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Investment Risk, Financial Slack, and Value of Capital: A Theoretical Examination

Du Du

Department of Economics and Finance, City University of Hong Kong, Hong Kong, China
Email: dudu22@cityu.edu.hk

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Abstract

Investment risk is a ubiquitous concern among entrepreneurs. In an influential paper, Albuquerque and Wang [1] show that investment risk creates the disutility effect because it is disliked by a risk-averse entrepreneur. In response, the value of installed capital rises above its replacement cost so as to induce investment. The finding of [1] is based on firms operated under autarky, which rules out the accumulation of financial reserves. By developing and solving a dynamic stochastic model that involves asset allocation, consumption, business liquidation, and investment-specific shocks, we show fundamentally different results for a firm that has built up a high degree of financial slack: The firm now gets affected by investment-specific shocks only when these shocks are correlated with the stock market. In that case, investment risk that exposes the firm more to the market simultaneously depresses corporate investment and the valuation of capital. We further show that a firm permanently cuts both its investment and the valuation of capital once it has broken out of autarky.

Keywords

Investment Risk, Autarky, Financial Slack, Liquid Wealth, Tobin's q on Capital

1. Introduction

Capital investment is the core operation to be conducted by any firm. What is less known is that capital investment is usually subject to various investment-specific shocks in addition to the capital adjustment cost. This idea of investment risk goes back to Keynes [2] who argues that firm-level production is subject to shocks on the marginal efficiency of investment. By interpreting such shocks as the investment-specific technology risks, which are then incorporated into the neo-classical framework on real business cycles, Greenwood, Hercowitz, and Huffman [3], Greenwood, Hercowitz, and Krusell [4], and Fisher [5], among others, show that investment-specific shocks play an important role in generating the aggregate volatility in the US economy.[1]

In a financial setting, Albuquerque and Wang (AW) [1] study the impact of investment-specific shocks at the firm-level where the firm is operated under autarky, in the sense that agents of the firm in combination exactly consume all firm's net outputs which rule out the possibility of building up financial slack.[2] In particular, they show that investment risks make the newly invested capital less desirable than the installed capital to a risk-averse agent. As a result, the Tobin's q (Brainard and Tobin [6]; Tobin [7]), defined as the ratio between the firm capital's market value to the replacement cost of its capital stock, is larger than unity even without the adjustment cost. Intuitively, this larger-than-unity q delivers valuation premium to the installed capital which encourages the agent to invest at the presence of the investment-specific shocks that she dislikes.

AW's [1] analyses raise the following questions: How would investment risk affect the accumulation of capital in a more realistic setting where the firm is able to build up its financial reserves? Related, how would financial slack affect the valuation of capital at the presence of investment risk? To provide an answer, we develop and solve a dynamic stochastic model for a firm featuring intertemporal asset allocation, consumption, costly busines liquidation, and investment-specific shocks. Our model breaks autarky by allowing the firm to borrow against its capital so as to take positions in the stock market, while the agent of the firm simultaneously makes decisions on consumption and capital investment by taking into account the potential liquidation.

Incorporating both asset allocation and investment-specific shocks into a model poses technical challenges. First and in contrast to AW [1] where capital stock serves as the unique state variable, the accumulated financial reserves also affect the firm's decisions giving rise to a partial differential equation (PDE) for the characterization of the model. Second, the investment and asset allocation policies are now intrinsically linked to one another and their interactions have direct impact on both firm policies and the resulting value function of the firm. We tackle these challenges by resorting to financial reasoning which allows a reduction of the problem to its one-dimensional version. We then solve the model

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[1] More specifically, Greenwood, Hercowitz, and Krusell [3] report that 30% of output fluctuations in the post-war U.S. period are generated by investment-specific shocks. Greenwood Hercowitz, and Krusell [4] document that these shocks account for 60% of growth and 30% volatility for the postwar U.S. economy. Using an econometric approach that relaxes the identification in [4], Fisher [5] shows that 50% of U.S. fluctuations are accounted for by investment-specific shocks.

