

Mean Reversion and Self-Valuation of European Common Stocks

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Abstract

This article presents empirical evidence that European stock markets independently determine the values of their listed common stocks. It shows that European common stocks establish their own discount rates, which in turn determine their equity values. The study models mean reversion in continuous time and demonstrates its minimal impact on prediction accuracy, indicating the efficiency of self-valuation. Mean reversion of current stock prices is an activity where investors assess the values of common stocks listed on an exchange. The speed of mean reversion of a current stock price determines the discount rate. The continuous-time first-order autoregressive model captures both the mean reversion of a current stock price and the volatility due to mean reversion, describing the dynamics of mean reversion in pricing. The closing prices of European common stocks are shown to be volatile, indicating the inherent risks of investing in common stocks.

Keywords

Demeaned Stock Price, Mean Reversion, First-Order Autoregression, Discount Rate, Prediction, Self-Valuation, Heuristic Derivation, STOXX Europe 600, Equity Value, Volatility, Efficiency

1. Introduction

In recent financial theory, Lee [1] has introduced the concept of the first-order autoregressive property of a current demeaned stock price to derive a discount factor for a lagged demeaned stock price. This development demonstrates that the stock market can self-value a firm's common stock, with empirical evidence supporting the self-valuation of securities in the S&P 500 stock market.

This article extends this theory to European stock markets, providing empirical evidence that these markets also self-value the common stocks listed on their stock

exchanges. By generalizing the theory of self-valuation, we can better understand the dynamics of European stock markets.

The European stock markets are encapsulated by the STOXX Europe 600 index, which offers a comprehensive measure of the European equity markets. This index covers a diverse range of seventeen countries and eleven industries across Europe and the United Kingdom, representing nearly 90% of the investable market.

This article delves deeper into the concept of mean reversion in continuous time, which is essential for self-valuation. In European stock self-valuations, the effect of mean reversion on prediction accuracy is minimal, demonstrating the efficiency of self-valuation.

The prediction error resulting from speculation is identical to the prediction error caused by mean reversion. Valuation is a discounting process. Closing European stock prices are shown to be volatile, indicating the risks of investing in common stocks.

This article renames the residual error of a continuous-time AR (1) process as the prediction error due to mean reversion. According to Shiller [2], the stock market processes information very inefficiently because the current stock price is highly speculative and volatile. The stock market's inefficiency stems from the fact that the current stock price not only reflects the equity value of a firm's common stock, as prediction error due to mean reversion must also be considered. Prediction error is commonly referred to as volatility. The equity value of a company's common stock is determined through the valuation of relevant information.

The model of mean reversion in this article differs significantly from those in the literature, which typically use stock returns as a modeling variable. For example, Poterba and Summers [3] focussed on testing stock returns for evidence of mean reversion to differentiate mean reversion from random walk. In contrast, the model in this article shows that the current stock price mean reverts.

Fama [4] [5] hypothesized an efficient stock market, assuming a current stock price was not volatile. However, test results show that the closing S&P 500 and the closing STOXX Europe 600 are volatile.

The process of information handling is integral to self-valuation. Mean reversion of current stock prices is the economic activity that enables the stock market to self-assess the common stocks listed on its stock exchange. Mean reversion of a current stock price is modeled in continuous time. The speed of mean reversion of a current stock price determines the discount rate.

The stock market self-determines the equity value of a firm's common stock. We can measure the efficiency of self-valuation, specifically, the extent to which equity value is created as a fraction of the current stock price.

The stock market does not independently set the current stock price; instead, prices tend to revert to the mean following economic shocks. These fluctuations, driven by mean reversion, are quantified as volatility. However, there is no established metric to assess the efficiency of the stock market or the accuracy of the current stock price.

Since mean reversion indicates that current stock prices rise and fall together due to common shocks, stock returns are automatically correlated. Empirical studies, such as those by Fama and French [6], which relied on the correlation of stock returns to make inferences, confirmed the mean reversion of current stock prices.

The first attempt at asset pricing was by Sharpe [7]. Sharpe's capital asset pricing model states that the expected return on a security equals the risk-free interest rate plus the security's beta times the market portfolio's excess return over the risk-free interest rate. Beta measures the covariance risk of the security. Sharpe derived his model from the mean-variance model of stock returns. However, this model does not specify a timeline, meaning Sharpe's model is not designed for prediction purposes. Additionally, converting a mean stock return to a current stock price is not feasible with this model. Other asset pricing models in the literature, which focussed on expected stock return, faced similar limitations (see, for example, Black [8], Ross [9] and Breeden [10]). Dessaint *et al.* [11] reported problems in test results for Sharpe's capital asset pricing model.

