Journal of Mathematical Finance, 2026, 16(1), 1-17
https://www.scirp.org/journal/jmf
ISSN Online: 2162-2442
ISSN Print: 2162-2434

Beyond Closed-Form Liquidity Models: An AI-Enhanced Quantitative Approach

Marcello Forcellini

Department of Human Sciences, Link Campus University, Rome, Italy
Email: m.forcellini@unilink.it

How to cite this paper: Forcellini, M. (2026) Beyond Closed-Form Liquidity Models: An AI-Enhanced Quantitative Approach. *Journal of Mathematical Finance*, **16**, 1-17.
https://doi.org/10.4236/jmf.2026.161001

Received: December 20, 2025
Accepted: February 2, 2026
Published: February 5, 2026

Abstract

Liquidity spreads are a fundamental manifestation of trading frictions in financial markets, with important implications for asset pricing, risk management, and market stability. Classical theoretical models of liquidity typically rely on restrictive assumptions and closed-form specifications that limit their ability to capture nonlinear and adaptive liquidity dynamics observed in modern markets. This paper proposes a novel mathematical framework for liquidity spread estimation that integrates artificial intelligence within a rigorously defined analytical structure. Liquidity spreads are modeled as nonlinear functionals of observable market states, represented by operator-based mappings embedded in suitable function spaces. Artificial intelligence enters the framework through flexible parameterizations of these operators, allowing for endogenous learning of complex liquidity dynamics while preserving theoretical properties such as well-posedness, stability, and interpretability. The model is analyzed using tools from functional analysis and approximation theory, and sufficient conditions for existence, continuity, and robustness are established. By reconciling analytical finance theory with AI-inspired functional representations, the proposed framework extends classical liquidity models and provides a theoretically grounded foundation for future research on liquidity, pricing, and risk in high-dimensional financial markets.

Keywords

Liquidity Spread, Market Liquidity, Mathematical Finance, Functional Analysis, Artificial Intelligence, Operator-Based Models, Nonlinear Approximation

1. Introduction

Liquidity, the ease with which an asset can be traded without significantly affecting its price, is a central concept in financial economics. In financial markets, liquidity conditions directly shape transaction costs, market efficiency, and risk management practices. One key manifestation of liquidity conditions is the liquidity spread, commonly proxied by bid-ask spreads and other transaction cost measures. The liquidity spread reflects, not only trading costs, but also the underlying frictions in market participation, asymmetric information, and transaction costs. Understanding and accurately estimating liquidity spreads is crucial for pricing, hedging and risk control, especially, in environments characterized by rapid market changes and stress [1].

Classical theoretical models of liquidity and liquidity spreads tend to build on well-established frameworks in market microstructure and asset pricing. For example, foundational theories trace illiquidity to market imperfections such as transaction costs, asymmetric information, funding constraints and participation costs, showing how these frictions elevate expected returns and widen liquidity spreads under certain conditions. Traditional models often employ equilibrium or structural approaches to derive analytical expressions for liquidity measures, embedding assumptions about rational agents, homogeneous information, or linear dynamics that facilitate tractable solutions. Though such models have been valuable in establishing baseline insights, they frequently struggle to capture the complex, nonlinear behavior of liquidity dynamics observed in real markets [2]. Indeed, one typical limitation of classical analytical approaches is their reliance on assumptions of stationary relationships and linear responses to market conditions. While analytically convenient, these simplifications may obscure important dynamics such as regime shifts, nonlinear feedback loops, and interactions among multiple liquidity determinants (e.g. order flow, volatility, and information arrival). Moreover, classical models seldom incorporate adaptive learning or data-driven adjustment mechanisms, which are increasingly relevant given the high dimensionality and speed of modern market data. As markets have evolved, with electronic trading, high-frequency trading and algorithmic strategies becoming dominant, liquidity patterns exhibit features poorly explained by static or linear models alone. This gap highlights the need for frameworks that can encapsulate complex relationships among liquidity drivers without sacrificing mathematical rigor [3].

In response to these limitations, this paper proposes a novel theoretical framework that integrates artificial intelligence (AI) with mathematical finance for liquidity spread estimation. Specifically, a model has been developed in which liquidity spreads are represented as a functional of observable market state variables and where this functional form is learned via an AI-inspired structure embedded within a rigorous mathematical framework. Unlike purely empirical machine learning approaches that prioritize prediction accuracy without interpretability, the proposed model seeks to retain analytical properties, such as well-posedness, stability, and theoretical grounding, while benefiting from AI's ability to approximate complex, nonlinear mappings. This hybridization enables us to articulate a mathematical object that is both expressive enough to capture intricate liquidity patterns and sufficiently structured for formal analysis [2].

The contribution of this theoretical work is twofold. First, it advances the literature on liquidity modeling by embedding AI mechanisms into a mathematically tractable framework, thereby extending classical theories with adaptive functional structures. Second, it provides formal insights into how such models can be analyzed in terms of existence, uniqueness, and stability properties. In doing so, we offer a bridge between analytical finance theory and modern AI methods, with implications for future research on liquidity, pricing, and risk in high-dimensional market environments.

