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The Gravitational Constant as the Function of the Cosmic Scale

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Abstract

This paper uses the cosmic evolution picture constructed by the principal and associated fiber bundles and, with the help of gauge invariance, systematically proposes the γ factor theory that the Newton's law of universal gravitation and the cosmological constant of Einstein's equation must be corrected in the large-scale space-time structure of the universe. That is, it is found that the calculated value of Newton's universal gravitation in space-time above the scale of galaxies must be multiplied by $1/\gamma$ to be consistent with the measured value, and the cosmological constant of Einstein's equation is no longer a constant but a function that increases with the increase of the scale of cosmic regions. Therefore, the cyclic hypothesis of cosmic evolution is proposed, and it is further found that the gravitational constant that people think is natural is not a constant but a function that changes with the scale of cosmic regions. Therefore, the reason for the dark matter and dark energy hypothesis may be that the gravitational constant is a variable. The existence of actual dark matter and dark energy may be just an illusory hypothesis, and their origin comes from the understanding that the gravitational constant is constant.

Keywords

The Gravitational Constant, The Einstein Equation, The Evolution of Universe

1. Introduction

In the previous exploration of the fundamental interactions of the large-scale universe, we have discovered the existence of a γ factor that affects Newton's universal gravitation or the cosmic constant Einstein's equations through the invariance of the gauge transformation of the principal and associated fiber bundles structure of the universe. It makes the calculated value of Newton's universal gravitation in space-time larger than the galaxy scale multiplied by $1/\gamma$ to match the measured value, and makes the cosmological constant of Einstein's equations no longer a constant but a function that increases with the increase of the scale of cosmic regions. The γ factor can be used to explain that the so-called dark matter or dark energy phenomenon is actually caused by the deviation caused by the fact that Newton's law of universal gravitation or Einstein's equations are no longer completely valid in space-time scales larger than or equal to galaxies [1]. Dark matter and dark energy do not actually exist. The reason why the dark matter and dark energy hypothesis was proposed began because humans did not understand the role of the γ factor [2]-[12].

But what is the physical meaning of the γ factor? The author of this article can first briefly answer this question: it reflects that the gravitational constant is not a fixed value but a variable! The gravitational constant is actually a function of the scale of cosmic regions! If we have such a conceptual shift, then the mystery of dark matter and dark energy can be easily explained.

The specific reasoning and analysis of the physical significance of the γ factor and the conclusion that it leads to a non-constant gravitational constant is precisely the task of this article. Below, the author will elaborate on the description.

2. The Gauge Transformation Corresponding to γ Factor

In references [1] [13] [14], the author proposed a differential geometric representation of the cosmological landscape of the principal and associated bundles. A connection on the principal bundle $P(M,G)$ is a 1-form field $\boldsymbol{\omega}_U$ that specifies a Lie algebra $\mathcal{G}$ value of C^∞ on U for a local trivial $T_U:\pi^{-1}[U]\to U\times G$, that is, a connection on $U\subset M$. In this case, if $T_V:\pi^{-1}[V]\to V\times G$ is another local trivial, that is, $U\cap V\neq\varnothing$, and the transition function from the local trivial T_U to T_V is g_{UV}, then the generalized Gauge equation holds [15],

$$\boldsymbol{\omega}_V(Y)=\mathcal{A}d_{g_{UV}(x)^{-1}}\boldsymbol{\omega}_U(Y)+L^{-1}_{g_{UV}(x)*}g_{UV*}(Y),\ \ \forall x\in U\cap V, Y\in T_xM \tag{1}$$

where $L^{-1}_{g_{UV}(x)}$ is the inverse mapping of the left translation $L_{g_{UV}(x)}$ generated by $g_{UV}(x)\in G$, and $L^{-1}_{g_{UV}(x)*}\equiv\left(L^{-1}_{g_{UV}(x)}\right)_*$.

Furthermore, the principal bundle structure is composed of the structure group, the principal bundle manifold, and the bottom manifold has a principal bundle section as the gauge potential, a curvature as the gauge field strength, and satisfies the curvature gauge transformation relation between two different regions U and V. For example, in general, if $g_{UV}:U\cap V\to G$ is a local trivial transition function from T_U to T_V, $\boldsymbol{\Omega}_V$ and $\boldsymbol{\Omega}_U$ are the curvatures of the bottom manifold in the V region or the U region, respectively, then on the bottom manifold $U\cap V$ has:

$$\boldsymbol{\Omega}_V=\mathcal{A}d_{g_{UV}^{-1}}\boldsymbol{\Omega}_U \tag{2}$$

where $\mathcal{A}d_{g_{UV}^{-1}}$ is defined as a linear transformation on the Lie algebra of the structure group G. If the structure group is a matrix group, then (2) can be further expressed as:

$$\boldsymbol{\Omega}_V = g_{UV}^{-1}\boldsymbol{\Omega}_U g_{UV} \tag{3}$$

In fact, from Equations (2) and (3), we have:

$$\boldsymbol{\Omega}_V = \mathcal{A}d_{g_{UV}^{-1}}\boldsymbol{\Omega}_U = g_{UV}^{-1}\boldsymbol{\Omega}_U g_{UV} \tag{4}$$

