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Is Control Friction Always Hurting Outside Investors? Implications from a Theoretical Study

Du Du

Department of Economics and Finance, City University of Hong Kong, Hong Kong, China
Email: dudu22@cityu.edu.hk

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Abstract

We analyze the financial and welfare implications of corporate control frictions. Our dynamic stochastic model features control-ownership wedge where outside investors have imperfect control over the decisions of their firm, and a rich opportunity set available to the firm that allows it to trade unconstrainedly in financial markets. The model makes numerous predictions. A deterioration of the protection for outside investors initially depresses but later on raises the firm's dividend payouts. The firm's controlling agent exploits the control friction by over-investing and taking more aggressive positions in the stock market. The empire building motive of the controlling agent at a higher degree of control friction may actually drive up the firm valuation. The controlling agent generally gains from a lower degree of investor protection and the implied utility gains are higher for a lower investment risk, a lower degree of risk aversion, and a lower equity risk premium.

Keywords

Control Friction, Control-Ownership Wedge, Investor Protection, Asset Allocation, Dividend Payout, Welfare Analyses

1. Introduction

It has been widely documented that for most publicly traded firms around the world, ownership and control often vest with dominant shareholders whose cash flow rights are substantially lower than their control rights (La Porta, Lopez-de-Silanes, and Shleifer [1]; Claessens, Djankov, and Lang [2]). Under imperfect investor protection[1], this control-ownership wedge gives rise to the "control

[1]Investor protection is defined by the extent to which the commercial law and its enforcement protect investors from expropriation by company insiders.

friction"—an agency problem between large controlling shareholders and minority shareholders from outside in that a controlling agent may choose activities to her own benefits, which are at the cost of other investors of the firm. While outside investors try to align the incentives of the controlling agent with their own goals through various corporate governance mechanisms, the implied control friction remain a persistent observed phenomenon (see, for example, La Porta, Lopez-de-Silanes, and Shleifer [1], Faccio and Lang [3], Lemmon and Lins [4], and Laeven and Levine [5]).

In this paper, we analyze the financial and welfare effects of control friction in a theoretical setting. Differentiating from previous studies, which usually impose extra constraints on the firm's opportunity set, we study an entrepreneurial firm which has the full access to capital investment, production, and financial trading. The controlling agent of the firm, sometimes referred to as the controlling shareholder, serves as the firm's entrepreneur who considers an intertemporal optimization problem by choosing a wide array of firm-level policies to her own benefits. With the unconstrained opportunity set available to the firm, our results show that corporate stealing at the presence of control friction has different temporal impact on the firm's dividend payouts, which may enhance the valuation of the firm. While the welfare effect of stealing is always negative for outside shareholders, its magnitudes exhibit substantial temporal variations as the firm's financial status evolves across the time.

More specifically, we integrate the control-ownership wedge into a dynamic stochastic model featuring asset allocation, consumption, costly business liquidation, and investment-specific shocks. Due to the wedge, the entrepreneur in our setup benefits from a stealing technology that allows her to extract private benefits from the firm when investor protection is imperfect. Put it another way, the entrepreneur at the presence of control friction effectively faces a call-option-like situation in that she enjoys more of the upside but bears less of the downside of her decisions with the firm. Different from previous studies on control friction (e.g., Albuquerque and Wang (AW) [6]), the entrepreneur in our setup benefits from both the productivity of the firm-held capital and the risk compensation from the stock market. Simultaneously, she is subject to 1) the productivity shocks; 2) the investment-specific shocks; and 3) the market risks.

Given the private benefits induced by control friction, the entrepreneur has the incentive to retain her control of the firm for as long as possible. Since firm-level risks cannot be perfectly hedged away, however, the entrepreneur faces the risk of losing all her benefits when the firm is forced into liquidation after a series of bad shocks. These two forces in combination give the entrepreneur a strong empire-building motive under which she cuts dividend payouts, over-invests, and takes a more aggressive position in the stock market. While re-allocating resources from dividend payouts to investment and asset allocation brings about a short-term sacrifice to the outside investors, it leads to higher payouts to all firm agents in the longer term. Financially, this is because the entrepreneur is equipped with an expanded opportunity set which allows her to take unconstrained positions in the stock market. With imperfect investor protection and given a sizable equity risk premium, the entrepreneur's empire-building motive prompts her to put gains from financial trading back to the stock market which reinforces the value-creation effect from the firm's asset allocation strategies. Consequently, the firm's dividend payout, albeit starting at a lower level due to the stealing, grows more rapidly when compared to the case with the perfect investor protection which enables a "long-term gain" for all investors.

We next examine whether this "shorter-term pain for long-term gain" profile helps enhance the valuation of the firm for outside investors. To our purpose, we first back out the state-dependent internal rate of return (IRR) for the firm by extending the procedure described in Wang, Wang, and Yang (WWY) [7] to the case with control friction. Using IRR as the discount rates, we then calculate the valuation of the firm as the present value of its dividend payouts that are obtained from a large sample of simulated firms. We show that firm valuation thus calculated is indeed higher when investor protection is imperfect. This result, with its emphasis on resource re-allocation effect when the entrepreneur is given an ample opportunity set, challenges the conventional wisdom that corporate stealing is always damaging to a firm.

While a deterioration of investor protection may enhance the firm's valuation, we show that it always depresses outside shareholders' welfare. This is because only the first moment information is used for valuing the firm, while higher-order moments also matter for welfare analyses. Indeed, both over-investment and a more aggressive position in the stock market expose the firm to higher volatilities which are disliked by a risk averse agent. For the entrepreneur, the implied negative volatility effect is dominated by the positive cash-flow effect because stealing enables her to consume uniformly more than under the perfect-protection case. In contrast, the volatility effect dominates for an outside investor to whom the cash-flow effect, in the form of "short-term pain for long-term gain", is much weakened. Consequently, an outside shareholder always suffers welfare losses when the degree of investor protection deteriorates.

We show that the implied welfare loss by an outside shareholder can be quite large when the firm is in financial distress. Intuitively, when the firm is in a bad financial status, the higher volatility further raises the downside by accelerating the firm's liquidation which is value-destroying to outside investors because their consumptions relies on the firm. For the same reason, the welfare gain by the entrepreneur drops rapidly when the firm falls into debt. Due to the offsetting cash-flow effect, however, the magnitude of welfare changes, quoted in percentage, is much lower for the entrepreneur when compared to that for an outside investor. Taken together, our results deliver a clear economic message: While enhancing investor protection (to reduce control friction) is indeed preferrable from the perspective of welfare analyses, it is particularly desirable when the firm is in financial distress.

In summary, we provide a negative answer to the question "Is control friction always hurting outside investors?" In fact, at the presence of control-ownership wedge firm valuation may rise following a deterioration of investor protection. We further show that outside investors' welfare loss under control friction is particularly large when the firm is in financial distress, whereas the controlling agents' welfare gain from control friction is lower for a higher investment risk, a higher degree of risk aversion, or a less constrained opportunity set for the firm. These new findings significantly expand the existing studies on control friction (e.g., La Porta, Lopez-de-Silanes, and Shleifer [1]; AW [6]; Basak, Chabakauri, and Yavuz (BCY) [8]).

Our paper is closely related to the literature that examines the economic and financial effects of control-ownership wedge. Using a two-period model, Shleifer and Wolfenzon [9] explains why firms are larger and more valuable with better investor protection. Like our paper, Dow, Gorton, and Krishnamurthy (DGK) [10] also examine the impact of managerial empire building motive at the presence of control friction. Their focus, however, is exclusively on over-investment so that the firm is effectively shut out of financial trading. Aslan and Kumar [11] present a three-period model in which the endogenous choice of ownership concentration affects the cost of borrowing and the probability of default. BCY [8] study the asset pricing implications of the control-ownership wedge when controlling shareholder can endogenously accumulates his control over a firm. For tractability, they adopt a myopic preference so that the controlling shareholder effectively considers a two-period problem for his optimization. None of these studies considers welfare analyses for firm agents which are emphasized in our paper. In addition, all these papers except for DGK focus on models that are (essentially) static. In contrast, our analyses emphasize the critical importance of intertemporally optimal decisions made by the controlling agent which are the key drivers of all our financial and welfare implications.

Some predictions of model are already confirmed empirically. For example, one key prediction from our model is the resource re-allocation effect which transfers dividend payouts to investment and asset allocation, and this effect is stronger for a lower degree of investor protection. Consistent with this prediction, Fama and French [12] show that firm with more investments pay fewer dividends. Duchin *et al.* [13] find that poor corporate governance is associated with larger investments in risky financial assets. Franzoni [14] and Chen, Lu, and Sougiannis [15] both report that entrepreneurs have stronger empire-building motives when their firms that are subject to more severe agency frictions.

The remainder of our paper is organized as follows. Section II presents the setup of the model. Section III characterizes the model and Section IV provides its numerical solution. Section V quantitatively analyze the impact of control friction on firm policies (V.A), firm valuation (V.B), and firm agents' welfare (V.C). Finally, Section VI concludes.

2. Model Setup

2.1. Capital Accumulation and Production

We assume that the firm's capital stock K_t evolves according to

$$\mathrm{d}K_t = \left(I_t - \delta_K K_t\right)\mathrm{d}t + \epsilon I_t \mathrm{d}Z_t^I, \tag{2.1}$$

where I_t is investment, $\delta_K > 0$ is the depreciation rate, $\epsilon > 0$ is a volatility parameter; Z_t^I is a standard Brownian motion. Since the firm's output comes from its capital stock, the specification of (2.1) implies that output fluctuations arise from shocks to the marginal efficiency of investment (Keynes [16]), that is, the investment-specific technology shocks. This specification is motivated by the growing literature that emphasizes the important role of investment-specific technology shocks as a source of aggregate volatility (e.g., Greewood, Hercowitz, and Huffman [17]; Fisher [18], among others).

The gross output of the firm over the period $(t, t + \mathrm{d}t)$ is given by $K_t \mathrm{d}A_t$ which is proportional to its time-t capital stock K_t. The firm's operating profit $\mathrm{d}Y_t$ over the same period is thus given by

$$\mathrm{d}Y_t = K_t \mathrm{d}A_t - I_t \mathrm{d}t - G\left(I_t, K_t\right)\mathrm{d}t, \tag{2.2}$$

where A_t denotes the productivity of the capital; $G(I, K)$ is the adjustment cost. Motivated by WWY [7], we assume that the firm is subject to productivity shocks in that

$$\mathrm{d}A_t = \mu_A \mathrm{d}t + \sigma_A \mathrm{d}Z_t^A, \tag{2.3}$$

where Z_t^A is another standard Brownian motion that is independent of Z_t^I; $\mu_A > 0$ is the mean of the productivity shock, and $\sigma_A > 0$ is the volatility of the productivity shock. Consistent with Hayashi [19], we assume that the adjustment cost $G(I, K)$ is convex in I and homogeneous of degree one in I and K. Specifically,

$$G(I, K) = g(i)K = \frac{\theta i^2}{2}K, \tag{2.4}$$

where $i = I/K$ denotes the investment-capital ratio, and the parameter θ measures the degree of adjustment cost.

2.2. The Stealing Technology

Following Shleifer and Vishny [20], La Porta *et al.* [21], and the literature on investor protection, we assume that the controlling agent is fully entrenched and has complete control over the firm's investment and payout policies. More details on how control rights differ from cash flow rights (via dual-class shares, pyramid-ownership structures, cross-ownership, etc.) can be found in Bebchuk, Kraakman, and Triantis [22], La Porta, Lopez-de-Silanes, and Shleifer [23], among others. Throughout the paper, we will use the names "controlling agent", "entrepreneur", or "controlling shareholder" interchangeably.

Building on Johnson *et al.* [24], La Porta *et al.* [21], and AW [6], we model private benefits via a stealing technology in that the entrepreneur may "steal" a s-fraction of the firm's average output $\mu_A K$, by incurring a convex cost in the amount of

$$H(s, AK) = \frac{\beta}{2} s^2 \mu_A K, \tag{2.5}$$

where the parameter β measures the degree of investor protection. Specifically, a higher β implies a larger marginal cost $\beta s \mu_A K$ of diverting cash for private benefits and hence a stronger protection for outside investors. Taking into account s and $H(.)$, the entrepreneur's consumption with the firm is given by

$$C_1 = yD + s\mu_A K - \frac{\beta}{2} s^2 \mu_A K, \tag{2.6}$$

where D denotes the firm's dividend payout; $y \in [0,1]$ which denotes the entrepreneur's cash-flow right with the firm; $s\mu_A K - \frac{\beta}{2} s^2 \mu_A K$ denotes her net benefits from stealing.