[2] In the setup of AW [1], certain types of firm agents are still allowed to take positions in the stock market whose capitalization is equal to the assembled valuation of a continuum of identical firms. With their design for modeling, however, taking positions in the stock market is equivalent to holding the agents' own firm because all firms are identical. For a given firm, AW [1] focus on the no-trade equilibrium under which there are no financial tradings and each agent consumes exactly her entitled dividends from the firm. Consequently, firm's net output is exactly consumed period by period which rules out the possibility of building up financial reserves for the firm.

numerically with reasonably imposed boundary conditions that characterize the firm's liquidation policy. Identifying a solution to this new model with investment risk is deemed as the first contribution of the paper, and it lays the foundation for the subsequent financial analyses.

As its second contribution, we gauge the different impact of investment risk on a firm with abundant financial reserve (or liquid wealth) vs. a firm operated under autarky. To this purpose, we first solve the problem for a self-sufficient firm by extending AW's [1] analyses to the case where the firm's agent has recursive preference and the firm-held capital is subject to adjustment cost.[3] We confirm AW's [1] insight that higher investment risk, as measured by the volatility parameter ϵ for firm-specific shocks, creates a more severe "disutility effect" to the risk-averse agent which depresses investment but raises the valuation of the installed capital. When the firm breaks out of autarky, however, it can potentially achieve a high degree of financial slack which fundamentally changes the financial impact of ϵ. In particular, we find that ϵ no longer affects investment nor the valuation of capital if investment risk is purely idiosyncratic. When investment-specific shocks are positively correlated with the stock market, a higher ϵ does depress investment but for a different logic than that of AW [1]. Indeed, a higher ϵ in this case exposes the firm more to the market risk which drives down not only the capital investment but also the valuation of capital. These results complement AW's [1] study on investment-specific shocks.

We further examine the "switch-on effect" when a firm under autarky has just gained the access to financial trading. From the lens of our model, such a firm immediately cuts its consumption and investment and assigns a lower valuation to its capital. Simultaneously, it takes a large position in the stock market for the purpose of harvesting the equity risk premium. With the full access to financial trading, our calibrated model indicates an efficient accumulation of financial slack which could potentially drive the consumption-capital ratio to infinity. Consequently, the cut in consumption is only temporary. While investment and valuation of capital also rise with the financial slack, they never get back to their autarky levels for most of the ϵ-realizations. These results point to a dominant value-creation effect from financial trading which support Warren Buffet's long-term advocate of using the stock market to create and preserve wealth.

Existing papers on entrepreneurial firm either study the state of autarky exclusively (e.g., AW [1]; Ai and Li [8]) or study firms will the full access to financial markets exclusively (e.g., Wang, Wang, and Yang [9]; Bolton, Wang, and Yang [10]). This paper fills the gap by making connections between self-sufficient firms and firms with the ability to accumulate financial slack. In particular, we show that access to financial markets enables a firm to substantially grow its value relative to that under autarky. This result, however, hinges on a healthy stock market that persistently provides a positive expected return. Indeed, it is not

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[3] The focus of AW [1] is to study the financial implications of the agency-friction attributable to the control-ownership wedge: An important economic issue that we abstract away from in this paper.

uncommon to observe that entrepreneurial firms avoid financial trading when facing under-developed stock markets that do not deliver reasonable returns to investors. Our result is thus broadly consistent with the literature that emphasizes the importance of financial development and investor protection on firm operations and growth (e.g., Rajan and Zingales [11]; Shleifer and Wolfenzon [12]; Valickova, Havranek, and Horvath [13]).

The rest of the paper is organized as follows. Section II presents the setup of the model. Section III characterizes the model by applying the principle of dynamic programming and imposing the financially sensible boundary conditions. Section IV provides the numerical solution to the model where the firm has the full access to financial markets, while Section V solves the problem for a firm operated under autarky. Based on these solutions, Section VI discusses the financial impact of investment risk and the switch-on effect for a firm which has just broken out of autary. Section VI concludes.