To prove the above formula, we only need to prove that the tangent vectors on both sides of the above formula at point e (corresponding to the unit element e of the Lie group at $t=0$) in the tangent space (*i.e.* Lie algebra) are the same. To this goal, we can calculate the following formula to prove it (although more detailed proof can be found in ref. [1]):

$$\begin{aligned}\text{Right tangent vector} &= \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\left[g_{UV}^{-1}\boldsymbol{\Omega}_U g_{UV}\right] = \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\left[I_{g_{UV}^{-1}}\boldsymbol{\Omega}_U\right] \\ &= I_{g_{UV}^{-1}*}\left[\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\boldsymbol{\Omega}_U\right] = \mathcal{A}d_{g_{UV}^{-1}}\boldsymbol{\Omega}_U = \text{Left tangent vector}\end{aligned} \tag{5}$$

Therefore, under the cross-sectional transformation σ, on the bottom manifold (that is, the world we assume, with $U \subset V$, for example, U represents the solar system region, and V represents the region greater than or equal to the galaxy), having $\boldsymbol{\omega}_U \to \boldsymbol{\omega}_V$, that is, Equation (1) holds, and here g_{UV}^{-1} and g_{UV} are related to a gauge transformation.

Furthermore, it can be proved that Equation (3) is equivalent to the following similarity transformation Equation (6) [13] [14]:

$$\boldsymbol{\Omega}_V = g_{UV}^{-1}\boldsymbol{\Omega}_U g_{UV} \Leftrightarrow \hat{F}'_{\mu\nu} = W\hat{F}_{\mu\nu}W^{-1} \tag{6}$$

where $\hat{F}'_{\mu\nu}$ is defined as the Lie algebraic matrix representation of the gravitational gauge field strength in region V on the bottom manifold; $\hat{F}_{\mu\nu}$ is defined as the Lie algebraic matrix representation of the gravitational gauge field strength in region U on the bottom manifold. W^{-1} and W can be regarded as the matrix expression of a certain gauge transformation, which can be called a gauge similarity transformation.

The author should emphasize that this gauge similarity transformation is affected by the regional scale, or depends on the change of regional scale. The scale of different regions U or V determines different gauge potentials or field strengths. The generalized gauge equation and the above gauge similarity transformation are the transformation relationship between the gauge potentials or field strengths of two different regions at the same space-time point of the bottom manifold (our universe). Not only that, after repeated consideration, the author also believes that: if the structure group is taken as the general linear group ($GL(m,\mathbb{C})$), then the generalized gauge transformation between the basic interactions corresponds to the gauge transformation between several large subgroups (for example, $U(1)$, $SU(2)$, $SU(3)$, $O(1,3)$), while the gauge transformation within a basic interaction (such as electromagnetic force) only corresponds to the gauge transformation within a "large" subgroup. For example, the gauge transformation within the electromagnetic interaction or weak interaction only involves the

$U(1)$ or $SU(2)$ group [16] [17], which are only subgroups of $GL(m,\mathbb{C})$. These components are just a kind of "representation" of the original connection or curvature on the principal bundle in the real world (bottom manifold); in this sense, "quantization" or "classicization" is just a natural "representation" expression of the original connection or curvature selection in different regions of the real world, and it is not that they can replace each other, or that any physical field can be or needs to be quantized, especially gravity. It should be emphasized that the original connection (gauge potential) or curvature (gauge field strength) on the principal bundle is gauge invariant, and the gauge potential or field strength on the bottom manifold is just the projection component mapped (gauge selection) by the original connection or curvature of the principal bundle choosing different cross sections. This is the physical meaning of the unification of the basic interactions in the world [1] [13] [14].

Based on the above assumption, this article believes that dark matter and dark energy are hypothetical hypotheses generated by the differences in gauge transformations of gravitational gauge potentials or gauge field strengths between different regional scales on the bottom manifold. There is no dark matter or dark energy, only changes in the gauge potential or field strength on the bottom manifold caused by gauge transformations. The original Einstein equations, Newton's second law or the law of universal gravitation may undergo changes in form under such transformations, and may differ from their original form. There exists a difference in the γ factor, which leads to the hypothesis of the existence of dark matter or dark energy [1].