Due to the limitations and lack of empirical support for modeling stock returns, I focus on modeling the current demeaned stock price. This approach allows for valuation by discounting, which is a fundamental principle in finance. Valuation by discounting follows the same principle as computing the present value by discounting. In a present-value calculation, discounting removes the embedded time value from a future cash flow. Similarly, when discounting a lagged demeaned stock price, it removes the embedded prediction error from the lagged demeaned stock price.

In Section 2, I provide a heuristic derivation of the pricing equation for a current demeaned stock price, X_t . X_t is a continuous-time first-order autoregressive model with a discounted lagged demeaned stock price and volatility inversely proportional to the current stock price, S_t . The methodology for estimating the parameters of the continuous-time AR (1) model is described.

I show that the risk of investing in a firm's common stock is partly the prediction error due to mean reversion and partly the net prediction error from making a forecast. The total risk of investing in a firm's common stock is the total prediction error or volatility.

Section 3 provides the source for the data used in the empirical study. A description of the data for the STOXX Europe 600 is included.

Section 4 presents the empirical results, which confirm that European stock markets self-value their listed common stocks. The closing prices of European common stocks exhibit mean reversion, indicating self-valuation. Additionally, the equity values are high, and the closing prices are shown to be volatile.

In Section 5, I refer to the limitations associated with modeling stock returns, and the advantages of modeling the current demeaned stock price in asset pricing. I also explore the market processes that support the derivation of the prediction model. The remainder of the conclusion summarizes the main results.

2. The $e^{-\theta}$ Model

To understand the pricing equation for a current demeaned stock price, X_t , we start by decomposing the current stock price, S_t , into its mean stock price, μ , and the current demeaned stock price, X_t : $S_t = \mu + X_t$.

The heuristic derivation of the pricing equation for X_t is based on prediction and mean reversion. Unlike future demeaned stock prices, which are random, a lagged demeaned stock price, X_{t-1} , is known and can be discounted.

To avoid dealing with random variables, we start modeling one period back. At $t-1$, the current demeaned stock price, X_{t-1} , predicts the future demeaned stock price, X_t , at t . The predictor, X_{t-1} , is discounted. At t , the current demeaned stock price, X_t , equals the discounted lagged demeaned stock price plus prediction error ε :

$$X_t = e^{-\theta} X_{t-1} + \varepsilon \quad (1)$$

where $e^{-\theta}$ is a discount factor in continuous time, and θ is a discount rate.

Prediction error, ε , in Equation (1) is commonly referred to as volatility:

$$\varepsilon = \lambda \nu S_t dz \quad (2)$$

where $\varepsilon \sim N\{0, (\lambda S_t)^2 \nu^2\}$, the volatility, ν , is inversely proportional to S_t , and dz is a unit normal random variable. The parameter, λ , is an integrating term, which indicates that the prediction error, ε , is introduced by mean reversion of X_t . See Karatzas and Shreve [12] for an alternative derivation of Equations (1) and (2) from Equation (3).

The prediction error, ε , is modeled by specifying the variables given in Equation (2).

In the pricing Equations (1) and (2), the current demeaned stock price, X_t , is a continuous-time first-order autoregressive process (AR (1)). The prediction error, $\lambda \nu$, of the continuous-time AR (1) process is introduced by mean reversion of X_t . This allows the prediction error, $\lambda \nu$, due to mean reversion of X_t to be estimated. Speculation's share of prediction error is $\lambda \nu$.

A first-order autoregression in continuous time generates a discount factor, $e^{-\theta}$.

The current demeaned stock price, X_t , is mean reverting towards zero as shown in

$$dX_t = -\theta X_t dt + \varepsilon. \quad (3)$$

The speed of mean reversion is θ . The discount rate in Equation (1) is equal to the speed at which the current demeaned stock price reverts to its mean. The error, ε , in dX_t is $\nu S_t dz$. X_t is volatile due to volatility, ν . The volatility, ν , is measured by changes in the current demeaned stock price, dX_t . Volatility, ν , is modeled by Equation (3).

Mean reversion is a market process. The integration of $dX_t = -\theta X_t dt + \varepsilon$ results in the continuous-time AR (1) process, as shown in Equations (1) and (2).

