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Assessing the Value Generated by Investor Discussion Boards for Short-Term Traders

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Abstract

Investor discussion boards (IDBs), such as Stocktwits and TheLion, are widely used by short-term traders. This paper adopts a novel approach to assess the value that IDBs can provide to these traders. Our model identifies four key elements that influence this value under specific assumptions: 1) the board's reputation mechanism, 2) the posters' posting rate, 3) the posters' learning ability, and 4) the board's poster growth rate. While the reputation mechanism determines the direction of the value, the combined impact of all four elements determines its magnitude. Understanding this value has significant implications for practitioners, researchers, and policymakers.

Keywords

Online Trading, Online Investors, Social Network, Investor Discussion Board, Reputation Mechanism

1. Introduction

Social networks, also known as new media, consist of internet-based applications that utilize the technological foundations of Web 2.0. These platforms allow users to create, share, and exchange information and ideas within virtual communities. The rapid expansion and widespread adoption of the internet have fundamentally transformed the way we interact with the market, significantly impacting society. Social networks have gained popularity and have had a notable influence on various aspects of our economy, including commerce, marketing, investing, and customer relations, among others [1].

As a special form of social networks, investor discussion boards (IDB, hereafter) play a critical role in securities trading and investment, especially for retail investors. Over the past two decades, we have seen a surge in electronic trading, a trend expected to continue. The rapid growth of IDBs like Reddit, Stocktwits, and Thelion, along with other electronic platforms for investors to share market information, highlights the increasing interest in electronic securities trading [2]. Not only has there been an increase in IDBs, but the number of users on these platforms has also grown significantly [3]. For example, in 2021, many retail online investors gathered on Reddit's WallStreetBets discussion board to challenge short-selling hedge funds and drive the shares of GameStop (GME) and several other "meme stocks" on a rollercoaster ride [4]. Consequently, the Securities and Exchange Commission (SEC), the Federal Trade Commission (FTC), and the Commodity Futures Trading Commission (CFTC) are particularly interested in monitoring activities on these social networks to protect investors' interests[1]. In the wake of the GME rally, hedge funds have intensified their scrutiny of online social media platforms to monitor retail traders' investment trends. With fears over further short squeezes, these funds have made substantial cuts to their short positions. This development has firmly established the influence of online retail investors, signalling a paradigm shift that Wall Street cannot afford to disregard.

Understanding how to measure the value that short-term traders can gain from the information shared on these platforms would allow us to assess their popularity among online investors, membership retention and survival rates, and their potential value in mergers and acquisitions. While IDBs have been around for some time, questions remain about how to quantify the value these platforms can generate for investors.

We fill the literature by proposing a closed-form equation to gauge the value that a platform can generate for short-term traders. According to our equation, the value that IDB can generate for short-term traders is determined by four board-level elements with certain assumptions: (i) the board's reputation mechanism, (ii) the posters' posting rate, (iii) the posters' learning ability, and (iv) the board's poster growth rate. Notably, the reputation mechanism alone determines the direction of the value (positive or negative), while all four elements together determine its magnitude.

The rest of this paper is structured into five sections. Section 2 provides a literature review, discussing relevant studies. The theoretical models are presented in Section 3, followed by discussions on related issues, which are stated in Section 4. Finally, the paper concludes in Section 5.

2. Related Work

The first comprehensive study on online IDBs was conducted by Peter D. Wysocki in 1998. In this study, Wysocki investigates whether changes in message-posting

[1]There are numerous cases related to online securities fraud. For instance, in September 2000, the SEC alleged that a 15-year-old individual used internet stock message boards to talk up, or manipulate, stock prices, and then unload his positions in a classic "pump-and-dump" operation. In 2007, in a lawsuit trying to block the Whole Foods grocery chain from acquiring Wild Oats Markets, the Federal Trade Commission (FTC) alleged that John Mackey, the CEO of Whole Foods, made anonymous attacks on Wild Oats Markets via internet message boards to push down its price so that Whole Foods could acquire it at a lower price.

volume are merely random fluctuations or if they are connected to underlying firm characteristics and stock market activity. His findings indicate that variations in overnight posting activity contain valuable information. Specifically, overnight message-posting volume can forecast changes in the next-day stock trading volume and returns, see [5].

Antweiler and Frank conduct the first study using a text classifier to quantify investor sentiments from online discussions, see [6]. Their research shows that the direction of trading volatility is mainly from online forums to the financial market, suggesting that online conversations help predict market volatility. Additionally, they find that increased disagreement among participants in online discussions is correlated with higher trading volume, and the volume of postings predicts subsequent negative returns. Consequently, they conclude that online discussions are not just noise but contain valuable information for investors. Das *et al.* also use language-processing algorithms to evaluate the intensity and dispersion of sentiment expressed by online participants, see [7]. Their study shows a significant correlation between posters' sentiment levels, posting activity, stock returns, and trading volumes. Specifically, they suggest a positive association between investor sentiment expressed online and current trading volume.

Heimer and Simon conduct a study that, while not directly using data from Facebook, focuses on a social networking website like Facebook, see [8]. They collect comprehensive trading records and communications from thousands of traders. Their findings suggest that within the trading community, individual investors are more likely to communicate with other investors who have achieved higher returns. Furthermore, upon learning about favorable returns, these investors tend to increase their trading activity. Park *et al.* conduct a study examining the impact of information from Naver.com on investors' trading decisions and investment performance, see [9]. Their findings suggest that active participation on the online discussion board is associated with a higher likelihood of making investment mistakes, which ultimately negatively affects investors' overall investment performance. In addition, SeekingAlpha.com, one of the largest investment-focused social networks in the United States, allows retail investors to contribute posts and publish their own articles. Chen *et al.* conduct a study and find that articles and comments on Seeking Alpha can predict future stock prices and earnings surprises across all time frames examined, ranging from one month to three years, see [10].

Using posts from HotCopper internet stock message board, Leung and Ton assess the influence of message board activity on the ASX financial market, see [11]. They document that small-cap stocks are significantly affected by message board activity, with increased posting correlating with positive abnormal returns and sustained trading volume. Conversely, large-cap stocks show no significant impact. Renault uses data from the microblogging platform StockTwits and provides empirical evidence that online investor sentiment helps forecast intraday stock index returns, see [12]. The study shows that the initial change in investor sentiment within the first half-hour predicts the subsequent return of the S&P 500 index ETF during the final half-hour of trading. Bartov *et al.* finds that the collective opinion derived from individual tweets on Twitter effectively predicts a firm's upcoming quarterly earnings and announcement returns, see [13]. These findings highlight the importance of considering the overall sentiment expressed in individual tweets when evaluating a stock's potential prospects and value.

More recently, Cookson and Niessner study the disagreement among online investors and introduce a novel measure of disagreement derived from the sentiment of retail investors on StockTwits, see [14]. It demonstrates that this measure correlates more strongly with abnormal trading volume compared to other alternatives, aligning with foundational theories of disagreement and trading. Despite its influence on trading activity, this disagreement measure doesn't impact stock returns. Also using data from StockTwits, Cookson *et al.* reveal selective exposure among 400,000 online users, with bullish individuals being five times more inclined to follow others sharing bullish sentiments about a stock, see [15]. This creates echo chambers, where bulls encounter significantly more bullish messages and fewer bearish ones compared to bears over a 50-day period. Finally, Hirshleifer *et al.* examine the impact of social networks on market reactions to earnings announcement news by analyzing data from StockTwits messages and household trading records, see [16]. Their findings reveal that increased social connectedness leads to more timely incorporation of news into stock prices, but it also results in greater opinion divergence and excessive trading.

Despite the first study on IDBs being published over two decades ago, academic research on the subject remains relatively new and continues to evolve. Currently, only a limited number of studies directly focus on IDBs, attracting interest from scholars in fields such as computer science, management information systems, finance, and applied statistics. However, to our knowledge, since Peter D. Wysocki's 1998 study, literature has lacked a theoretical framework for measuring the value that an online IDB can provide for investors.

3. Theoretical Framework

We develop our theoretical framework by first measuring the value that a single message can generate. Next, we model the value each poster contributes through his or her messages. Since the IDB consists of active posters, we ultimately model the overall value the platform generates for short-term traders.

3.1. The Value a Message *m* Can Generate: w_t^m

We start our theoretical framework by assuming that a message posted on the IDB includes an identification of the asset being discussed, like a stock symbol, along with an opinion expressed by the poster or assigned by a third-party agent[2]. The opinion, such as "Strong Buy" or "Short", reflects the poster's sentiment and at-

[2]For example, a text classifier uses its algorithm to assign a sentiment score to a non-self-disclosed online message. Details of text classifiers are discussed in Antweiler and Frank's study [6].

tempt to forecast the future movement of the underlying asset's price, often referred to as expected return[3].