2. Preliminaries and Problem Setting

We consider a continuous-time financial market defined on a filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\geq 0}$, $\mathbb{P}$), where ($\mathcal{F}_t$) represents the information available to market participants up to time *t*. Trading occurs over a finite horizon $[0,T]$, and asset prices are adapted to the filtration, reflecting the standard no-anticipation condition. Market participants observe a vector of state variables $X_t \in \mathbb{R}^d$, which may include prices, trading volume, volatility proxies, order flow imbalance, or other liquidity-relevant indicators. This information structure is consistent with classical microstructure and asset-pricing frameworks [4].

Liquidity is modeled as an endogenous market characteristic arising from trading frictions. We define the liquidity spread L_t as a non-negative process representing the instantaneous cost of immediacy, encompassing bid-ask spreads, price impact, and execution costs. Formally, L_t is treated as a measurable functional of the underlying market state

$$L_t = \mathcal{L}(x_t)$$

where $\mathcal{L}$ is an unknown mapping that captures the joint effect of market conditions on liquidity. This abstract definition generalizes classical measures such as quoted spreads or effective spreads, allowing for a broader and more flexible representation of liquidity costs [5].

The central problem addressed in this paper is the theoretical estimation of the liquidity spread functional $\mathcal{L}$. Given the high dimensionality and potential nonlinearity of the state space, closed-form analytical expressions for $\mathcal{L}$ are generally unavailable or overly restrictive. We therefore frame the estimation problem as the identification of a functional $\mathcal{L}_\theta$ within a suitably chosen function class, parameterized by θ, such that

$$\mathcal{L}_\theta \approx \mathcal{L}$$

in an appropriate functional norm. Unlike traditional parametric approaches, the proposed framework allows $\mathcal{L}_\theta$ to be represented by an AI-inspired structure, while remaining embedded within a mathematically well-posed setting. This formulation lays the foundation for the theoretical development of AI-enhanced liquidity modeling in subsequent sections.

3. A Brief Literature Review: Classical Liquidity Spread Models

Classical liquidity spread models originate primarily from the market microstructure literature, where trading frictions and information asymmetries are central determinants of transaction costs. One of the earliest and most influential benchmark models is due to Kyle (1985), who introduces a framework in which liquidity emerges endogenously from the interaction between informed traders, noise traders, and a market maker. In this setting, the bid-ask spread and price impact reflect the adverse selection risk faced by liquidity providers. Although analytically elegant, the Kyle model assumes linear price impact and Gaussian information structures, which limit its ability to capture nonlinear liquidity dynamics [4].

Another foundational strand of the literature focuses on inventory-based models, where liquidity spreads arise from the risk borne by market makers who hold inventory. The seminal work of Amihud and Mendelson (1980 and 1986) links bid-ask spreads to asset pricing, demonstrating that illiquidity commands a return premium. These models provide closed-form expressions for spreads under simplifying assumptions regarding risk aversion, order arrival processes, and market equilibrium. While analytically tractable, they typically rely on static or quasi-static equilibria and do not account for rapid changes in market conditions [6].

More recent theoretical contributions embed liquidity into asset pricing and equilibrium models, treating liquidity as a state variable that affects expected returns and volatility [7].

These frameworks succeed in explaining liquidity risk premia and systemic liquidity effects, but liquidity spreads themselves are often modeled indirectly or as reduced-form quantities. As a result, these models provide limited guidance for capturing fine-grained, time-varying liquidity costs at high frequency. Despite their conceptual importance, according to the available literature, classical liquidity spread models share several structural and analytical limitations. First, they typically assume linear relationships between liquidity, order flow, and prices, which conflicts with empirical evidence of nonlinear and regime-dependent liquidity behavior. Second, the reliance on closed-form solutions necessitates restrictive assumptions about distributions, agent behavior, and market structure. Third, classical models generally lack adaptive mechanisms, treating liquidity as either exogenously specified or slowly evolving, rather than dynamically learned from data.

These limitations become particularly pronounced in modern electronic markets, where liquidity is shaped by high-dimensional interactions among heterogeneous agents and rapidly changing information flows. Consequently, while classical models provide valuable theoretical benchmarks, they struggle to accommodate the complexity and nonlinearity observed in contemporary liquidity dynamics. This motivates the development of alternative frameworks (such as AI-enhanced mathematical models) that preserve theoretical rigor while allowing for greater functional flexibility in liquidity spread estimation.

4. AI-Enhanced Mathematical Model

This section introduces an AI-enhanced mathematical framework for liquidity spread estimation that preserves analytical structure while allowing for flexible, nonlinear representations. The model is constructed as a functional mapping from observable market states to liquidity spreads, embedding artificial intelligence mechanisms within a rigorously defined mathematical setting.

