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Entropy as an Indicator for Risk Sharing Pool Quality

Miwaka Yamashita

Department of Accounting and Finance, Faculty of Business Administration, Toyo University, Tokyo, Japan
Email: yamashita063@toyo.jp

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Abstract

This study investigates several pooling risk sharing schemes and the measurement of their pool quality as peer to peer (P2P) insurance platforms. The P2P insurance is an emerging and growing insurance scheme that helps each other compensate the pool's total losses by funding and risk sharing ex-post with a certain rule among the pool participants. The new scheme comes from preliminary empirical research regarding existing P2P insurance. In the end, the schemes are characterized by the rule and the adjacency matrix of the pool, which leverages network theory, and the new risk sharing rule looks like adjusted version of the typical network adjacency matrix, which identifies indicators of pool quality of a new scheme. While assessing effectiveness among several risk sharing rules, it finds that, especially for the new risk sharing scheme, those matrix entropy serves as an indicator of pool quality like volatility and network centrality. Examples illustrate the effectiveness of the proposed model in assessing pool quality. This study contributes to the literature by proposing a new risk sharing scheme, which is based on preliminary empirical research, considering pool quality, and introducing the adjusted version of adjacency matrix as a rule and a tool to model relative relationships among pool participants. It expands the understanding of risk sharing mechanisms in P2P insurance platforms and provides valuable insights for pool management and quality assessment. The study highlights the importance of considering relative relationships among pool participants.

Keywords

Entropy, Adjacency Matrix, Peer to Peer, Pooling, Risk Sharing

1. Introduction

Globally, numerous peer to peer (P2P) insurance platforms have emerged recently (e.g., Frendsurance in Germany and Lemonade in the US), and they expanded many traditional mutual aid schemes, such as Takaful and Rotating Savings and Credit Association ([1]-[3]). In the scheme, pool participants help each other in order to compensate for the total losses of the pool by funding ex-post along with a certain formula among the pool participants, such as dividing by total participant number. For example, in smartphone repair costs insurance case, if there are 100 pool participants, at the end of one year, the smartphone repair costs for all pool participants are added up and shared among the 100 participants. There exists no insurance company and each participant helps each other, paying one 100^{th} of the total cost. There is a variation of the formula, and it is that a participant will pay one 99^{th} of the total cost other than himself. This study investigates the latter, although the former is used for general example explanation for simplicity.

Academic research in this area has thus far focused on modelling traditional risk sharing mechanisms, developing P2P risk sharing formulas, and seeking optimal risk-pooling schemes, as well as determining optimal conditions in terms of deduction or excess loss reinsurance levels ([4] [5] and papers cited in Section 2.1). Moreover, research has examined the superiority of P2P risk sharing pools in terms of insurance pool stability ([6]), the mitigation of moral hazard the establishment of trust among pool members ([7]), and others. In addition, Pareto Optimality and its conditions are discussed in [8]-[13], and this paper does not discuss from the point of view.

In such a situation, preliminary empirical research was performed. The pool could be more attractive for existing and potential pool participants if the pool participants are only the well-known participants each other. Generally, there are several concerns regarding P2P insurance platform, like moral hazard (not sure if all platform participants be cooperative to save the loss) and adverse selection (more possibility to cause losses, more willingly to join the pool), because minute pool participant information is unknown. In order to take such trust issues into account, from the empirical research, a new risk sharing could be suggested, and it is that the pool participants share their total risk only among well-known participants.

The remainder of this paper is organized as follows. Section 2 introduces the basic risk sharing and network theories, as well as preliminary empirical research. Section 3 describes the problem setting regarding risk sharing pool, along with previous related research, and explains the model, the concept of adjusted version of adjacency matrix (hence force, adjacency matrix is denoted by AM and the adjusted version of AM is denoted by adjusted adjacency matrix (AAM), which detailed in Section 3.1), and the entropy indicator. In Section 4, numerical examples are presented, followed by a summary in Section 5.

2. Preparation

2.1. Risk Sharing Theory

The universal principles of risk sharing schemes were examined and summarized recently by [14] and [15], who proposed an ideal framework for such rules [16],

[17]-[19] analysed the general characteristics of risk sharing. According to [19], a standard risk sharing theory involves forming a pool comprising n individuals, with X_i representing the stochastically determined loss of individual i (i = 1 to n). The losses incurred by pool individuals over a certain period are shared among n individuals based on predefined rules. X_i is a random variable in a general probability space (Ω, F, P), with a finite mean value. The loss vector $\boldsymbol{X}_n = (X_1, X_2, \cdots, X_n)$ constitutes the pool and $S_n = \sum_{i=1}^{n} X_i$ denotes the total pool loss. Individual's pre-determined loss to bear is denoted by H_i, which is realized after one period according to the risk sharing rule, expressed as follows:

$$\boldsymbol{H}(\boldsymbol{X}_n) = (H_1(\boldsymbol{X}_n), H_2(\boldsymbol{X}_n), \cdots, H_n(\boldsymbol{X}_n)),$$

typically satisfying $\sum_{i=1}^{n} H_i(\boldsymbol{X}_n) = \sum_{i=1}^{n} X_i = S_n$.