In fact, the derivation of the γ factor can be achieved by constructing gauge transformations as follows:

$$\begin{array}{ccc} \hat{F}'_{\mu\nu} & = & W\hat{F}_{\mu\nu}W^{-1} \\ \hat{F}'_{\mu\nu} \equiv \text{gauge field strength in } V & \Leftrightarrow & \hat{F}_{\mu\nu} \equiv \text{gauge field strength in } U \\ & \downarrow\downarrow & \\ & \hat{F}'_{\mu\nu} = \gamma\left(\dfrac{|a|}{a_0}\right)\hat{F}_{\mu\nu} & \end{array} \tag{7}$$

where, $\gamma\left(\frac{|a|}{a_0}\right)$ can be defined as a function of $\frac{|a|}{a_0}$, which the author suggests is called the gauge similarity transformation factor. $|a|$ is the absolute value of the acceleration related to the region, which can be defined as the acceleration of the particle relative to a basic frame determined by distant matter in the universe. a_0 is an acceleration constant related to our galaxy. Experimental findings [18] show that it is approximately $a_0 \sim cH_0 = 2\times10^{-8}\ \text{cm}\cdot\text{s}^{-2} \sim 5\times10^{-8}\ \text{cm}\cdot\text{s}^{-2}$. Surprisingly, the value of a_0 of 2×10^{-8} cm·s^{-2} is very consistent with the measured value of the acceleration of the solar system in the gravitational field of the Milky Way $(2.32 \pm 0.16) \times 10^{-8}$ cm·s^{-2} [19], indicating that the regional scale corresponding to a_0 is the scale of our solar system. Therefore, the solar system can be used as a reference system to determine the scope of application of relativity and Newtonian mechanics. Generally speaking, as the regional scale corresponding to a_0 increases, it should be more difficult for the overall regional matter to change its acceleration $|a|$ under the gravitational field of the galaxy. So, on average, if the acceleration a_0 is taken as the benchmark, the larger the galaxy scale, the smaller its overall acceleration $|a|$ should be. For example, if the regional scale of the solar system corresponding to a_0 is U_{sun} and the galaxy is V_{uni}, there should be an inverse relationship,

$$\frac{|a|}{a_0} \sim \frac{U_{sun}}{V_{uni}} \tag{8}$$

Obviously, when the galaxy scale V_{uni} approaches the universe scale, the above formula tends to 0. If the galaxy scale shrinks to the U_{sun} scale, the above formula is 1. When the galaxy scale is very smaller than the reference system scale, the above formula tends to ∞. The corresponding acceleration shows that when $|a| \ll a_0$, $\gamma\left(\frac{|a|}{a_0}\right) \to \frac{|a|}{a_0} \to 0$, and when $|a| = a_0$, $\gamma\left(\frac{|a|}{a_0}\right) \to \gamma(1)$, and when $|a| \gg a_0$, $\gamma\left(\frac{|a|}{a_0}\right) \to 1$, so the gauge similarity transformation factor γ varies between 0 and 1. In order to reflect this law, we speculatively choose the $\gamma\left(\frac{|a|}{a_0}\right) \equiv th\left(\frac{|a|}{a_0}\right)$ function to "simulate" this change. However, this does not take into account the fact that Einstein's law of gravity and Newton's second law or the law of universal gravitation is valid within the scale of the solar system. Therefore, the author generally believes that when $|a| \ge a_0$, $th\left(\beta\frac{|a|}{a_0}\right) \to 1$ should hold. At this time, the reference system should include the scale of our solar system, and a suitable coefficient β should be introduced to make $|a| \ge a_0$,

$$th\left(\beta\frac{|a|}{a_0}\right) \to 1 \tag{9}$$

An important question now is whether the guage transformation corresponding to the γ coefficient constructed in this way really exists, that is, the transformation formula,

$$\hat{F}'_{\mu\nu} = W\hat{F}_{\mu\nu}W^{-1} = \gamma\left(\frac{|a|}{a_0}\right)\hat{F}_{\mu\nu} \tag{10}$$

Do the guage transformations (matrices or operators) W and W^{-1} in the equation exist reasonably? To this end, by multiplying both sides of Equation (10) by W, one can obtain:

$$W\hat{F}_{\mu\nu} = \gamma\left(\frac{|a|}{a_0}\right)\hat{F}_{\mu\nu}W \tag{11}$$

Which can deduce:

$$\left[W,\hat{F}_{\mu\nu}\right]=W\hat{F}_{\mu\nu}-\hat{F}_{\mu\nu}W=\gamma\left(\frac{|a|}{a_0}\right)\hat{F}_{\mu\nu}W-\hat{F}_{\mu\nu}W=\left(\gamma\left(\frac{|a|}{a_0}\right)-1\right)\hat{F}_{\mu\nu}W \tag{12}$$