4.1. Functional and Operator-Based Representation

Let $X_t \in \mathbb{R}^d$ denote the vector of observable market state variables at time *t*, adapted to the filtration $(\mathcal{F}_t)_{t\geq 0}$. These variables may include prices, returns, volatility measures, order flow, and other indicators associated with liquidity conditions. We model the liquidity spread L_t as a measurable functional of the state process,

$$L_t = \mathcal{L}(x_t)$$

where $\mathcal{L}: \mathbb{R}^d \to \mathbb{R}_+$ is an unknown nonlinear mapping. This formulation generalizes classical parametric specifications by allowing liquidity to depend on the joint configuration of multiple market variables rather than on a small set of linear predictors [2].

To facilitate theoretical analysis, we interpret *L* as an element of a suitable function space $\mathcal{H}$, such as a Banach or Hilbert space equipped with norm $\|\cdot\|_{\mathcal{H}}$. In this setting, the liquidity spread estimation problem becomes the identification of a functional operator that maps the state process into liquidity costs. This operator-based perspective aligns with recent developments in functional approximation theory and operator learning, where complex economic relationships are modeled as mappings between infinite-dimensional spaces [8].

4.2. Integration of Artificial Intelligence into the Model Structure

Artificial intelligence enters the framework through a parameterized approximation of the liquidity functional. Specifically, we consider a family of operators $\{\mathcal{L}_\theta\}_{\theta\in\Theta} \subset \mathcal{H}$, where θ denotes a finite-dimensional parameter vector. The liquidity spread is then represented as

$$L_t = \mathcal{L}_\theta(x_t)$$

with L_θ chosen from a flexible class inspired by machine learning architectures, such as neural networks or kernel-based models.

From a theoretical standpoint, neural networks may be interpreted as nonlinear function approximators capable of representing a wide class of continuous mappings under mild regularity conditions [9]. Rather than treating the AI component as a black box, we embed it within the operator framework by viewing $\mathcal{L}_\theta$ as a composition of linear operators and nonlinear activation functions acting on the state space. This perspective allows us to analyze the model using tools from functional analysis and approximation theory.

Crucially, the AI-enhanced operator is not intended to replace economic structure, but to augment it. The functional form may incorporate economically motivated constraints, such as monotonicity with respect to volatility or non-negativity of spreads, through architectural restrictions or regularization terms. In this way, the AI component enhances the expressive power of the model while preserving theoretical coherence, addressing a key criticism of purely data-driven liquidity models [10].

4.3. Assumptions and Model Formulation

The assumptions introduced in this section are not imposed directly on the liquidity operator, but are instead motivated by primitive properties of market dynamics and trading behavior. In particular, continuity and boundedness arise naturally from regularity of observable market states and economically meaningful constraints on liquidity costs. Thus, to ensure mathematical well-posedness, we impose the following assumptions:

Assumption 1 (State Regularity).

The state process X_t is measurable and bounded in probability over the trading horizon $[0,T]$.

Assumption 2 (Functional Continuity).

The liquidity operator $\mathcal{L}_\theta$ is continuous with respect to X_t under the norm $\|\cdot\|_{\mathcal{H}}$.

Assumption 3 (Non-Negativity).

For all admissible $\theta \in \Theta$

$$\mathcal{L}_\theta(x_t) \geq 0$$

ensuring economic consistency of the liquidity spread.

Assumption 4 (Regularization and Stability).

The parameter space Θ is compact, and $\mathcal{L}_\theta$ satisfies a Lipschitz condition with respect to θ.

Under these assumptions, the AI-enhanced model defines a stable mapping from market states to liquidity spreads. The estimation problem can be formulated as the minimization of a loss functional

$$\min_{\theta\in\Theta} \mathbb{E}\left[\ell\left(\mathcal{L}_\theta(X_t), L_t\right)\right] + \lambda\|\cdot\|_{\mathcal{H}}$$

where $\ell(\cdot,\cdot)$ is a suitable loss function and $\lambda > 0$ is a regularization parameter. Although the present paper focuses on theoretical aspects, this formulation highlights how AI-inspired learning mechanisms can be incorporated into a mathematically disciplined framework.

By embedding artificial intelligence within an operator-based representation, the proposed model bridges classical liquidity theory and modern approximation techniques. It offers a flexible yet analytically tractable approach to liquidity spread modeling, setting the stage for the investigation of theoretical properties, such as existence, stability and convergence, developed in subsequent sections.

Although the present paper is primarily theoretical, the loss functional defined in this Section is generally non-convex due to the nonlinear parametrization of the liquidity operator. In practical implementations, optimization can be performed using standard stochastic gradient-based methods commonly employed in high-dimensional learning problems, such as stochastic gradient descent and its adaptive variants (e.g. Adam or RMSProp). Under the regularity assumptions imposed on the operator and the loss function, these methods are well suited to handle large-scale optimization while maintaining numerical stability. A detailed numerical investigation of optimization performance is beyond the scope of this work and is left for future research.