Alternatively, the current stock price, S_t , is mean reverting towards μ as shown in

$$dS_t = \theta(\mu - S_t) dt + \varepsilon. \quad (4)$$

The error, ε , in dS_t is $\nu S_t dz$. S_t is volatile due to volatility, ν . Volatility, denoted

as ν , is quantified by the variation in the current stock price, represented by dS_t .

The current demeaned stock price, X_t , and the current stock price, S_t , are both volatile.

The parameter, λ , arising from the mean reversion of X_t , in a formal derivation of the pricing equation for X_t is given as

$$\begin{aligned}\lambda &= \int_{t-1}^t e^{-\theta(t-q)} dz(q) \\ \lambda &= \sqrt{(1 - e^{-2\theta})/2\theta}\end{aligned}\quad (5)$$

Since λ is a function of the speed, θ , of mean reversion of X_t , the prediction error due to mean reversion is $\lambda\nu$. The parameter, λ , identifies the prediction error of the continuous-time AR (1) process as the prediction error due to mean reversion. The net prediction error is $(1-\lambda)\nu$, and the total prediction error is volatility, ν .

The parameter $\lambda < 1$ if X_t is mean reverting and $\lambda = 1$ if X_t is not mean reverting. The current stock price, S_t , is a random walk if $\lambda = 1$.

Speculators drive mean reversion. The prediction error due to speculation is $\lambda\nu$. During the COVID-19 pandemic, the speed, θ , of mean reversion and the prediction error, $\lambda\nu$, due to speculation were exceptionally high (Lee [1]). Mean reversion produces substantial random changes in a current stock price. When speculation is high, mean reversion causes a current stock price to fall and rise sharply.

The lagged demeaned stock price, X_{t-1} , has embedded prediction error. Discounting it removes the embedded prediction error. The discount due to embedded prediction error is $(1 - e^{-\theta})X_{t-1}$. There is no embedded prediction error in the term, $e^{-\theta}X_{t-1}$.

The discounted valuation model given by Equations (1) and (2) works for a dividend-paying firm and a non-dividend paying firm. This is an essential generality.

The pricing equation for X_t in Equations (1) and (2) can be elaborated further as follows:

$$S_t - \mu = e^{-\theta}(S_{t-1} - \mu) + \lambda\nu S_t dz \quad (6)$$

One term of the equity value is $e^{-\theta}(S_{t-1} - \mu)$. Equity value consists of two terms, $\mu + e^{-\theta}(S_{t-1} - \mu)$.

The speed, θ , of mean reversion of X_t as given by $dX_t = -\theta X_t dt + \varepsilon$, determines the discount rate, θ , derived from the discount factor, $e^{-\theta}$. The equity value, $e^{-\theta}(S_{t-1} - \mu)$, is determined by self-valuation.

The lagged stock price, S_{t-1} , was a transaction stock price at $t-1$. $(S_{t-1} - \mu)$ is a forecast in modeling. A future stock price is unknown. The previous transaction stock price is known. The previous transaction stock price provides a reference point for determining the current stock price.

The prediction model is based on two key market mechanisms: forecasting and mean reversion. The residual prediction error is prediction error due to mean re-

version of $(S_t - \mu)$.

The current stock price, S_t , is the sum of equity value and prediction error due to mean reversion.

Estimation

The boundary conditions for the continuous-time AR (1) model given by Equation (6) are that $\theta > 0$ and $\nu > 0$. The parameters $\{\theta, \nu\}$ are parameterized to estimate them in continuous time. The original parameters $\{\theta, \nu\}$ are transformed so that $\theta = e^{-x_1}$ and $\nu = e^{-x_2}$. The mean $\mu = x_3$.

A log-density function is derived from Equation (6).

$$ld = \ln \left\{ \frac{1}{(\lambda \nu S_t \sqrt{2\pi})} \right\} - \frac{\{S_t - \mu - e^{-\theta} (S_{t-1} - \mu)\}^2}{2(\lambda \nu S_t)^2} \quad (7)$$

Maximum likelihood estimates of $\{x_1, x_2, x_3\}$ are obtained by maximizing a log-likelihood function based on the above log-density function. See GAUSS [13] and Maxlik [14].