(When consider above function, the variable be not necessarily S_n as there might be an individual not directly related to S_n).

For instance, in the case of equal risk sharing rule among all individuals (denoted by H_i^u), it is expressed as follows:

$$H_i^u(\boldsymbol{X}_n) = \frac{S_n}{n},$$

where $\boldsymbol{E}\left[\sum_{i=1}^{n} H_i^u(\boldsymbol{X}_n)\right] = \boldsymbol{E}\left[\sum_{i=1}^{n} \frac{S_n}{n}\right] = \boldsymbol{E}\left[\sum_{i=1}^{n} X_i\right]$.

However, in this case, each individual's expected loss value differs before and after risk sharing.

$$\boldsymbol{E}\left[H_i^u(\boldsymbol{X}_n)\right] = \frac{S_n}{n} \neq \boldsymbol{E}[X_i]$$

[20]-[22] demonstrate the benefits of such risk sharing rule.

2.2. Network Theory

Among network theory, studies related to risk sharing pool properties using AM have been conducted by [1] [17]-[19] [22]-[27]. Some studies also have focused on moral hazard countermeasures and enhancing moral hazard mitigation functions. Although details are not discussed but see a typical textbook like [28], the AM centrality is employed to measure the pool quality of n individuals. Each element of adjacency matrix (denoted as $\boldsymbol{A}$ ($= A_{i,j}$) is either 0 or 1, subject to the following conditions:

$A_{i,j} = 1$ if i and j have a (trusted) relationship. Self-loops ($i = j$) are assigned a value of 0.

$A_{i,j} = 0$ for all other cases (including when $i = j$).

(i and j are called "node" and the relationship between nodes are called "link."

Just in case, X is denoted as a loss and here it is not discussed.)

Among the various indicators of the pool quality, the simplest is the number of trusted pairs within the pool, denoted by d and described below:

$$d = \sum_{i,j}^{n,n} A_{i,j} / 2.$$

This represents the number of relationship pairs and is referred to as degree centrality. Another measurement is eigenvalue centrality, which serves as an indicator of the degree of importance and connectivity of each node X_i to other nodes in the pool, reflecting its relationships with other nodes. Eigenvalue centrality is calculated as follows:

$$\boldsymbol{AX} = \lambda \boldsymbol{X}, \quad \boldsymbol{X} = {}^{t}\left(X_1, X_2, \cdots, X_n\right)\left(= {}^{t}\boldsymbol{X}_n\right)$$

where $\boldsymbol{A}$ represents the AM, $\boldsymbol{X}$ is the eigenvector (column vector), and λ is the eigenvalue, (scalar). The eigenvalue shows how nodes are connected, serving as a measure of the transitive influence of nodes. It reflects the influence originating from nodes with higher connectivity, indicating that nodes with more connections contribute more to the significance of a node than connections from nodes with lower connectivity. For individual i, the i-th element of the eigenvalue vector $\boldsymbol{X}$ represents the individual's score. Degree centrality and eigenvalue centrality serve as measurements of the risk sharing pool quality, emphasizing the importance of existing relationships (see [28]). Interestingly, the Peron–Frobenius theorem states that when all elements of the matrix are zero or positive, only one eigenvector exists with all positive elements corresponding to the largest eigenvalue. Matrices AM and AAM satisfy this condition.

2.3. Related Papers

[6] introduced a risk sharing rule through a stochastic matrix to enhance insurance pool stability. They used the pool's insurance loss volatility for better pool sustainability, along with optimizing combinations of insurance, deductible levels, and excess loss cover levels. Although this approach does not use AM, it employs a similar matrix, as illustrated below.

First, the risk sharing rule is established as $\boldsymbol{CX}$, a linear combination of each individual's risk, where $\boldsymbol{C}$ represents the stochastic matrix (here, C is used instead of H for the same description of [6]), and $\boldsymbol{X} = {}^{t}\left(X_1, X_2, \cdots, X_n\right)\left(= {}^{t}\boldsymbol{X}_n\right)$. Second, the total volatility of the risk sharing mechanism $\boldsymbol{CX}$ is computed (see **Appendix A**). In this setting, the burden on each individual is weighted and not necessarily equal among the pool.

[29] proposed an original (penalty type) P2P risk sharing formula considering pool quality using network AM and discussed the eigenvalue of the AM for better measurement. The risk sharing rule, H^{NAM}, is defined as follows:

$$H^{NAM}\left(\boldsymbol{X}_n\right) = \left(H_1^{NAM}\left(\boldsymbol{X}_n\right), H_2^{NAM}\left(\boldsymbol{X}_n\right), \cdots, H_n^{NAM}\left(\boldsymbol{X}_n\right)\right)$$

$$H_i^{NAM}\left(\boldsymbol{X}_n\right) = \frac{S_n}{n} + k\left[\frac{X_i^2}{2n} - \frac{1}{2n^2(n-1)}\sum_{i,j}^{n,n} A_{i,j} X_i X_j\right]$$

where $A_{i,j}$ represents network AM, $A_{i,i} = 0$, and k denotes a constant. (Original paper used G instead of A, but in this paper, using A is more consistent with other research result description.) This formula incorporates penalties or incentives for better pool payoff, aiming to cultivate better pool participants (reduces losses and lowers loss volatility) upon joining and afterward. Although superficially, $\sum_i H_i^{NAM}\left(\boldsymbol{X}_n\right) \neq \sum_i X_i$ may appear unfair, this method accounts for incentives and penalties. The features of the proposed risk sharing rules are as follows:

Encouragement of individual efforts (as, in addition to the normal pool loss burden, a burden proportional to the individual's (X_i^2) is imposed).