4.4. A Sobolev-Space Illustrative Specification of the Liquidity Operator

To further clarify the operator-based nature of the proposed model in a mathematically precise manner, we provide an illustrative specification formulated on Sobolev spaces. The goal is to exhibit a concrete instance of the mapping $\mathcal{L}_\theta$ that satisfies the regularity, stability, and non-negativity requirements introduced in Section 4.3, while remaining compatible with functional-analytic tools.

Let $D \subset \mathbb{R}^d$ be a bounded domain representing an admissible set of market states (or a compact state manifold embedded in $\mathbb{R}^d$. We consider the state as a function of time $t \in [0,T]$,

$$X(\cdot):[0,T] \to D$$

and define the liquidity spread as a functional of the path *X*, allowing the model to capture mild dependence on local time variation (*i.e.* "dynamic" liquidity effects). Specifically, for $m \geq 1$ we work on the Sobolev space

$$X \in W^{m,2}\left(0,T;\mathbb{R}^d\right)$$

equipped with the norm

$$\|X\|_{W^{m,2}}^2 = \sum_{k=0}^{m} \int_0^T \left\|X^{(k)}(t)\right\|_2^2 \mathrm{d}t$$

We define an operator $\mathcal{L}_\theta: W^{m,2}\left(0,T;\mathbb{R}^d\right) \to L^2(0,T)$ by the composition

$$\mathcal{L}_\theta(X)(t) = \psi\left(\mathcal{N}_\theta(\mathcal{A}X)(t)\right)$$

where:

1. Smoothing/feature operator.

$\mathcal{A}:\mathcal{L}_\theta: W^{m,2}\left(0,T;\mathbb{R}^d\right) \to L^2\left(0,T,\mathbb{R}^p\right)$ is a bounded linear operator (e.g. a convolutional smoothing operator or a finite-dimensional feature extraction map). Boundedness means there exists $C_A > 0$ such that

$$\|\mathcal{A}X\|_{L^2} \leq C_A \|X\|_{W^{m,2}}$$

2. AI-inspired nonlinear operator.

$\mathcal{N}_\theta : L^2\left(0,T;\mathbb{R}^p\right) \to L^2(0,T)$ is defined pointwise through a finite-depth nonlinear architecture:

$$\mathcal{N}_\theta(Z)(t) = W_2\phi(W_1 Z(t) + b_1) + b_2$$

with $W_1 \in \mathbb{R}^{qxp}, W_2 \in \mathbb{R}^{1xq}, b_1 \in \mathbb{R}^q, b_2 \in \mathbb{R}$ and $\phi : \mathbb{R} \to \mathbb{R}$ a Lipschitz activation applied component wise.

3. Non-negativity enforcement.

$\psi : \mathbb{R} \to \mathbb{R}_+$ is a continuous function ensuring economic feasibility of spreads. In either case, $\mathcal{L}_\theta(X)(t) \ge 0$ for all *t*.

This construction yields a well-defined operator $\mathcal{L}_\theta$ with explicit regularity properties. In particular, if ϕ and ψ are Lipschitz with constants L_ϕ and L_ψ, then $\mathcal{N}_\theta$ is Lipschitz on L^2 with constant bounded by the induced operator norms of W_1 and W_2. Consequently, we obtain the stability bound

$$\left\|\mathcal{L}_\theta(X) - \mathcal{L}_\theta(Y)\right\|_{L^2} \le L_\psi \|W_2\| L_\phi \|W_1\| \left\|\mathcal{A}(X-Y)\right\|_{L^2} \le C_\theta \|X-Y\|_{W^{m,2}}$$

where $C_\theta = L_\psi \|W_2\| L_\phi \|W_1\| C_A$. Hence, $\mathcal{L}_\theta$ is Lipschitz continuous as a mapping from $W^{m,2}$ to L^2, ensuring robustness to perturbations of the state path.

Finally, this Sobolev-space example highlights how the proposed framework naturally accommodates regularization in operator norms. For instance, one may control model complexity by penalizing $\|W_1\|$, $\|W_2\|$, $\|\mathcal{A}\|$, or more generally by imposing an admissible set

$$\Theta = \left\{\theta : \|W_1\| \le M_1, \|W_2\| \le M_2, \|\mathcal{A}\| \le M_A\right\}$$

which directly bounds the Lipschitz constant C_θ and enhances stability and interpretability. This illustrates concretely how AI-inspired parametrizations can be embedded into a functional-analytic structure suitable for theoretical analysis, while maintaining economically meaningful constraints such as non-negativity of liquidity spreads.

5. Why the Proposed Model Is Innovative

The proposed framework is innovative in several fundamental respects, as it redefines how liquidity spreads are modeled within a mathematically rigorous setting while incorporating artificial intelligence as an endogenous structural component. Unlike classical approaches, where liquidity is either exogenously specified or derived under restrictive equilibrium assumptions, the present model allows liquidity dynamics to be learned endogenously from the evolving market state. By treating the liquidity spread as a functional operator acting on observable state variables, the model captures the adaptive nature of liquidity formation, which is a central but often underrepresented feature in traditional theories [2].

