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Mathematics of Stock Valuation: Why the Potential Payback Period (PPP) Outperforms the P/E and PEG Ratios

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Abstract

This article introduces the Potential Payback Period (PPP), a valuation model initiated by the author, as a mathematically rigorous and conceptually richer alternative to traditional ratios. While the Price-to-Earnings (P/E) and PEG ratios have long been used to assess stock value, they suffer from critical limitations: the P/E ignores growth and discounting, while the PEG applies a simplistic linear adjustment and neglects risk. The PPP corrects these shortcomings by incorporating earnings growth, interest rates, and risk (via CAPM-based discounting) into a unified logarithmic structure. Using tools such as the Gordon Growth Model, Taylor expansion, and L'Hospital's Rule, the paper shows that the P/E and PEG ratios are special cases of the PPP [1]-[3]. As a result, the PPP offers a more consistent, interpretable, and forward-looking metric that aligns with modern financial theory and investor needs.

Keywords

Potential Payback Period (PPP), Price/Earnings Ratio (P/E), PEG Ratio, Taylor Expansion, Gordon Growth Model (GGM), L'Hospital's Rule, Stock Valuation

1. Introduction

Traditional stock valuation relies heavily on the Price-to-Earnings (P/E) ratio and its derivative, the PEG ratio (Price-to-Earnings relative to earnings Growth). While these metrics are valued for their simplicity, they fall short of capturing key financial realities such as the time value of money, risk-adjusted discounting, and nonlinear earnings growth [4] [5]. These limitations have become more apparent in today's dynamic financial environment, where companies exhibit high volatility, variable growth rates, and significant risk asymmetries.

In response to these challenges, this article introduces and develops the Potential Payback Period (PPP), a new valuation model initiated by the author [6] [7]. The PPP generalizes and mathematically subsumes both the P/E and PEG ratios by embedding them in a more comprehensive framework based on temporal valuation logic, growth-adjusted compounding, and discounted cash recovery. At its core, the PPP answers the fundamental question of equity valuation: How many years of future discounted earnings would it take to theoretically recover the price paid for a stock today?

By leveraging formal tools such as the Gordon Growth Model [8], Taylor series approximation [9], and L'Hospital's Rule [9], this paper demonstrates that the P/E and PEG ratios emerge as simplified special cases within the PPP structure, valid only when earnings growth and discounting are ignored. The PPP thus offers not only a more robust financial foundation but also an interpretive bridge between stock and bond valuation through the derivation of a Stock Internal Rate of Return (SIRR).

2. Mathematical Formulations

2.1. The Price/Earnings (P/E) Ratio

$$\frac{P}{E_0}$$

This classic valuation ratio assumes constant annual earnings and no discounting, implying a direct recovery of the stock price over time through static earnings.

Here, E_0 is the most recent annual earnings per share.

This simplification provides an intuitive but incomplete measure of a stock's value.

2.2. The PEG Ratio

$$\text{PEG} = \frac{P/E}{g}$$

where:

- g is the expected earnings growth rate.

The PEG ratio introduces growth into valuation by dividing the P/E by g. However, this is a linear adjustment that neither reflects compounding effects nor the time value of money, and it makes no provision for discounting or risk. It thus provides a crude and often misleading representation of valuation under non-static conditions.

2.3. The Potential Payback Period (PPP)

$$\text{PPP} = \frac{\log\left(\dfrac{P/E\cdot(g-r)}{1+r}+1\right)}{\log\left(\dfrac{1+g}{1+r}\right)}$$

where:

- g = earnings growth rate (assumed to decline gradually toward r).
- r = discount rate (including both the risk-free rate and a risk premium via CAPM [10]).

This formula calculates the number of years theoretically required for the present value of growing earnings to equal the current stock price, assuming full earnings distribution. Unlike the P/E and PEG ratios, the PPP captures nonlinear growth, incorporates the cost of capital, and yields a time-based valuation metric that reflects both earning power and risk-adjusted return expectations.