Incentives for all individuals to act as a pool (because a higher level of trusted relationships within the pool leads to a smaller risk burden for each individual).

Ultimately, greater trust among pool members leads to greater benefits for both individuals and the pool as a whole (see **Appendix B**).

The scalar k could be decided ex post to be this risk sharing platform fair.

In this setting, the burden on each individual varies among pool participants, with incentives and penalties based on their network connections within the pool.

2.4. Entropy

This study considers the matrix entropy of the AM as a measure of pool quality rather than relying solely on the eigenvalue centrality of the matrix. Although the definition of entropy varies (see [30]), the standard Shannon entropy formula (denoted as H), is defined for probability p_i ($i = 1, \cdots, n$) incident as follows:

$$H = -\sum_i^n p_i \log\left(p_i\right)$$

Here, the matrix $\boldsymbol{A}$'s entropy (denoted as H_M) is expressed as:

$$H_M = -\sum_{i,j}^{n,n} A_{i,j} \log\left(A_{i,j}\right)$$

From an economic viewpoint, entropy is related to decision making (see **Appendix D**), and from an insurance perspective, to the Esscher transform ([31] [32]).

2.5. Preliminary Empirical Research

The aim of the preliminary empirical research is to know whether current standard P2P risk sharing (e.g., equal risk sharing rule) insurance can be improved. The study was conducted by asking 38 university students. Detailed conditions are described in the last part of Appendices, Preliminary Empirical Research. Results and indications are that standard P2P risk sharing is sold and contracted to an unspecified number of people, and the pool members are generally unknown each other. They dislike this situation, and they rather prefer to share the risk among the participants who know well each other. At the end, such a scheme is a pool with subgroups (gathering of acquaintances only), which means that the total pool consists of the subgroup, but risk sharing is only among the subgroups. No relation or risk sharing with other subgroups. The number of students is 38 and it is a small survey, but to generate ideas, it is still worth thinking about, for the next section.

3. The New Risk Sharing Model

From the preliminary empirical research, the following new risk sharing rule is suggested and introduced. The scheme seems like that several subgroups are gathering and just it is. However, gathering saves administrative costs. Moreover, a subgroup might have opportunity to exchange information regarding the risk with other subgroups, and this also a merit for gathering several subgroups.

3.1. Model Setting

Define AM for the new model whose AM comes from $G_{i,j}$. The new risk sharing rule will use the adjusted version of AM comes form $A_{i,j}$ so in this paper calls it adjusted adjacency matrix (AAM).

The definition of risk sharing pool and its rule are as follows:

The pool comprises n individuals.

If individual *i* has a trust relationship with *j*, then set matrix (AM) elements such that: $A_{i,j} = A_{j,i} = 1$. All other elements are set to zero (including $A_{i,i}$).

In the context of network theory with n nodes, this configuration is typical in AM representation, where a value of 1 indicates a link between nodes *i* and *j*, denoting a trusted relationship.

In order to obtain AM of the new model, additional adjustments were made to the model. Just in case, it is called AAM. If all $A_{i,j} \neq 0$ for all *i* of specific *j*, $G_{i,j}$ is set as follows.

$$G = G_{i,j} = \begin{pmatrix} 0 & \frac{A_{1,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{1,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{1,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{2,1}}{\sum_{i=1}^{n} A_{i,1}} & 0 & \frac{A_{2,3}}{\sum_{i=1}^{n} A_{i,n}} & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{3,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{3,2}}{\sum_{i=1}^{n} A_{i,2}} & 0 & \cdots & \frac{A_{3,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{A_{n,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{n,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{n,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & 0 \end{pmatrix},$$

where, if $A_{i,j} = 0$ for all *i* of specific *j*, $G_{i,j} = A_{i,j} = 0$.

The risk sharing formula is determined by this $G_{i,j}$ as follows:

Risk sharing of *i*: $\boldsymbol{H}^{AAM} = \boldsymbol{GX}$, $H_i^{AAM}(\boldsymbol{X}) = \sum_j^n G_{i,j} X_j$,

where $\boldsymbol{X}$ is the risk exposure ($\boldsymbol{X} = {}^t(X_1, X_2, \cdots, X_n)\left(= {}^t\boldsymbol{X}_n\right)$, $\boldsymbol{X}_n$ is the row vector of risk).

This risk sharing mechanism implies that the individual *i*'s underwriting risk contributes to the person *j*'s loss if *i* has a (trusted) relationship with *j*. However, this contribution is not solely attributed to *j*; instead, it is shared among all individuals who have a relationship with *j*. Therefore, $G_{i,j}$ serves as a relative relationship version of $A_{i,j}$, and for an individual with multiple relationships within the pool, each relationship with another individual carries a weight of $1/k$, where k is the number of relationships.