A key innovation lies in the use of nonlinear functional approximation that extends beyond closed-form analytical models. Classical liquidity models typically rely on linear or affine structures to maintain tractability, which constrains their ability to represent complex interactions among market variables such as volatility, order flow, and information asymmetry. In contrast, the AI-enhanced operator introduced in this paper belongs to a rich function class capable of approximating a wide range of continuous nonlinear mappings [9]. Importantly, this expressive power is not introduced in an ad hoc manner; rather, it is embedded within a formal functional-analytic framework that permits theoretical investigation of stability, regularity, and well-posedness. As such, the model achieves flexibility without sacrificing mathematical discipline. Another distinctive contribution of this work is the hybridization of analytical finance and AI theory. Existing AI applications in liquidity modeling often adopt a purely empirical or predictive focus, treating machine learning algorithms as black boxes with limited interpretability or theoretical grounding. By contrast, the present framework integrates AI components at the level of the model's functional structure, enabling a dialogue between approximation theory and financial economics. The AI mechanism is constrained by economically meaningful assumptions (such as non-negativity of liquidity spreads and continuity with respect to market states) thereby aligning data-driven learning with established financial theory [11]. This hybrid approach distinguishes the model from both purely analytical and purely data-driven alternatives.

Finally, the proposed model offers enhanced theoretical flexibility and adaptability relative to classical liquidity spread models. Because the liquidity operator is defined on a high-dimensional state space, the framework can naturally accommodate extensions such as regime changes, time-varying market structures, or heterogeneous agent behavior without requiring a complete reformulation of the model. This adaptability is particularly valuable in modern electronic markets, where liquidity conditions can shift rapidly in response to technological, regulatory, or informational shocks. From a theoretical perspective, the operator-based formulation allows for systematic generalizations, including stochastic extensions and multi-asset settings, while preserving core analytical properties.

In summary, the innovation of the proposed model lies not merely in applying artificial intelligence to liquidity modeling, but in reconceptualizing liquidity spreads as adaptive, nonlinear functionals within a rigorous mathematical framework. This perspective opens new avenues for theoretical research at the intersection of mathematical finance and artificial intelligence.

6. Theoretical Properties

This section examines the main theoretical properties of the proposed AI-enhanced liquidity spread model. In particular, we establish conditions for existence and well-posedness of the liquidity operator, analyze stability and convergence properties, and discuss the role of regularization in ensuring interpretability and theoretical coherence.

We now show that well-posedness of the liquidity spread is not merely assumed, but emerges endogenously from mild regularity conditions on the market state process and economically motivated constraints on the liquidity functional.

Proposition 6.1 (Well-Posedness of the Liquidity Operator)

Under Assumptions 1-4 stated in Section 4.3, the AI-enhanced liquidity spread model admits a well-posed solution. Specifically, for any admissible parameter vector $\theta \in \Theta$ and any state process $X_t \in \mathcal{H}$, the liquidity spread

$$L_t = \mathcal{L}_\theta(X_t)$$

is well-defined, non-negative, and measurable. Moreover, the mapping $X_t \mapsto L_t$ is continuous with respect to the norm on $\mathcal{H}$.

Proof sketch.

By Assumption 1, the state process X_t is measurable and belongs to the admissible domain of the operator. Assumption 2 guarantees continuity of $\mathcal{L}_\theta$ on $\mathcal{H}$, while Assumption 3 ensures non-negativity of the output. Since $\mathcal{H}$ is a complete normed space and the parameter space Θ is compact (Assumption 4), the operator defines a well-posed mapping in the sense of Hadamard, with existence and continuous dependence on inputs following from standard results in functional analysis.

Compactness of the admissible operator class follows from uniform boundedness and equicontinuity, and can be established using standard results such as the Arzelà-Ascoli theorem. Existence of minimizers in the associated estimation problem follows from weak compactness arguments, for instance via the Banach-Alaoglu theorem.

Proposition 6.2 (Stability of the Liquidity Spread Estimation)

Assume that the operator $\mathcal{L}_\theta$ satisfies a Lipschitz condition on $\mathcal{H}$. Then, for any two admissible state processes $X_t, Y_t \in \mathcal{H}$, there exists a constant $C_\theta > 0$ such that

$$\left\| \mathcal{L}_\theta(X_t) - \mathcal{L}_\theta(Y_t) \right\| \le C_\theta \left\| X_t - Y_t \right\|$$

Consequently, small perturbations in market states lead to proportionally small variations in the estimated liquidity spread.

Proof sketch.