Then, except for the limit point $\gamma\left(\frac{|a|}{a_0}\right)=1$, $\left(\gamma\left(\frac{|a|}{a_0}\right)-1\right)\neq 0$, and in general $\hat{F}_{\mu\nu}W\neq 0$, because if it is 0, then $W\hat{F}_{\mu\nu}=0$ is obtained from Equation (11), and $\left[W,\hat{F}_{\mu\nu}\right]$ is meaningless under this condition. Therefore, it can be deduced that Equation (12) is not commutative under the condition of Equation (11):

$$\left[W,\hat{F}_{\mu\nu}\right]\neq 0 \tag{13}$$

This is very logical, because W has the properties of an operator and $\hat{F}_{\mu\nu}$ belongs to the gravitational field strength. If they are commutative, it means that the gravitational field strength given by the gauge similarity transformation is unchanged. This is possible in the solar system, but on a space-time scale greater than or equal to the galaxy, we now know that this is usually not true, otherwise, there would be no hypothesis of dark matter.

Furthermore, using Equation (10), $\hat{F}'_{\mu\nu}=W\hat{F}_{\mu\nu}W^{-1}=\gamma\hat{F}_{\mu\nu}$, we can reversely deduce the matrix representation of the gauge transformation (conversion function) as follows:

First, consider that there are many matrices expressing the gravitational strength that is diagonal matrix of the 2nd-order metric tensor [1] [13] [14], that is,

$$\left\{\hat{F}_{\mu\nu}\right\}=\left\{g_{jj}\right\}\equiv\begin{pmatrix} g_{00} & 0 & 0 & 0\\ 0 & g_{11} & 0 & 0\\ 0 & 0 & g_{22} & 0\\ 0 & 0 & 0 & g_{33}\end{pmatrix} \tag{14}$$

and the matrix expression of the gauge transformed gravitational strength is:

$$\left\{\hat{F}'_{\mu\nu}\right\}\equiv\gamma\begin{pmatrix} g_{00} & 0 & 0 & 0\\ 0 & g_{11} & 0 & 0\\ 0 & 0 & g_{22} & 0\\ 0 & 0 & 0 & g_{33}\end{pmatrix} \tag{15}$$

This means that the gravitational strength is a diagonal matrix expression of the second-order metric tensor corresponding to a diagonal matrix $\left\{g_{jj}\right\}$ of $\left\{\hat{F}_{\mu\nu}(x)\right\}$. So we can always find such a matrix similarity transformation $\{W\}$, which transforms the diagonalization matrix of $\hat{F}_{\mu\nu}(x)$ into $\gamma\left\{g_{jj}\right\}$, and the similarity transformation expression is deduced as follows:

$$\begin{gathered}\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03}\\ w_{10} & w_{11} & w_{12} & w_{13}\\ w_{20} & w_{21} & w_{22} & w_{23}\\ w_{30} & w_{31} & w_{32} & w_{33}\end{pmatrix}\begin{pmatrix} g_{00} & 0 & 0 & 0\\ 0 & g_{11} & 0 & 0\\ 0 & 0 & g_{22} & 0\\ 0 & 0 & 0 & g_{33}\end{pmatrix}\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03}\\ w_{10} & w_{11} & w_{12} & w_{13}\\ w_{20} & w_{21} & w_{22} & w_{23}\\ w_{30} & w_{31} & w_{32} & w_{33}\end{pmatrix}^{-1}\\ =\gamma\begin{pmatrix} g_{00} & 0 & 0 & 0\\ 0 & g_{11} & 0 & 0\\ 0 & 0 & g_{22} & 0\\ 0 & 0 & 0 & g_{33}\end{pmatrix}\end{gathered} \tag{16}$$

where defining

$$\{W\} \equiv \begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{pmatrix} = \begin{pmatrix} W_0 & W_1 & W_2 & W_3 \end{pmatrix} \quad (17)$$

Then, one can get:

$$\begin{pmatrix} w_{00}g_{00} & w_{01}g_{11} & w_{02}g_{22} & w_{03}g_{33} \\ w_{10}g_{00} & w_{11}g_{11} & w_{12}g_{22} & w_{13}g_{33} \\ w_{20}g_{00} & w_{21}g_{11} & w_{22}g_{22} & w_{23}g_{33} \\ w_{30}g_{00} & w_{31}g_{11} & w_{32}g_{22} & w_{33}g_{33} \end{pmatrix} = \gamma \begin{pmatrix} g_{00}w_{00} & g_{00}w_{01} & g_{00}w_{02} & g_{00}w_{03} \\ g_{11}w_{10} & g_{11}w_{11} & g_{11}w_{12} & g_{11}w_{13} \\ g_{22}w_{20} & g_{22}w_{21} & g_{22}w_{22} & g_{22}w_{23} \\ g_{33}w_{30} & g_{33}w_{31} & g_{33}w_{32} & g_{33}w_{33} \end{pmatrix} \quad (18)$$