2. Setup

2.1. Capital Accumulation and Production

The firm's output is given by AK_t, where K_t is the firm's capital stock and the constant A denotes its productivity. We assume that K_t evolves according to

$$\mathrm{d}K_t = \left(I_t - \delta_K K_t\right)\mathrm{d}t + \sigma_K K_t \mathrm{d}Z_t + \epsilon I_t \mathrm{d}Z_t^I, \tag{2.1}$$

where I_t is investment; $\delta_K > 0$ is the depreciation rate; σ_K and ϵ are the volatility parameters; Z_t and Z_t^I are two standard Brownian motions that are mutually independent. Specifically, $\sigma_K K_t \mathrm{d}Z_t$ captures the usual capital depreciation shock (e.g., Bolton, Wang, and Yang [10]), while $\epsilon I_t \mathrm{d}Z_t^I$ governs the investment-specific shocks. The specification of $\epsilon I_t \mathrm{d}Z_t^I$ is first introduce by AW [1] in a setup featuring autarky, and it is motivated by the growing literature that emphasizes the important role of investment-specific technology shocks as a source of aggregate volatility (see [3]-[5], among others). Economically, firm's output comes from its capital stock and the specification of $\epsilon I_t \mathrm{d}Z_t^I$ implies that a large part of output fluctuations arise from shocks to the marginal efficiency of investment ([2]).

The corporate investment I induces the usual adjustment cost (e.g., Hayashi [14]). Therefore, the firm's free cash flow after paying the investment-related costs (but before consumption) is given by

$$Y_t = AK_t - I_t - G\left(I_t, K_t\right), \tag{2.2}$$

where the adjustment cost $G\left(I_t, K_t\right)$ takes the form of

$$G\left(I, K\right) = g\left(i\right)K = \frac{1}{2}\theta i^2 K \tag{2.3}$$

where $i \equiv I/K$.

2.2. The Preferences

The infinitely lived agent (entrepreneur) of the firm has a recursive preference (e.g., Epstein and Zin [15]; Dufie and Epstein [16]) as follows:

$$J_t = E_t\left[\int_t^{\infty} f\left(C_s, J_s\right)\mathrm{d}s\right], \tag{2.4}$$

where $f\left(C, J\right)$ is known as the normalized aggregator for consumption C and the agent's utility J.[4] Dufie and Epstein [16] show that $f\left(C, J\right)$ for Epstein-Zin non-expted homothetic recursive utility is given by

$$f\left(C, J\right) = \frac{\zeta}{1-1/\psi}\frac{C^{1-1/\psi} - \left(\left(1-\gamma\right)J\right)^{\chi}}{\left(\left(1-\gamma\right)J\right)^{\chi-1}}, \tag{2.5}$$

where

$$\chi = \frac{1-1/\psi}{1-\gamma}. \tag{2.6}$$

In (2.5) and (2.6), The parameter $\zeta > 0$ is the agents' subjective discount rate, the parameter $\psi > 0$ measures the elasticity of substitution (EIS), and the parameter $\gamma > 0$ is the coefficient of relative risk aversion. The widely used constant-relative-risk-averse (CRRA) utility is a special case of the Duffie-Epstein-Zin-Weil recursive utility specification with EIS set to the inverse of γ, *i.e.*, $\psi = \gamma^{-1}$.