To measure the value a message could generate at a given time t, denoted as w_t^m, we assign a unit value of +1 if the sentiment expressed in the message correctly predicts the subsequent one-period return of the asset. Conversely, if the sentiment is incorrect, we assign a unit value of −1. The term "subsequent one-period" can vary depending on how the platform's developer sets it, how the researcher defines it, or how the discussion board user perceives it. Although it can span any timeframe, previous studies predominantly employ one trading day as one period[4].

To illustrate, consider the following scenario: If a message expresses optimistic sentiment and the subsequent stock price indeed rises, the message provides certain value to traders, so that the value assigned to that message would be +1. This indicates that any investor who followed this sentiment and purchased the security after reading the message would have made a profitable trade. Similarly, if a pessimistic sentiment aligns with a subsequent decrease in the stock price, this also yields a positive value. In this case, investors who heeded the sentiment and shorted (sold) the stock would have made money (or avoided losses). However, in a rare case where a message conveys neutral sentiment and the subsequent stock price remains unchanged, the sentiment is also considered correct, resulting in a positive value for that message because the follower who did not trade would save money for other better opportunities. Neutral sentiment is considered incorrect if the subsequent stock price changes significantly, either upward or downward, as the follower would miss the opportunity to make a profitable trade using either a long or short position. Thus, we list the assigned values as follows:

$$w_t^m = \begin{cases} +1, & \text{if Sentiment is } Optimistic_t \text{, and } R_{t+1} > 0 \\ +1, & \text{if Sentiment is } Pessimistic_t \text{, and } R_{t+1} < 0 \\ +1, & \text{if Sentiment is } Neutral_t \text{, and } R_{t+1} = 0 \\ -1, & \text{if Sentiment is } Optimistic_t \text{, and } R_{t+1} < 0 \\ -1, & \text{if Sentiment is } Pessimistic_t \text{, and } R_{t+1} > 0 \\ -1, & \text{if Sentiment is } Neutral_t \text{, and } R_{t+1} \neq 0 \end{cases} . \quad (1)$$

In Equation (1), R_{t+1} represents the subsequent return of the security after the sentiment was posted at time t. It is important to note that an irrelevant message is considered to have little value[5]. As a result, irrelevant messages are generally discouraged, and forum administrators often take necessary actions to remove

[3]Many platforms provide a sentiment indicator for posters to explicitly disclose their sentiment on a voluntary basis. For instance, Yahoo! Finance allows a poster to choose one of the following sentiments: Strong Buy, Buy, Hold, Sell, Strong Sell, or not disclose (by default). TheLion.com offers two more sentiments: Short and Scalp. The Short sentiment suggests short selling while the Scalp sentiment implies buying and selling quickly with the intent of a day-trade profit but without any specific sentiment. Raging Bull also allows their posters to specify his or her short-term and long-term sentiments, see [17].

[4]Prior literature indicating that online posters' sentiments have predictive power for up to two trading days, see [18] and [19].

[5]A user can always "Report Abuse" to the administrator if the message contains inappropriate content.

them. We do not consider such messages in our model.

3.2. The Value a Poster *A* Can Generate: w_t^A

To clarify, we must first differentiate between posters or authors, users or participants, and online traders. A poster or author, denoted as *A*, is an individual who actively contributes to the IDB by posting messages in text form. This is distinct from a viewer, who consumes content by reading messages but does not post. The term user or member encompasses both posters and viewers, representing the broader community of platform participants. In contrast, online traders refer to individuals who make trades based on the information available on the IDB. Online IDB traders may be posters, who both post and trade, or viewers, who trade without posting. Therefore, IDB traders are a subset of users who utilize the platform's information to inform their trading decisions. This distinction is critical for understanding the dynamics and value generation within IDBs.

The contribution of poster *A* is evaluated based on the value of total messages he or she has posted on IDB at time *t*. Let us assume that poster *A* has posted a total of *m* messages at time *t*, out of which *p* messages were correct and each yielded a value of +1, while *q* messages were incorrect, and each had a value of −1. Hence, we can express this relationship as follows:

$$m_t^A = p_t^A + q_t^A \tag{2}$$

As an example, poster *A* has posted a total of 10 messages at time *t*, out of which 6 were determined to be correct and 4 were wrong later at time *t* + 1. Consequently, the accuracy of poster *A*'s sentiment forecast accuracy is calculated as 6/10 = 0.6, or 60 percent. The general formula for measuring the probability of poster *A*'s forecast accuracy at time *t*, denoted as $Pr_{Right}^{A,t}$, is expressed as follows:

$$Pr_{Right}^{A,t} = \frac{p_t^A}{p_t^A + q_t^A} = \frac{p_t^A}{m_t^A} \tag{3}$$

Based on $Pr_{Right}^{A,t}$, we define two types of posters or authors: superior authors (SA) and inferior authors (IA). In a random process with two possible outcomes, 50% accuracy is expected. However, SA exhibits forecast accuracy higher than 0.5, resulting in a positive contribution to the community[6]. Conversely, any poster with a forecast accuracy not higher than 0.5 is categorized as an IA. As a result, IAs either have no or negative contribution to the community. We then propose the following formula to measure the net contribution of *A* at time *t*:

$$w_t^A = right_t^A - wrong_t^A = Pr_{Right}^{A,t} \cdot m_t^A - \left(1 - Pr_{Right}^{A,t}\right) m_t^A = \left(2Pr_{Right}^{A,t} - 1\right) m_t^A \tag{4}$$

For example, 6 correct and 4 wrong postings would yield a net value of 2 units which can be calculated as (2 * 0.6 − 1) * 10 = 2. Initially, we assume that there are

[6]According to [20], opinion leaders voluntarily disclose their sentiments in their messages. Audiences who trust these sentiments are likely to be influenced by the opinion leaders and may adjust their investment decisions accordingly. An opinion leader or influential poster may or may not be a superior author (SA). It depends on whether the discussion board is effective (leading to a virtuous cycle) or ineffective (leading to a vicious cycle).

no costs associated with posting messages online. However, it is important to acknowledge that posters typically incur various tangible and/or intangible costs, including research expenses, personal time, membership fees, information-acquiring expenses, and opportunity costs. Therefore, the net contribution of a poster could be expanded to include these costs:

$$w_t^A = right_t^A - wrong_t^A - cost_t^A \tag{5}$$

where $cost_t^A$ represents the collective costs incurred by poster *A* at time *t*. We will discuss Equation (5) further in Section 4.3. To keep it simple here, we use Equation (4) in which SA would yield a positive value if $Pr_{Right}^{A,t} > 0.5$, while IA would have a negative or zero value if $Pr_{Right}^{A,t} \le 0.5$.

Next, we discuss the variables m_t^A and $Pr_{Right}^{A,t}$ in Equation (4). The variable m_t^A represents the total number of messages posted by poster *A* at time *t*. It is evident that m_t^A is a function of time *t* and must be a nonnegative integer {0, 1, 2, 3, ...}. To begin, we will consider a simple linear relationship between m_t^A and time *t*, which can be expressed as (nonlinear models are discussed in Section 4.1 below):

$$m_t^A = m_i^A + b_A t \tag{6}$$

where m_i^A represents poster *A*'s number of messages posted at the time of initial registration, implying that $m_i^A = 0$. Here, b_A signifies poster *A*'s posting rate on the board, and b_A is set to be a positive value[7].

By taking the first derivative with respect to time *t*, we obtain $b_A = \frac{\mathrm{d}m_t^A}{\mathrm{d}t}$, which represents poster *A*'s marginal posting rate. To keep it simple here, this marginal posting rate is assumed to be constant, as it is dependent on the poster's posting habits (nonlinear models are discussed in Section 4.1 below). Substituting Equation (6) into Equation (4), we have the following expression:

$$w_t^A = \left(2Pr_{Right}^{A,t} - 1\right) b_A t \tag{7}$$

Next, we introduce the concept of rank or rating, denoted as *r*, which serves as a measure of a poster's reputation on a IDB platform *F*. Intuitively, a higher rank corresponds to a higher forecast accuracy for the poster on the board. As we are aware, a random process would yield an accuracy of 50% when there are two possible outcomes (right or wrong), we define $Pr_{Right}^{A,t}$ is a function of rank *r*, expressed as:

$$Pr_{Right}^{A,t} = 0.5 + a_F r_t^A \tag{8}$$

where Equation (8) can be interpreted as $Pr_{Right}^{A,t}$ = (random accuracy) + (forecast ability) by poster *A* at time *t*. The threshold value of 0.5 is important, as any probability greater than 0.5 would yield a positive value in Equation (7). For a new author, their rank is initially set to 0, meaning their accuracy is not different from

[7]It is possible for a poster to have one or more messages deleted by the administrator due to various reasons. However, it is unlikely for a poster's messages to be consistently deleted over a considerable amount of time by the administrator because this type of poster would be blocked by the administrator in the first place. Thus, we do not consider a case with a negative b_A in the model.

the random accuracy of 0.5, at least from the view of other existing users. The coefficient, a_F, signifies the effectiveness of a reputation mechanism employed by the board *F*.