2.3. Firm Agents' Preferences

The entrepreneur has the standard CRRA utility over her consumption as follows:

$$J_t = E_t\left[\int_t^\infty \zeta e^{-\zeta(v-t)} U(C_{1,s}) \mathrm{d}s\right]. \tag{2.7}$$

In (2.7), ζ denotes her subjective discount rate;

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \tag{2.8}$$

where $\gamma(>0)$ denotes the (constant) coefficient of relative risk aversion.

All outside investors are identical and they have the same preference as the entrepreneur and the preferences are defined on their consumption streams with the firm[2]. Thus, an outside shareholder i's utility derived from the firm can be written as:

$$J_{2,t} = E_t\left[\int_t^\infty \zeta e^{-\zeta(v-t)} \frac{\left(C_{2,s}^i\right)^{1-\gamma}}{1-\gamma} \mathrm{d}s\right], \tag{2.9}$$

where $C_{2,s}^i = \phi^i D_s$ with ϕ^i denoting his cash-flow right with the firm. In combination, outside shareholders own a total of $1-y$ fraction of the firm.

2.4. Financial Markets and the Firm's Liquid Wealth

In addition to consumption and investment, the entrepreneur can also invest in a risk-free asset which pays a constant rate of interest r and the risky market portfolio (Merton [25]). Assume that the incremental return $\mathrm{d}R_t$ of the market portfolio over the time period $\mathrm{d}t$ is i.i.d. as follows:

$$\mathrm{d}R_t = \mu_R \mathrm{d}t + \sigma_R \mathrm{d}B_t, \tag{2.10}$$

[2]While we do not exclude the possibility that an outside investor also obtains incomes from other sources, in the paper we focus on his consumption that come from the firm so as to facilitate our welfare analyses.

where μ_R and σ_R are constant mean and volatility parameters of the market portfolio return process; B_t is a standard Brownian which correlates with Z_t^I and Z_t^A by ρ_I and ρ_A, respectively[3].

Let W and X denote the firm's liquid wealth and the amount invested in the risky asset, respectively. Their difference, $W-X$, is thus invested in the risk-free asset. Out of its liquid asset, the firm pays the cost for capital investment, distribute dividend (y -fraction of which is paid to the entrepreneur), and runs the stealing cost to the entrepreneur's private benefit. Thus, the firm's liquid wealth evolves according to

$$\mathrm{d}W_t = \left[rW_t + \eta\sigma_R X_t - D_t - s_t\mu_A K_t\right]\mathrm{d}t + X_t\sigma_R \mathrm{d}B_t + \mathrm{d}Y_t, \tag{2.11}$$

where

$$\eta \equiv \frac{\mu_R - r}{\sigma_R}, \tag{2.12}$$

which denotes the market Sharpe ratio; $\mathrm{d}Y_t$ is given by (2.2). Note that W can be negative, under which the firm borrows against its capital stock[4].

3. Characterizing the Model

3.1. The Optimization Problem

Accounting for her cash-flow right, we write the entrepreneur's value function, which is introduced in (2.7), as $J(K,W;y)$. By the standard dynamic programming argument, the entrepreneur solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\zeta J(K_t,W_t;y) = \max_{D,s,X,I} \zeta U(C_{1t}) + E_t\left[\mathrm{d}J(K_t,W_t;y)\right], \tag{3.1}$$

where $E_t[.]$ the expectation conditional on information available at time t. In this original version of HJB, the left-hand side (LHS) $\zeta J(K_t,W_t;y)$ reflects the required rate of return for the entrepreneur to hold the firm (per unit of time); the right-hand side (RHS) is the expected gain to the entrepreneur from holding the firm which involves two components: 1) her utility derived from the firm and 2) the expected change of her value function. By Ito's lemma and making use of (2.1), (2.2), (2.3), and (2.11), we obtain

$$\begin{aligned} 0 = \max_{D,s,X,I} & -\zeta J(K,W;y) + \zeta U(C_1) + (I - \delta_K K) J_K \\ & + \left[rW_t + \eta\sigma_R X_t + \mu_A K_t - I_t - G(I_t,K_t) - D_t - s_t\mu_A K_t\right] J_W \\ & + \frac{\epsilon^2 I^2}{2} J_{KK} + \rho_I \epsilon I \sigma_R X J_{KW} + \frac{\sigma_A^2 K^2 + 2\rho\sigma_A K\sigma_R X + \sigma_R^2 X^2}{2} J_{WW}, \end{aligned} \tag{3.2}$$

[3]We assume that ρ_I, $\rho_A < 1$ so that the firm-level risks cannot be fully hedged away by taking positions in the stock market.

[4]By pledging its capital as the collateral, the firm-level borrowing is without any risk so it is also assessed at the constant risk-free rate of r.

where C_1 is given by (2.6)[5]. The entrepreneur chooses the firm's optimal dividend payout according to

$$\zeta U'(C)\frac{\partial C}{\partial D} = J_W(K,W), \tag{3.3}$$

which is the usual condition that marginal utility of consumption is equated with the marginal value of wealth, with the adjustment that one unit reduction of W only raises the entrepreneur's consumption by the units of $y(<1)$ which characterizes her cash-flow right with the firm. The first-order condition (FOC) for the diversion (or stealing) s is given by

$$\zeta U'(C)\frac{\partial C}{\partial s} = J_W(K,W)\mu_A K. \tag{3.4}$$

Combining (3.4) with (3.3) and making use of (2.6) to evaluate the C-derivatives, we can explicitly solve out the optimal stealing as follows:

$$s^* = \frac{1-y}{\beta} \tag{3.5}$$

(3.5) indicates that the entrepreneur steals more when investor protection deteriorates or when there is a high degree of control friction as indicated by a lower y.

The entrepreneur further chooses the firm's investment and asset allocation policies that are optimally determined according to

$$\left[1+G_I(I,K)\right]J_W - \epsilon^2 I J_{KK} - \rho_I \epsilon \sigma_R X J_{KW} = J_K \tag{3.6}$$

and

$$X = -\frac{\mu_R - r}{\sigma_R^2}\frac{J_W}{J_{WW}} - \frac{\rho\sigma_A}{\sigma_R}K - \frac{\rho_I \epsilon \sigma_R I}{\sigma_R^2}\frac{J_{KW}}{J_{WW}}, \tag{3.7}$$

respectively. Du (2024) provides the financial interpretations of the above FOCs for I and X under the context of a special case when the control-ownership wedge is shut down.

3.2. Controlling Shareholder's Shadow Valuation of the Firm

We conjecture (and verify later) that the controlling shareholder's value function $J(K,W;y)$ takes the following form:

[5]More specifically,

$$\mathrm{d}J(K_t,W_t;y) = J_K E_t[\mathrm{d}K_t] + J_W E_t[\mathrm{d}W_t] + \frac{1}{2}J_{KK}E_t\left[(\mathrm{d}K_t)^2\right] + \frac{1}{2}J_{WW}E_t\left[(\mathrm{d}W_t)^2\right] + J_{KW}E_t[\mathrm{d}K_t\mathrm{d}W_t].$$

Making use of (2.1) and (2.11), $E_t[\mathrm{d}K_t] = I_t - \delta_K K_t$; $E_t[\mathrm{d}W_t] = rW_t + \eta\sigma_R X_t - D_t - s_t\mu_A K_t + E_t[\mathrm{d}Y_t]$; $E_t\left[(\mathrm{d}K_t)^2\right] = \epsilon^2 I_t^2$; $E_t\left[(\mathrm{d}W_t)^2\right] = \sigma_R^2 X_t^2 + E_t\left[(\mathrm{d}Y_t)^2\right] + 2E_t[X_t\sigma_R\mathrm{d}B_t\mathrm{d}Y_t]$; $E_t[\mathrm{d}K_t\mathrm{d}W_t] = E_t\left[\epsilon I_t \mathrm{d}Z_t^I X_t\sigma_R\mathrm{d}B_t\right]$. By further making use of (2.2) and (2.3), we obtain $E_t\left[(\mathrm{d}W_t)^2\right] = \sigma_R^2 X_t^2 + \sigma_A^2 K_t^2 + 2\rho\sigma_A K_t\sigma_R X_t$ and $E_t[\mathrm{d}K_t\mathrm{d}W_t] = \rho_I\epsilon I_t\sigma_R X_t$. Substituting the obtained $E_t[.]$-formulas into (3.1) yields (3.2) where we omit the time-dependences for simplicity. Also note that the formulas for the value function's derivatives are provided in (3.12)-(3.16).

$$J(K,W;y)=\frac{\left(bP(yK,yW)\right)^{1-\gamma}}{1-\gamma}, \tag{3.8}$$

where the constant b is given by

$$b=\zeta\left[1+\frac{1-1/\gamma}{\zeta}\left(r-\zeta+\frac{\eta^2}{2\gamma}\right)\right]^{\frac{\gamma}{\gamma-1}}, \tag{3.9}$$

which is independent of y [6]. In (3.8), we let the arguments of $P(.)$ to be yK and yW which emphasizes that the entrepreneur's cash-flow right with the firm applies to both the liquid wealth W and the capital stock K. We interpret $P(yK,yW)$ as the certainty-equivalent (CE) valuation of the firm by the entrepreneur: It denotes the minimum dollar amount that she would demand to permanently give up her entitlement to the firm (including her privileges with the firm through the stealing technology) and retire as a Merton-style consumer.

To establish the form of $P(.)$, we exploit the model's homogeneity property to reduce the entrepreneur's problem to one dimension. Specifically, we treat K_t as the scaling factor, and we use lower case letters to denote the following variables: firm's liquid wealth $w_t=W_t/K_t$, the entrepreneur's CE valuation of the firm $p_t=P_t/K_t$, consumption $c_t=c_t/K_t$, investment $i_t=I_t/K_t$, and risky ass *et al.* location $x_t=X_t/K_t$. Using these notations, we can express $P(yK,yW)$ from (3.8) as

$$P(yK,yW)=(yK)\cdot p(w;y)=y\cdot\left[Kp(w;y)\right], \tag{3.10}$$

where $Kp(w;y)$ is interpreted as the entrepreneur's shadow valuation of the firm with w being the measure of the firm's financial status. Due to her incomplete ownership with the firm, the entrepreneur's CE valuation of the firm, $P(yK,yW)$, is only y-fraction of this shadow valuation as indicated by (3.10)[7]. In view of (3.8)-(3.10), we can write the entrepreneur's value function more explicitly as:

$$J(K,W;y)=\frac{\left[byKp(w)\right]^{1-\gamma}}{1-\gamma}. \tag{3.11}$$

3.3. The Implied Ordinary Differential Equations (ODEs)

By (3.11) we have the following expressions for J-derivatives:

$$J_W=(by)^{1-\gamma}\left(p(w)K\right)^{-\gamma}p'(w), \tag{3.12}$$

[6]The formula of b is inferred from the solution of Merton's [25] consumption and portfolio choice problem which can be treated as a simplified version of our model when capital investment, stochastic production, and the control-ownership wedge are all shut down.

[7]We can further write the scaled shadow valuation of the firm $p(.)$ by $p(w;y)=w+q(w;y)$, where w denotes the liquid wealth per unit of capital and $q(.)$ denotes the liquid-wealth valuation of the firm-held capital which is usually referred to as "the average q" (Tobin [26]). $q(.)$ in general varies with w which reflects the time-varying effect of financial slack on the valuation of capital when the firm faces the liquidation risk.

$$J_K = (by)^{1-\gamma}\left(p(w)K\right)^{-\gamma}\left(p(w) - wp'(w)\right), \tag{3.13}$$

$$J_{WW} = (by)^{1-\gamma}\left(p(w)K\right)^{-\gamma-1}\left[p(w)p''(w) - \gamma\left(p'(w)\right)^2\right], \tag{3.14}$$

$$J_{KW} = (by)^{1-\gamma}\left(p(w)K\right)^{-\gamma-1}\left[-wp(w)p''(w) - \gamma p'(w)\left(p(w) - wp'(w)\right)\right], \tag{3.15}$$

$$J_{KK} = (by)^{1-\gamma}\left(p(w)K\right)^{-\gamma-1}\left[w^2 p(w)p''(w) - \gamma\left(p(w) - wp'(w)\right)^2\right]. \tag{3.16}$$

By substituting J from (3.8) and (3.10) into (3.3) and simplifying, we obtain the following consumption rule:

$$c_1^*(w;y) = y\zeta^{\frac{1}{\gamma}} b^{\frac{1-\gamma}{\gamma}} p'(w;y)^{-\psi} p(w;y). \tag{3.17}$$

where b is given by (3.9). Due to the control-ownership wedge, the entrepreneur's consumption is naturally discounted by a factor of $y(<1)$ which reflects her partial cash-flow right with the firm. By (2.6), the firm's dividend policy at the presence of control friction is

$$D^*(w;y) = \zeta^{\frac{1}{\gamma}} b^{\frac{1-\gamma}{\gamma}} p(w;y)p'(w;y)^{-\psi} K - \frac{1-y^2}{2\beta y}\mu_A K, \tag{3.18}$$

where we've made use of the entrepreneur's optimal stealing policy given in (3.5). Expressed in terms of D^*, the entrepreneur's optimal consumption can be written as

$$C_1^*(w;y) = yD^*(w;y) + \frac{1-y^2}{2\beta}\mu_A K. \tag{3.19}$$

Relative to her cash-flow right, the entrepreneur consumes more by the amount of $\frac{1-y^2}{2\beta}\mu_A K$ which is attributed to the stealing technology[8].