2.4. Main Features of the PPP Methodology

2.4.1. Derivation of the PPP Formula

To make the PPP model's foundation transparent, we derive it from fundamental financial valuation principles. Let a stock's earnings grow at a rate g and be discounted at a rate r. Assuming full earnings distribution and starting earnings of E_0, the present value of future earnings, represented as an infinite series, is:

$$P = \sum_{t=1}^{\infty} \frac{E_0 (1+g)^t}{(1+r)^t}$$

This infinite geometric series becomes:

$$P = E_0 \sum_{t=1}^{\infty} \left(\frac{1+g}{1+r}\right)^t = E_0 \cdot \frac{\left(\frac{1+g}{1+r}\right)}{1-\left(\frac{1+g}{1+r}\right)} = E_0 \cdot \frac{1+g}{r-g}$$

This result applies only when $g < r$. When $g \approx r$, we assume that g declines linearly toward r over time to maintain convergence.

The PPP formula generalizes this relationship by transforming the infinite series into a finite one. It removes the limiting condition $g < r$ and defines the number of years N required for the present value of growing earnings to equal the price P, using a finite horizon:

$$P = \sum_{t=1}^{N} \frac{E_t}{(1+r)^t}$$

Solving this sum algebraically to isolate N is complex and does not lead to a simple formula. To overcome this difficulty, we use a logarithmic approximation method. This approach assumes that earnings grow geometrically and are discounted geometrically over time, meaning each year's earnings are multiplied by a growth factor $(1 + g)$ and discounted by a factor $(1 + r)$. This leads to a clean formula for PPP, expressed in logarithmic terms.

Assuming $E_t = E_0 (1+g)^t$, and simplifying algebraically, the PPP becomes:

$$\text{PPP} = \frac{\log\left(\frac{P/E \cdot (g-r)}{1+r} + 1\right)}{\log\left(\frac{1+g}{1+r}\right)}$$

2.4.2. Growth g Declining toward r

The assumption that the earnings growth rate g gradually declines toward the discount rate r reflects the real-life cycle of companies. While young firms may experience rapid growth, this growth typically slows as they mature, eventually converging toward the cost of capital. This assumption promotes conservative valuation practices and helps prevent the overestimation of long-term growth by avoiding unrealistic extrapolations.

Below is a sensitivity table showing how PPP varies as the relationship between g and r changes, with g linearly declining toward r:

P/E	g (%)	r (%)	PPP
30	20	4	**16.14**
30	10	4	**21.92**
30	4.001	4	**30.00**

The linear decline from g to r is part of the realistic modeling assumption built into the PPP methodology.

Using L'Hôpital's Rule, we will demonstrate in Section 4.3 (*Stability of PPP Under Limiting Conditions: A L'Hôpital's Rule Approach*) that the PPP smoothly converges to the P/E ratio when the growth rate and discount rate are equal, confirming that the P/E ratio is a special and limiting (or degenerate) case of the PPP.

2.4.3. Estimating g and r

Practitioners can estimate from consensus analyst forecasts (e.g., FactSet, Yahoo Finance), or historical earnings compound annual growth rates (CAGR). The discount rate is best derived via the Capital Asset Pricing Model (CAPM) [10]:

$$r = r_f + \beta\left(R_m - r_f\right)$$

where r_f is the risk-free rate, R_m is the market return, and β measures a stock's market sensitivity. Estimation errors may arise from forecast variability, changing market conditions, and incorrect beta assumptions.

3. Revisiting the P/E Ratio through the Gordon Growth Model

The Gordon Growth Model (GGM) evaluates a stock as the present value of an infinite stream of dividends growing at a constant rate:

$$P = \frac{D_1}{r - g}$$

where:

- P: Stock price.

- D_1: Dividend expected in year 1.
- r: Required rate of return (discount rate).
- g: Constant dividend/earnings growth rate.