By taking the first derivative with respect to rank *r* in Equation (8), we obtain $a_F = \frac{\mathrm{d}Pr_{Right}}{\mathrm{d}r}$, which represents the marginal forecast accuracy per unit of increased rank. It is important to note that a_F is a board-level mechanism which applies to the entire board *F*, not just a single poster. Whether to assign a higher rank based on forecast accuracy is determined by the system or algorithm. Each IDB is different and may have its own reputation mechanism that associates with a unique a_F. We argue that if $a_F > 0$, the IDB is effective and generates positive value for short-term traders. A positive value implies that the platform assigns higher ranks to authors with higher forecast accuracy. With an effective reputation mechanism in place, information asymmetry and adverse selection can be reduced. A negative a_F implies that higher ranks assigned by the platform are associated with lower forecast accuracy. Conversely, if $a_F \le 0$, the board *F* would be considered ineffective and produce negative or no value for short-term traders[8].

Like b_A in Equation (6), a_F is also assumed to be a constant determined by the reputation mechanism employed by board *F*. It is worth noting that different discussion boards may adopt different reputation mechanisms, some provide positive value (flourishing) and others provide negative value (withering), depending on how their rating systems function in practice. Thus, a_F plays a crucial role in determining the fate of the board. Substituting Equation (8) into Equation (7), we obtain the following expression:

$$w_t^A = \left[2\times\left(0.5 + a_F r_t^A\right) - 1\right] b_A t = 2a_F r_t^A b_A t \tag{9}$$

where it is evident that $a_F > 0 \Rightarrow w_t^A \ge 0$ given r_t^A is a nonnegative term, b_A is set to be a positive and time *t* is nonnegative. Therefore, $a_F > 0$ implies a virtuous cycle for board *F*, indicating it generates positive value. On the other hand, $a_F \le 0$ leads to a vicious cycle, indicating it generates negative value. We will further discuss the concepts of virtuous and vicious cycles in Section 4.2.

We additionally propose that rank, r_t^A, itself is a function of time *t*. While the coefficients a_F and b_A are assumed to be constant in Equation (9), r_t^A is a variable. It is reasonable to assume that a poster's rank generally increases over time. This is because posters possess the ability to learn from experienced peers and their own past investing experiences, see [8] and [21]. We propose a simple linear relationship between r_t^A and time *t* (nonlinear models are discussed in Section 4.1 below):

$$r_t^A = r_i^A + c_A t \tag{10}$$

where $r_i^A = 0$ because a poster is initially assigned no rank upon registering on

[8]A reputation mechanism could be implemented either mandatorily by the forum designer/creator or voluntarily by all participants within the online community. An effective voluntary reputation mechanism would occur when a poster makes a correct forecast and other participants or the system grant him or her positive credit/reviews, such as the rating system used by Ebay.com and Thelion.com.

the board *F*. It is important to note that rank r_t^A must be a nonnegative integer. The term c_A represents poster *A*'s credit-building rate, which is determined by the poster's learning ability[9]. We assert that c_A is a positive constant because it would be counterintuitive for a poster, in a competitive environment, to become more skillful and knowledgeable yet deteriorate in forecast accuracy[10].

In Equation (10), by taking the first derivative with respect to time *t*, we obtain $c_A = \frac{\mathrm{d}r_t^A}{\mathrm{d}t}$, which represents *A*'s marginal learning ability or rate of rank growth. Substituting Equation (10) into Equation (9) gives us the value of poster *A* at time *t* as:

$$w_t^A = 2a_F b_A c_A t^2 \tag{11}$$

where a_F represents the reputation mechanism implemented by board *F*, b_A denotes *A*'s marginal posting rate, and c_A signifies *A*'s marginal learning ability. Therefore, a_F, b_A, and c_A are all constants in Equation (11), illustrating that *A*'s contribution is a quadratic function of time *t*. Since b_A and c_A are both nonnegative, the sign of w_t^A is solely determined by the sign of a_F. A positive (negative) value of a_F implies the reputation mechanism employed by board *F* is effective (ineffective) where higher rank aligns with higher (lower) forecast accuracy.

3.3. The Total Value Generated by Board *F*: W_t^F

Equation (11) formulates the value generated by a single poster at time *t*. We further assume there are N_t posters at time *t*, and if all posters have the same duration of activity as the board *F* (assuming all posters registered at the beginning of the board when $t = 0$, and no new posters have been added since), we will have:

$$W_t^F = \sum_{A=1}^{N_t} w_t^A = \sum_{A=1}^{N_t} 2a_F b_A c_A t^2 = 2a_F t^2 \sum_{A=1}^{N_t} b_A c_A \tag{12}$$

We will later relax the assumption of equal activity duration for all posters. First, let us discuss the value W_t^F generated by board *F* under two scenarios: (1) when $a_F > 0$ and (2) when $a_F \le 0$ [11]. In the scenario where $a_F > 0$, indicating an effective board that accurately rewards posters' forecast accuracy, we assume that all posters will eventually converge to the board-level posting rate, b_F, and to the board-level learning ability, c_F, under a virtuous cycle[12]. Considering N_t as the total number of posters on board *F* at time *t*, we therefore have:

[9]A new registered author is given the lowest threshold rank $r = 0$. Negative rank is not considered because any poster who has a negative rank attracts no follower and therefore would alternatively reregister a new account with a different avatar and start it over again.

[10]Note that b_A and c_A are assumed to be uncorrelated because b_A is based on the poster's posting habit, while c_A is based on the poster's forecasting ability.

[11]The case when $a_F \le 0$ is covered in Section 4.2.

[12]We acknowledge the limitation of assuming a uniform board-level posting rate and learning ability across all posters. This assumption implies that, in an ideal scenario, all posters will eventually converge to a common level of engagement and knowledge acquisition. In a more realistic scenario, the posting rate could reflect an average rate among posters, while learning ability might also be based on an average level across the poster base.

$$b_1 = b_2 = \cdots = b_N = b_F \text{, and } c_1 = c_2 = \cdots = c_N = c_F \text{,} \tag{13}$$

and

$$b_1 c_1 = b_2 c_2 = \cdots = b_N c_N = b_F c_F \text{,} \tag{14}$$

thus, we also have $b_F = \frac{\sum_{A=1}^{N} b_A}{N}$ and $c_F = \frac{\sum_{A=1}^{N} c_A}{N}$. Therefore, we have the following expression:

$$b_F \cdot c_F = \frac{\sum_{A=1}^{N} b_A}{N} \cdot \frac{\sum_{A=1}^{N} c_A}{N} = \frac{N b_F}{N} \cdot \frac{N c_F}{N} \tag{15}$$

thus, the total value the board F can generate is rewritten as:

$$W_t^F = \sum_{A=1}^{N_t} w_t^A = 2a_F t^2 \left(b_1 c_1 + b_2 c_2 + \cdots + b_N c_N\right) = 2a_F t^2 N_t b_F c_F \tag{16}$$

Now, let us assume the number of posters is a linear function of time given by:

$$N_t = N_0 + d_F t \tag{17}$$

where the initial value N_0 is set to 0 due to no poster initially. Thus, at time t, under an unrealistic scenario, if all posters registered at the debut of board F and no new poster thereafter, the total value generated by board F at time t would be:

$$W_t^F = 2a_F b_F c_F t^2 \cdot d_F t = 2a_F b_F c_F d_F t^3 \tag{18}$$

in which we can see that at the very beginning of the board, when $t = 0$, there were no members, so the initial value was zero. Based on Equation (18), the total value generated by board F is a cubic function of time and increases as t gets larger.