By substituting (3.12) and (3.15)-(3.14) into (3.6)-(3.7) and simplifying, we obtain

$$i^*(w) = \frac{p - (w+1)p' - \rho_I \epsilon p' \frac{\gamma p - wh}{p}\left(\eta\frac{p}{h} - \rho\sigma_A\right)}{\theta p' + \left(1-\rho_I^2\right)\epsilon^2 \frac{p'}{p}\frac{(\gamma p - wh)^2}{h} - \gamma\epsilon^2\frac{p^2 p''}{hp'}}, \tag{3.20}$$

$$x^*(w;i) = \frac{\eta}{\sigma_R}\frac{p}{h} - \frac{\rho\sigma_A}{\sigma_R} - \rho_I\frac{\epsilon i}{\sigma_R}\left(\frac{\gamma p}{h} - w\right), \tag{3.21}$$

where

$$h(w;y) \equiv \gamma p'(w;y) - \frac{p(w;y)p''(w;y)}{p'(w;y)} \tag{3.22}$$

By substituting optimal policies of (3.5), (3.17), (3.20), and (3.21), as well as (3.12)-(3.16) into (3.2), scaling by K wherever is necessary, and simplifying, tedious algebra gives

[8]More specifically, the entrepreneur's net benefit from stealing, as given by the last term of (3.19), is equal to the diverted amount minus the convex cost of stealing (see Equation (2.5)).

$$
\begin{aligned}
0 = & \frac{\zeta^{\psi} b^{1-\psi} p\left(p'\right)^{1-\psi} - \zeta \psi p}{\psi - 1} + \mu_A p' - \rho \eta \sigma_A p' + \left(r + \delta_K\right) w p' \\
& - \delta_K p + \frac{1}{2} \frac{\eta^2 p p'}{h} \psi_y p' \mu_A - \frac{1}{2}\left(1 - \rho^2\right) \sigma_A^2 \frac{p' h}{p} \\
& + \frac{1}{2} \frac{\left[p - (w+1) p' - \rho_I \epsilon p' \frac{\gamma p - w h}{h}\left(\eta \frac{p}{h} - \rho \sigma_A\right)\right]^2}{\theta p' + \left(1 - \rho_I^2\right) \epsilon^2 \frac{p'}{p} \frac{\left(\gamma p - w h\right)^2}{h} - \gamma \epsilon^2 \frac{p^2 p''}{h p'}},
\end{aligned}
\tag{3.23}
$$

where

$$
\psi_y \equiv \frac{(1-y)^2}{2 y \beta}. \tag{3.24}
$$

3.4. Boundary Conditions

The firm gets liquidated when w becomes sufficiently negative, upon which the firm-held capital stock yields the terminal value of lK_t for $l \in (0,1)$. Let τ be the (stochastic) liquidation time optimally chosen by the entrepreneur so that the firm's liquidation value can be written as $W_\tau + lK_\tau$. Upon liquidation, the entrepreneur collects her entitled liquidation payment and retires as a Merton consumer (Merton [21]) where her value function takes the form of

$$
J^M\left(y[W + lK]\right) = \frac{\left(by[W + lK]\right)^{1-\gamma}}{1-\gamma}, \tag{3.25}
$$

where b is given by (3.9); and the expression $y[W + lK]$ again reflects the entrepreneur's partial cash-flow right with the firm. Let W_d denote the firm's liquidation boundary with $w_d \equiv W_d / K$. Since W_d is optimally chosen, we have the following value matching and smooth-pasting conditions:

$$
J(K, W_d; y) = J^M\left(y[W_d + lK]\right), \tag{3.26}
$$

$$
\left.\frac{\partial J(K, W; y)}{\partial W}\right|_{W=W_d} = \left.\frac{\partial J^M\left(y[W + lK]\right)}{\partial W}\right|_{W=W_d}. \tag{3.27}
$$

Simplifying (3.26)-(3.27) by making use of (3.8), (3.10), and (3.25), we obtain

$$
p(w_d; y) = w_d + l, \tag{3.28}
$$

$$
p'(w_d; y) = 1. \tag{3.29}
$$

At the other end when $w \to \infty$, the firm is no longer concerned about the potential liquidation under which

$$
\lim_{w \to \infty} p(w; y) = w + \hat{q}(y), \tag{3.30}
$$

where $\hat{q}(y)$ denotes the constant valuation of the firm-held capital for the given y. To identify $\hat{q}$, we simplify (3.20) and (3.23) at $w \to \infty$ by making use of (3.30) so as to obtain[9]

[9]Under $p(w) = w + \hat{q}$, h defined by (3.22) degenerates to γ. Furthermore, the definition of b by (3.9) implies that all terms that are loading on w get cancelled out. Du [27] provides the detailed derivations for such simplications when there is no control-ownership wedge at $y = 1$.

$$\hat{i} = \frac{\hat{q} - 1 - \rho_I \epsilon \eta \hat{q}}{\theta} \tag{3.31}$$

and

$$0 = (1 + \psi_y)\mu_A - \hat{i} - \frac{1}{2}\theta i^2 - \rho\eta\sigma_A - (r + \delta_K)\hat{q} + (1 - \rho_I \epsilon \eta)\hat{i}\hat{q}, \tag{3.32}$$

where ψ_y is given by (3.24). (3.31)-(3.32) imply a quadratic equation on $\hat{i}$ and we pick the root so that $\hat{i}$ as the optimal investment in the limiting case is increasing in the firm's expected productivity μ_A, *i.e.*,

$$\hat{i}(y) = \frac{r + \delta_K}{1 - \rho_I \varepsilon \eta} - \sqrt{\left(\frac{r + \delta_K}{1 - \rho_I \varepsilon \eta}\right)^2 - \frac{2}{\theta}\left[(1 + \psi_y)\mu_A - \frac{r + \delta_K}{1 - \rho_I \varepsilon \eta} - \rho\eta\sigma_A\right]}. \tag{3.33}$$

Substituting (3.33) back into (3.31) identifies $\hat{q}$ [10]. (3.28)-(3.29) combined with (3.30) thus provide the three boundary conditions required to solve the second-order ODE of (3.23) on $p(w; y)$ with the free boundary w_d. For simplicity, in the following we generally omit the y-dependences unless there is a need to highlight the control-ownership wedge.

4. Numerical Solution to the Model

Table 1 summarizes the parameter values used in our baseline calibration. For parameters governing the return process of the financial assets, the risk-free interest rate r equals 4.6%, the market equity risk premium $\mu_R - r$ equals 6%, the volatility of the market portfolio return σ_R equals 20%, and consequently the market Sharpe ratio $\eta = (\mu_R - r)/\sigma_R$ equals 0.3. Following WWY [7], the firm agents' subjective discount rate is set at the same level as r; γ is set at 2 which implies a reasonably low risk aversion for the entrepreneur; the capital depreciation rate δ_K is set at 0.125; ρ_A is set at 0 which implies that the firm-level productivity shock is purely idiosyncratic. For production-related parameters, we use the estimates in Eberly, Rebelo, and Vincent [28] as the guideline and set the average productivity μ_A to 20% and volatility of productivity shocks σ_A to 15%. Consistent with the estimate by Whited [29], we take the adjustment cost parameter θ to be 2. As suggested by Hennessy and Whited [30], we choose the capital liquidation price l to be 0.9. Consistent with Du [27], we set ρ_I at 0.3 implying an investment risk that is positively correlated with the stock market. Du [27] allow ϵ, denoting the volatility of investment-specific shocks, to vary between 0 and 1 and we choose its midpoint 0.5 as the baseline value for ϵ.

The two key parameters to our setup are the investor protection parameter, β, and the controlling shareholder's cash-flow right, y. AW [6] estimate that β (which is denoted by η in their paper) is 28.44 in South Korea and 2325 in

[10]Financially, $\hat{q}$ denotes the entrepreneur's shadow valuation of the firm-held capital after the firm has achieved a very high degree of financial slack. Numerically, we find that both $\hat{q}(y)$ and $\hat{i}(y)$ are decreasing in y which is intuitive: When y decreases which aggravates the control friction, the entrepreneur can steal more from the firm which 1) raises her valuation of the firm and 2) prompts her to invest more so that she can gain more in the future (recall that the amount of diversion is proportional to the firm's capital stock).

Table 1. Baseline parameterization.

Panel A: Market environment					
$r = 0.046$	$\sigma_R = 0.2$	$\eta = 0.3$	$\rho_A = 0$	$\rho_I = 0.3$	
Panel B: Preferences					
$\zeta = 0.046$	$\gamma = 2$				
Panel C: Investment and production					
$\mu_A = 0.2$	$\sigma_A = 0.15$	$\theta = 2$	$l = 0.9$	$\delta_K = 0.125$	$\epsilon = 0.5$
Panel D: Related to control friction					
$y = 0.3$	$\beta = 20$				

This table summarizes the baseline parameterization to our model. Panel A describes the market-related parameters, where r, σ_R, η, ρ_I, and ρ_A denote, respectively, the risk-free rate, the volatility of the market portfolio, market Sharpe ratio, the correlation between the market portfolio returns and investment-specific shocks, and the correlation between the market portfolio returns and productivity shocks. Panel B reports preference parameters, where ζ and γ denote the subjective discount rate and the degree of risk aversion, respectively. Panel C calibrates firm-related parameters, where μ_A, σ_A, θ, l, δ_K, and ϵ denote, respectively, the average productivity, volatility of productivity shock, adjust cost parameter, capital liquidation price, the rate of capital depreciation, and the volatility of investment-specific shocks. Panel D reports parameters related to control friction, where y and β denote the controlling shareholder's cash-flow right and the coefficient for the cost of stealing which measures the degree of investor protection, respectively. All parameters are annualized.

the US. We consider two scenarios of β throughout our quantitative analyses: $\beta = 20$ which is its baseline value for imperfect investor protection; and $\beta = \infty$ indicating the perfect investor protection. We allow the entrepreneur's cash-flow right y to vary from 0.1 to 1 in Section V. A so as to study the policy impact of control friction. The baseline level of y is set at 0.3.

Figure 1 plots the marginal value of liquid wealth P_W as the function of the firm's financial slack $w \equiv W/K$ for the two scenarios of β, where $P_W = p'(w)$ which is calculated from the entrepreneur's perspective. First of all, $p'(w) \geq 1$ because one unit of increase of W raises P at least by its nominal value. Specifically, P_W approaches one when w approaches either its lower (w_d) or its upper end (∞). In between, P_W stays above one because raising the financial slack provides the extra benefit that lowers the probability of the costly liquidation. The incremental impact of W on P also depends on β. Specifically, when there is perfect investor protection at $\beta = \infty$ (dashed line), the entrepreneur obtain less benefit from the firm than that at $\beta = 20$ (solid line). Consequently, she lowers P_W across all levels of w and she liquidates the firm earlier at a higher w_d.