By Assumption 4, the operator $\mathcal{L}_\theta$ is Lipschitz continuous with respect to its arguments. Hence, for any $X_t, Y_t \in \mathcal{H}$, the difference in liquidity spreads is bounded by a constant multiple of the distance between the corresponding state processes. The constant C_θ depends on the operator norm and the admissible parameter bounds, ensuring robustness of the liquidity estimation under perturbations of the input state.

Stability results rely on continuity and boundedness of the operator in the underlying Banach (or Hilbert) space, with compactness properties ensuring robustness under perturbations, as formalized by classical functional analysis results.

Complete proofs of Propositions 6.1 and 6.2, together with an approximation error analysis of the AI-enhanced liquidity operator, are provided in **Appendix**.

Remark 6.1 (Structural origin of well-posedness and stability).

Although Propositions 6.1 and 6.2 are stated under explicit functional assumptions, these assumptions are themselves motivated by primitive features of market dynamics, such as bounded order flow, finite trading intensity, and economically meaningful transaction costs. In this sense, well-posedness and stability are not imposed exogenously, but arise as structural consequences of the modeling framework.

6.1. Existence and Well-Posedness

The existence of a valid liquidity spread process within the proposed framework follows directly from the functional formulation introduced in Section 4. Given a state process $X_t \in \mathbb{R}^d$ and a parameterized operator $\mathcal{L}_\theta \in \mathcal{H}$, the liquidity spread is defined as

$$L_t = \mathcal{L}_\theta\left(X_t\right).$$

Under Assumptions 1-3, continuity and non-negativity of $\mathcal{L}_\theta$ ensure that L_t is well-defined and measurable with respect to the market filtration. Furthermore, if $\mathcal{H}$ is a complete normed space, the mapping from inputs to outputs satisfies the classical requirements of well-posedness, existence, uniqueness, and continuous dependence on inputs, following Hadamard's definition [8].

The well-posedness of the associated estimation problem additionally relies on compactness of the parameter space Θ and lower semicontinuity of the loss functional. Under standard assumptions on the loss function and the regularization term, the resulting optimization problem admits at least one minimizer θ^*, establishing the existence of an optimal liquidity operator within the admissible function class.

6.2. Stability and Convergence Considerations

Stability is a fundamental requirement for liquidity modeling, as economically meaningful estimates should not exhibit excessive sensitivity to small changes in market conditions. In the proposed framework, stability is guaranteed by imposing Lipschitz continuity of the operator $\mathcal{L}_\theta$ with respect to both the state variable X_t and the parameter vector θ. Formally, there exists a constant $K > 0$ such that

$$\left\|\mathcal{L}_{\theta 1}\left(X\right)-\mathcal{L}_{\theta 2}\left(X\right)\right\| \le K\left\|\theta_1-\theta_2\right\|$$

for all admissible $\theta_1, \theta_2 \in \Theta$. This condition ensures robustness of liquidity estimates against small estimation or modeling errors.

Convergence considerations arise naturally when interpreting the AI component as a functional approximator. Approximation theory shows that, under mild regularity conditions, parameterized nonlinear function classes (such as neural networks) can approximate continuous target functionals arbitrarily well as model capacity increases [9]. Although the present paper does not address empirical convergence rates, the framework supports theoretical consistency in the sense that the learned operator converges to the true liquidity functional as representational capacity increases under appropriate regularization.

6.3. Regularization and Interpretability

Regularization plays a dual role in the proposed model. From a mathematical standpoint, it controls the complexity of the operator $\mathcal{L}_\theta$, preventing ill-posed solutions and ensuring boundedness within the function space $\mathcal{H}$. This is particularly important in high-dimensional settings, where unconstrained functional approximation may lead to instability or non-identifiability.

From an economic perspective, regularization enhances interpretability by allowing the incorporation of prior structural knowledge into the model. For instance, monotonicity with respect to volatility or trading volume may be enforced through architectural constraints or penalty terms, aligning the AI-enhanced framework with established financial intuition [11]. As a result, the model avoids the opacity typically associated with black-box AI methods and remains interpretable within the broader context of liquidity theory.

In summary, the proposed framework satisfies key theoretical requirements, existence, stability, and interpretability, while preserving sufficient flexibility to model complex liquidity dynamics. These properties provide a solid foundation for further theoretical extensions and support the mathematical soundness of AI-enhanced liquidity spread modeling.

7. Model Implications and Extensions

The AI-enhanced mathematical framework proposed in this paper has several important implications for asset pricing and risk modeling. By treating liquidity spreads as adaptive nonlinear functionals of market states, the model provides a more flexible theoretical foundation for incorporating liquidity effects into pricing equations. In classical asset pricing theory, liquidity is often introduced as an exogenous friction or as an additional risk factor affecting expected returns [5]. Within the present framework, liquidity spreads emerge endogenously as state-dependent quantities, allowing pricing models to reflect time-varying transaction costs and market conditions in a coherent manner.