The original parameters θ and ν are recovered from their exponential functions. For instance, dropping the numbering of x , the point estimate, $\bar{\theta} \cong e^{-\bar{x}}$. The maximum likelihood estimate of x is \bar{x} . The asymptotic standard error of $e^{-\bar{x}}$

is $\left| \frac{d(e^{-\bar{x}})}{d\bar{x}} \right| \times \text{S.E.}(\bar{x})$, where $\text{S.E.}(\bar{x})$ is the standard error of \bar{x} . The t-statistic

for $\bar{\theta}$ is $1/\text{S.E.}(\bar{x})$. By eliminating the approximate mean $e^{-\bar{x}}$, the t-statistic for $\bar{\theta}$ depends on the standard error of the exponent \bar{x} of the mean transformed parameter $e^{-\bar{x}}$.

The exponent x of e^{-x} has a normal distribution with mean \bar{x} and variance σ^2 . In the derivation of a t-statistic for the point estimate $\bar{\theta}$, the numerical approximation of e^{-x} is used because it has a normal distribution with mean $e^{-\bar{x}}$ and variance $e^{-2\bar{x}}\sigma^2$.

3. Data

The historical data for the STOXX Europe 600 index used in this empirical study were obtained online from Yahoo! Finance. The website is <https://ca.finance.yahoo.com/quote/%SESTOXX/history/>. The ticker symbol for the STOXX Europe 600 is ^STOXX.

The STOXX Europe 600 is a comprehensive equity index that represents the prices of common stocks traded on stock exchanges across Europe and the United Kingdom. The samples for testing self-valuation consist of daily closing prices of the STOXX Europe 600, adjusted for stock splits. The data cover three years: 2022, 2023, and 2024.

4. Empirical Results

The empirical results show that European stock markets accurately valued their

listed common stocks in 2022, 2023, and 2024. Speculators help drive closing stock prices towards equilibrium, and these prices exhibit mean reversion, indicating self-valuation. The continuous-time first-order autoregressive model captures mean reversion of a current stock price ($\theta > 0$) and the volatility (λv) accounted for by mean reversion. These two factors explain the dynamics of mean reversion in pricing. Additionally, equity values range from 98.67% to 99.19% of closing prices, indicating the high efficiency of self-valuation. Prediction errors are less than 1.5% of closing prices, falling within a small margin of error. The closing prices of European common stocks were found to be volatile, indicating the inherent risks of investing in common stocks.

The test equation used for the empirical study is Equation (6), which models the current demeaned stock price, $(S_t - \mu)$, as normally distributed. The discount rate, θ , is estimated from the exponent of the autoregressive coefficient, $e^{-\theta}$, while the estimated volatility, v , is measured by changes in the current demeaned stock price, dX_t . The estimated mean, μ , is the average of the current stock price, S_t .

The point estimates of the daily parameters, $\{\theta, v, \mu\}$, of the $e^{-\theta}$ model are shown in **Table 1**. The data consist of daily closing STOXX Europe 600 adjusted for stock splits, covering the years 2022, 2023, and 2024. The daily discount rate, θ , and the daily volatility, v , are restricted from being negative.

Table 1. Estimates of daily parameters for the $e^{-\theta}$ model.

Security	Period	n	θ	v	μ
STOXX 600	2022	254	0.0378 (2.15)	0.0136 (22.12)	434 (39.19)
STOXX 600	2023	251	0.1032 (3.41)	0.0102 (21.27)	456 (167.74)
STOXX 600	2024	250	0.0429 (2.26)	0.0083 (21.88)	507 (86.56)

Notes

Observations are daily ex-dividend closing STOXX Europe 600 adjusted for stock splits. The number of observations is n . The table shows the point estimates of the parameters for the $e^{-\theta}$ model. The daily discount rate is θ and the daily volatility is v . The parameter, μ , is the mean of closing STOXX Europe 600. The t-statistics for the parameter point estimates are shown in parentheses below the corresponding estimates.

The critical value of the t-statistic for θ and v is 1.64 at the 5% level of significance.

The parameters, $\{\theta, v, \mu\}$, shown in **Table 1** are significantly greater than zero, confirming that European stock markets self-value the common stocks listed on their stock exchanges.

The $e^{-\theta}$ model shows that the equity value, $e^{-\theta}(S_{t-1} - \mu)$, is determined by mean reversion and prediction. The discount factor is influenced by mean rever-

sion, and the lagged demeaned stock price is a forecast that is then discounted.

Mean reversion of a current demeaned stock price or a current stock price stabilizes the pricing system. The modeling variable is a current demeaned stock price, which is normally distributed. Estimation theory requires a normally distributed variable with volatility.