5. Quantitative Implications

5.1. Impact of *y*

We first analyze the impact of control friction on firm's optimal policies and the

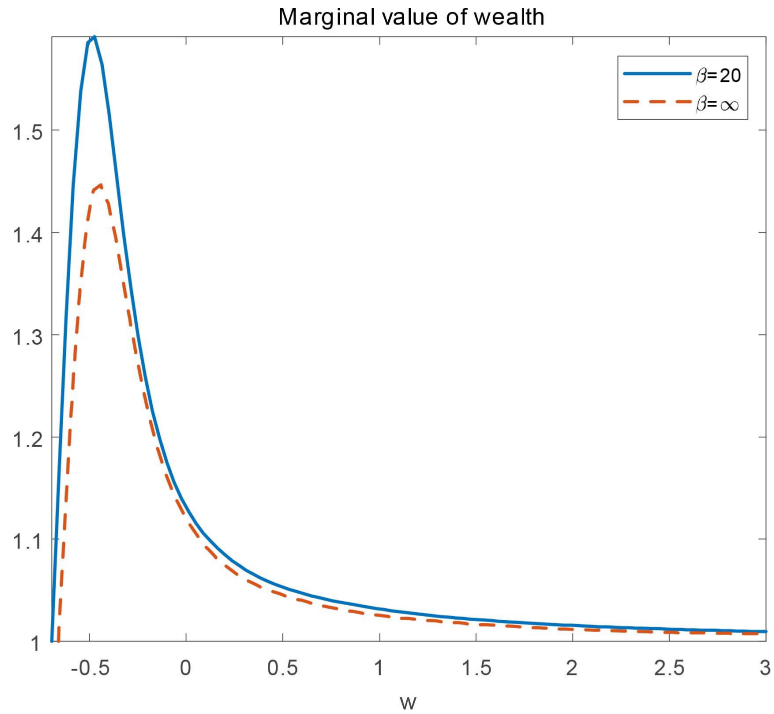

Figure 1. Marginal value of liquid wealth at the presence of control-ownership wedge. **Figure 1** plots the numerical solutions to the entrepreneur's problems in terms of $p'(w)$ ($=P_W$) which denotes the firm's marginal value of wealth, where $w \equiv W/K$ which measures the firm's financial status. The plots are with respect to two β -scenarios: $\beta = 20$ which indicates the imperfect investor protection (sold line) and $\beta = \infty$ indicating the perfect investor protection that shuts down the control friction (dashed line). All other model parameters are at their baseline levels that are reported in **Table 1**.

results are plotted in **Figure 2**[11]. As we lower the entrepreneur's cash-flow right y for the given $\beta < \infty$, the degree of control friction gets higher and Panel A of **Figure 2** shows that it facilitates stealing which is quite intuitive. Since the entrepreneur's consumption C_1 depends on her cash-flow right, a lower y mechanically reduces the optimal consumption-capital ratio c_1^* (see its formula by (29)) as plotted in Panel B. We further consider C_1/y which is interpreted as the entrepreneur's consumption per unit of her cash-flow right with the firm, or "PU-consumption" in short. As shown in Panel C of **Figure 2**, the optimal PU consumption-capital ratio, c_1^*/y, is monotonically decreasing in y which reflects the simple fact that more stealing under a higher degree of control friction enables the entrepreneur to consume more for each unit of her cash-flow right.

The optimal investment-capital ratio i^* and optimal portfolio allocation x^* are plotted in Panel D&E, respectively. Like c_1^*/y, they are both decreasing in y. In contrast, Panel F of **Figure 2** shows that the optimally chosen liquidation boundary w_d is monotonically increasing as y rises. Taken together, the

[11]Since policies plotted in Panel B-D of **Figure 2** also load on the firm's financial status, we evaluate their dependences on y at $w = 0$. We confirm in unreported exercises that the implications remain qualitatively unchanged when w is set at different levels.

entrepreneur at a higher degree of control friction, as indicated by a lower y, steals more (Panel A), consumes more in the "per-unit" sense (Panel C), over-invests (Panel D), and takes a more aggressive position in the stock market (Panel E)[12]. This way, she grows the firm bigger in the longer-term which delays its liquidation (Panel F) and thus allows her to retain the control for a longer time.

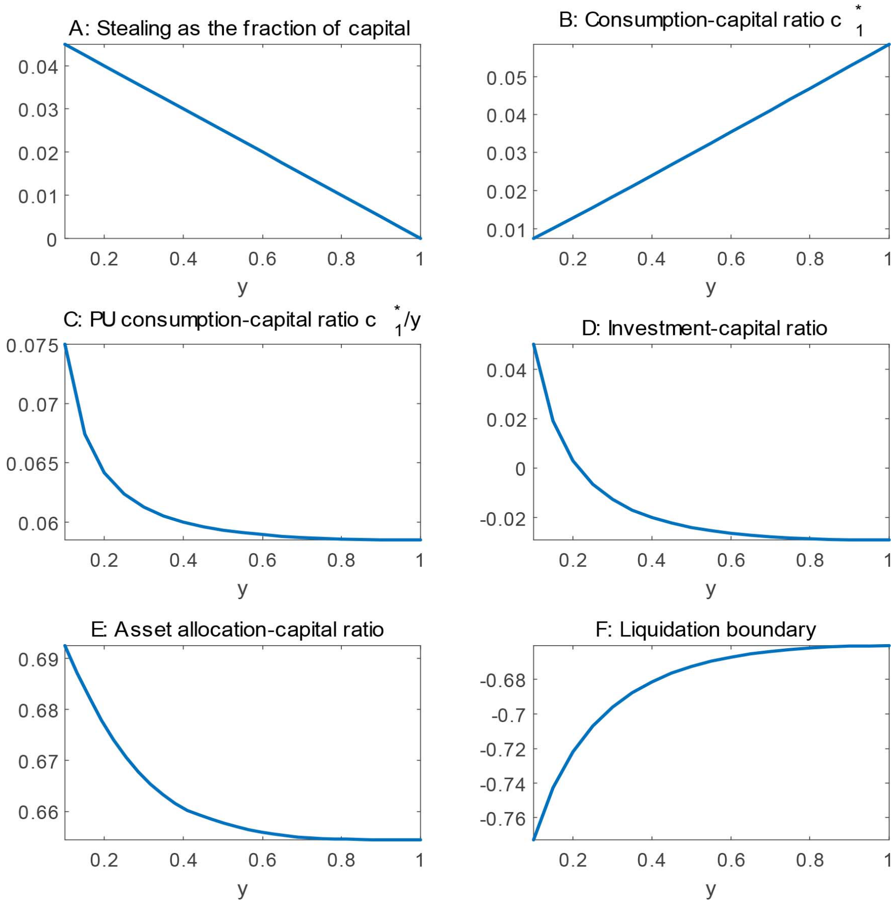

Figure 2. Impact of the controlling shareholder's cash-flow right y. **Figure 2** plot the various firm-level policies set by the entrepreneur when her cash-flow right with the firm, as captured by y, varies from 0.1 to 1. These policies are optimal stealing s^* (Panel A), optimal consumption-capital ratio the c_1^* (Panel B), optimal consumption-capital ratio per unit of her cash-flow right c_1^*/y (Panel C), optimal investment-capital ratio i^* (Panel D), optimal asset allocation-capital ratio x^* (Panel E), and optimal liquidation bondary w_d (Panel F).

5.2. Valuation of the Firm by Outside Shareholders

5.2.1. Internal Rate of Return

We next consider the valuation of the firm. This valuation is different than

[12]In particular, the resource re-allocation effect is stronger at a lower y. Intuitively, a lower y, which indicates a wider control-ownership wedge, strengthens the entrepreneur's empire building motives since she bears less of the downside. Consequently, she over-invests even more and takes a more aggressive position in the stock market which on average allows the firm to grow even bigger.

$P(W,K)$, the firm's shadow valuation by the entrepreneur, which takes into account her privileges from the stealing technology. In contrast, the firm valuation we want to calculate here is viewed by outside investors so it can be treated as the valuation of the firm in the open market. To this purpose, we need to calculate the present value (PV) of dividend payouts to outside shareholders with the appropriate discounting. Since the firm involves various risks, a discounting by the risk-free rate doesn't seem appropriate. While we can calculate the implied risk premium demanded by the firm according to its market exposure, previous researches (e.g., Moskowitz and Vissing-Jorgensen [31]; Mueller [32]; WWY [7]) show that entrepreneurial firms also claim idiosyncratic risk premium. To account for both the systematic and the additional idiosyncratic risk premium for non-diversifiable firm-level risk, we follow WWY [7] to calculate the cost of capital ξ, which is deemed as the internal rate of return (IRR) for the firm, as follows:

$$Q(K_0,W_0)=q_0K_0=E\left[\int_0^\tau e^{-\xi(w_0)t}\mathrm{d}Y_t+e^{-\xi(w_0)\tau}lK_\tau\right], \tag{5.1}$$

where τ denotes the stochastic liquidation time. $\xi(.)$ thus defined is state-dependent and Panel A of **Figure 3** plots the implied ξ as the function of w for two scenarios of β where the scenario at $\beta=\infty$, by ruling out stealing, shuts down the control friction.

Consistent with WWY [7], the implied IRR is monotonically decreasing and it rises substantially as w drops towards w_d. In comparison, the IRRs backed out at $\beta=20$ (solid line), denoted by $\xi^\beta(w)$, stay below their perfect-protection counterparts at $\beta=\infty$ (dashed line), denoted by $\xi^\infty(w)$, for all w which implies a lower cost of capital for the entrepreneur when the degree of investor protection deteriorates. Intuitively, the entrepreneur gains privileges from the firm (through stealing) when $\beta<\infty$. Consequently, she is willing to run the firm even at a relatively low level of ξ.

5.2.2. Dividends Payouts

We simulate the firm's dividend payouts $\{D_t\}$ as the averages from 10,000 simulated firms where each firm is subject to three types of shocks: the investment-specific shocks of Z_t^I, the productivity shock of Z_t^A, and the B_t-shock for market risks. More specifically, we obtain a sequence of independent draws from a trivariate normal distribution with correlation coefficient being ρ_I for $\{Z_t^I,B_t\}$, ρ for $\{Z_t^A,B_t\}$, and 0 for $\{Z_t^I,Z_t^A\}$. The path of $\{Z_t^I,Z_t^A,B_t\}$ are then mapped into a path of the state variables (K_t,W_t) that corresponds to one particular firm, from which we calculate the firm's optimal policies that help drive the evolution of (K,W) next period. Whenever a firm's financial slack falls below w_d, it gets liquidated, delivers $(1-y)(w_d+l)K_t$ to outside shareholders in next period, and then is removed from simulation[13]. We start at $w_0=0$ for all simulated firms and without the loss of generality we further normalize their initial capital K_0 to one.

[13]Recall that the entrepreneur receives $y(W+lK)$ upon the firm's liquidation. She then retires as a Merton consumer with the initial wealth for her retirement given by $y(w_d+l)K_t$.

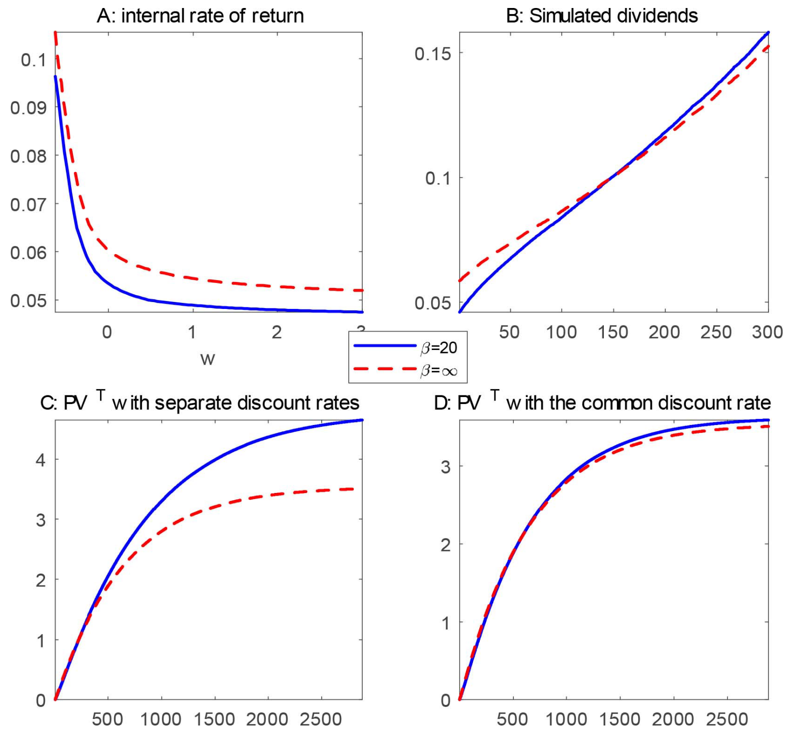

Figure 3. Firm valuation from outside shareholders' perspective. **Figure 3** plots variables related to firm valuation for two scenarios of β: $\beta = 20$ and $\beta = \infty$. Panel A plots the internal rate of return (IRR) for the firm as the function of the firm's financial slack w. Panel B plots the simulated average dividend payouts D which are based on 10,000 simulated firms for a time horizon of 300 months, where all simulated firms start at $w_0 = 0$. Panel C&D plot PV^T, the present value of firm's dividend payout up to period T when T varies from 0 to 2880 months. In Panel C, we pair the backed-out IRRs with the $\{D_s\}_{s=0}^{2880}$ that are simulated under the same β-scenario, while in Panel D we uses IRRs backed out at $\beta = \infty$ as the common discount rates to calculate the implied PV^T under both β-scenarios.