That is:

$$\{\hat{F}_{\mu\nu}\}\begin{pmatrix} W_0 & W_1 & W_2 & W_3 \end{pmatrix} = \begin{pmatrix} g_{00}W_0 & g_{11}W_1 & g_{22}W_2 & g_{33}W_3 \end{pmatrix} \quad (19)$$

The above formula can be simplified as:

$$\{\hat{F}_{\mu\nu}\}W_i = g_{ii}W_i,\ i = 0,1,2,3 \quad (20)$$

Now, we can associate $\{\hat{F}_{\mu\nu}\}$ with the eigenvalues and eigenvectors of its matrix, *i.e.* g_{ii} is the *i*-th eigenvalue of the matrix $\{\hat{F}_{\mu\nu}\}$, and W_i is the eigenvector corresponding to g_{ii}. Because $\{W\}$ must be a reversible matrix, the eigenvectors of $\{\hat{F}_{\mu\nu}\}$ need to be linearly independent, that is, the necessary and sufficient condition for the 4-order square matrix $\{\hat{F}_{\mu\nu}\}$ to be similar to the diagonal matrix $\{g_{jj}\}$ is that the eigenvectors W_0, W_1, W_2 and W_3 of $\{\hat{F}_{\mu\nu}\}$ are linearly independent of each other. Of course, under the condition of the Equation (16) this is satisfied, hence the linearly independent eigenvectors W_0, W_1, W_2 and W_3 of $\{\hat{F}_{\mu\nu}\}$ can be constructed. For example, from Equation (18), by selecting:

1) $w_{01} = 1 = w_{02} = w_{03}$, one can obtain:

$$w_{00} = \frac{-g_{11} - g_{22} - g_{33} + 3\gamma g_{00}}{(1-\gamma)g_{00}} \quad (21)$$

2) $w_{10} = 1 = w_{12} = w_{13}$, one can obtain:

$$w_{11} = \frac{-g_{00} - g_{22} - g_{33} + 3\gamma g_{11}}{(1-\gamma)g_{11}} \quad (22)$$

3) $w_{20} = 1 = w_{21} = w_{23}$, one can get:

$$w_{22} = \frac{-g_{00} - g_{11} - g_{33} + 3\gamma g_{22}}{(1-\gamma)g_{22}} \quad (23)$$

4) $w_{30} = 1 = w_{31} = w_{32}$, one can get:

$$w_{33} = \frac{-g_{00} - g_{11} - g_{22} + 3\gamma g_{33}}{(1-\gamma)g_{33}} \quad (24)$$

Finally, one can arrive at:

$$W_0 = \begin{pmatrix} \dfrac{-g_{11} - g_{22} - g_{33} + 3\gamma g_{00}}{(1-\gamma) g_{00}} \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{25}$$

$$W_1 = \begin{pmatrix} 1 \\ \dfrac{-g_{00} - g_{22} - g_{33} + 3\gamma g_{11}}{(1-\gamma) g_{11}} \\ 1 \\ 1 \end{pmatrix} \tag{26}$$

$$W_2 = \begin{pmatrix} 1 \\ 1 \\ \dfrac{-g_{00} - g_{11} - g_{33} + 3\gamma g_{22}}{(1-\gamma) g_{22}} \\ 1 \end{pmatrix} \tag{27}$$

$$W_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \dfrac{-g_{00} - g_{11} - g_{22} + 3\gamma g_{33}}{(1-\gamma) g_{33}} \end{pmatrix} \tag{28}$$

The above results allow us to obtain a 4th-order gauge transformation matrix expression: $\{W\} = (W_0\ W_1\ W_2\ W_3)$, and this expression is non-degenerate, so it is reasonable, indicating that this gauge transformation that extends with the scale of the universe can reasonably exist, and is actually determined or selected by the intrinsic properties and structure of the principal and associated bundles of the universe. The γ factor derived from this gauge transformation invariance is actually a function of the space-time scale. If the solar system is defined as a standard unit space-time scale, then γ decreases as the space-time scale increases relative to the scale of the universal region in units of this standard scale, which can be simulated by the hyperbolic tangent, that is, the gauge similarity transformation equation can now be expressed as:

$$\hat{F}'_{\mu\nu} = W\hat{F}_{\mu\nu}W^{-1} = th\left(\frac{|a|}{a_0}\right)\hat{F}_{\mu\nu} \tag{29}$$

Here, the region U where $\hat{F}_{\mu\nu}$ is located is smaller or much smaller than the region V where $\hat{F}'_{\mu\nu}$ is located, or physically speaking, U ~ Solar System, V ≥ "Galaxy". Therefore, it can be understood that the invariance of the gauge transformation caused by the choice of this kind of internal gauge in the universe finally determines γ, which is essentially determined by the internal space-time structure of the principal and associated fiber bundles of the universe. The most powerful evidence supporting this "theory of the determination of the intrinsic properties of the universe" is that the universe is indeed observed to be expanding at an accelerated rate in experiments, from which it can be obtained that a γ factor does have an impact on Newton's gravitational formula [18] [19].