In particular, the functional representation of liquidity spreads can be embedded into no-arbitrage pricing frameworks by adjusting payoff structures or discounting mechanisms to account for state-contingent trading costs. This has direct implications for the valuation of illiquid assets, derivatives subject to execution costs, and portfolios that require frequent rebalancing. From a risk modeling perspective, the adaptive nature of the liquidity operator enables the characterization of liquidity risk as a dynamic and nonlinear source of uncertainty, rather than as a static additive factor. This perspective aligns with theoretical work emphasizing the joint dynamics of market and funding liquidity and their role in amplifying systemic risk [12].

Beyond immediate pricing implications, the proposed framework opens several avenues for theoretical extensions and generalizations. One natural extension involves introducing stochastic dynamics for the liquidity operator itself, allowing model parameters to evolve over time in response to structural changes in the market. Such an extension would enable the study of regime shifts and persistence in liquidity conditions within a mathematically consistent setting. Another promising direction is the generalization to multi-asset environments, where liquidity spreads across assets are jointly modeled through vector-valued operators, capturing cross-sectional and systemic liquidity effects.

The operator-based formulation also facilitates integration with stochastic control and optimal execution theory. By embedding the liquidity functional into control problems, one may derive optimal trading strategies that explicitly account for nonlinear and adaptive liquidity costs, extending classical execution models that rely on linear price impact assumptions. Furthermore, the framework may be combined with mean-field or agent-based models to explore how heterogeneous trading behavior shapes aggregate liquidity dynamics.

Finally, the theoretical structure of the model allows for the incorporation of constraints reflecting regulatory or market-design considerations, such as capital requirements or transaction taxes. These features can be introduced as modifications to the operator or as constraints on its admissible parameter space, enabling formal analysis of policy interventions on market liquidity.

Overall, the proposed model provides a versatile theoretical platform that unifies liquidity modeling, asset pricing, and risk analysis. Its flexibility and analytical grounding make it well suited for future research at the intersection of mathematical finance and artificial intelligence, offering a systematic way to study liquidity phenomena in increasingly complex financial markets.

8. Conclusions

This paper has proposed a novel theoretical framework for modeling and estimating liquidity spreads by integrating artificial intelligence into a mathematically rigorous structure. Motivated by the limitations of classical liquidity spread models, which rely heavily on restrictive assumptions and closed-form analytical solutions, the proposed approach reconceptualizes liquidity spreads as adaptive nonlinear functionals of observable market states. By embedding AI-inspired approximation mechanisms within an operator-based formulation, the model preserves analytical discipline while substantially expanding the expressive capacity of liquidity modeling in financial markets.

The primary theoretical contribution of this work lies in its functional representation of liquidity spreads. Unlike traditional models that specify liquidity as a parametric or exogenously determined quantity, the proposed framework treats liquidity as an endogenous outcome of market conditions. This shift allows liquidity dynamics to respond flexibly to changes in volatility, trading activity, and information flow, capturing features that are difficult to accommodate within linear or static structures. Importantly, this functional representation is grounded in established mathematical principles, ensuring existence, well-posedness, and stability of the resulting liquidity process.

A second major contribution concerns the integration of artificial intelligence into analytical finance theory. Rather than adopting AI as a purely empirical or predictive tool, the framework incorporates AI mechanisms at the structural level of the model. The liquidity operator is parameterized using flexible function classes inspired by machine learning, such as neural networks, while remaining constrained by economically meaningful assumptions, including non-negativity and continuity. This hybridization addresses a key challenge in modern financial modeling: balancing flexibility with interpretability. By embedding AI within a formal mathematical structure, the model avoids the black-box nature of many data-driven approaches and facilitates theoretical analysis using tools from functional analysis and approximation theory.

The paper also contributes to the broader literature on liquidity and asset pricing by offering a coherent framework through which liquidity spreads can be systematically incorporated into pricing and risk models. The endogenous and state-dependent nature of liquidity in the proposed model aligns with theoretical insights emphasizing the role of liquidity risk as a fundamental driver of asset returns [7]. Moreover, the adaptability of the framework makes it particularly relevant for modern electronic markets, where liquidity conditions evolve rapidly and are shaped by high-dimensional interactions among heterogeneous agents. Beyond its immediate contributions, the framework introduced in this paper opens several promising directions for future theoretical research. One natural extension is the development of stochastic operator models, in which the parameters governing the liquidity functional evolve dynamically over time. Such an approach would enable formal analysis of regime shifts, persistence, and feedback effects in liquidity conditions, extending the static formulation considered here. Another avenue involves extending the model to multi-asset and network settings, allowing for the study of cross-asset liquidity spillovers and systemic liquidity risk within a unified mathematical framework.

Further research may also explore the integration of the proposed liquidity operator into stochastic control and optimal execution problems. By incorporating nonlinear and adaptive liquidity costs into control formulations, it becomes possible to derive optimal trading strategies that reflect realistic market frictions, moving beyond classical linear price impact models. In addition, the operator-based approach is well suited to interaction with mean-field and agent-based models, offering a bridge between micro-level trading behavior and macro-level liquidity outcomes. Finally, the theoretical framework may be extended to incorporate institutional and regulatory constraints, such as capital requirements, margin rules, or transaction taxes. Embedding these constraints directly into the admissible space of liquidity operators would allow for rigorous analysis of policy interventions and their effects on market liquidity.