As a precaution, H^{AAM} serves as a reasonable risk sharing mechanism because

$\sum_{i=1}^{n} H_i^{AAM}\left(\boldsymbol{X}\right) = \sum_{i=1}^{n} X_i$.

3.2. Better Risk sharing and Better Pool Quality

The model's characteristics are summarized in Table 1 alongside other risk sharing rules for comparison. The AAM model can be viewed as a form of relationship-weighted risk sharing, in which, greater interconnectedness among participants leads to a lighter individual burden of risk within the pool, benefiting the collective rather than individuals in isolation. In this sense, the pool tends to attract participants with stronger relationships (ex-ante selection), and pool participants are incentivized to act prudently to avoid placing undue strain on the pool in the event of a claim (ex-post behaviours). These factors lead to pool quality management incentives and influence the preference for pool quality indicators.

Comparing those risk sharing rules, the rational is more like qualitative. The equal weighted risk sharing rule (Uniform) does not take care of trust among pool members. Weighted and Penalty/Incentive think about the trust but need a formula to decide it. Inverse of Relationship might be the most natural way to take trust into account.

Table 1. Risk sharing characteristics.

	Uniform	Weighted	Penalty/Incentive	Inverse of Relationship (This Paper)
Risk sharing rule	Total loss of the pool is equally divided among the pool participants.	Total loss of the pool is distributed based on certain weights.	Total loss of the pool is equally divided among the pool participants with penalty/incentive.	Total loss of the pool is distributed based on the relationship among the pool participants.
Stability indicator	Volatility member average	Volatility weighted average	-	Volatility relationship weight average
Fairness and its indicator	Equal burden itself	Weight burden itself	Individual penalty and pool penalty Note: The sum of each may not always match the sum of the losses.	Relationship weight burden
Pool quality indicator	-	-	How relationship is not centralized by specific individuals.	How relationship is not centralized by specific individuals.
Pool management	-	Depends on how the weights are decided.	Moral hazard and Adverse Selection protection effect	Moral hazard and Adverse Selection protection effect

3.3. Pool Quality Indicators for the New Model

Quantitative comparison of risk sharing method or pool quality is discussed. This is for among the same type risk sharing rule. The comparison of different risk sharing rules are in the previous section and it is qualitative comparison. The model pool quality is examined using [6]'s loss volatility, fairness, centrality, and matrix entropy. Eigen value is useful for pool quality measure. It is applied to AM and it does not related to the risk sharing rule. Here is the idea. Since the new risk sharing rule uses AMM and its eigen value could be the measure of the rule. However, as shown in **Table 2** and **Appendix C**, the largest eigen value of AAM is always 1 for any networks, and entropy is introduced to differentiate.

3.3.1. Loss Volatility

The lower the expected loss of the pool's result outcome volatility, the better the pool quality. The expected loss of the pool as a whole depends on the risk sharing formula or matrix, although each individual loss occurs independently. For the new model, loss volatility depends on AM components ($A_{i,j}$), and the optimization for minimizing loss volatility is expressed as follows in the case of all individual losses are the same distribution:

$$\min \sum_{i,j}^{n,n} A_{i,j}^2 .$$

The optimal solution (the best matrix) for this minimization is when $A_{i,j} = \frac{1}{n-1}$ for all i, j . ($A_{i,i} = 0$). It will be the same discussion for AMM ($G_{i,j}$).

3.3.2. Fairness

While the expected loss of the entire pool is independent of the matrix, individual losses may not be distributed fairly. The pool is fair as a whole ($\sum_{i=1}^{n} H_i^{AAM}\left(\boldsymbol{X}\right) = \sum_{i=1}^{n} X_i$), but not individually ($\boldsymbol{E}\left[H_i^{AAM}\left(\boldsymbol{X}\right)\right] \neq \boldsymbol{E}\left[X_i\right]$). Additionally, no incentives or penalties are based on individual losses and their probability distributions. It will be the same discussion for AMM ($G_{i,j}$).

3.3.3. Degree Centrality (Total Degree)

An optimal network for pool quality comprises all $A_{i,j} = A_{j,i} = 1$ except when $i = j$ (in which case, it is 0). This implies that:

$A_{i,j} = \frac{1}{n-1}$ for all i, j and degree centrality is 1 ($\frac{n-1}{n-1}$) for all nodes.

It will be the same discussion for AMM ($G_{i,j}$).

3.3.4. Eigenvalue Centrality

The eigenvalues of the AAM always include one, which is the eigenvalue of its eigenvalue centrality (see **Appendix C**). Unlike AM cases, the eigenvalue of AMM does not reflect the quality of the pool.

Entropy

For AM, this metric is $-\sum_{i,j}^{n,n} A_{i,j} \log\left(A_{i,j}\right)$, and for AMM, denoted by H_M^{AAM}, the matrix entropy is defined as $H_M^{AAM} = -\sum_{i,j}^{n,n} G_{i,j} \log\left(G_{i,j}\right)$. In an optimal quality network, where all $A_{i,j} = A_{j,i} = 1$ or $G_{i,j} = G_{j,i} = 1$, except when $i = j$ (in this case, it is 0). In that case, for example, for AMM, the entropy is:

$$H_M^{AAM} = \frac{n}{n-1}\log(n-1), \text{ where } G_{i,j} = \frac{1}{n-1} \text{ for all } i, j.$$

Table 2 summarizes the differences between AM and AAM in terms of pool quality.