Panel B of **Figure 3** reports the simulated dividend payouts under the two β-scenarios. Compared to $\{D_t^\infty\}$ that are simulated under the perfect investor protection (dashed line), the simulated $\{D_t^\beta\}$ at $\beta = 20$ (solid line) falls below initially but rises above in the longer term, *i.e.*, roughly after the first 160 months. The initial depression of D is quite intuitive: It is due to the entrepreneur's stealing which reduces the firm's available resources to make payouts. On the other hand, stealing in our setup also provides the empire building incentive for the entrepreneur since she can enjoy a higher privilege with the firm when it grows bigger. With the expanded opportunity set that allows her to take unconstrained positions in the stock market, the entrepreneur has a strong incentive to re-allocate resources from payouts to asset allocation which strengthens the value-creation effect leading to even more aggressive asset allocation policies in the future[14]. The rising W (liquid wealth) and the rising X (asset allocation) reinforces

[14]In unreported simulation exercise, we find that the firm's asset allocation X keeps growing across the time and it is always higher at $\beta = 20$ than at $\beta = \infty$.

each other which gives rise to the higher dividend payouts in the longer term.

5.2.3. Firm Valuation with IRRs as the Discount Rates

We now examine whether the "short-term pain for long-term gain" pattern of dividend payouts actually benefit the outside investors. To this purpose, we combine the firm's IRR calculated from Section V.B.1 with the simulated dividends payouts from Section V.B.2, and calculate the valuation of the firm by

$$PV\left(w_0\right) \equiv \frac{1}{1-y} E\left[\int_0^{\tau} \mathrm{e}^{-\xi\left(w_s\right)s}\left(1-y\right)D_s \mathrm{d}s + \mathrm{e}^{-\xi\left(w_\tau\right)\tau}\left(1-y\right)\left(W_\tau + lK_\tau\right)\right], \quad (5.2)$$

where $1-y$ denotes the aggregated cash-flow right by outside investors; τ denotes the firm's (stochastic) liquidation time. To implement the definition of (5.2), we introduce the trancated version of PV, denoted as PV^T, which is calculated as follows:

$$PV^T\left(w_0\right) \equiv \frac{1}{1-y} E\left[\int_0^{\min\{\tau,T\}} \mathrm{e}^{-\xi\left(w_s\right)s}\left(1-y\right)D_s \mathrm{d}s \right. \\ \left. + \mathrm{e}^{-\xi\left(w_\tau\right)\tau} 1_{\{\tau>T\}}\left(1-y\right)D_T + \mathrm{e}^{-\xi\left(w_\tau\right)\tau} 1_{\{\tau<T\}}\left(1-y\right)\left(W_\tau + lK_\tau\right)\right]. \quad (5.3)$$

In (5.3), $1_{\{.\}}$ denotes the indicator function which equals 1 when what is within {.} is correct and 0 otherwise. Apparently, $PV = PV^T$ when $T \to \infty$. For a finite T, PV^T can be treated as the valuation of a firm which "expires" at T. Given T, we calculate PV^T through a large sample of simulated firms. Upon its "expiration" before the liquidation, a firm delivers $\left(1-y\right)D_T$ to outside shareholders. However, if the firm is liquidated before its "expiration", it delivers $\left(1-y\right)\left(W_\tau + lK_\tau\right)$ and is then excluded from the rest of the simulation. As indicated by its definition, we explicitly account for the state-dependence of the discount rates, as indicated by $\xi\left(w_s\right)$, in our calculations of PV^T.

To ensure that PV^T converges to PV, we simulate a long time series of dividend payouts up to 240 years so that a further increase of the time span has little impact on the implied PV^T. We thus use the value of PV^T obtained at the end of the 240th year as the (accurate) approximation of PV. To study the impact of control friction on firm valuation, we plot in Panel C&D of **Figure 3** the implied $PV^T s$ from $T=0$ up to the 240th year under two scenarios of β, where the initial financial slack is set at zero without loss of generality.

In Panel C, we pair $\xi\left(.\right)s$ that are backed out according to (5.1) with the simulated dividend payouts for the same β-scenario. Relative to its perfect-protection counterpart which shuts down the control friction (dashed line), PV^T at $\beta = 20$ (solid line) falls below initially but quickly rises above. It then stays above with the implied discrepancy getting wider and wider as time goes by. In unreported exercises, we confirm that this result holds as we vary w_0. Thus, the "short-term pain for long-term gain" pattern of dividend payouts, which is induced by the control friction at the presence of imperfect investor protection, indeed translate into a higher valuation of the firm.

To reinforce our result, we further plot in Panel D of the **Figure 3** the implied

PV^T where we use the common discounting scheme of $\xi^{\infty}(.)$ for its calculation under both β-scenarios. Compared to plots in Panel C, the implied $PV^T s$ at $\beta = \infty$ (dashed line) remain unchanged but the implied $PV^T(\beta = 20)$ (solid line) falls below because the underlying dividend streams are now discounted by the higher discount rates implied from $\xi^{\infty}(.)$ (see Panel A of **Figure 3** that shows $\xi^{\infty} > \xi^{\beta}$). Naturally, this higher discounting with $\xi^{\infty}(.)$ keeps $PV^T(\beta = 20)$ below $PV^T(\beta = \infty)$ for a longer time. Still, the strengthened value-creation effect attributed to the entrepreneur's empire building motives at $\beta < \infty$ eventually drives $PV^T(\beta = 20)$ above $PV^T(\beta = \infty)$, albeit by a smaller margin when compared to that plotted in Panel C.

5.3. Welfare Analyses

In Section V.B, we show that control friction enables a higher firm valuation for outside shareholders. One caveat for the analysis is that it focuses on mean-only information. Indeed, the definition of (5.2) indicates that only information from the first moment of dividend payouts is needed for the calculation of PV. As shown in Panel A&B of **Figure 4**, the simulated dividend payouts are more volatile at $\beta = 20$ (solid line) than at $\beta = \infty$ that shuts down the control friction (dashed line), either along the dimension of time (Panel A) or along the dimension of the firm's initial financial slack w_0 (Panel B). Intuitively, the entrepreneur's empire building motives facilitated by control friction at $\beta < \infty$ leads to over-investment and a more aggressive position in the stock market relative to the case at $\beta = \infty$. Consequently, the firm is subject to more of the investment and market risks which is captured by a more volatile dividend stream. The higher volatility is disliked by a risk averse agent which could lead to a lower utility at $\beta < \infty$ despite of the higher average payouts. In this section, we examine this logic by performing the welfare analyses for the two types of firm agents.

5.3.1. Numerically Calculating Utilities

Since all firm agents are endowed with the same CRRA utility function given by (2.8), we calculate their accumulated utilities from the firm from period 0 up to period T by

$$V_1^T(w_0) \equiv E\left[\int_0^{\min\{\tau,T\}} \mathrm{e}^{-\zeta s} U(C_{1,s})\,\mathrm{d}s + \mathrm{e}^{-\zeta\tau} 1_{\{\tau>T\}} U(C_T)\right] \tag{5.4}$$

for the entrepreneur, and by

$$\begin{aligned} V_2^T(w_0) \equiv E\Big[&\int_0^{\min\{\tau,T\}} \mathrm{e}^{-\zeta s} U((1-y)D_s)\,\mathrm{d}s + \mathrm{e}^{-\zeta\tau} 1_{\{\tau>T\}} U((1-y)D_T) \\ &+ \mathrm{e}^{-\zeta\tau} 1_{\{\tau<T\}} U((1-y)(W_\tau + lK_\tau))\Big] \end{aligned} \tag{5.5}$$

for outside investors. In (5.4)-(5.5), τ denotes the firm's (stochastic) liquidation time; $1_{\{\cdot\}}$ denotes the indicator function; ζ denotes the (constant) subjective discount; $U(.)$ is given by (2.8). In the former case with V_1^T, the entrepreneur derives her utility solely from her consumption stream $\{C_{1,s}\}_{s=0}^T$ when the firm is alive, where C_1 is given by (3.19). Upon the firm's liquidation, she obtains the

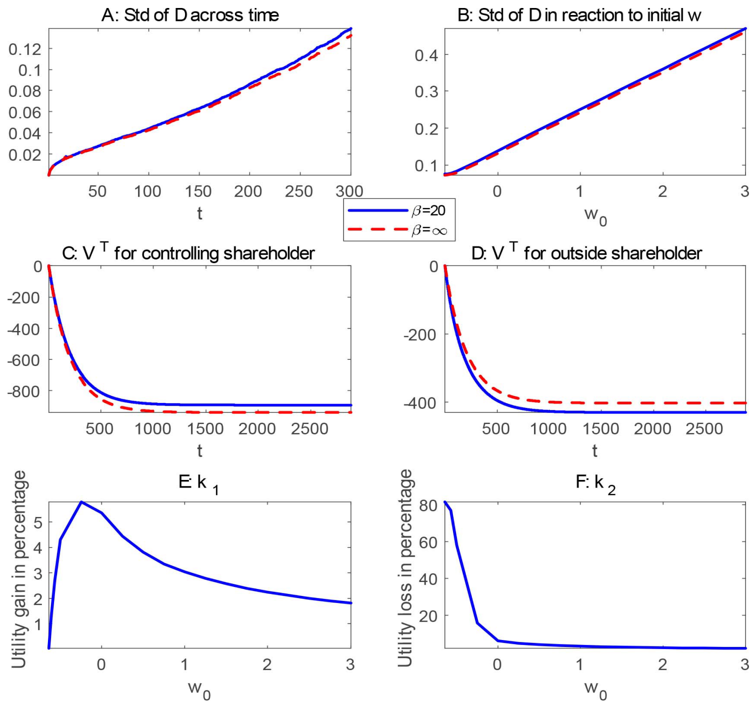

Figure 4. Utility analyses. Panel A&B plot the volatility of the simulated dividend payouts along the dimension of the time and along the dimension of the firm's initial financial slack w_0, respectively, where $w_0 = 0$ for Panel A and the time span is 300 months for Panel B. Panel C&D plot V_1^T and V_2^T, respectively, which denote utilties that are accumulated from the firm up to to period T by the controlling shareholder (V_1^T) and by outside shareholders (V_2^T). Variables in Panel A-D are all plotted under two β-scenarios: $\beta = 20$ and $\beta = \infty$. Panel E&F plot the percentage welfare gain for the controlling shareholder, denoted by k_1, and the percentage welfare loss for outside shareholders, denoted by k_2, respectively, when the degree of investor protection deteriorates from $\beta = \infty$ to $\beta = 20$.

lump-sum payment of $y(W_\tau + lK_\tau)$ which, instead of being consumed, is used as the initial wealth for her retirement as the Merton consumer. In the latter case with V_2^T, outside investors derive their utilities from the firm according to their cash-flow right which include 1) the dividend payouts of $(1-y)D_s$ up to T conditional on that the firm is alive; and 2) the liquidation payment of $(1-y)(W_\tau + lK_\tau)$ conditional on a firm liquidation that occurs before T. Given V_j^T for $j = 1,2$, the welfare that the j th type of agents derives from the firm is its limit V_j when $T \to \infty$.

To ensure that V_j^T converges to V_j, we again simulate a large sample of firms from which we generate a long time series of $\{C_{1,s}\}_{s=0}^T$ and $\{D_s\}_{s=0}^T$ that covers a period of 2880 months. The details on simulation are described in Section V.B.2. Without the loss of generality, we set w_0 to 0 and calculate V_1^T and V_2^T according to (5.4)-(5.5) by making use of the simulated streams of consumptions and dividend payouts. Panel C&D of Figure 4 plot the implied V_1^T and V_2^T, respectively, for the two β-scenarios. Both V_1^T and V_2^T converge faily quickly in that the implied curves all start to flatten out after the first 1000 months. Given that the baseline value of γ is greater than one, (2.8) implies that $U(C)$ is negative at any positive C. Consequently, the implied $V_1^T s$ and $V_2^T s$ are all negative.