3. γ Factor Influence on the Formula of Gravity

In order to ensure that Einstein's law of gravitational attraction remains unchanged and Newton's law of universal gravitation (and second law) remains unchanged within the scale of the solar system, the gauge similarity transformation should be given by:

$$\hat{F}'_{\mu\nu} = W\hat{F}_{\mu\nu}W^{-1} = \hat{F}_{\mu\nu} \tag{30}$$

In the case of a galaxy greater than or equal to galaxies, for example, $|a| < a_0$, then $0 < \gamma\left(\frac{|a|}{a_0}\right) < 1$, and the gauge transformation above changes to:

$$\hat{F}'_{\mu\nu} = \gamma\left(\frac{|a|}{a_0}\right)\hat{F}_{\mu\nu} < \hat{F}_{\mu\nu} \tag{31}$$

Thus, Newton's second law from the perspective of our solar system becomes:

$$m\gamma\left(\frac{|a|}{a_0}\right)a < ma = F \tag{32}$$

Newton's second law needs to be revised, which is consistent with MOND's view [18] [20]-[22].

In fact, from the point of view that the gauge field strength is the curvature of the manifold at the bottom of the principal fiber bundle, considering the linear Einstein equations under the Newtonian approximation, we can have [23] [24]:

$$m\frac{\mathrm{d}^2x^i}{\left(\mathrm{d}x^0\right)^2} = -m\Gamma^i{}_{00} \tag{33}$$

where, $\Gamma^i{}_{00}$ is the Christoffel symbol. According to Newton's approximation, $g_{ab} = \eta_{ab} + h_{ab}$, etc., we can get:

$$m\frac{\mathrm{d}^2x^i}{\left(\mathrm{d}x^0\right)^2} = -m\frac{1}{2}h_{00,i} \tag{34}$$

and considering in the classical gravitational field, Newton's second law can be written as:

$$m_I\frac{\mathrm{d}^2x^i}{\mathrm{d}t^2} = m_g\left(-\frac{\partial\varphi}{\partial x^i}\right) \tag{35}$$

where φ is the Newtonian gravitational potential. Comparing the above two equations, we get:

$$\varphi = \frac{1}{2}h_{00} + \text{const} \tag{36}$$

Assume that the gravitational field at infinity disappears as $\varphi = 0$, then the metric should return to the Minkowski metric, $h_{00} = 0$, so the constant in the above formula is 0, which allows one to obtain:

$$g_{00} = -1 + h_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) \tag{37}$$

However, considering the gauge transformation and Newton approximation:

$$\hat{F}_{\mu\nu} \to \gamma \hat{F}_{\mu\nu} \to \gamma \hat{F}_{\mu\nu} \to \gamma \Gamma^{i}{}_{00} \to \gamma g_{00} \to \gamma h_{00} \to \gamma \varphi \tag{38}$$

in the classical gravitational field above, Newton's second law becomes:

$$m\left(-\frac{\partial \gamma \varphi}{\partial x^i}\right) = m\gamma a < ma = F \tag{39}$$

This once again shows that the difference in gravity is reflected in the γ factor, which is the origin of the dark matter hypothesis.

In addition, if the scale of cosmic space-time region V_{uni} $\gg$ "reference system scale, or $\gg$ galaxy scale", then $\frac{|a|}{a_0} \ll 1$, $th\left(\beta \frac{|a|}{a_0}\right) \to 0$, its physical meaning is that the gravitational gauge field strength is $\hat{F}'_{\mu\nu} = th\left(\beta \frac{|a|}{a_0}\right) \hat{F}_{\mu\nu} \to 0$, which corresponds to the gravitational gauge field strength of the manifold region on the cosmic scale is 0. Equation (39) means that Newton's second law becomes:

$$\lim_{\frac{|a|}{a_0} \to 0} \left[mth\left(\beta \frac{|a|}{a_0}\right) a \right] \to 0 \tag{40}$$

This means that the curvature of the entire region of the universe is zero, and the attraction disappears. The author speculates that at this scale, the universe may be driven by the energy conservation law of the initial kinetic energy of the Big Bang and the disappearance of the attraction, and it may also produce repulsive force. This part of the conversion energy of the disappearance of gravity plus the initial kinetic energy left by the original Big Bang may be the origin of the so-called dark energy [1].

