2}$ on the surface of the Sun is approximatively equal to 2.12×10^{-6}. For which, according to the right sides of the formulas (58) and (59), it is possible to measure the contribution due to the corrections made to the Newtonian gravitational force both in the case of the commonly accepted theory and in the case of the same theory corrected according to this article, if we measure the distance from the centre of the Sun with sufficient accuracy (and, of course, we know the mass m quite precisely). Obviously, in order to do a crucial experiment between the commonly accepted theory and the same theory corrected according to this article we would also need to measure the gravitational forces with sufficient accuracy.

Moreover, the best way to carry out such a crucial experiment would be to perform a series of measurements of the gravitational force at different distances from the centre of the Sun, in order to also measure the trend of the gravitational force as a function of these distances.

Therefore, in theory, another crucial experiment could be done to discriminate between the commonly accepted theory and the same theory corrected according to this article by means of measurements of the gravitational force near the Sun.

6. General Conclusions

In this article we started from our correction to the traditional Schwarzschild metric [1], which correction has been performed by assuming that the General Theory of Relativity is valid, differently from other articles [24]-[26] that deal with the same topics of this article and that instead start from proposals to change the General Theory of Relativity.

In this way we have obtained new expressions for the gravitational potential and the gravitational force in the case described by the Schwarzschild metric.

As we have seen, these new expressions do not entail any event horizon and, consequently, any black hole.

Therefore, this article confutes all the physics that on the basis of the Schwarzschild solution foresees the possibility of the existence of event horizons and black holes [1]-[3] [5] [27].

Moreover, we have noted that these new expressions are in accordance with the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [1] [5]-[10], which symmetry excludes the possibility of event horizons, and therefore of black holes, in general [1].

On the other hand, we have seen that there is no available experimental data that can discriminate between the commonly accepted theory and the same theory corrected according to this article.

However, we have noted that, in theory, is possible to perform in the solar system some crucial experiments, that discriminate between the commonly accepted theory and the same theory corrected according to this article. Therefore, it would be appropriate to try to make one of such crucial experiments.

Finally, according to this article, all the physics that is based on the incorrect Schwarzschild solution, in particular on the incorrect expressions of the gravitational potential and of the gravitational force, should be modified on the basis of the correct formulas that we have calculated.

Obviously, the introduction of such new formulas for the gravitational potential and the gravitational force in the case described by the Schwarzschild metric can have many applications both in the gravitational physics and in the analysis of astronomical data.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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