Different than the plots in **Figure 3** on firm valuation, the implied welfare for outside investors, calculated as V_2^T in its convergence region (Panel D of **Figure 4**), is lower with the control friction at $\beta = 20$ (solid line) than without the friction at $\beta = \infty$ (dashed line) which restores the usual conclusion that corporate stealing is damaging to minority shareholders from outside. Despite of the higher volatility of dividend payouts at a lower degree of investor protection (Panel A&B of **Figure 4**), the implied welfare for the entrepreneur is still higher with the control friction at $\beta = 20$ than without (Panel C). To understand the difference, notice that there are two offsetting effects that determine the welfare impact of control friction. Relative to its perfect-protection counterpart, control friction at a lower β induces the empire building motive that gives rise to both the higher dividend payouts in the longer term (cash-flow effect) and a more volatile stream of payouts (volatility effect). While cash-flow effect enhances welfare, the volatility effect depresses it. In the case with outside investors, the volatility effect dominates so that a lower degree of investor protection delivers a welfare loss. In contrast, the volatility effect is dominated by a strengthened cash-flow effect in the case with the entrepreneur because the stealing technology enables a consumption stream that stays uniformly above its perfect-protection counterpart.

5.3.2. Welfare Effects of Imperfect Investor Protection

We further conduct welfare analyses to quantify the impact of imperfect investor protection on both the entrepreneur and outside investors. More specifically, we calculate welfare gain (or loss) for the two types of firm agents when the degree of investor protection deteriorates from $\beta = \infty$ to $\beta = 20$. To this purpose, we account for the dependences of V_1 and V_2 on their respective consumption stream by explicitly writing their formulas as follows:

$$V_1\left(\left\{C_{1,s}\right\}; w_0\right) \equiv E\left[\int_0^\tau \mathrm{e}^{-\zeta s} U\left(C_{1,s}\right) \mathrm{d}s\right] \tag{5.6}$$

$$V_2\left(\left\{C_{2,s}\right\}; w_0\right) \equiv E\left[\int_0^\tau \mathrm{e}^{-\zeta s} U\left(C_{2,s}\right) \mathrm{d}s + \mathrm{e}^{-\zeta \tau} U\left(C_{2,\tau}\right)\right]. \tag{5.7}$$

where $C_{1,s}$ is given by (3.19); $C_{2,s} = (1-y) D_s$ and $C_{2,\tau} = (1-y)(W_\tau + lK_\tau)$. It is easy to see that (5.6)-(5.7) are the limiting cases of (5.4)-(5.4) when $T \to \infty$ [15].

[15]We emphasize the difference between V_1 and the entrepreneur's value function J which is defined by (2.7). While J accounts for all utilities that entrepreneur accumulates, either from the firm or as a Merton consumer after the liquidation of the firm, V_1 defined by (5.6) focuses on her utilities obtainable from the firm before its liquidation. In particular, V_1 would be quite different than J when liquidation is imminent which is the case in our simulation when the initial financial slack w_0 is fairly close to the liquidation boundary w_d. For our welfare analyses, we are interested in calculating the entrepreneur's utilities with the firm so it seems appropriate to choose V_1 over J. In addition, simultaneously calculating V_1 and V_2 according to (5.6)-(5.7) enables us to gauge the welfare effect of control friction (or equivalently, investor protection) on different types of firm agents in an internally consistent manner.

To quantify the welfare effects, we calculate the percentage change in consumptions under the perfect investor protection that is required to maintain a firm agent's welfare level when the degree of investor protection deteriorates. Consider the entrepreneur first. For the given w_0, define a constant number k_1 such that

$$V_1^{\infty}\left(\left\{(1+k_1)C_{1,s}^{\infty}\right\};w_0\right)=V_1^{\beta}\left(\left\{C_{1,s}^{\beta}\right\};w_0\right) \tag{5.8}$$

In (5.8), the superscript " ∞ " denotes the case when control friction is shut down at $\beta=\infty$ while the superscript " β " denotes the case with the control friction at a finite β. In particular, $\left\{C_{1,s}^{\infty}\right\}$ and $\left\{C_{1,s}^{\beta}\right\}$ denote the entrepreneur's optimal consumption stream at $\beta=\infty$ and at $\beta<\infty$, respectively. The entrepreneur is likely to gain from the control friction which implies that $V_1^{\infty}\left(\left\{C_{1,s}^{\infty}\right\};w_0\right)<V_1^{\beta}\left(\left\{C_{1,s}^{\beta}\right\};w_0\right)$. k_1 thus measures the welfare gain for the entrepreneur, in terms the percentage increase of her consumption from its perfect-protection level, when the degree of investor protection deteriorates which introduces the control friction. Note that an increase of $1+k_1$ applies to the entire consumption stream.

Turning to outside investors, we similarly define a constant k_2 for the given w_0 such that

$$V_2^{\infty}\left(\left\{(1-k_2)C_{2,s}^{\infty}\right\};w_0\right)=V_2^{\beta}\left(\left\{C_{2,s}^{\beta}\right\};w_0\right) \tag{5.9}$$

where the superscripts " ∞ " and " β " are defined in a similar way as that for V_1. As shown in Panel D of **Figure 4**, an outside investor achieves the higher utility when control friction is shut down at $\beta=\infty$. Therefore, to maintain the welfare level when the degree of investor protection deteriorates, we need to adjust his consumption at $\beta=\infty$ downward from the optimal level of $\left\{C_{2,s}^{\infty}\right\}$. In other words, an outside investor feels indifferent when β falls below ∞ only if his consumption stream prior to the change is actually below $\left\{C_{2,s}^{\infty}\right\}$. The implied k_2, as defined in (5.9), thus measures his welfare loss, in terms of the percentage decrease of his consumption stream, when the degree of investor protection deteriorates.

To gauge the impact of the firm's initial financial status on the implied welfare effects, we vary the initial w_0 from −0.64 to 3 when calculating k_1 and k_2. We plot the resulting $k_1(w_0)$ and $k_2(w_0)$ in Panel E&F of **Figure 4**, respectively, where both k_1 and k_2 are quoted in percentage. As w_0 rises, the firm's dividend payouts become more attributable to the accumulated liquid wealth W and less attributable to the firm's capital K because a higher financial slack, as measured by w, allows a more aggressive asset allocation strategy which enables the firm to better exploit the value-creation effect from the stock market. Given that the amount of stealing is proportional to K, a higher w_0 thus implies, in a relative sense, a lower degree of damage that stealing imposes on outside investors. This decreasing stealing effect implies a lower welfare loss so that $k_2(w_0)$ plotted in Panel F of **Figure 4** is monotonically decreasing. Specifically, k_2 can rise as high as 81.5% when w_0 drops to a level that is very close to w_d. Intuitively, a higher firm-level volatility, which is attributed the deterioration in the degree of investor protection (see Panel A&B of **Figure 4**), is quite costly to outside shareholders when it can easily trigger the firm's liquidation at a sufficiently low w_0.

In comparison, the entrepreneur's welfare gain due to the control friction is much lower (Panel E). The difference in magnitudes between k_1 and k_2 reflects the power of the control right which allows the entrepreneur to "smooth out" her welfare to β-variations. Unlike k_2, k_1 is not monotonic in w_0 and the dividing point is roughly at $w_0 = -0.28$. When $w_0 > -0.28$, the decreasing stealing effect explained in the last paragraph is dominant which also drives a decreasing $k_1(w_0)$. When w_0 is relatively low, on the other hand, the resource re-allocation effect, which means that the entrepreneur exploits the control friction to re-allocate resources from dividend payouts to investment and asset allocation that helps raise her consumption in the future, becomes the dominant force. A higher w_0 depresses the concern for liquidation and thus enhances the resource re-allocation effect which raises the entrepreneur's welfare. Consequently, we observe an increasing $k_1(w_0)$ in the region between w_d and −0.28. In summary, our model provides the first theoretical study that links the welfare effect of investor protection to the firm's financial status. By doing so, it predicts that 1) the entrepreneur's welfare gain following a deterioration of investor protection is hump-shaped; and 2) investor protection is more important for outside investors when the firm is in a worse financial condition.

5.3.3. Comparative Analysis on Welfare Effects

In this subsection, we perform comparative analysis with respect to the implied welfare gains (or losses) when we vary some key model parameters. Panel A-C of **Figure 5** plot the results with respect to k_1. Panel A shows the impact of ϵ which controls the volatility of investment-specific shocks. A higher ϵ means a higher firm-level volatility that comes from investment risk which drives up the volatility of the simulated dividend payouts. Consequently, it drives up the volatility of the consumption stream which is disliked by the risk-averse entrepreneur. While a deterioration of investor protection still generates the welfare gain for the entrepreneur at a higher ϵ, the implied consumption stream $\left\{C_{1,s}^{\beta}\right\}$ at $\beta < \infty$ becomes less desirable because of the extra volatility induced by ϵ. Consequently, the welfare gain from control friction as measured by k_1 is lower at $\epsilon = 1$ (dashed line) than at $\epsilon = 0$ (solid line).

We next allow the firm agents' degree of risk aversion γ to vary and plot the implications on $k_1(w_0)$ in Panel B of **Figure 5**. At a higher $\gamma = 4$, the higher volatility of $\left\{C_{1,s}^{\beta}\right\}$ under the control friction is more disliked by the entrepreneur which implies a lower welfare gain across all w_0. An interesting observation is that k_1 at $\gamma = 4$ apparently turns negative when w_0 gets close to the implied liquidation boundary at $\gamma = 4$, which implies welfare losses instead of gains to the entrepreneur. Intuitively, a drop of w_0 towards w_d raises the concern about the negative effect of the higher volatility at $\beta < \infty$ because it can trigger the value-destroying liquidation more easily. When the entrepreneur is more risk

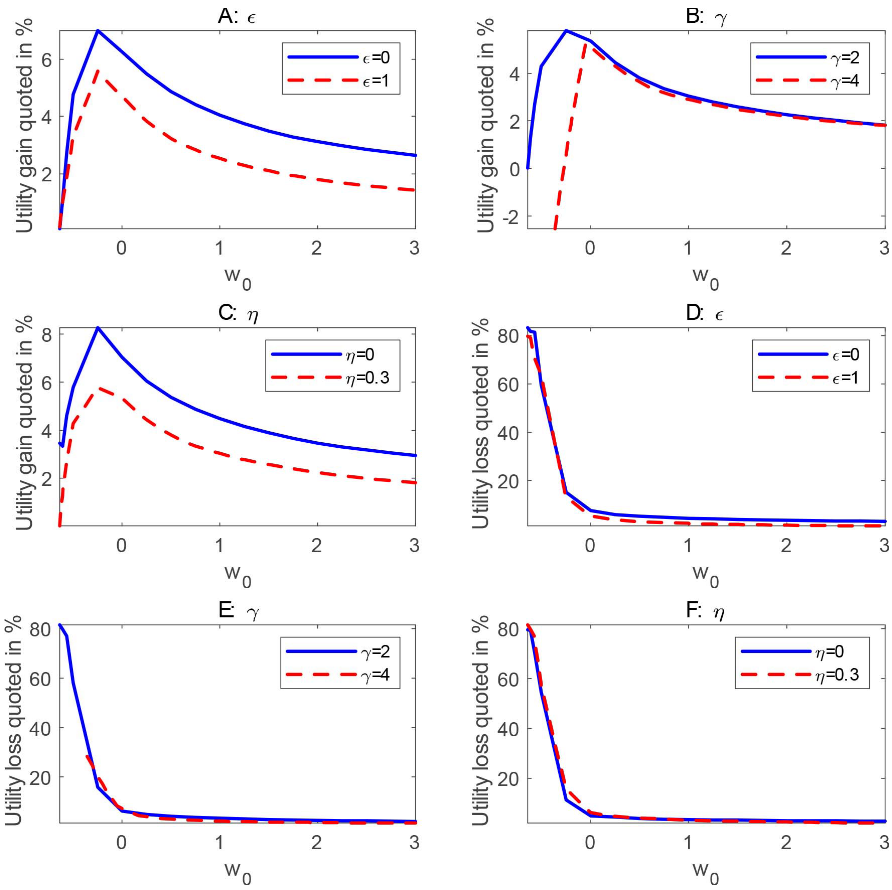

Figure 5. Comparative analysis with respect to k_1 and k_2. k_1 and k_2 denotes the welfare gain for the controlling shareholder and the welfare loss for outside shareholders, respectively, when the degree of investor protection deteriorates from $\beta = \infty$ to $\beta = 20$. Panel A-C plot comparative statics with respect to k_1 when we vary the volatility parameter for investment-specific shocks ϵ (Panel A), the firm agents' degree of risk aversion γ (Panel B), and the market Sharpe ratio η which determines the equity risk premium (Panel C). Panel D-F plots comparative statics with respect to k_2 when we vary ϵ (Panel D), γ (Panel E), and η (Panel F), respectively.

averse, the volatility effect is substantially enhanced and may even dominate the positive cash-flow effect (induced by stealing at the presence of control friction) when w_0 is very close to w_d. Consequently, she may also suffer a welfare loss following a deterioration of investor protection.