Table 2. Pool quality indicators.

	AM (No relation to risk sharing rule, but AM of the pool network is valuable.)	AMM (The new risk sharing uses AAM as a rule. So this is a hybrid of pool network and the rule) (It is called AAM since it looks like the adjusted version of AM)
Degree centrality	Maximization calculation. More relationship, better pool.	Minimization calculation. More diversified relationship, better pool.
Eigenvalue centrality (Eigenvalue and Eigenvalue Centrality)	Eigenvalue: Maximization calculation. Larger eigenvalue, more densely connected.	Eigenvalue: The eigenvalue is always 1.
Matrix entropy	Always entropy = 0.	Maximization calculation. Larger entropy, more diversified.

3.4. Discussion

This section discusses how to set up a P2P risk sharing pool considering pool quality and rules, along with associated pool quality issues and measurement methods. Initially, setting the risk sharing rule uniformly among pool participants as outlined in Section 2.1 with $H_i^u(\boldsymbol{X}_n) = \frac{S_n}{n}$, ($S_n = \sum_{i=1}^n X_i$), appears natural. This approach is particularly suitable for cases where no individual information is available for pool participants. However, in practice, the expected burden of the risks (losses) for the entire pool may not be equal to the expected individual loss ($\boldsymbol{E}\left[H_i^u(\boldsymbol{X}_n)\right] = \frac{S_n}{n} \neq \boldsymbol{E}[X_i]$). Consequently, weighted risk sharing mechanism are often considered, such as the approach proposed by [6], which utilizes the matrix $\boldsymbol{C}$ for the rule explained in Section 2.3. While their matrix is not exactly the AM of network theory, their suggested loss volatility serves as a robust indicator of pool and scheme quality. Incorporating a trust relationship perspective into the pool and leveraging AM for this purpose could enhance risk sharing pool concepts.

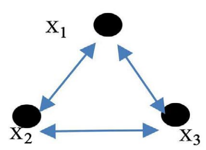
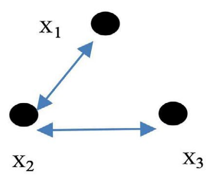
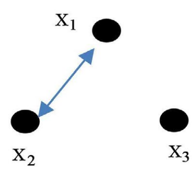
Another approach, demonstrated by [29], utilizes the AM for penalty relaxation in terms of the P2P risk sharing formula. In this scheme, unfortunately, degree centrality and eigenvalue centrality do not emerge as useful indicators of pool quality. Still, the scheme introduces incentives and penalties for pool participants as a whole, both before and after joining. However, superficial fairness is not satisfied ($\sum_i H_i^{NAM}(\boldsymbol{X}_n) \neq \sum_i X_i$) without the adjustment of the scalar k.

To reconcile the usefulness of AM in depicting trust relationships with the requirement of superficial fairness, a new risk sharing rule is proposed based on preliminary empirical research, incorporating human trust weighted risk sharing rule by with AAM. As the AAM eigenvalue for eigenvalue centrality is always 1, rather than using eigenvalue centrality, matrix entropy is introduced, as a superior indicator of pool quality. Prevention of moral hazard and adverse selection is also expected in this AMM use.

4. Numerical Example

In this section, a numerical example with three individuals in the pool is presented, illustrating different relationship patterns (excluding the no-relationship case), as depicted in Table 3. In Table 3, there are three cases of Case 1, Case 2 and Case4. For each of the case, there are three members depicted by a black dot. Arrows indicate that there is a relationship. Quality indicators are also provided. Adjacency matrices, degree centrality, eigenvalue of adjacency matirices, eigenvalue vectors are shown.

Table 3. Pool quality indicators: Example of a pool with three individuals.

Case 1	Case 2	Case 3
		

AM:

$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$D(x_1, x_2, x_3) = (2,2,2)$	$D(x_1, x_2, x_3) = (2,2,2)$	$D(x_1, x_2, x_3) = (1,1,0)$

(Case 1 is the most connected.)

Eigenvalue = 2, −1, −1 Eigenvalue = $\sqrt{2}$, 0, $-\sqrt{2}$ Eigenvalue = 1, 0, −1

(Case 1 has the largest.)

Eigenvalue vector elements:

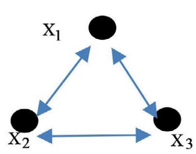
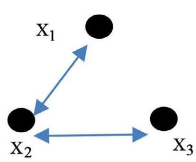
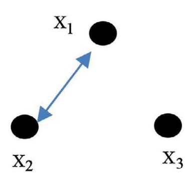
$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ x \\ x \end{pmatrix} = 2\begin{pmatrix} x \\ x \\ x \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ \sqrt{2}x \\ x \end{pmatrix} = \sqrt{2}\begin{pmatrix} x \\ \sqrt{2}x \\ x \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ x \\ 0 \end{pmatrix} = 1\begin{pmatrix} x \\ x \\ 0 \end{pmatrix}$

(Case 1 is the most connected.)