2.3. Financial Markets and the Firm's Liquid Wealth

The agent of the firm can invest in a risk-free asset which pays a constant rate r and a risky asset denoting the risky market portfolio (Merton, 1971). Assume that the incremental return $\mathrm{d}R_t$ of the market portfolio over the time period $\mathrm{d}t$ is given by,

$$\mathrm{d}R_t = \mu_R \mathrm{d}t + \sigma_R \mathrm{d}B_t, \tag{2.7}$$

where μ_R and σ_R are constant mean and volatility parameters of the market portfolio return process; B_t is a standard Brownian which correlates with Z_t and Z_t^I by ρ and ρ_I, respectively.[5] The market Sharpe ratio is defined as

$$\eta = \frac{\mu_R - r}{\sigma_R}. \tag{2.8}$$

In the following, we generally use η in place of μ_R, which turns out to be more convenient for our derivations.

Let W and X denote the firm's liquid wealth (or financial reserve) and the amount invested in the risky asset, respectively. Their difference, $W - X$, is thus invested in the risk-free asset. Out of its liquid asset, the firm pays the investment cost and consumes. Thus, W_t evolves according to

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[4] C here denotes the firm-level consumption whereas J denotes the utility for the entrepreneurs who makes all the decisions on behalf of the firm. In this study, we abstract from the commonly observed agency friction of the control-ownership wedge, and simply assume that the entrepreneur works for the best of the firm.

[5] We assume that $\rho, \rho_I < 1$. Consequently, firm-level risks cannot be fully hedged away by taking positions in the stock market.

$$\mathrm{d}W_t = \left[rW_t + \eta\sigma_R X_t + AK_t - I_t - G\left(I_t, K_t\right) - C_t \right]\mathrm{d}t + \sigma_R X_t \mathrm{d}B_t. \tag{2.9}$$

Note W can be negative under which firm borrows against the its capital stock.[6] Section III.D describes the firm's debt capacity, *i.e.*, the maximum amount that it can borrow, through a set of boundary conditions that characterize the firm's liquidation policy.

3. Characterizations of the model

3.1. Dynamic Programming

Let $J(K,W)$ denote the value function for the agent of the firm. By the principle of dynamic programming, the agent chooses consumption C, capital investment I, and the risky asset allocation X to solve the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} 0 = \max_{C,X,I} f\left(C_t, J\right) &+ \left[rW_t + \eta\sigma_R X_t + AK_t - I_t - G\left(I_t, K_t\right) - C_t \right] J_W \\ &+ \left(I - \delta_K K\right) J_K + \frac{\epsilon^2 I^2 + \sigma_K^2 K^2}{2} J_{KK} \\ &+ \left(\rho_I \epsilon I + \rho\sigma_K K\right)\sigma_R X J_{KW} + \frac{\sigma_R^2 X^2}{2} J_{WW}, \end{aligned} \tag{3.1}$$

where J_W, J_K, J_{KK}, J_{KW} and J_{WW} denote partial derivatives.

The first-order condition (FOC) for consumption C is given by

$$f_C\left(C_t, J\right) = J_W\left(K, W\right), \tag{3.2}$$

(3.2) is the usual optimal condition that the marginal utility of consumption is set to equate with the marginal utility of wealth. The FOC with respect to portfolio choice X is given by

$$X = -\frac{\left(\mu_R - r\right)}{\sigma_R^2}\frac{J_W}{J_{WW}} - \frac{\left(\rho\sigma_K K + \rho_I \epsilon I\right)\sigma_R}{\sigma_R^2}\frac{J_{KW}}{J_{WW}}. \tag{3.3}$$

On the right-hand side (RHS) of (3.3), the first term denotes the usual myopic demand, while the second term denotes intertemporal hedging demand attributed to the changing investment opportunities that are induced by i) the capital depreciation shocks; and ii) the investment-specific shocks. In particular, the component of hedging demand induced by investment risk, $\frac{\rho_I \epsilon I \sigma_R}{\sigma_R^2}\frac{J_{KW}}{J_{WW}}$, is proportional to I which implies an intrinsic link between I and X. To my best knowledge, this paper is the first to theoretically characterize such an interaction between the firm's investment policy and its asset allocation policy.