We further examine the impact of equity risk premium which is measured by the market Sharpe ratio η holding σ_R unchanged. With the market risk compensation shut down at $\eta = 0$, the value-creation effect is gone so that the firm at $\beta < \infty$ no longer takes aggressive positions in the stock market. This has two effects on the entrepreneur's welfare. First, the volatility of her consumption stream comes down since control friction facilitated by the imperfect investor protection no longer induces her to increase the firm's market exposure. Second, without the opportunity from the stock market, the resource re-allocation effect now implies that the firm is allocating even more resources to investment. The further strengthened over-investment enables a faster growth of the firm's capital stock K which allows the entrepreneur to steal even more because the amount that she can divert is proportional to K. Both effects work in the same direction of driving up the welfare gain. Consequently, $k_1(w_0)$ rises uniformly when η is reduced from its baseline level of 0.3 to 0 as plotted in Panel C of **Figure 5**[16].

We now turn to Panel D-F of **Figure 5** which plots the results of comparative statics for outside investors. Unlike k_1, k_2 remains positive irrespective of parametric changes which means that control friction always hurts outside shareholders. The dominant force that determines k_2 is apparently the firm's initial financial slack w_0. In particular, when $w_0 < 0$ so that the firm is in debt, a drop of w_0 towards the firm's liquidation boundary substantially raises k_2 implying a huge welfare loss to outside investors that is attributed to the control friction. The impact of all other determinants of k_2 are relatively minor because they're subject to offsetting forces. For example, a higher γ aggravates the volatility effect which tends to raise k_2 implying a larger welfare loss. This is indeed the case when w_0 is relatively small as plotted in Panel E of **Figure 5**. However, a higher γ also prompts the entrepreneur to adopt less aggressive policies on investment and asset allocation which reduces firm-level risks as indicated by a smoother dividend payout scheme. This consumption-smoothing effect is more pronounced when the firm is in a better financial status[17] leading to a lower $k_2(w_0)$ at the higher γ when w_0 is relatively high.

To summarize, welfare gains for the entrepreneur that results from stealing under imperfect investor protection is relatively small. Such gains may decrease as firm comes closer to its liquidation and they may even turn negative under certain circumstances. In contrast, outside investors suffer substantial welfare losses from a deterioration of investor protection when the firm is initially in debt (as indicated by a negative w_0). Variations in parameters that govern investment risk, risk preference, and risk compensation deliver clear effects on welfare changes for the entrepreneur, whereas their welfare implications are mixed for outside investors. Overall, our results deliver a clear economic message: While enhancing investor protection is indeed preferrable from the perspective of welfare analyses, it

[16]It is worth noticing that the shape of the implied $k_1(w_0)$ in Panel C is more nuanced at $\eta = 0$ (solid line) in that it also decreases a bit when w_0 is very close to the liquidation boundary (equaling -0.705 at $\beta = \infty$). Since w_d^β at $\beta = 20$ is lower than $w_d^\infty = -0.705$, a deterioration from perfect investor protection to $\beta = 20$ enables the entrepreneur to gain the breathing room when the firm is very close to its liquidation an instant of time before the change in β. Apparently, the implied liquidation relief effect becomes stronger as w_0 drops towards w_d^∞. Still, this effect is uniformly dominated by the resource re-allocation effect in the case with $\eta = 0.3$. When the market's risk compensation is shut down at $\eta = 0$, however, gaining the breathing room following the deterioration of investor protection becomes more valuable because it simultaneously points to a higher percentage welfare gain as discussed in the main text. Consequently, the aforementioned liquidation relief effect is further strengthened and our numerical result indicates that it can become dominant when w_0 becomes sufficiently close to w_d^∞ leading to an increased k_1 as w_0 further drops towards w_d^∞ (leftmost of the solid line plotted in Panel C of **Figure 5**).

[17]This is because the entrepreneur now is less concerned about the potential liquidation, which allows her to focus more on intertemporal smoothing.

is particularly desirable when the firm is in financial distress.

6. Discussions

Our robust result that imperfect investor protection raises firm valuation for outside investors is quite surprising because it seems to suggest that corporate stealing, despite of its destructive effect in earlier periods, can actually benefit outside shareholders. We emphasize that this result relies critically on the value-creation effect which hinges on a sizable equity risk premium and an expanded opportunity set for the firm that allows it to take unconstrained positions in the stock market. Indeed, we show in the top two panels of **Figure 6** that PV is lower at $\beta = 20$ than at $\beta = \infty$ [18] when we either shut down the market Sharpe ratio η (Panel A) or when we impose an asset allocation constraint of $x \le \bar{x} = 1$ (Panel B) where x denotes the asset allocation-capital ratio[19]. Intuitively, when there is no risk compensation for loading on the stock market or when the firm's asset allocation strategy is severely constrained, the value-creation effect from stock trading is effectively (or largely) shut down so that it can no longer efficiently accumulate its financial wealth. Consequently, the entrepreneur's empire building motives at the presence of control friction (when $\beta < \infty$) can no longer deliver a higher firm valuation to outside investors.

In our calculations for PV, we use the cost of capital ξ, referred to as the internal rate of return (IRR) for firm-held capital, as the (time-varying) discount rates for firm-generated dividend payouts. Since the entrepreneurial firm in our setup is 100% equity financed (from both the controlling shareholder and all the outside shareholders), using the cost of capital as the discount rates seems appropriate. Empirically, however, IRR is primarily used for free cash flows instead of for the dividend payouts D [20]. In our setup, the firm's free cash flows are naturally proxied by its accumulated liquid wealth W. As a robustness check, we use

[18]As explained in Section V.B.1, $\xi^{\beta}(w)$ at $\beta = 20$ takes into account the entrepreneur's privileges with the firm. In other words, $\xi^{\beta}(w)$ is a discounting scheme solely from the entrepreneur's perspective while $\xi^{\infty}(w)$ at $\beta = \infty$ reflects the more objective measure of the firm's IRR. Consequently, in these exercises we use the common discounting scheme of $\xi^{\infty}(.)$ to calculate the implied $PV^{T}s$ under both β-scenarios. Also recall that PV is calculated as PV^{T} in its convergence region.

[19]Imposing the asset allocation constraint poses extra challenges for solving the entrepreneur's problem. To address the technical challenge, we raise the adjustment cost parameter θ from its baseline level of 2 to 5 when we impose $x \le \bar{x} = 1$. We find that raising θ efficiently restores the accuracy of our numerical solution for the case with the asset allocation constraint. Simultaneously, we find that raising θ leaves firm valuations in the baseline case virtually unaffected which confirms that variations in θ has little impact on the implied $PV^{T}s$. These unreported results are available upon request.

[20]One may also be concerned that IRR is typically used as the criterior that helps a firm make its investment decisions. In other words, IRR is the end result of valuation, not the means of it. As will be discussed in next three paragraphs, however, the backed-out IRRs naturally account for both the systematic component and the idiosyncratic component of risk premia demanded by the firm and both components are vital for an appropriate discounting scheme on firm-generated cash flows. In addition, the adoption of IRR for the discounting scheme helps uncover new insights about firm-level risk compensations at the presence of control friction.

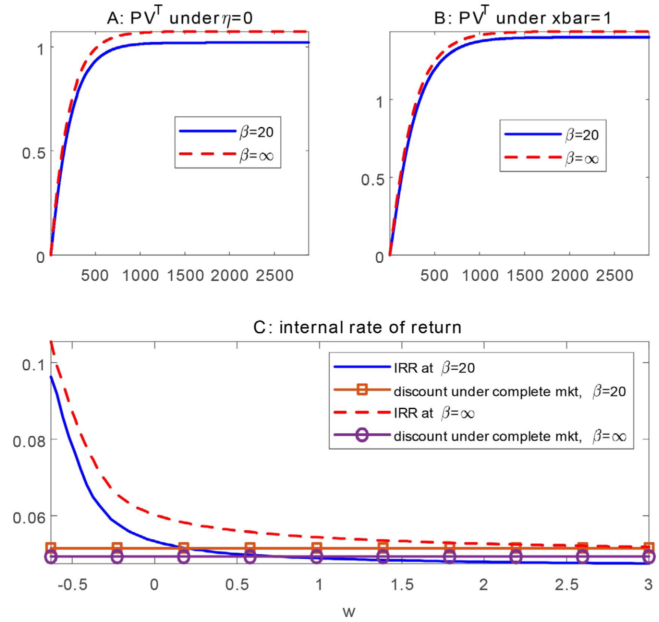

Figure 6. Firm valuation under different model variants and more results on the internal rate of return (IRR). The top two panels plot PV^T, the present value of firm's dividend payouts up to period T, when equity risk premium is shut down and when we impose the asset allocation constraint of $x \le \bar{x} = 1$, respectively, where x denotes the asset allocation-capital ratio. The bottom panel plots the internal rate of return (IRR) for the firm as the function of the firm's financial slack $w(\equiv W/K)$. For comparison, we plot in the same panel the firm-level discount rates under complete markets. All the plots are with respect to two scenarios of β: $\beta = 20$ and $\beta = \infty$ (which shuts down the control friction).

IRR as the discount rates for W instead of for D. Calculated in this way, we find that a firm with the unrestricted opportunity set from the stock market still exhibits a higher valuation with the control friction than without. Intuitively, the accumulated liquid wealth in our setup in the main driver for firm-level payouts so they exhibit the same temporal variations of "short-term pain for long-term gain" leading to the very similar implications on firm valuation.

Admittedly, IRR is only one of many choices for the firm's discounting scheme. Another choice is to use the traditional cost of capital model (that captures the market risk part) with an addition of idiosyncratic risk represented as a shock with zero expected value and a constant volatility[21]. However, making the compensation adjustment for idiosyncratic risk is in general hard to get done. Compared to some ad hoc way of adding such "idiosyncratic risk premium", our choice with IRR has the advantage of naturally accounting for both the systematic and the additional idiosyncratic risk premium for firm-levels risk. Furthermore, it helps generate new insights on firm-level compensations when firm agents are subject to the control friction. To illustrate the point, recall from Section V.B.2 that IRR

[21]We thank a referee for making this point. In unreported exercises, we find that our model's implications on firm valuations remain largely unchanged when we use the traditional cost of capital model to deduce the discount rates without accounting for the idiosyncratic component of firm-level risks.

backed out at $\beta = 20$ (with control friction) $\beta = \infty$ (without control friction) are denoted by $\xi^{\beta}(w)$ and $\xi^{\infty}(w)$, respectively. To identify the idiosyncratic component of ξ, we let $w \to \infty$ so that firm-level risks is no longer a concern and the market becomes effectively complete. We calculate the implied $\hat{\xi}(\beta)$ (for $\beta = 20$) and ξ^{FB} (for $\beta = \infty$) under this special case[22] which are then plotted in the bottom panel of **Figure 6** together with $\xi^{\beta}(w)$ and $\xi^{\infty}(w)$.

As shown in Panel C of **Figure 6**, both $\hat{\xi}(\beta)$ and ξ^{FB} are invariant to w. This is because IRR under complete markets simply reflects the (constant) risk-free rate r plus the compensation for the firm's (time-invariant) market exposure where the market risk compensation $\mu_R - r$ is also a constant. In comparison, $\hat{\xi}(\beta)$ (line with square marker) stays above ξ^{FB} (line in circle) which reflects the entrepreneur's over-investment under control friction that exposes the firm more to the market risk. Intuitively, the entrepreneur gains privileges from the firm through stealing when $\beta < \infty$. Consequently, she is willing to run the firm even at a relatively low level of ξ. This implication is supported by Jiang, Lee, and Yue [33] who use Chinese data to show that controlling shareholders' private rents may not be fully included in normal expected returns. A similar point is also made in BCY [8] who show that the controlling shareholder hoards shares of the firm even if the realized risk premium is low because he is compensated not only by the risk premium but also by the fraction of the diverted output.