However, not all posters have the same durations of activity. Since they join the board at different times, each poster has a different time variable or duration of activity. Some posters have shorter durations of activity or registered later, while others have longer durations due to earlier registration. According to Equation (17), there are d_F additional posters at each incremental time interval. Meanwhile, the longer the duration of activity, the higher the poster's contribution or value w_t^A, as stated in Equation (11). Thus, the total value board F can generate at a specific time t is as follows (see Appendix 1 for details of the derivation):

$$W_t^F = 2a_F b_F c_F d_F \cdot \frac{t \cdot (t+1) \cdot (2t+1)}{6} \tag{19}$$

where t is known, it represents the duration from the beginning of board F to current. Thus, we can predict the expected total value board F will generate at any future time $t + n$ (n represents the number of periods after current time t) as:

$$W_{t+n}^F = \frac{a_F b_F c_F d_F}{3} \cdot (t+n) \cdot \left[(t+n)+1\right] \cdot \left[2(t+n)+1\right] \tag{20}$$

Thus, the present value of W_{t+n}^F is calculated as $\frac{W_{t+n}^F}{(1+k)^n}$. Therefore, the sum of all future values that board F will generate, discounted to the present time t using a discount rate k, would be (excluding current value W_t^F):

$$PV_t^F = \sum_{n=1}^{+\infty} \frac{a_F b_F c_F d_F}{3} \cdot (t+n)(t+n+1)(2t+2n+1) \cdot \frac{1}{(1+k)^n} \tag{21}$$

where PV_t^F is the present value at time t based on all discounted future values that board F will generate. Again, time t is known in the equation representing the duration of the board from its debut to current, n ranges from 1 to infinity to represent future period (if $n = 0$, $PV_t^F = W_t^F$), k is the discount rate, representing the short-term traders' required rate of return. Firstly, we can prove that this series converges as n approaches infinity (see Appendix 2 for the proof). Secondly, the closed-form solution for Equation (21) is as follows (see Appendix 3 for derivation):

$$PV_t^F = \frac{a_F b_F c_F d_F}{3} \cdot \left(\frac{2t^3 + 9t^2 + 13t + 6}{k} + \frac{6t^2 + 24t + 24}{k^2} + \frac{12t + 30}{k^3} + \frac{12}{k^4} \right) \tag{22}$$

where a_F, b_F, c_F and d_F are constant values that can be estimated through empirical analysis using statistical methods.

Continuous compounding represents the ultimate mathematical boundary for compound interest, achieved if it were calculated and reinvested into an investor's account balance over an infinite number of periods. Although this is infeasible in real-world scenarios, the idea of continuously compounded interest is crucial as it provides the lowest present value that board F would generate at the current time based on all discounted future values. Therefore, the equation below offers a method to gauge the minimum theoretical present value, assuming it continuously generates value for its short-term investors and e is the Euler's number ≈ 2.718 (please see Appendix 4 for derivation):

$$\min\left(PV_t^F\right) = \frac{a_F b_F c_F d_F}{3} \cdot \left[\frac{2e^{3k} + 8e^{2k} + 2e^k}{\left(e^k - 1\right)^4} + \frac{(6t+3)\cdot\left(e^{2k} + e^k\right)}{\left(e^k - 1\right)^3} + \frac{\left(6t^2 + 6t + 1\right)\cdot e^k}{\left(e^k - 1\right)^2} + \frac{2t^3 + 3t^2 + t}{e^k - 1} \right] \tag{23}$$

In summary, with some assumptions, the value that an IDB can generate for short-term traders is determined by four board-level elements besides time t: i) a_F, which represents the board's reputation mechanism; ii) b_F, a nonnegative number indicating the posting rate of all posters; iii) c_F, a nonnegative number representing the learning ability of all posters; iv) d_F, a nonnegative number signifying the board's poster growth rate. With these four elements, we can further calculate the value that the board can generate at any specific time and its present value at current. Importantly, all four board-level elements can be estimated through empirical analysis employing statistical methods.

It is clear that a_F determines the sign of the value, while all four elements jointly determine the magnitude of the value. The proposed model outlined in this study carries various practical implications. The model provides insights into the construction and maintenance of an effective discussion board. It suggests that a reputation mechanism can be implemented not only by peer users voluntarily (as observed on platforms like eBay.com and TheLion.com) but also by the platform designer, particularly in the context of IDBs (such as Motley Fool's CAPS system, see [1]).

Not all IDBs have delivered positive value; some, such as Big-Boards.com, Cake Financial, Currensee, and Stockpickr, eventually disappeared due to low-quality information and declining posting activity. Among the remaining active boards, certain platforms are more active or popular than others. Our model provides a theoretical explanation for these observed phenomena and identifies the key factors that influence platform success and longevity.

4. Discussion

4.1. Nonlinear Setup

First, several researchers have demonstrated the exponential growth of online activities, coinciding with the exponential growth of internet usage and internet-based security trading, see [3] and [22]. Consequently, the linear functions presented in Equation (6) can be substituted with a nonlinear exponential function represented by $m_t^A = \left(b'_A\right)^t$, where b' signifies the willingness of all posters to post messages. It is assumed that $b'_A > 1$.

Similarly, Equation (10) can be replaced with $r_t^A = \left(c'_A\right)^t$ where c' represents the marginal learning ability among all posters, and it is assumed that $c'_A > 1$. The time variable, *t*, is an integer ranging from 0 to positive infinity. Consequently, the value a poster can provide in Equation (11) would be expressed as:

$$w_t^A = 2a_F\left(b'_A\right)^t\left(c'_A\right)^t = 2a_F\left(b'_A c'_A\right)^t \tag{24}$$

Secondly, in an alternative nonlinear configuration, Equation (6) can be substituted with a more conservative diminishing marginal utility function $m_t^A = b''_A\sqrt{t}$, where b'' remains as the willingness of all posters to leave messages. Equation (10) can also be replaced with $r_t^A = c''_A\sqrt{t}$, where c'' is also the marginal learning ability among all posters. Consequently, the value a poster can provide as presented in Equation (11) would be transformed to:

$$w_t^A = 2a_F b''_A\sqrt{t}c''_A\sqrt{t} = 2a_F b''_A c''_A t \tag{25}$$

Thirdly, if Equation (8) is replaced by a standardized rank measure $Pr_{Right}^{A,t} = 0.5 + a_F\left(\frac{r_t^A}{r_t^A+1}\right)$, where $\frac{r_t^A}{r_t^A+1} \in [0,+1]$ and $r_t^A \in [0,+\infty)$, the Equation (11) becomes:

$$w_t^A = \frac{2a_F b_A c_A t^2}{c_A t + 1} \tag{26}$$

Similarly, Equation (24) can be further rewritten as:

$$w_t^A = \frac{2a_F\left(b'_A c'_A\right)^t}{\left(c'_A\right)^t + 1} = 2a_F\frac{\left(b'_A c'_A\right)^t}{\left(c'_A\right)^t + 1} \tag{27}$$

By the same token, Equation (25) can be further written as:

$$w_t^A = \frac{2a_F b_A'' \sqrt{t} c_A'' \sqrt{t}}{c_A'' \sqrt{t} + 1} = 2a_F \frac{b_A'' c_A''}{c_A'' \sqrt{t} + 1} t \tag{28}$$

A more realistic model would involve nonlinearity, but the previous illustration in Section 3 is presented in a linear form as a simplified foundational model. Importantly, our conclusions remain unchanged in a nonlinear setup, and particularly, it would not affect the significance of a_F in determining the value that an IDB can provide.

4.2. Virtuous versus Vicious Cycles

In Section 3.2, we propose the existence of two types of posters: superior authors (SAs) and inferior authors (IAs). In a virtuous cycle, the reputation of each poster, as measured by their reputation score, becomes public to all participants in the community. With an effective and transparent reputation mechanism in place, a higher rank indicates greater forecasting accuracy, thus fostering a virtuous cycle. Consequently, the long-term evolvement of an IDB under a virtuous cycle can be observed through the following five stages:

1) SAs play a crucial role by recommending securities with explicit sentiments based on their research, see [20]. They aim to build their reputation within the community and profit from the trading activities of other members who act on their recommendations, see [23]. Higher liquidity and/or narrower bid-ask spreads contribute to lower transaction costs, creating additional incentives for SAs, see [24]. On the other hand, IAs struggle to attract followers due to their low reputation. Their recommendations receive little attention from readers, and investors are unwilling to trade based on IA's opinions.

2) The potential for larger trading gains, along with increased liquidity and lower transaction costs, incentivizes more users to share their opinions and recommendations. SAs continue to benefit from their influence, while IAs strive to improve and elevate their status. IAs must build their reputation by consistently providing valuable information to the community. Only valuable and accurate forecasts are rewarded, allowing IAs to earn reputation and eventually transition into SAs. As a result, the board becomes dominated by SAs over time.

3) SAs continue to contribute by posting valuable information, driven by the incentives of higher trading gains and lower transaction costs. Meanwhile, IAs persist in sharing their opinions through legitimate accounts. In this competitive environment, IAs must invest more time and effort into research to improve their forecast accuracy and earn recognition from their audience. IAs acknowledge that with higher ranks, they would attract more viewers. An IA cannot fraudulently become an SA by using methods such as self-promotion or purchasing ranks. Both SAs and IAs compete and have strong incentives to share opinions and provide accurate forecasts, resulting in the prevalence of high-quality messages with high accuracy.

4) The information posted on the board provides significant value and accuracy. As the user base expands, more people are willing to participate and share their opinions and information. The presence of an effective and reliable reputation mechanism ensures that higher-ranked posters produce higher-quality messages and demonstrate greater forecast accuracy. With the publicized rank of each poster, information asymmetry can be mitigated. Messages posted by highly ranked posters reach a larger audience, potentially leading to possible herding behaviours in trading.