The FOC for investment I is given by

$$\left[1 + G_I\left(I, K\right)\right] J_W = J_K + \epsilon^2 I J_{KK} + \rho_I \epsilon \sigma_R X J_{KW}. \tag{3.4}$$

By raising capital stock by one unit, the firm needs to forgo $1 + G_I\left(I, K\right)$ units

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[6] By pledging its capital as the collateral, the firm-level borrowing is risk-free which is assessed at the constant rate of r as well.

of wealth which translate into a loss in marginal utility of $\left[1+G_I\left(I,K\right)\right]J_W$. The disutility effect of investment risk is captured by the second term on the RHS of (3.4), $\epsilon^2 IJ_{KK}$, which is proportional to I,[7] and it serves to depress the usual marginal benefit of investment as captured by J_K. Finally, the third term on the RHS of (3.4) reflects the interactions between X and I and it can be either positive or negative depending on the sign of ρ_I.

3.2. Reduction to One Dimension

(3.1) subject to (3.2) - (3.4) is a PDE subject to a set of stochastic controls which in general is very hard to solve. In what follows, we use financial reasoning to simplify the problem. First, we conjecture (and verify later through its numerical soluton) that the value function $J\left(K,W\right)$ takes the following form:

$$J\left(K,W\right)=\frac{\left(bP\left(K,W\right)\right)^{1-\gamma}}{1-\gamma}, \tag{3.5}$$

where the constant coefficient b is given by (3.28);[8] $P\left(K,W\right)$ is interpreted as the certainty-equivalent (CE) valuation of the firm by the entrepreneur working as the firm's agent. Financially, it denotes the minimum dollar amount that the entrepreneur would demand to permanently give up the firm.

To establish the form of $P(.)$ which involves two state variables, we exploit the model's homogeneity property to reduce the problem to one dimension. Specifically, we treat the firm's capital K_t as the scaling factor, and we use lower case letters to denote the following variables: firm's liquid wealth $w_t=W_t/K_t$, the agent's CE valuation of the firm $p_t=P_t/K_t$, consumption $c_t=C_t/K_t$, investment $i_t=I_t/K_t$, and risky asset allocation $x_t=X_t/K_t$. Using these notations, $P\left(K,W\right)$ is financially rewritten as

$$P\left(K,W\right)=K\cdot p\left(w\right)=K\left[w+q\left(w\right)\right], \tag{3.6}$$

where w denotes the liquid wealth per unit of capital and $q(.)$ denotes the liquid-wealth valuation of capital which serves as the illiquid asset owned by the firm. In the following analyses, we use w as the measure of the firm's financial slack where a higher w indicates a higher degree of slack. Due to the unhedgeable firm-level risks (see footnote 5), the firm is subject to liquidation. Consequently, $q(.)$ varies with w in general which reflects the effect of financial slack on the valuation of capital when the firm faces the liquidation risk.

3.3. Optimal Policies and HJB for the One-Dimensional Problem

Using (3.6), we rewrite (3.5) as

$$J\left(K,W\right)=\frac{b^{1-\gamma}}{1-\gamma}\left[Kp\left(w\right)\right]^{1-\gamma}. \tag{3.7}$$

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[7] Note that $\epsilon^2 IJ_{KK}<0$ because $J_{KK}<0$ (see Eq. (3.8)) and we focus on the usual case where $I>0$. In its numerical solution, we find that I could turn negative when the firm is sufficiently close to its liquidation, which reflects a fire sale of the firm's capital to avoid the costly liquidation.

[8] We formally identify b in Section III.E when we pursue a limiting case of (3.5).