The connection or curvature of large-scale spacetime can certainly affect the dynamical equations. In fact, the physical essence reflected by the gauge similarity transformation Formula (4) can be realized by introducing a factor that varies with the space-time scale into the cosmological term of the Einstein equations, for example:

$$G_{ab} + \Lambda\left(\frac{|a|}{a_0}\right) g_{ab} = 8\pi T_{ab} \tag{41}$$

where G_{ab} is the Einstein tensor, g_{ab} is the metric tensor, T_{ab} is the energy-momentum tensor, Λ was originally the cosmological constant, and is now related to $\gamma\left(\frac{|a|}{a_0}\right)$, that is,

$$-\Lambda g_{ab} = 8\pi \left(\frac{1-\gamma}{\gamma}\right) T_{ab} \tag{42}$$

For a detailed proof, please refer to [1]. In fact, according to the similarity gauge transformation and Einstein equation, let $\gamma = th\left(\beta\frac{|a|}{a_0}\right)$, we have:

$$R_{ab} - \frac{1}{2}Rg_{ab} = \frac{8\pi}{\gamma}T_{ab} \tag{43}$$

Further, consider the Einstein equation with the cosmological constant Λ:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi T_{ab} \tag{44}$$

can be transformed into:

$$R_{ab} = 8\pi\left(T_{ab} - \frac{1}{2}Tg_{ab}\right) + \Lambda g_{ab} \tag{45}$$

with

$$R = -8\pi T + 4\Lambda \tag{46}$$

then by substituting three Equations (44)-(46) into Equation (43), one can obtain:

$$\frac{8\pi}{\gamma}T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab} - \Lambda g_{ab} \tag{47}$$

So, we have Equation (42), and under this condition, Equation (43) and Einstein's equation with cosmological constant (41) can be derived from each other.

The gauge similarity transformation $R_{ab} \to \gamma R_{ab}$ (note that according to the principal bundle theory, curvature is equivalent to field strength) corresponding to condition (42) is equivalent to adding the cosmological constant Λ to the Einstein equations. This shows that in a sense, adding a certain cosmological constant to the Einstein equations is equivalent to applying a certain gauge transformation to its curvature. The cosmological constant is no longer a constant and is related to the coefficient γ of the gauge field strength after the gauge similarity transformation. This is the physical meaning of the cosmological constant. Moreover, Λ is affected by $\gamma\left(\frac{|a|}{a_0}\right)$, and Λ is also a function of the space-time scale, that is, it becomes very large as the space-time scale increases, which naturally solves the "cosmological constant problem of physicists or astronomers", that is, the cosmological constant increases to a very large extent as the universal region scale increases [15]; when the space-time scale tends to the universe scale, $\gamma\left(\frac{|a|}{a_0}\right) \to \frac{|a|}{a_0} \to 0$, $|\Lambda| \to \infty$, the gauge gravitational field strength is 0, and the Einstein equation enters the chaotic region; when the space-time scale is smaller than the solar system scale, $\gamma\left(\frac{|a|}{a_0}\right) \to 1$, $\Lambda \to 0$, the gravitational field strength remains unchanged after the gauge transformation, the original Einstein equation is held, and Newton's second law is held. The conditions that must be satisfied for the specific change of Λ are determined by Equation (42).

4. γ Influence to Cosmological or Gravitational Constant

In short, from the above analysis of the cosmological constant of the Einstein equation, the author found that since the cosmological constant now changes with the factor $\gamma\left(\frac{|a|}{a_0}\right)$ and is no longer a constant, but a function that changes with the scale of the cosmic region, or is related to the regional acceleration ratio $\frac{|a|}{a_0}$, the Einstein equation will change with the size of the cosmic region and present a different form, so that the Newton's law or law of universal gravitation observed at the same point in the bottom manifold (that is, our universe) in the U region may not be the same as the Newton's law or law of universal gravitation observed at the same point in the V region. The transformation between them satisfies the similar gauge transformation (or generalized gauge equation); and it is the difference between both that creates the fictitious origin of dark matter and dark energy.