5) In the long run, the board succeeds. Abundant high-quality messages and an expanding user base are expected to emerge under a virtuous cycle. SAs dominate the board, while each IA follows a healthy and ambitious learning path. IDB traders generate trading profits by following the recommendations of SAs, resulting in a win-win situation. Over time, the board not only maintains a high membership retention rate but also attracts new members, leading to substantial trading gains for traders. The influx of credible posters further contributes to the board's thriving state. As a result, the value generated by the board remains positive. Under a virtuous cycle, as shown in Equation (19), the positive values of a_F, b_F, c_F, and d_F jointly contribute to the positive value of W_t^F.

In the opposite scenario of a vicious cycle, we assume that each poster's reputation score or credibility is not publicized due to the lack of an effective and transparent reputation mechanism. This makes it unlikely for users to differentiate SAs from IAs. The reputation of each poster cannot be accurately determined, as the information is either unavailable or misled by the ineffective reputation mechanism. Under such conditions, a higher rank does not indicate higher forecast accuracy, leading to a vicious cycle. Therefore, we propose that the long-term evolution of a board under a vicious cycle would go through the following five stages:

1) Both SAs and IAs recommend securities with explicit sentiments based on their opinions. The motivation behind disclosing their sentiments is to build a reputation in the community and hope for larger trading gains from their followers. However, in a vicious cycle, it is impossible to differentiate between SAs and IAs.

2) Without a reputation mechanism, inferior authors (IAs) who put minimal or no effort in research leave recommendations on securities, hoping that viewers will follow their hype. It is common for IAs to register multiple accounts (or bot accounts) to manipulate vulnerable securities, such as those with low trading prices and low institutional holdings. They may also utilize fake accounts to spread false news or misinformation in hopes of maximizing their trading gains.

3) Without the ability to differentiate SAs from IAs, some follower investors are lured into investing in "bad" securities (such as lottery-like stocks, see [25]) recommended by IAs. As a result, SAs do not receive the same level of attention and respect as they would in a virtuous cycle. The liquidity of securities recommended by SAs is not significantly enhanced, and transaction costs remain unchanged or even worse. Consequently, SAs are discouraged from participating and eventually

leave the board, opting to switch to other, better peer platforms. However, IAs, who invest little or no effort with limited forecast ability, dominate the board. Market manipulation becomes more prominent in a vicious cycle, with small-cap or micro-cap securities often becoming victims of the online pump-and-dump scheme.

4) Due to the lack of forecast ability, followers who trade securities based on IAs' recommendations experience losses. With little incentive for SAs to contribute, few high-quality messages are posted. As IAs dominate the board, low-quality messages prevail, resulting in a noisy board with little value. This perpetuates the vicious cycle and creates a "lemon market" scenario in the online financial community, see [26].

5) Over time, traders who follow IAs' hype lose money. Fewer people follow IAs' recommendations, and more users exit the platform with losses. It becomes a lose-lose situation for both SAs and IAs. The lack of SAs and the abundance of IAs render the community ultimately worthless. Consequently, the value that the board provides becomes negative, representing the sum of all losses from traders.

Under a vicious cycle, the parameter a_F is expected to be non-positive due to the ineffectiveness of the board's reputation mechanism. However, the parameter b_F is still expected to be positive due to the influx of postings from IAs. The credit-building rate c_F is also expected to be positive, as the ranks of IAs increase over time, although this increase is incorrectly rewarded by the system. Additionally, the parameter d_F, poster growth rate, is also expected to be positive, even though active IAs dominate the board and SAs become inactive. We assume inactive members do not delete their accounts, even if they no longer use the board. Therefore, the value of W_t^F, as measured by Equation (19), becomes negative over time under such a vicious scenario.

4.3. Moral Hazard

Moral hazard is the risk that one party will take excessive risks because they do not bear the full consequences of their actions, causing others to suffer negative outcomes. A problem of moral hazard may arise if the IDB lacks supervision by its administrator and/or market regulators. Moral hazard can occur in both virtuous and vicious cycles, although it is more prominent in a virtuous cycle. By incorporating posting costs into Equation (4), we obtain the following equation:

$$w_t^A = \left(2Pr_{Right}^{A,t} - 1\right)m_t^A - cost_t^A \tag{29}$$

where $cost_t^A \geq 0$ represents the total costs that poster A has incurred at time t. To maximize w_t^A, poster A would seek to minimize their cost regardless of the sign and magnitude of $\left(2Pr_{Right}^{A,t} - 1\right)m_t^A$. In other words, posters would aim to minimize cost so to maximize w_t^A regardless of their forecast ability.

Any poster can freely and irresponsibly recommend securities without incurring any cost. However, such behavior is unlikely to earn IA a good reputation within the community. Since IAs typically lack followers under a virtuous cycle, the likelihood of moral hazard occurring with them is low. On the other hand, SAs, who are assumed to have higher posting ethics, generally do not engage in price manipulation. However, we cannot eliminate the possibility of moral hazard when an SA or a group of SAs becomes extremely influential in the community.

In a scenario involving the opinion leader effect, where a highly ranked poster becomes extremely influential in the community, their recommendations on any security will attract many follower traders who will trade based on those opinions. This influential poster understands that by focusing on a security that is easily manipulable (such as a stock with a low price per share and low trading volume), they can attract a large following and generate substantial trading profits with minimal risk and negligible costs or consequences. An example is "Roaring Kitty" in the case of GME on the Reddit forum. As a result, certain influential posters may exploit their power to manipulate security prices, see [27]. This gives rise to moral hazard, wherein these influential posters leverage their influence in the community to manipulate vulnerable securities, maximizing their financial profits with minimal or no cost, while their followers bear the costs in the form of financial losses.

Moral hazard leads to market inefficiencies and information asymmetries, posing a threat to the interests of other investors. Influential posters or opinion leaders, seeking to maximize their trading profits while minimizing costs, may especially target vulnerable stocks with weak fundamental and technical aspects, see [18]. Due to the lack of broad ownership and thin floating shares, many low-priced stocks are susceptible to price manipulation, which can be significantly amplified by influential posters, see [5]. In the presence of moral hazard, regulatory bodies such as the SEC and the FTC are expected to intervene. In extreme cases of rampant market manipulation beyond control, the discussion board may face potential shutdown if legal action is taken by policymakers against its users or the discussion board. Instances of lawsuits against stock manipulators provide evidence of this[13]. Therefore, the implementation of internal and external monitoring mechanisms can mitigate the prevalence of moral hazard on investor discussion boards.

4.4. Reputation Mechanism

An effective and reliable reputation mechanism is a game-changer and the most crucial factor in the model. In an anonymous context, a poster's credibility can only be determined through their ranks, ideally provided by a reputation mechanism. An effective reputation mechanism plays a vital role in reducing asymmetric information among board members and preventing adverse selection, contributing

[13]For example, on January 27, 2021, the Discord server associated with WallStreetBets was taken down for violating community guidelines, including hate speech and spreading misinformation, during the GameStop (GME) short squeeze frenzy. Another instance is the action taken by the UK's Financial Conduct Authority (FCA) in 2020, which issued warnings and shut down several websites and forums found to be promoting false or misleading information to manipulate stock prices.

to the development of a sustainable online financial community. In the future, a poster's reputation may not only be generated by peer ratings or reviews (as seen on platforms like TheLion.com and eBay.com) but also by the social network systems (as seen on platforms like Motley Fool CAPS, as discuss by Avery *et al.* [28]).

For instance, it is feasible to implement an algorithm that objectively ranks a poster based on their sentiment about a security and its contemporaneous and/or subsequent performance. Such a rating system would be fairer and more objective than the purely peer-given rating systems commonly used by most IDBs today. In a pure peer-given rating system, there is still a possibility for people to register multiple accounts to promote their primary account, see [29].

When self-promotion takes place, it introduces bias into a poster's credibility, ultimately detrimentally affecting the quality of information and the value of the board. As of today, there are hardly any IDBs that utilize subsequent security returns as a rewarding method, except for the CAPS platform. Therefore, we recommend a hybrid rating system that combines various measures. In this suggested hybrid system, for instance, 60 percent of the reputation score is determined by the accuracy of predictions, 20 percent by the precision of same-day reports, and the remaining 20 percent by peer reviews[14]. Predictive ability is the dominant factor in this system. The specific allocation of 60/20/20 can vary depending on the intentions of the platform developers. As a result, a higher reputation score, indicating higher forecasting ability, would contribute to a virtuous cycle as discussed earlier. In addition, the reputation score could be accompanied by an accuracy percentage, indicating the poster's forecasting accuracy. Ultimately, with an effective reputation mechanism in place, the value generated by an IDB should increase due to the higher quantity and quality of messages and more members. In contrast, a board lacking an effective and reliable reputation mechanism will experience a vicious cycle, resulting in fewer users and eventually leading to inactive boards and potential shutdown. We predict that in the future, any successful investor discussion board will, at a minimum, incorporate an effective reputation mechanism with a positive a_F, which should be measurable and monitored by using statistical methods.