It includes networks, degree centrality and eigenvalue for AM and their eigenvalue vectors.

When the risk sharing rule is Uniform (equal weighted), there might still be the issue of pool quality. **Table 3** shows the quality difference of the pool Case 1, Case 2 and Case 3 for the Uniform rule and, the quality difference among Case 1, Case 2 and Case 3 is measured by, for instance, degree centrality (D). The figures show Case 1 has the largest D and is most connected (better quality). The largest AM egenvalue of those cases also shows it also. These discussions are regarding the pool network and not the rule. For instance, the rule of Uniform is expressed by the matrix with all elements are $1/n$ (n is the number of the members) and this matrix is the same for any pools. However, in the new risk sharing rule, the rule matrix of Case 1, Case 2 and Case 3 are different, and volatility, D and eigenvalue could be indicators for better quality of risk sharing. In **Table 4**, in the similar form of **Table 3**, there are three cases of Case 1, Case 2 and Case4. For each of the case, there are three members depicted by a black dot. Arrows indicate that there is a relationship. **Table 4** is for the new risk sharing rule. Adjacency matrices, degree centrality, eigenvalue of adjacency matirices, eigenvalue vectors are shown as well as volatility of the pool payoff as a whole and entropy. As in **Table 4**, volatility shows that Case 1 has the smallest and it is most diversified (better). Also, D shows the connectedness and Case 1 is the most connected. Regarding eigenvalue, all cases have 1 as the largest eigenvalue. So, entropy is useful for measure the quality of the new risk sharing rule. Case 1 has the largest entropy and most diversified.

Table 4. Pool quality indicators (AAM): Example of a pool with three individuals.

Case 1	Case 2	Case 3
		
	AAM:	
$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{2} & \frac{0}{1} \\ \frac{1}{1} & 0 & \frac{1}{1} \\ \frac{0}{1} & \frac{1}{2} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$D(x_1,x_2,x_3)=(1,1,1)$	$D(x_1,x_2,x_3)=(1/2,2,1/2)$	$D(x_1,x_2,x_3)=(1,1,0)$

(Case 1 is the most connected)

Eigenvalue = 1, $1/\sqrt{2}$, $-1/\sqrt{2}$ Eigenvalue = 1, 0, −1 Eigenvalue = 1, 0, −1

(Always the largest eigenvalue is 1)

Eigenvalue vector elements:

Continued

$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} x \\ x \\ x \end{pmatrix}=1\begin{pmatrix} x \\ x \\ x \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 1 & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} x \\ 2x \\ x \end{pmatrix}=1\begin{pmatrix} x \\ 2x \\ x \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ x \\ 0 \end{pmatrix}=1\begin{pmatrix} x \\ x \\ 0 \end{pmatrix}$
Volatility = 3/2	Volatility = 5/2	Volatility = 2
	(Case 1 is the smallest.)	
Entropy = 3log2	Entropy = log2	Entropy = 0
	(Case 1 is the largest.)	

It includes rule degree centrality and eigenvalue for AAM and their eigenvalue vectors, volatility of the pool (individual volatility = 1), and matrix entropy of AAM.

5. Summary

This study introduced a new novel and innovative new P2P risk sharing scheme considering pool quality and presented an indicator of pool quality. By leveraging network theory, the new risk sharing rule looks like an adjusted version of typical risk sharing AM and it could be called AMM. It was constructed from AM with adjustments to reflect relative relationships among pool participants and the AMM can be used for the new P2P risk sharing scheme. In this situation, the entropy emerged as a suitable measure for assessing pool quality.

Building upon existing research, this study investigates how the new risk sharing scheme is influenced by the pool quality and how to measure this quality. Pool quality is typically modelled using network theory, where pool participants are represented as nodes and their relationship statuses as links (see Section 2.2). While cultivation of trust among pool members is partially addressed in this study, considering trust relationships that can themselves be realized through these links, is crucial. Information is consolidated in the AM of the pool, which is pivotal for determining quality factors such as stability, fairness, and trust relationships. This study establishes an AAM as a relative relationship matrix, and it can be said from the other side that it devises the new risk sharing rule based on AAM, and identifies indicators of pool quality. Pay-out volatility and centrality of the matrix, commonly used to investigate network characteristics (degree and eigenvalue points of view), are examined as pool quality indicators. Additionally, entropy, defined in various ways, is investigated, with matrix entropy being an appropriate quality indicator for the new risk sharing model.

In terms of pool management, using the network matrix, the model cultivates moral hazard and adverse selection protection within the pool, both at the time of joining the pool and afterward. Moreover, using matrix entropy proves to be an indicator of pool quality as well as volatility and network centrality. Future challenges lie in devising more complex formulas beyond simple linear combinations for risk sharing mechanisms.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendices

Appendix A

[6] proposed a toolkit to analyse P2P insurance products based on the explicit structure of (P2P) networks rather than individual characteristics. They set the risk sharing formula to be decided by the matrix $C_{i,j}$, where C is a column stochastic matrix with $\sum_{j=1}^{n} C_{i,j} = 1$. Risk sharing of individual *i* with original risk x_i is represented as $\xi = CX$, with the characteristic of $\sum_{i=1}^{n} \xi_{ji} = \sum_{i=1}^{n} X_i$.