Assuming full earnings distribution (*i.e.*, a 100% payout ratio), we substitute:

$$P = \frac{E_1}{r-g} \Rightarrow \frac{P}{E_1} = \frac{1}{r-g}$$

Since $E_1 = E_0(1+g)$, we express the price in terms of current earnings E_0, yielding the growth-adjusted P/E ratio:

$$\frac{P}{E_0} = \frac{1+g}{r-g}$$

This formulation introduces growth and discounting, improving upon the static P/E model. However, it remains dependent on idealized assumptions, including:

- Perpetual constant growth,
- Full earnings payout (no reinvestment),
- A static discount rate, and
- No explicit indication of the recovery period for the investment.

Moreover, the Gordon Growth Model suffers from a serious mathematical limitation: the requirement that $g < r$ in order for the formula to be valid and convergent. This constraint severely restricts its applicability in high-growth scenarios, precisely when a more robust valuation tool is needed.

In contrast, the Potential Payback Period (PPP) provides a more flexible and realistic structure. It explicitly connects valuation to time-based recovery, risk, and nonlinear earnings dynamics, and does not suffer from the same restrictive condition. Thanks to its finite-horizon framework and logarithmic formulation, the PPP remains valid even when $g \geq r$, accommodating real-world cases where short- to medium-term growth can exceed the discount rate.

4. Taylor Expansion and Linear Approximations

4.1. Taylor Expansion in Valuation Metrics

The PPP formula relies on logarithmic expressions that can be expanded via the Taylor series:

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

While the PEG ratio uses only the first-order term, the PPP retains the full structure, incorporating higher-order compounding effects. This makes the PPP more accurate and better aligned with how earnings accumulate over time under growth and risk.

4.2. Taylor Approximation of the PPP Denominator

The denominator of the PPP is:

$$\log\left(\frac{1+g}{1+r}\right)=\log(1+g)-\log(1+r)$$

Applying Taylor series expansions:

$$\log(1+g)\approx g-\frac{g^2}{2},\quad \log(1+r)\approx r-\frac{r^2}{2}$$

Thus:

$$\log\left(\frac{1+g}{1+r}\right)\approx(g-r)-\frac{g^2-r^2}{2}$$

This shows that the PPP implicitly includes nonlinear terms that reflect compound effects, whereas the PEG ratio oversimplifies the growth-discounting relationship by omitting these corrections.

4.3. Stability of PPP under Limiting Conditions: A L'Hospital's Rule Approach

As $g \to r$, the PPP becomes an indeterminate form:

$$\text{PPP}=\frac{0}{0}$$

Applying L'Hospital's Rule:

$$\lim_{g\to r}\text{PPP}=\frac{P/E}{1+r}\cdot(1+r)=\frac{P}{E}$$

Thus, the PPP smoothly converges to the P/E ratio when growth and discount rates are equal, confirming that P/E is a special (or degenerate) case of PPP. In contrast, the PEG ratio diverges as $g \to 0$, producing meaningless results in low-growth contexts and demonstrating its instability.

5. Conceptual Implications

5.1. Time Value, Risk, and Growth: A Comprehensive Incorporation

The PPP captures the core elements of modern financial theory:

- Discounting of future earnings via r,
- Risk adjustment through a CAPM-derived discount rate,
- Earnings compounding through g.

By embedding these components directly into its structure, the PPP offers a more realistic and theory-consistent valuation model.

5.2. Temporal Interpretability and Investor Relevance

The PPP provides a clear time-based interpretation: how many years of discounted earnings are required to recover the current stock price. This interpretation is absent in both P/E and PEG, making PPP uniquely valuable for investors focused on long-term earning power and capital recovery.

5.3. A Unified and Generalized Valuation Framework

- P/E is a degenerate case of PPP when $g = r = 0$.
- PEG is a linear simplification of the full PPP model.
- PPP is a generalized, logarithmic, growth- and risk-adjusted valuation metric.