Table 1 shows that closing STOXX Europe 600 was volatile in 2022, 2023, and 2024. The t-statistics for volatility, ν , are around 21, which are higher than the critical value of 1.64 at the 5% level of significance. The values of volatility, ν , range from 0.83% to 1.36% of closing STOXX Europe 600, indicating that the volatilities are less than 1½% of closing STOXX Europe 600.

A volatile current stock price shows the common stock is risky. The volatility, ν , is a measure of investment risk. The pricing Equations (1) and (2) have volatility, ν , as a risk measure and do not mention the risk-free interest rate. Covariance of stock return with the return on a market stock portfolio is the conventional measure of investment risk. This risk measure is related to the pricing equation in vogue.

STOXX Europe 600



Figure 1. Changes in the current STOXX Europe 600 on May 12, 2025.

A screenshot of the movements in the current STOXX Europe 600 on May 12, 2025, is shown in **Figure 1**. The chart illustrates the mean reversion of the STOXX Europe 600, with small fluctuations observed throughout the day.

The current stock price, S_t , shown in Equation (4) is mean reverting. The prediction error due to mean reversion is $\lambda \nu$. The net prediction error is $(1 - \lambda) \nu$. The breakdown of prediction errors due to mean reversion and net prediction errors is shown in **Table 2**. The daily prediction errors due to mean reversion range from 0.81% to 1.33% of closing STOXX Europe 600. Prediction errors due to mean reversion are within a small margin of error.

Equity values, $\{\mu + e^{-\theta}(S_{t-1} - \mu)\}$, range from 98.67% to 99.19% of closing STOXX Europe 600, indicating the high efficiency of self-valuation.

Table 2. Breakdown of prediction errors.

Security	Period	n	λ	$\lambda\nu$	$(1-\lambda)\nu$
STOXX 600	2022	254	0.9812	0.0133	0.0003
STOXX 600	2023	251	0.9505	0.0096	0.0006
STOXX 600	2024	250	0.9789	0.0081	0.0002

Notes

Observations are daily ex-dividend closing STOXX Europe 600 adjusted for stock splits. The number of observations is n . The parameter, λ , is calculated from Equation (5). The prediction error due to mean reversion is $\lambda\nu$. The net prediction error is $(1 - \lambda)\nu$.

Net prediction errors, $(1 - \lambda)\nu$, are exceptionally low, ranging from 0.02% to 0.06% of the closing STOXX Europe 600 prices. This indicates that the closing STOXX Europe 600 prices often exhibit mean reversion and are influenced by speculative activities.

5. Conclusions

The challenge of deriving an equation for the current stock price has persisted for researchers over the years. Sharpe's [7] model was a pioneering effort in asset pricing, focusing on expected or mean stock return. Subsequent researchers faced difficulties in modeling observable phenomena such as mean reversion and volatility of current stock prices, which fluctuate significantly due to speculation and prediction. Speculation often increases volatility.

Economists struggled to model the discounting process under uncertainty until Lee [1] introduced the concept of a current demeaned stock price as a modeling variable. Lee's model integrates the equation for mean reversion, resulting in a continuous-time first-order autoregressive current demeaned stock price. This approach discounts a lagged demeaned stock price, aligning with the financial principle of valuation by discounting, as recognized by economists like Shiller [15], LeRoy and Porter [16], and Fama and French [6].

In a heuristic derivation, it is demonstrated that the current demeaned stock price is the sum of changing equity value and prediction error due to mean reversion. Changing equity value is a discounted lagged demeaned stock price. This derivation highlights that market mechanisms of mean reversion and prediction are based on real-world events, such as substantial rises and falls in current stock prices and constant changes. Empirical results support mean reversion and prediction, indicating that the current stock price is influenced by both processes, independent of stock portfolio diversification. The risk of investing in a firm's common stock is volatility, but mean reversion ensures a stable pricing system,

ruling out a random-walk model. Market speculation often causes the current stock price to revert to its mean.

The parameters, $\{\theta, \nu, \mu\}$, of the pricing equation were estimated for daily closing demeaned STOXX Europe 600 for 2022, 2023, and 2024. The parameters were significantly different from zero, confirming the pricing equation for European common stocks. European stock markets self-value the common stocks listed on their stock exchanges.

The daily prediction errors caused by mean reversion fluctuate between 0.81% and 1.33% of the daily closing value of the STOXX Europe 600. These errors remain within a narrow range, indicating minor prediction variations. This suggests that the self-valuation of European common stocks is highly efficient.

Conflicts of Interest

The author declares no conflicts of interest.

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