4.5. Herding Behavior

Decades ago, investment professionals often dismissed online investor discussion boards as mere rumors or noise. However, in today's financial landscape, characterized by event-driven trading strategies and herding behaviors, a single online message, such as a tweet from Tesla's CEO Elon Musk, can significantly influence the prices of certain securities[15]. This opinion leader effect is especially pronounced when the poster has a large following and garners widespread attention,

[14]Predictions can be set for one day, multiple days, or even longer terms.

[15]Derwent Capital Markets started a hedge fund in 2012 with $40 million under management and a strategy based on signals derived from Twitter. In 2012, Thomson Reuters MarketPsych Indices (TRMIs), which are based on analysis of news and social media, provided insight into emotion and sentiment associated with specific countries, commodities, currencies, and economic sectors. In 2013, Bloomberg introduced a feature on its financial data terminals that incorporates a stream of relevant Twitter messages delivered to hedge funds, investment banks, and other investors.

see [30]. Another example involves a poster named "Roaring Kitty" who posted bullish commentary about GameStop (GME) on the WallStreetBets discussion board on Reddit.com. This potentially contributed to a significant short squeeze, leading to several hedge funds losing an estimated $5 billion in 2021[16]. Even without influential posters, when numerous posters collaborate and share similar opinions about security, it can create the impression that this sentiment reflects the broader view among investors, as seen with meme stocks like AMC Entertainment Holdings, Inc. (AMC). In such situations, IDB traders often align with the prevailing sentiment and adjust their trading strategies accordingly. This phenomenon is particularly noticeable in online financial communities that attract fresh investors with limited trading experience.

Herding behavior refers to instances when traders make investment decisions based on the actions of others, rather than their own analysis. It is often considered irrational as it ignores fundamental data and can lead to large, unsustainable buy-ins or sell-offs. Social networks, where news and rumors spread quickly and can be magnified by investors' overreactions, often reinforce herding behavior, see [31] [17]. One consequence of herding behavior is that online investors tend to overreact to market news or rumors, causing security prices to deviate further from their fundamental value and become more volatile. In the internet era, information spread quickly and can be exaggerated by the collective actions of online investors on social networks. When a larger number of traders engage in herding, volatility is likely to increase, introducing more uncertainty and trading risk, see [32].

Unarguably, herding behaviour can potentially lead to higher trading liquidity and/or narrower bid-ask spreads, implying lower transaction costs. This is because when many traders make similar trades simultaneously, there is likely an increase in trading volume and liquidity. However, these apparent benefits of herding behavior are relatively small and can be detrimental in the long run, contributing to market inefficiencies, potential price distortions and financial losses.

Numerous studies have observed herding behavior in stocks that receive significant attention on IDBs. Sabherwal *et al.* argue that online message boards can facilitate herding, leading to temporary increases in stock prices followed by reversals, resulting in higher-than-normal trading volume and excess volatility, see [18]. Hirschey *et al.* conduct two studies examining the market responses to stock recommendations posted on The Motley Fool's discussion board, see [33] and [34]. Both studies find evidence of herd-like behavior among online investors and conclude that the stock advice provided on Motley Fool's platform garners more attention and impact than buy-sell recommendations in traditional media. These

[16]"Roaring Kitty" better known as Keith Gill, gained widespread recognition in 2021 for his active presence on the WallStreetBets subreddit and YouTube, advocating for GameStop (GME) with a bullish outlook. Gill frequently shared insights on GameStop's potential for growth, culminating in his testimony before Congress regarding the significant short squeeze in January 2021, orchestrated by a legion of individual investors.

[17]https://www.sec.gov/oiea/investor-alerts-and-bulletins/ib_sentimentinvesting.

findings suggest that IDBs significantly influence investor behavior and market dynamics.

Digital resources and the vast online financial communities offer investors valuable tools for success in stock trading. By engaging with finance-focused social networks, investors access a wealth of information not readily available through other channels. When a significant number of individuals participate in these platforms, the collective knowledge shared becomes immense. Seasoned investors who actively participate in these online communities often learn about attention-worthy securities and market developments before they are widely reported by conventional news outlets. While there are no guarantees of profits, these boards provide valuable market insights, keeping investors well-informed about ongoing events. IDBs not only connect online investors but also enhance their efficiency and profitability. It is worth noting that herding behavior is not inherently detrimental if all participants can make informed decisions and profit accordingly, see [35].

In a virtuous cycle, herding in security trading tends to occur more frequently than in a vicious cycle. This is primarily because messages posted by highly ranked SAs or opinion leaders reach a larger audience, leading to more pronounced herding behaviors in security trading. It is natural to expect that investors are less likely to follow the recommendations of IAs, resulting in reduced instances of herding in a vicious cycle.

5. Concluding Remarks

The internet has profoundly influenced how investors gather information and engage with others. In recent years, several online investor discussion boards have gained popularity, such as Motley Fool, Reddit, Stocktwits, and TheLion among others. However, not all IDBs have provided positive value to their users; some, like Big-Boards.com and Stockpickr, disappeared due to low-quality information and eventually a lack of activity. To understand why some online trading platforms succeed while others fail, it is crucial to develop a theoretical framework for assessing the value these platforms can generate for investors.

To address this gap in literature, we argue that a discussion board will survive and flourish if it can generate positive value; otherwise, it will decline and vanish. According to our model, such value is determined by four elements with certain assumptions: i) the board's reputation mechanism, ii) the poster's posting rate, iii) the poster's learning ability, and iv) the board's poster growth rate. Among these elements, the reputation mechanism plays a crucial role in determining the fate of the board (sign of the value), while all four elements jointly influence the magnitude of the value. Additionally, we propose a closed-form equation to calculate such value the board generates at any given time, as well as its present value.

To relax some of our assumptions, we extend our analysis to non-linear frameworks and demonstrate that the conclusions remain consistent. In our discussion, we compare the virtuous scenario with the vicious scenario, arguing that herding behavior in online trading is more common in a virtuous cycle, while instances of fake news and fake accounts are more prevalent in a vicious cycle. Additionally, market manipulation tends to be more prominent in a vicious cycle. Lastly, while moral hazard can occur in both scenarios, it is more likely to manifest in a virtuous cycle.

Academic research on investor discussion boards has been growing. The ability to estimate the value that these platforms can generate for online traders lays the foundation for future studies and offers practical insights into their survival rate, popularity among online investors, and their impacts on financial markets, among other aspects. The proposed model allows us to compare the values generated by different investor discussion boards, which can further assist in decision-making for mergers and acquisitions. The proposed model can also serve as an initial approach for selecting an online trading platform based on the value it can generate for traders. We believe that gaining a deeper understanding of the value these platforms can generate over time will benefit practitioners, researchers, and policymakers alike.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix 1. Derivation of the W_t^F

According to Equation (18) in the text, at time *t*, if all the posters registered at the debut of the board *F*, the value that board *F* can provide for short-term traders at time *t* would be:

$$W_t^F = 2a_F b_F c_F t^2 \cdot d_F t = 2a_F b_F c_F d_F t^3 \tag{A1-1}$$

However, as we know that not all posters have the same durations of activity. Since they join the board at different times, each poster has a different time variable. Some posters have shorter durations of activity or registered later, while others have longer durations due to earlier registration. According to Equation (17) in the text, there are d_F additional new posters register at each incremental time interval. Meanwhile, the longer the duration of activity (*t*), the higher the poster's value w_t^A, as stated in Equation (11) in the text. For example, at time $t = 1$, all the posters registered at the beginning of the board (when $t = 0$), so the value that the board can generate based on these posters' contributions is as follows, according to Equation (A1-1):

$$W_1^F = 2a_F b_F c_F 1^2 \cdot d_F \tag{A1-2}$$

Then at time $t = 2$, there are $2d_F$ posters, among which d_F are older posters and d_F are newly registered posters. Posters who registered earlier had larger contributions of $2a_F b_F c_F 2^2 d_F$, while newly registered posters had smaller contributions of $2a_F b_F c_F 1^2 d_F$. Thus, the total value that board *F* can generate at $t = 2$ are:

$$W_2^F = 2a_F b_F c_F 2^2 d_F + 2a_F b_F c_F 1^2 d_F \tag{A1-3}$$

Following this logic, we can determine the total value that board *F* can create at a specific time *t* as follows where *t* is a known time at current:

$$W_t^F = 2a_F b_F c_F d_F \cdot \left[t^2 + (t-1)^2 + (t-2)^2 + \cdots + 2^2 + 1^2 \right] \tag{A1-4}$$

Since the sum of squares of *n* natural numbers can be calculated using the formula as:

$$t^2 + (t-1)^2 + (t-2)^2 + \cdots + 2^2 + 1^2 = \frac{t(t+1)(2t+1)}{6} \tag{A1-5}$$