In fact, from Equation (41), the Einstein equation containing the cosmological term becomes:

$$R_{ab}-\frac{1}{2}Rg_{ab}=8\pi T_{ab}-\Lambda g_{ab}=8\pi\frac{T_{ab}}{\gamma\left(\frac{|a|}{a_0}\right)} \tag{48}$$

that is, it can be abbreviated as:

$$G_{ab}=8\pi\frac{T_{ab}}{\gamma\left(\frac{|a|}{a_0}\right)} \tag{49}$$

Here, G_{ab} certainly is the Einstein tensor. So, the Einstein equations for the universe on a large scale are suggested to be given by:

$$G_{ab}=R_{ab}-\frac{1}{2}Rg_{ab}+\Lambda\left(\frac{|a|}{a_0}\right)g_{ab}=8\pi T_{ab} \tag{50}$$

Therefore, the evolution law of Einstein's equations in large-scale cosmic regions is affected by $\Lambda\left(\frac{|a|}{a_0}\right)$ or $\gamma\left(\frac{|a|}{a_0}\right)$ and is expressed as follows:

$$\begin{cases} G_{ab}=8\pi T_{ab},\ |a|\ll a_0,\ \gamma\to 1,\ \text{Einstein equation, Newton gravity valid region} \\ G_{ab}=8\pi\frac{T_{ab}}{\gamma},\ 0<\gamma<1,\ \text{Einstein, Newton gravity deviation region, vertual dark matter}\uparrow \\ -\Lambda g_{ab}=8\pi\left(\frac{1-\gamma}{\gamma}\right)T_{ab}\to\infty,\ |a|\gg a_0,\ \gamma\to 0,\ \text{Chaotic region of gravity}\downarrow\text{ or repulsion}\uparrow \end{cases} \tag{51}$$

In detail, the mechanism of the above picture at the two extreme points of curvature 0 and 1 can be explained as follows:

1) The 0 curvature interval is a fluctuation zone or a chaotic zone where gravity disappears completely or repulsion increases. At the 0 limit point, Einstein's

equations and Newton's gravitational law do not hold. The repulsion energy converted from the disappearance of gravity is the source of dark energy.

2) Whether the universe will continue to evolve depends on which part of the force disappears more. According to the principle of energy conservation, the author speculates that: from the first branch of the simulation function $\gamma = th\left(\beta \frac{|a|}{a_0}\right)$ to point 0, the gravitational force disappears more, the repulsive force will be generated more, and the universe may continue to evolve towards the second branch (see details in ref. [1]). On the contrary, it will rebound and shrink towards the first branch with a high probability. It is not impossible that the universe will repeatedly explode and shrink in the first branch with a certain probability to oscillate and evolve. The driving force of evolution ultimately comes from the principle of energy conservation. In the process of the first branch, universe curvature tending to 1, as $|a| \ll a_0$, the universe appears as a closed 4-dimensional spherical surface, and its space-time scale is gradually compressed until it becomes a singularity. Then the Big Bang begins a new curvature evolution from $1 \to 0$; at the limit point +1, Einstein's equation and Newton's law of universal gravitation are both valid. During the first branch: $0 < \gamma < 1$, Einstein's equation and Newton's law of universal gravitation are only "partially" valid compared with that in the solar system. The difference between the solar system and beyond galaxy on the Einstein equation and Newton's law of universal gravitation is just the source of the illusory dark matter. At this time, Newton's law of universal gravitation is corrected to: $F = G\frac{mM}{\gamma r^2}$, and the Einstein field equation is corrected to: $G_{ab} = 8\pi \frac{T_{ab}}{\gamma}$.

Obviously, γ makes Newton's gravitational constant no longer a constant. Can we say that γ is ultimately determined by dark matter or dark energy? This statement seems to be divorced from the γ factor, the gauge gravitational potential or the gauge field strength, which is essentially a reflection of the principal bundle connection or curvature. At present, it has not been truly measured whether dark matter and dark energy really exist, let alone that both dark matter and dark energy are some kinds of gauge field. In fact, the existence of γ can be simply explained by the fact that the gravitational constant is no longer a constant but a function of the evolution of the scale of the universal region. This function becomes smaller as the scale of the universal region increases in units of the solar system, that is,

$$F = \frac{\mathrm{G}}{\gamma}\frac{mM}{r^2} = \mathrm{G}'\frac{mM}{r^2} \to \mathrm{G}' \equiv \frac{\mathrm{G}}{\gamma} \tag{52}$$

Therefore, the above formula once again clearly draws the most important conclusion of this article: the gravitational constant $\mathrm{G} \to \mathrm{G}'$ is not a constant but a function that changes with the size of the universal region! The produced reason for the dark matter and dark energy hypothesis may be because the gravitational constant is a variable!

5. Conclusion and Outlook

Based on the cosmic evolution picture constructed by the author using the principal and associated fiber bundles, we systematically proposed the γ factor theory that the Newton's law of universal gravitation and the cosmological constant of Einstein's equation must be corrected in the large-scale space-time structure of the universe with the help of gauge invariance. From this, the cyclic hypothesis of cosmic evolution is proposed, and further finds that the gravitational constant that people think is natural is not a constant but a function of the scale of the universal region! Therefore, the produced reason for the dark matter and dark energy hypothesis may be that the gravitational constant is a variable. The existence of dark matter and dark energy may be just an illusory hypothesis, and their origin comes from the understanding that the gravitational constant is constant.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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