They examined the volatility of the pool as a whole risk after the risk sharing, and while C is not an AM, it also considers the trust relationship among pool individuals to some extent. The analysis is more aligned with that of [33] doubly stochastic matrix and weak ordering.

Appendix B

The risk sharing rule in [29]'s model can be expressed as:

$$\boldsymbol{H}^{NAM}\left(\boldsymbol{X}_n\right) = \left(H_1^{NAM}\left(\boldsymbol{X}_n\right), H_2^{NAM}\left(\boldsymbol{X}_n\right), \cdots, H_n^{NAM}\left(\boldsymbol{X}_n\right)\right)$$

$$H_i^{NAM}\left(\boldsymbol{X}_n\right) = \frac{S_n}{n} + k\left[\frac{X_i^2}{2n} - \frac{k1}{2n^2\left(n-1\right)} \sum_{i,j}^{n,n} A_{i,j} X_i X_j\right]$$

where $A_{i,j}$ denotes network AM, $A_{i,i} = 0$, and k is a constant. (Original paper used G instead of A, but in this paper, using A is more consistent with other research result description.) The second term represents the penalty for each individual loss and the third term acts to suppress it. $\sum_{i=1}^{n} H_i\left(\boldsymbol{X}_n\right) = \sum_{i=1}^{n} X_i$ is not necessarily satisfied; however, if the network is trustworthy among all other pool members and all the elements of $\boldsymbol{X}_n$ are identical, the penalty disappears, and $\sum_{i=1}^{n} H_i\left(\boldsymbol{X}_n\right) = \sum_{i=1}^{n} X_i$ is also satisfied (in other cases, this portion represents a positive profit for the pool, and may be used for business expenses).

$$H_i^{NAM}\left(\text{Full Network}\right) = \frac{S_n}{n} + k\left[\frac{X_i^2}{2n} - \frac{1}{2n^2\left(n-1\right)} \sum_{i,j}^{n,n} \left(1-\delta_{i,j}\right) X_i X_j\right]$$

$$\boldsymbol{E}\left[\sum_{i=1}^{n} H_i^{NAM}\left(\text{Full Network}\right)\right] = \boldsymbol{E}\left[\sum_{i=1}^{n} \frac{S_n}{n}\right] + 0 = \boldsymbol{E}\left[\sum_{i=1}^{n} X_i\right]$$

This penalizes individuals based on the size of their losses, and it fosters a network of trust and cooperation aimed at loss prevention and providing incentives. Examining the relationship with the largest eigenvalue (eigenvalue centrality) reveals that when all individual trust relationships are established ($A_{i,j} = 1$, $A_{i,i} = 0$), the (maximum) eigenvalue is $n - 1$.

$$\begin{pmatrix} 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ x \\ \vdots \\ x \end{pmatrix} = \left(n-1\right) \begin{pmatrix} x \\ x \\ \vdots \\ x \end{pmatrix}$$

$$\left(n-1\right)\begin{pmatrix} x & x & \cdots & x \end{pmatrix}\begin{pmatrix} x \\ x \\ \vdots \\ x \end{pmatrix}=n\left(n-1\right)x^{2}$$

The term, $\frac{1}{2n^{2}\left(n-1\right)}\sum_{i,j}^{n,n} A_{i,j}X_{i}X_{j}$, cancels out with the term, $\frac{X_{i}^{2}}{2n}$.

Appendix C

As the sum of each column element of $G_{i,j}$ is 1 for AAM, the following can be inferred:

$$\text{transpose}\left[\begin{pmatrix} -1 & \frac{A_{1,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{1,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{1,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{2,1}}{\sum_{i=1}^{n} A_{i,1}} & -1 & \frac{A_{2,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{3,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{3,2}}{\sum_{i=1}^{n} A_{i,2}} & -1 & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{A_{n,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{n,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{n,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & -1 \end{pmatrix}\right]\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

So, denoting determinate as det( ),

$$\det\left(\text{transpose}\left[\begin{pmatrix} -1 & \frac{A_{1,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{1,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{1,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{2,1}}{\sum_{i=1}^{n} A_{i,1}} & -1 & \frac{A_{2,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{3,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{3,2}}{\sum_{i=1}^{n} A_{i,2}} & -1 & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{A_{n,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{n,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{n,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & -1 \end{pmatrix}\right]\right)=0$$

In general, for a matrix $\boldsymbol{D}$, $\det\left(\boldsymbol{D}\right)=\det\left({}^{t}\boldsymbol{D}\right)$, implying:

$$\det\left(\begin{pmatrix} -1 & \frac{A_{1,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{1,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{1,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{2,1}}{\sum_{i=1}^{n} A_{i,1}} & -1 & \frac{A_{2,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \frac{A_{3,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{3,2}}{\sum_{i=1}^{n} A_{i,2}} & -1 & \cdots & \frac{A_{2,n}}{\sum_{i=1}^{n} A_{i,n}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{A_{n,1}}{\sum_{i=1}^{n} A_{i,1}} & \frac{A_{n,2}}{\sum_{i=1}^{n} A_{i,2}} & \frac{A_{n,3}}{\sum_{i=1}^{n} A_{i,3}} & \cdots & -1 \end{pmatrix}\right)=0.$$