Thus, we have the total value that board *F* can generate at the specific time *t* is as follows:

$$W_t^F = 2a_F b_F c_F d_F \cdot \frac{t(t+1)(2t+1)}{6} = \frac{a_F b_F c_F d_F}{3} \cdot t(t+1)(2t+1) \tag{A1-6}$$

where *t* represents the time from the beginning of board *F* to the current, and *t* is known. Since a_F, b_F, c_F and d_F are constants, we can denote $\theta = \frac{a_F b_F c_F d_F}{3}$. For example, if the current time is the 100th day since board *F* started (assuming each time interval is one day), the total value at the current time *t* is:

$$W_{100}^F = \theta \cdot 100(101)(201) = 2030100 \cdot \theta \tag{A1-7}$$

We can then predict the expected total value created by board F on the next day as follows:

$$W_{101}^{F} = \theta \cdot (100+1)\left[(100+1)+1\right]\left[2(100+1)+1\right] = 2091306 \cdot \theta \quad \text{(A1-8)}$$

And the present value of W_{101}^{F} is $\frac{W_{101}^{F}}{(1+k)^{1}}$ given that k is discount rate. Following this logic, we can predict the expected total value board F can provide for short-term traders at any future time $t+n$ (n represents the number of periods after today) as:

$$W_{t+n}^{F} = \theta \cdot (t+n)(t+n+1)(2t+2n+1) \quad \text{(A1-9)}$$

Thus, the present value of W_{t+n}^{F} is $\frac{W_{t+n}^{F}}{(1+k)^{n}}$. Therefore, the sum of all future values, discounted to the present using a discount rate k, would be (excluding current value W_{t}^{F}):

$$PV_{t}^{F} = \sum_{n=1}^{+\infty} \theta \cdot (t+n)(t+n+1)(2t+2n+1) \cdot \frac{1}{(1+k)^{n}} \quad \text{(A1-10)}$$

where PV_{t}^{F} is the present value generated by board F at time t. Again, time t is known in the equation representing the duration of the board from its debut to current, n ranges from 1 to infinity to represent future time (if $n = 0$, PV_{t}^{F} would be W_{t}^{F}), k is the discount rate, which is assumed to range from 0 to 1, representing the board traders' required rate of return.

Appendix 2. Proof of Convergence

The present value that a discussion board F can generate at time t can be calculated as:

$$PV_{t}^{F} = \sum_{n=1}^{+\infty} \frac{a_F b_F c_F d_F}{3} \cdot (t+n)(t+n+1)(2t+2n+1) \cdot \frac{1}{(1+k)^{n}} \quad \text{(A2-1)}$$

where t represents the time from the board's debut to current and it is known, and n is from 1 to infinity. Since all a_F, b_F, c_F and d_F are constant, we denote $\theta = \frac{a_F b_F c_F d_F}{3}$, denote $x = \frac{1}{1+k}$, where $0 < k < 1$, and denote $a = (t+n)$ so that we have the simplified value equation at time t as:

$$PV_{t}^{F} = \sum_{n=1}^{+\infty} \theta \cdot a(a+1)(2a+1) \cdot x^{n} \quad \text{(A2-2)}$$

where $0 < x < 1$ because $x = \frac{1}{1+k}$ and $0 < k < 1$. Thus, we can prove that this series is convergent as follows:

For testing convergence for $0 < x < 1$, we will apply the Ratio Test, which states that a convergent series must have:

$$\lim_{t \to +\infty} \left| \frac{a_{t+1}}{a_t} \right| < 1 \quad \text{(A2-3)}$$

First, compute a_{t+1} where $a = t + 1$:

$$a_{t+1} = \theta \cdot (t+1)(t+2)(2t+3) \cdot x^{t+1} \tag{A2-4}$$

Second, compute a_t where $a = t$:

$$a_t = \theta \cdot t(t+1)(2t+1) \cdot x^t \tag{A2-5}$$

Third, find the ratio $\frac{a_{t+1}}{a_t}$:

$$\frac{a_{t+1}}{a_t} = \frac{\theta \cdot (t+1)(t+2)(2t+3) \cdot x^{t+1}}{\theta \cdot t(t+1)(2t+1) \cdot x^t} \tag{A2-6}$$

Simplify this equation, we have:

$$\frac{a_{t+1}}{a_t} = \frac{(t+2)(2t+3) \cdot x}{t(2t+1)} \tag{A2-7}$$

For large t, the dominant terms in the numerator and denominator are both t^2, thus we have

$$\frac{a_{t+1}}{a_t} = \frac{(t+2)(2t+3) \cdot x}{t(2t+1)} \approx \frac{2t^2 x}{2t^2} = x \tag{A2-8}$$

Thus,

$$\lim_{t \to +\infty} \left| \frac{a_{t+1}}{a_t} \right| = x < 1 \tag{A2-9}$$

According to the Ratio Test, the series converges if this limit is less than 1, which holds since $0 < x < 1$. Therefore, we conclude that this series converges when n is approaching infinity.

Appendix 3. Total Value at Current

The present value that board F can generate is as follows given k represents the IDB traders' required rate of return:

$$PV_t^F = \sum_{n=1}^{+\infty} \frac{a_F b_F c_F d_F}{3} \cdot (t+n)(t+n+1)(2t+2n+1) \cdot \frac{1}{(1+k)^n} \tag{A3-1}$$

Since all a_F, b_F, c_F and d_F are constant, we can first denote $\theta = \frac{a_F b_F c_F d_F}{3}$, and denote $x = \frac{1}{1+k}$, where $0 < k < 1$, so that we have:

$$\begin{aligned} PV_0^F &= \theta \cdot \sum_{n=1}^{+\infty} (t+n)(t+n+1)(2t+2n+1) x^n \\ &= \theta \cdot \sum_{n=1}^{+\infty} \left[2n^3 + (6t+3)n^2 + (6t^2+6t+1)n + (2t^3+3t^2+t) \right] \cdot x^n \end{aligned} \tag{A3-2}$$

and this series can be broken down into the form:

$$\begin{aligned} PV_0^F = \theta \cdot \Big[& \sum_{n=1}^{+\infty} 2n^3 x^n + \sum_{n=1}^{+\infty} (6t+3) n^2 x^n + \sum_{n=1}^{+\infty} (6t^2+6t+1) n x^n \\ & + \sum_{n=1}^{+\infty} (2t^3+3t^2+t) x^n \Big] \end{aligned} \tag{A3-3}$$

We know the sum of a geometric series $\sum_{n=0}^{+\infty} x^n = \frac{1}{1-x}$ where $|x|<1$, so that

$$\left(\sum_{n=1}^{+\infty} x^n\right) - x^0 = \frac{1}{1-x} - 1 = \frac{x}{1-x} \tag{A3-4}$$

So that we have $\sum_{n=1}^{+\infty} x^n = \frac{x}{1-x}$. Now both sides take the first derivative with respect to *x*, we have:

$$\sum_{n=1}^{+\infty} n \cdot x^{n-1} = \frac{1}{(1-x)^2} \tag{A3-5}$$

then both sides times *x*, we have:

$$\sum_{n=1}^{+\infty} n \cdot x^{n} = \frac{x}{(1-x)^2} \tag{A3-6}$$

then both sides take the first derivative again with respect to *x*, we have:

$$\sum_{n=1}^{+\infty} n^2 \cdot x^{n-1} = \frac{1+x}{(1-x)^3} \tag{A3-7}$$

then both sides times *x* again, we have:

$$\sum_{n=1}^{+\infty} n^2 \cdot x^{n} = \frac{(1+x)x}{(1-x)^3} \tag{A3-8}$$

then both sides take the first derivative again with respect to *x*, we have:

$$\sum_{n=1}^{+\infty} n^3 \cdot x^{n-1} = \frac{1+4x+x^2}{(1-x)^4} \tag{A3-9}$$

finally, both sides times *x* again, we have:

$$\sum_{n=1}^{+\infty} n^3 \cdot x^{n} = \frac{\left(1+4x+x^2\right)x}{(1-x)^4} \tag{A3-10}$$

Now let us use these sums to find the sum of the series and we have:

$$\begin{aligned} PV_0^F = \theta \cdot \Bigg[& 2 \cdot \frac{\left(1+4x+x^2\right)x}{(1-x)^4} + (6t+3) \cdot \frac{(1+x)x}{(1-x)^3} \\ & + \left(6t^2+6t+1\right) \cdot \frac{x}{(1-x)^2} + \left(2t^3+3t^2+t\right) \cdot \frac{x}{1-x} \Bigg] \end{aligned} \tag{A3-11}$$

Now replace *x* with $\frac{1}{1+k}$, the first term can be simplified as:

$$\theta \cdot 2 \cdot \frac{\left[1+4\cdot\left(\frac{1}{1+k}\right)+\left(\frac{1}{1+k}\right)^2\right]\left(\frac{1}{1+k}\right)}{\left[1-\left(\frac{1}{1+k}\right)\right]^4} = \theta \cdot \left(\frac{2}{k^1} + \frac{14}{k^2} + \frac{24}{k^3} + \frac{12}{k^4}\right) \tag{A3-12}$$