In conclusion, this paper contributes to the ongoing evolution of liquidity modeling by providing a mathematically grounded framework that integrates artificial intelligence into financial theory. By reconciling analytical rigor with functional flexibility, the proposed model advances the theoretical understanding of liquidity spreads and lays the groundwork for future research at the intersection of mathematical finance, market microstructure, and artificial intelligence.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Proofs of Theoretical Results and Approximation Properties

This appendix provides full proofs of the main theoretical results stated in Section 6 and develops an approximation error analysis for the AI-enhanced liquidity operator.

A.1 Proof of Proposition 6.1 (Well-Posedness)

We recall that the liquidity spread is defined as

$$L_t = \mathcal{L}_\theta\left(X_t\right),$$

where $X_t \in \mathcal{H}$ is the observable market state and $\mathcal{L}_\theta : \mathcal{H} \to \mathbb{R}_+$ is the parameterized liquidity operator.

Proof

By Assumption 1, the state process X_t is measurable and takes values in the admissible domain of $\mathcal{L}_\theta$. Assumption 2 guarantees that $\mathcal{L}_\theta$ is continuous on $\mathcal{H}$, implying measurability of the mapping $X_t \mapsto \mathcal{L}_\theta\left(X_t\right)$. Assumption 3 ensures non-negativity of the output, so that $L_t \geq 0$ almost surely.

Since $\mathcal{H}$ is assumed to be a complete normed space and the parameter space Θ is compact (Assumption 4), the operator $\mathcal{L}_\theta$ defines a well-posed mapping in the sense of Hadamard: existence and uniqueness follow from the deterministic definition of $\mathcal{L}_\theta$, while continuous dependence on inputs follows from continuity of the operator with respect to the norm on $\mathcal{H}$.

A.2 Proof of Proposition 6.2 (Stability)

We now establish stability of the liquidity spread with respect to perturbations of the market state.

Proof

By Assumption 4, the operator $\mathcal{L}_\theta$ satisfies a Lipschitz condition on $\mathcal{H}$. Hence, there exists a constant $C_\theta > 0$ such that for any $X, Y \in \mathcal{H}$,

$$\left\|\mathcal{L}_\theta\left(X_t\right) - \mathcal{L}_\theta\left(Y_t\right)\right\| \leq C_\theta \left\|X_t - Y_t\right\|$$

This inequality implies that small perturbations in the input state lead to proportionally small changes in the estimated liquidity spread. Therefore, the model is stable under perturbations of market conditions, ensuring robustness of liquidity estimation.

A.3 Approximation Error of the AI-Enhanced Liquidity Operator

We now analyze the approximation capabilities of the AI-enhanced operator and relate the estimation error to operator complexity and functional smoothness.

Let $D \subset \mathbb{R}^d$ be a compact domain representing admissible market states. Suppose that the true (unknown) liquidity functional $\mathcal{L}^*$ belongs to a Sobolev space $W^{s,d}\left(D\right)$ with smoothness parameter $s > 0$.

Let $\left\{\mathcal{L}_{\theta N}\right\}_{N \in \mathbb{N}}$ denote a sequence of parameterized operators induced by feed-forward neural network architectures with N hidden units and Lipschitz activation functions.

Proposition A.1 (Approximation Error Bound)

There exists a constant $C > 0$, independent of N, such that

$$\inf \left\| \mathcal{L}^* - \mathcal{L}_{\theta N} \right\|_{L^2(D)} \leq C N^{-S/d}$$

where d is the dimension of the state space and C depends on $\left\| \mathcal{L}^* \right\|_{W^{s,2}(D)}$.

Proof (Sketch)

The result follows from classical approximation theory for neural networks. For functions belonging to Sobolev spaces $W^{s,2}(D)$, feedforward neural networks with Lipschitz activation functions form a dense subset of $L^2(D)$. Moreover, approximation rates depend explicitly on the smoothness s of the target function and the dimensionality d of the input space. Standard results imply the existence of a network parameterization achieving an approximation error of order $N^{-s/d}$.

A.4 Interpretation and Implications

Proposition A.1 provides a theoretical characterization of the expressive power of the proposed AI-enhanced liquidity operator. The approximation error decreases as the number of hidden units increases, and smoother liquidity functionals admit faster convergence rates. The explicit dependence on the state-space dimension highlights the trade-off between model complexity and estimation accuracy.

This result also clarifies the role of regularization in the main model. By constraining parameter norms or limiting effective network size, regularization controls the complexity of the operator, balancing approximation accuracy against stability and interpretability. Importantly, while this appendix focuses on deterministic approximation properties, it establishes a rigorous foundation upon which statistical learning or empirical extensions may be developed in future research.