This $\det(\boldsymbol{G}-\boldsymbol{I})=0$ suggests that $\boldsymbol{GX}=\lambda\boldsymbol{X}$ ($\boldsymbol{G}=G_{i,j}$) is true when $\lambda=1$ for certain non-zero vector $\boldsymbol{X}$, and, that one of the eigenvalues of AAM $G_{i,j}$ is always 1. Naturally, its eigenvalue vector elements are all non-negative, and the eigenvalue is for eigenvalue centrality.

Appendix D

To incorporate [34]'s framework, consider incidents x_i ($i=1,2,\cdots,K$). Let us also consider solving the following maximization problem, where h is a specific state.

$$\max -\sum_{i=1}^{K} p_i \log(p_i),$$

subject to $\sum_{i=1}^{K} p_i = 1$, $\sum_{i=1}^{K} p_i f_h(x_i) = F_h$.

The solution is:

$$p_i = \frac{1}{Z(\lambda_1,\lambda_2,\cdots,\lambda_m)} \mathrm{e}^{\left[-\lambda_1 f_1(x_i)-\lambda_2 f_2(x_i)-\cdots-\lambda_m f_m(x_i)\right]},$$

where $Z(\lambda_1,\lambda_2,\cdots,\lambda_m)=\sum_{i=1}^{K}\mathrm{e}^{\left[-\lambda_1 f_1(x_i)-\lambda_2 f_2(x_i)-\cdots-\lambda_m f_m(x_i)\right]}$ and λ represents the Lagrangian multiplier.

Here $\sum_{i=1}^{K} p_i f_h(x_i) = F_h$ denotes the constraints imposed by the h factor.

Preliminary Empirical Research

In order to research how P2P insurance platform are felt like by users, voluntarily and agreed, university students were asked and answered the below questions. The empirical research was based on asking questions and getting reply. Data shows how current standard P2P risk sharing insurance could be improved. The number of the answered university students were 38 and they are aged from 20 to 22, who are supposed to be unbiased with almost the same gender ratio.

The structure is as follows. Questions regarding to participate in P2P risk sharing scheme. For example, it is an insurance for smartphone repair cost and there are 100 participants. At the end of one year, the total smartphone repair cost of all participants is calculated and shared among the 100. More specifically, there supposed to be two kinds of scheme, 1) and 2):

1) The risk sharing is sold and contracted to an unspecified people, and the method of division is to divide by the total number of participants. (In the above example case, dividing by 100. However, more specifically here the total cost is defined by the total other than the cost of participant itself. Naturally, each participant pays one 99$^{\text{th}}$ of the total cost other than himself.)

2) The total pool consists of several subgroups of gathering of acquaintances. The method of risk sharing is to divide by the number of subgroup participants. (In the above example case and assuming subgroups are made up of 10, 30, 40, and 20 participants respectively, divide by the number of each subgroup participants for each subgroup. Also, more specifically here the total cost is defined by the total other than the cost of participant itself.)

- Question 1. Among the below from A to E, which two are appropriate for your individual opinion?

A. I'm worried because there is no information about what kind of people are there.

B. There may be more people than usual who use the smart phone roughly.

C. There may be more people than usual who are careless and use the smart phone roughly because of exactly having the insurance.

D. There may be more people than usual who make fraudulent claims.

E. There may be more people than usual who think they should get insurance because they break the smart phone a lot.

- The result is the below. (Some answered nothing, or no second most important reason.)

As the most important reason: D is picked by 12 students for the most worried, A and B is by10, E is by 3, and C is by 2.

As the second most important reason: A is picked by 4 students, B is by 5, C is by 9, D is by 7, E is by 5.

- Question 2. Which risk sharing scheme would you like to join, 1) or 2)?

Result: Number of Yes for 1) is 11, for 2) is 27.

- Question 3. Are you not resistant to new members joining? For each of 1) and 2).

Result: Number of Yes for 1) is 20, for 2) is 20. No for 1) is 12, for 2) is 16. (Some answered nothing, or either.)

- Question 4. In 2), the need for the subgroups to become one group could be due to reducing administrative costs. What else other benefits, do you think there are?

Result: Basically, one kind of merit is answered by 5 students, and it is the opportunity to exchange information regarding the risk. Other answers are minor.

- Discussions:

According to the replays of Question 1 and Question 2, they indicate 2) is preferable to 1) and the problem of the scheme 1) is mostly by the reason D, the risk of fraudulent claims. The replays of Question 3, how you get new members, shows they are happy to increase members and sure to invite persons who know them each other. Just in case the choice B is describing more substantial activity for concern, C is for moral hazard risk, and E is for adverse selection risk. However, theoretically B, C, and E is understandable, it could be vague and rare for the students. Even we agree to set the pool as several groups which among them they know each other, what the meaning to gather subgroups and be a single pool? They said that other than because of administrative cost savings, information exchange could be useful.