And the second term can be simplified as:

$$\theta \cdot (6t+3) \cdot \frac{\left[1+\left(\frac{1}{1+k}\right)\right]\left(\frac{1}{1+k}\right)}{\left[1-\left(\frac{1}{1+k}\right)\right]^3} = \theta \cdot \left(\frac{6t+3}{k^1} + \frac{18t+9}{k^2} + \frac{12t+6}{k^3}\right) \quad \text{(A3-13)}$$

And the third term can be simplified as:

$$\theta \cdot \left(6t^2+6t+1\right) \cdot \frac{\left(\frac{1}{1+k}\right)}{\left[1-\left(\frac{1}{1+k}\right)\right]^2} = \theta \cdot \left(\frac{6t^2+6t+1}{k^1} + \frac{6t^2+6t+1}{k^2}\right) \quad \text{(A3-14)}$$

And the fourth term can be simplified as:

$$\theta \cdot \left(2t^3+3t^2+t\right) \cdot \frac{\left(\frac{1}{1+k}\right)}{1-\left(\frac{1}{1+k}\right)} = \theta \cdot \frac{2t^3+3t^2+t}{k^1} \quad \text{(A3-15)}$$

Thus, adding four terms together, we have:

$$V_0^F = \theta \cdot \left(\frac{2}{k^1} + \frac{14}{k^2} + \frac{24}{k^3} + \frac{12}{k^4} + \frac{6t+3}{k^1} + \frac{18t+9}{k^2} + \frac{12t+6}{k^3} + \frac{6t^2+6t+1}{k^1} + \frac{6t^2+6t+1}{k^2} + \frac{2t^3+3t^2+t}{k^1}\right) \quad \text{(A3-16)}$$

Thus, we have the final simplified present value generated by board F as:

$$PV_t^F = \theta \cdot \left(\frac{2t^3+9t^2+13t+6}{k} + \frac{6t^2+24t+24}{k^2} + \frac{12t+30}{k^3} + \frac{12}{k^4}\right) \quad \text{(A3-17)}$$

where $\theta = \frac{a_F b_F c_F d_F}{3}$, we have the final closed-from equation for the present value that board F can generate as follows:

$$PV_t^F = \frac{a_F b_F c_F d_F}{3} \cdot \left(\frac{2t^3+9t^2+13t+6}{k} + \frac{6t^2+24t+24}{k^2} + \frac{12t+30}{k^3} + \frac{12}{k^4}\right) \quad \text{(A3-18)}$$

where k is the discussion board traders' required rate of return and ranges from 0 to 1 and a_F, b_F, c_F and d_F are all constant values that can be estimated through empirical analysis employing statistical methods. For example, $a_F = 0.001$, $b_F = 0.1$, $c_F = 0.5$, and $d_F = 0.3$ as arbitrary numbers, $k = 0.1$, and $t = 100$, we have the value of $PV_{100}^F \approx 142.53$ units.

Appendix 4. Present Value Generated by Board *F* Using Continuous Compounding

The present value generated by board F is as follows given k represents the IDB traders' required rate of return and e is the Euler's number approximately equals 2.718:

$$PV_t^F = \sum_{n=1}^{+\infty} \frac{a_F b_F c_F d_F}{3} \cdot (t+n)(t+n+1)(2t+2n+1) \cdot \frac{1}{e^{kn}} \tag{A4-1}$$

Since all a_F, b_F, c_F and d_F are constant, we can first denote $\theta = \frac{a_F b_F c_F d_F}{3}$, and denote $x = \frac{1}{e^k}$, where $0 < k < 1$, so that we have:

$$PV_0^F = \theta \cdot \sum_{n=1}^{+\infty} \left[2n^3 + (6t+3)n^2 + (6t^2+6t+1)n + (2t^3+3t^2+t) \right] \cdot x^n \tag{A4-2}$$

and like the deviation in Appendix 3, this series can be broken down into the form:

$$PV_0^F = \theta \cdot \left[\sum_{n=1}^{+\infty} 2n^3 x^n + \sum_{n=1}^{+\infty} (6t+3) n^2 x^n + \sum_{n=1}^{+\infty} (6t^2+6t+1) n x^n + \sum_{n=1}^{+\infty} (2t^3+3t^2+t) x^n \right] \tag{A4-3}$$

Since $0 < k < 1$, we have $x < 1$, so that we have the following expression when n ranges from 1 to infinity:

$$\sum_{n=1}^{+\infty} x^n = \frac{x}{1-x} \tag{A4-4}$$

$$\sum_{n=1}^{+\infty} n \cdot x^n = \frac{x}{(1-x)^2} \tag{A4-5}$$

$$\sum_{n=1}^{+\infty} n^2 \cdot x^n = \frac{(1+x)x}{(1-x)^3} \tag{A4-6}$$

$$\sum_{n=1}^{+\infty} n^3 \cdot x^n = \frac{(1+4x+x^2)x}{(1-x)^4} \tag{A4-7}$$

Now let us use these sums to find the sum of the series, we have:

$$PV_0^F = \theta \cdot \left[2 \cdot \frac{(1+4x+x^2)x}{(1-x)^4} + (6t+3) \cdot \frac{(1+x)x}{(1-x)^3} + (6t^2+6t+1) \cdot \frac{x}{(1-x)^2} + (2t^3+3t^2+t) \cdot \frac{x}{1-x} \right] \tag{A4-8}$$

Now replace x with $\frac{1}{e^k}$, the first term can be simplified as:

$$\theta \cdot 2 \cdot \frac{\left[1 + 4 \cdot \left(\frac{1}{e^k} \right) + \left(\frac{1}{e^k} \right)^2 \right] \left(\frac{1}{e^k} \right)}{\left[1 - \left(\frac{1}{e^k} \right) \right]^4} = 2\theta \cdot \frac{e^{3k} + 4e^{2k} + e^k}{(e^k - 1)^4} \tag{A4-9}$$

And the second term can be simplified as:

$$\theta \cdot (6t+3) \cdot \frac{\left[1 + \left(\frac{1}{e^k} \right) \right] \left(\frac{1}{e^k} \right)}{\left[1 - \left(\frac{1}{e^k} \right) \right]^3} = \theta (6t+3) \cdot \frac{e^{2k} + e^k}{(e^k - 1)^3} \tag{A4-10}$$

And the third term can be simplified as:

$$\theta\cdot\left(6t^2+6t+1\right)\cdot\frac{\left(\frac{1}{e^k}\right)}{\left[1-\left(\frac{1}{e^k}\right)\right]^2}=\theta\left(6t^2+6t+1\right)\cdot\frac{e^k}{\left(e^k-1\right)^2} \tag{A4-11}$$

And the fourth term can be simplified as:

$$\theta\cdot\left(2t^3+3t^2+t\right)\cdot\frac{\left(\frac{1}{e^k}\right)}{1-\left(\frac{1}{e^k}\right)}=\theta\left(2t^3+3t^2+t\right)\cdot\frac{1}{e^k-1} \tag{A4-12}$$

Thus, adding four terms together, we have:

$$PV_0^F=\theta\cdot\left[\frac{2\cdot\left(e^{3k}+4e^{2k}+e^k\right)}{\left(e^k-1\right)^4}+\frac{(6t+3)\cdot\left(e^{2k}+e^k\right)}{\left(e^k-1\right)^3}+\frac{\left(6t^2+6t+1\right)\cdot e^k}{\left(e^k-1\right)^2}+\frac{2t^3+3t^2+t}{e^k-1}\right] \tag{A4-13}$$

where $\theta=\frac{a_F b_F c_F d_F}{3}$, we have the final closed-from equation for the minimum present value that board F can generate using continuous compounding method as follows:

$$\min\left(PV_t^F\right)=\frac{a_F b_F c_F d_F}{3}\cdot\left[\frac{2e^{3k}+8e^{2k}+2e^k}{\left(e^k-1\right)^4}+\frac{(6t+3)\cdot\left(e^{2k}+e^k\right)}{\left(e^k-1\right)^3}+\frac{\left(6t^2+6t+1\right)\cdot e^k}{\left(e^k-1\right)^2}+\frac{2t^3+3t^2+t}{e^k-1}\right] \tag{A4-14}$$

where k is the discussion board traders' required rate of return and ranges from 0 to 1, e is the Euler's number, t is a known constant representing the current time since the board's debut, and a_F, b_F, c_F and d_F are all constant values that can be estimated through empirical analysis employing statistical methods. For example, $a_F=0.001$, $b_F=0.1$, $c_F=0.5$, and $d_F=0.3$ as arbitrary numbers, k = 0.1, and t = 100, we have the minimum value of $PV_{100}^F\approx 133.42$ units.