By ignoring the control friction, WWY [7] interprete the differences between $\xi^{\infty}(w)$ (dashed line) and ξ^{FB} as the idiosyncratic risk premium which they show is always positive. While this is also the case in our plots at $\beta = \infty$[23], we show a fundamentally different result when control friction is present at $\beta < \infty$: In that case, idisyncratic risk premium is similarly defined as $\xi^{\beta}(w) - \hat{\xi}(\beta)$ where $\xi^{\beta}(w)$ is plotted in solid line, and this difference is positive only for $w < 0.2$. Financially, the cost of capital under imperfect investor protection is determined by two offsetting forces: 1) the nondiversifiable firm-level risk which drives up the cost; 2) the entrepreneur's willingness to accept a lower IRR when

[22]At $\beta = \infty$ that rules out control friction, the firm achieves its first-best (FB) when $w \to \infty$, under which $\xi^{FB} = \beta_A^{FB}(\mu_R - r) + \beta_i^{FB}(\mu_R - r)$ where $\beta_A^{FB} = \frac{\rho\sigma_A}{\sigma_R}\frac{1}{q^{FB}}$ and $\beta_i^{FB} = \frac{\rho_I \epsilon i^{FB}}{\sigma_R}$. Financially, the expected rate of return from firm-held capital under FB is characterized by a two-factor version of capital asset pricing (CAPM) model where the two risk factors are 1) the productivity shock with its volatility governed by σ_A; and 2) the investment-specific shock with its volatility governed by ϵ. Specifically, the expression of ξ^{FB} degenerates to Equation (23) in WWY [7] when there is no investment risk at $\epsilon = 0$.

[23]As plotted in Panel C of **Figure 6**, the implied $\xi^{\infty}(w)$ (dashed line) stays above ξ^{FB} (line in circle) for ws up to 3. We find that $\xi^{\infty}(w)$ may actually fall below ξ^{FB} for a even larger w. We show the reason lies in the investment-specific shocks which is not considered in WWY. Indeed, in an unreported exercise we shut down ϵ and confirms the finding of WWY that the implied $\xi^{\infty}(w)$ always stays above ξ^{FB}. Intuitively, investment-specific shocks depresses capital investment which prompts the entrepreneur to re-allocate more resources to the stock market for a better exploitation of the value-creation effect. Consequently, she demands a lower rate of return from the firm-held capital.

she also derives privilege from the firm through stealing, which drives down the cost. The first force is dominant when the firm is close to its liquidation as indicated by the substantial increase of $\xi^{\beta}(w)$ when w nears w_d. For $w > 0.2$, however, the second force dominates so that $\xi^{\beta}(w)$ falls below the corresponding $\hat{\xi}(\beta)$ at $\beta = 20$. This result complements WWY [7] by showing that the so-called idisyncratic risk premium may easily change sign when the firm is subject to control friction.

In sum, our theoretical analyses highlight the key financial mechanism that an expanded opportunity set combined with control friction strengthens the entrepreneur's empire building motives which would lead to higher firm valuations even to outside shareholders who appear to suffer a loss in the earlier periods. If we treat the firm as if it is the entire economy where the entrepreneur works as the central government that runs the economy, then we can easily find its empirical support in the literature on growth. For example, Castro, Clementi, and MacDonald [34] show that South Korea experienced a much higher economic growth than India during the 1967-1996 even though Indian investors during this period have enjoyed better protection than their Korean counterparts. As another example, there is a wide acceptance that China's rapid economic growth in the past few decades has a lot to do with a powerful central government that direct resources to productive economic activities. Indeed, while resource re-allocation at China leads to the suffering of its older generation, the individuals of its latter generation, irrespective of their control rights in the country, all benefit from China's rapid economic growth.

7. Conclusion and Suggestions

We develop a dynamic stochastic model that integrates the control-ownership wedge into a framework featuring intertemporal asset allocation, consumption, costly business liquidation, and investment-specific shocks. The model makes numerous testable predictions. The entrepreneur has a stronger incentive to grow the firm at a lower degree of investor protection which prompts her to re-allocate resources from dividend payouts to more productive firm-level activities. Simultaneously, she demands a lower internal rate of return (IRR) for holding the firm. Value-creation effect from financial trading which is enhanced by the strong empire building motives may actually drive up the firm valuation. Deterioration of investor protection induces welfare losses to outside investors whose magnitudes rise substantially when the firm is in financial distress. Despite of the higher firm-level risks that results from empire building activities, the entrepreneur in general obtains welfare gain from the control friction. The magnitudes of welfare gains are higher for a lower investment risk, a lower degree of risk aversion, and a lower equity risk premium. We provide evidences documented in existing literature that support some of the predictions.

In our model, an outside investor, who lives indefinitely, experiences the whole process of "short-term pain for long-term gain" and we show in Section V.C.2 that the implied cash-flow effect is dominated by the volatility effect so that he always suffers a welfare loss when the degree of investor protection deteriorates. In reality, however, the implied "short-term pain for long-term gain" is usually a combined effect that applies to different generations in the population. Thus, while the older generation of firm agents suffers a pure loss (a lower payout plus a higher volatility), the latter generation may actually benefit if the enhanced cash-flow effect in the longer term dominates the volatility effect. We confirm this benefitting effect in an unreported simulation exercise where we calculate outside shareholders' utilities only after the simulated average dividend payouts have risen above their perfect-protection counterparts. An interesting extension of our model is thus to introduce inter-generational friction where 1) each firm agent is expected to live for finite amount of time; 2) there are conflicts between the different generations of the controlling agent in that the current generation cares less about cash flows incurred under the management of future generations. Such an extension would allow for a meaningful study on the interplays between the control friction and the inter-generational friction which we leave to future work.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] La Porta, R., Lopez-De-Silanes, F. and Shleifer, A. (1999) Corporate Ownership around the World. *The Journal of Finance*, **54**, 471-517. https://doi.org/10.1111/0022-1082.00115

[2] Claessens, S., Djankov, S. and Lang, L.H.P. (2000) The Separation of Ownership and Control in East Asian Corporations. *Journal of Financial Economics*, **58**, 81-112. https://doi.org/10.1016/s0304-405x(00)00067-2

[3] Faccio, M. and Lang, L.H.P. (2002) The Ultimate Ownership of Western European Corporations. *Journal of Financial Economics*, **65**, 365-395. https://doi.org/10.1016/s0304-405x(02)00146-0

[4] Lemmon, M.L. and Lins, K.V. (2003) Ownership Structure, Corporate Governance, and Firm Value: Evidence from the East Asian Financial Crisis. *The Journal of Finance*, **58**, 1445-1468. https://doi.org/10.1111/1540-6261.00573

[5] Laeven, L. and Levine, R. (2007) Complex Ownership Structures and Corporate Valuations. *Review of Financial Studies*, **21**, 579-604. https://doi.org/10.1093/rfs/hhm068

[6] Albuquerue, R. and Wang, N. (2008) Agency Conflicts, Investment, and Asset Pricing. *The Journal of Finance*, **63**, 1-40. https://doi.org/10.1111/j.1540-6261.2008.01309.x

[7] Wang, C., Wang, N. and Yang, J. (2012) A Unified Model of Entrepreneurship Dynamics. *Journal of Financial Economics*, **106**, 1-23.

https://doi.org/10.1016/j.jfineco.2012.05.002

[8] Basak, S., Chabakauri, G. and Yavuz, M.D. (2019) Investor Protection and Asset Prices. *The Review of Financial Studies*, **32**, 4905-4946. https://doi.org/10.1093/rfs/hhz038

[9] Shleifer, A. and Wolfenzon, D. (2002) Investor Protection and Equity Markets. *Journal of Financial Economics*, **66**, 3-27. https://doi.org/10.1016/s0304-405x(02)00149-6

[10] Dow, J., Gorton, G. and Krishnamurthy, A. (2005) Equilibrium Investment and Asset Prices under Imperfect Corporate Control. *American Economic Review*, **95**, 659-681. https://doi.org/10.1257/0002828054201422

[11] Aslan, H. and Kumar, P. (2012) Strategic Ownership Structure and the Cost of Debt. *Review of Financial Studies*, **25**, 2257-2299. https://doi.org/10.1093/rfs/hhs062

[12] Fama, E.F. and French, K.R. (2001) Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay? *Journal of Financial Economics*, **60**, 3-43. https://doi.org/10.1016/s0304-405x(01)00038-1

[13] Duchin, R., Gilbert, T., Harford, J. and Hrdlicka, C. (2017) Precautionary Savings with Risky Assets: When Cash Is Not Cash. *The Journal of Finance*, **72**, 793-852. https://doi.org/10.1111/jofi.12490

[14] Franzoni, F. (2009) Underinvestment Vs. Overinvestment: Evidence from Price Reactions to Pension Contributions. *Journal of Financial Economics*, **92**, 491-518. https://doi.org/10.1016/j.jfineco.2008.06.004

[15] Chen, C.X., Lu, H. and Sougiannis, T. (2011) The Agency Problem, Corporate Governance, and the Asymmetrical Behavior of Selling, General, and Administrative Costs. *Contemporary Accounting Research*, **29**, 252-282. https://doi.org/10.1111/j.1911-3846.2011.01094.x

[16] Keynes, J.M. (1936) The General Theory of Employment, Interest and Money. Macmillan.

[17] Greenwood, J., Hercowitz, Z. and Huffman, G.W. (1988) Investment, Capacity Utilization and the Real Business Cycle. *American Economic Review*, **78**, 402-417.

[18] Fisher, J.D.M. (2006) The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Journal of Political Economy*, **114**, 413-451. https://doi.org/10.1086/505048

[19] Hayashi, F. (1982) Tobin's Marginal Q and Average Q: A Neoclassical Interpretation. *Econometrica*, **50**, 213-224. https://doi.org/10.2307/1912538

[20] Shleifer, A. and Vishny, R.W. (1997) A Survey of Corporate Governance. *The Journal of Finance*, **52**, 737-783. https://doi.org/10.1111/j.1540-6261.1997.tb04820.x

[21] La Porta, R., Lopez-De-Silanes, F., Shleifer, A. and Vishny, R. (2002) Investor Protection and Corporate Valuation. *The Journal of Finance*, **57**, 1147-1170. https://doi.org/10.1111/1540-6261.00457

[22] Bebchuk, L.A., Kraakman, R. and Triantis, G.G. (2000) Stock Pyramids, Crossownership, and Dual Class Equity. In: Morck, R.K., Ed., *Concentrated Corporate Ownership*, University of Chicago Press, 295-315.

[23] La Porta, R., Lopez-de-Silanes, F., Shleifer, A. and Vishny, R.W. (1998) Law and Finance. *Journal of Political Economy*, **106**, 1113-1155. https://doi.org/10.1086/250042

[24] Johnson, S., Boone, P., Breach, A. and Friedman, E. (2000) Corporate Governance in the Asian Financial Crisis. *Journal of Financial Economics*, **58**, 141-186. https://doi.org/10.1016/s0304-405x(00)00069-6

[25] Merton, R.C. (1971) Optimum Consumption and Portfolio Rules in a Continuous-

Time Model. *Journal of Economic Theory*, **3**, 373-413. https://doi.org/10.1016/0022-0531(71)90038-x

[26] Tobin, J. (1969) A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, **1**, 15-29. https://doi.org/10.2307/1991374

[27] Du, D. (2024) Investment Risk, Financial Slack, and Value of Capital: A Theoretical Examination. *Journal of Mathematical Finance*, **14**, 365-387. https://doi.org/10.4236/jmf.2024.144021

[28] Eberly, J.C., Rebelo, S. and Vincent, N. (2009) Investment and Value: A Neoclassical Benchmark. Working Paper, Northwestern University and HEC Montreal.

[29] Whited, T.M. (1992) Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data. *The Journal of Finance*, **47**, 1425-1460. https://doi.org/10.1111/j.1540-6261.1992.tb04664.x

[30] Hennessy, C.A. and Whited, T.M. (2007) How Costly Is External Financing? Evidence from a Structural Estimation. *The Journal of Finance*, **62**, 1705-1745. https://doi.org/10.1111/j.1540-6261.2007.01255.x

[31] Moskowitz, T.J. and Vissing-Jørgensen, A. (2002) The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? *American Economic Review*, **92**, 745-778. https://doi.org/10.1257/00028280260344452

[32] Mueller, E. (2010) Returns to Private Equity—Idiosyncratic Risk Does Matter! *Review of Finance*, **15**, 545-574. https://doi.org/10.1093/rof/rfq003

[33] Jiang, G., Lee, C.M.C. and Yue, H. (2010) Tunneling through Intercorporate Loans: The China Experience. *Journal of Financial Economics*, **98**, 1-20. https://doi.org/10.1016/j.jfineco.2010.05.002

[34] Castro, R., Clementi, G.L. and MacDonald, G. (2004) Investor Protection, Optimal Incentives, and Economic Growth. *The Quarterly Journal of Economics*, **119**, 1131-1175. https://doi.org/10.1162/0033553041502171