This hierarchy reveals how PPP subsumes and improves upon both traditional metrics, offering a unified conceptual and mathematical framework for equity valuation.

5.4. Numerical Example, NVIDIA

Example – NVIDIA (as of October 2024):

- $P/E = 69.31$
- $g = 57.38\%$
- $r = 4.03\%$

$$g_{\text{avg}} = \frac{g+r}{2} = \frac{0.5738 + 0.0403}{2} = 0.30705$$

g_{avg} is the earnings average growth rate over the PPP period.

$$\text{PPP} = \frac{\log\left(\frac{69.31\cdot(0.30705 - 0.0403)}{1+0.0403} + 1\right)}{\log\left(\frac{1+0.30705}{1+0.0403}\right)} \approx 12.85 \text{ years}$$

PPP is the number of years required for future earnings—growing at an average rate of 30.705% and discounted at a rate of 4.03%—to equal the stock's current price corresponding to a P/E ratio of 69.31 (concept of "potential payback period").

$$\text{PEG} = \frac{P/E}{g} = \frac{69.31}{57.38} = 1.21$$

This shows how the Price-to-Earnings to Growth (PEG) ratio—here at a level above the overvaluation threshold of 1—may falsely signal overvaluation, while the PPP indicates a fast payback period and robust earnings power.

6. Conclusions

This article has proposed the Potential Payback Period (PPP) as a mathematically structured and conceptually integrative approach to stock valuation, one that builds upon and extends the logic behind the traditional P/E and PEG ratios. By embedding growth, discounting, and risk into a unified logarithmic framework, the PPP addresses several limitations of existing metrics and offers a time-based interpretation of valuation: the number of years theoretically required to recover a stock's price through future discounted earnings.

The analysis shows that the P/E ratio can be viewed as a limiting case of the PPP, valid when growth and discounting are absent. Similarly, the PEG ratio appears as a simplified linear approximation of the PPP's more complete structure.

In this sense, the PPP may be understood as a generalization that mathematically subsumes these widely used indicators.

An important feature of the PPP is its ability to capture a company's underlying earnings power, that is, the capacity to generate profits over time, which remains the principal determinant of a stock's intrinsic value [11]. By linking price to earnings in a time-adjusted and risk-aware manner, the PPP provides a clearer and more consistent foundation for assessing equity attractiveness.

While further empirical testing and refinement are needed, the PPP offers a promising framework that may complement or enhance traditional valuation heuristics. Its capacity to incorporate compounding effects, the time value of money, and risk makes it particularly relevant in contexts involving high-growth firms, long-term investments, or volatile environments. The author hopes that this contribution will stimulate further research and discussion on how stock valuation can evolve through more rigorous, transparent, and theoretically grounded methods.

Limitations and Future Research

While the PPP model improves upon the P/E and PEG ratios by integrating earnings growth, discounting, and risk, it still relies on certain simplifying assumptions for analytical clarity. Notably, the earnings growth rate g is modeled as declining linearly toward the discount rate r over the payback period, and the discount rate is assumed to remain constant. These assumptions, while facilitating a tractable and robust formulation, may not fully reflect the complexity of evolving growth trajectories or changing risk conditions in real-world markets.

Regarding the model's use of full earnings distribution, it should be noted that this is a theoretical construct rather than a literal assumption. The PPP does not depend on actual dividend payouts but rather uses the concept of full distribution to measure a firm's total earning power, which underpins intrinsic value, whether earnings are retained or distributed. In this sense, undistributed profits still contribute to a firm's capacity to generate value, and thus the model remains economically consistent.

Future research may extend the PPP by:

- Exploring multi-stage growth structures beyond the linear decline of g,
- Incorporating variable discount rates reflecting shifting macro and firm-specific conditions,
- Conducting empirical testing across sectors, geographies, and market cycles to validate the model's applicability and precision.

By addressing these areas, future developments could further refine the PPP as a theoretically grounded yet practically adaptable tool for modern equity valuation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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