Pursuing (3.7), we have the following expressions for J-derivatives:

$$J_W = (by)^{1-\gamma} \left(p(w) K \right)^{-\gamma} p'(w), \tag{3.8}$$

$$J_K = (by)^{1-\gamma} \left(p(w) K \right)^{-\gamma} \left(p(w) - wp'(w) \right), \tag{3.9}$$

$$J_{WW} = (by)^{1-\gamma} \left(p(w) K \right)^{-\gamma-1} \left[p(w) p''(w) - \gamma \left(p'(w) \right)^2 \right], \tag{3.10}$$

$$J_{KW} = (by)^{1-\gamma} \left(p(w) K \right)^{-\gamma-1} \left[-wp(w) p''(w) - \gamma p'(w) \left(p(w) - wp'(w) \right) \right], \tag{3.11}$$

$$J_{KK} = (by)^{1-\gamma} \left(p(w) K \right)^{-\gamma-1} \left[w^2 p(w) p''(w) - \gamma \left(p(w) - wp'(w) \right)^2 \right]. \tag{3.12}$$

Using (3.7) to simplify (2.5) and (2.6) which are then substituted into (3.2), we obtain the following consumption rule after making use of (3.8):

$$c^*(w) = \zeta^{\psi} b^{1-\psi} p'(w)^{-\psi} p(w). \tag{3.13}$$

By substituting (3.8)-(3.12) into (3.3) and (3.4) and performing necessary manipulations, the optimal asset allocation and investment policies are explicitly given by

$$x^*(w;i) = \frac{\eta}{\sigma_R} \frac{p}{h} - \frac{\rho_I \epsilon i + \rho \sigma_K}{\sigma_R} \left(\frac{\gamma p}{h} - w \right), \tag{3.14}$$

$$i^*(w;x) = \frac{p - (w+1) p' - \rho_I \epsilon \sigma_R x p' \left(\gamma - \frac{wh}{p} \right)}{\theta p' + \epsilon^2 \frac{\gamma p (p - 2wp') + w^2 p' h}{p}}, \tag{3.15}$$

where

$$h \equiv \gamma p' - \frac{pp''}{p'}; \text{[9]} \tag{3.16}$$

and we have used the definition of G in (2.3) for (3.25). By substituting (2.5) and (2.6), optimal policies of (3.13)-(3.15), and the expressions of (3.8)-(3.12) into (3.1), tedious algebra gives the following second-order nonlinear ODE to $p(w)$:

$$\begin{aligned} 0 = {} & \frac{\zeta^{\psi} b^{1-\psi} (p')^{1-\psi} - \zeta \psi}{\psi - 1} p + (rw + A) p' + \frac{1}{2} \frac{\eta^2 pp'}{h} - \delta_K (p - wp') \\ & - \frac{1}{2} (1 - \rho^2) \sigma_K^2 \frac{p'p}{h} \left(\gamma - \frac{wh}{p} \right)^2 - \rho \sigma_K \eta p' \left(\frac{\gamma p}{h} - w \right) + \frac{1}{2} \gamma \sigma_K^2 p^2 \frac{p''}{hp'} \\ & + \frac{1}{2} \frac{\left[p - (w+1) p' - \rho_I \epsilon p' \frac{\gamma p - wh}{p} \left(\eta \frac{p}{h} - \rho \sigma_K \left(\frac{\gamma p}{h} - w \right) \right) \right]^2}{\theta p' + (1 - \rho_I^2) \epsilon^2 \frac{p'}{p} \frac{(\gamma p - wh)^2}{h} - \gamma \epsilon^2 \frac{p^2 p''}{hp'}}, \end{aligned} \tag{3.17}$$

where h is defined in (3.16).

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[9] h is interpreted as the entrepreneur's effective risk aversion. In particular, under first best where $p(w)$ can be written as $p(w) = w + q$ for a constant q, h degenerates to γ.

3.4. Boundary Conditions at the Lower End

We now specify the firm's liquidation policy which also serves as boundary conditions to (3.17) at the lower end. Intuitively, the firm gets liquidated when w becomes sufficiently negative. Liquidation is irreversible and gives the firm-held capital stock a terminal value of lK_t, where $l \in (0,1)$ indicating a costly liquidation. Let τ^l be the (stochastic) liquidation time optimally chosen by the agent so that the firm's liquidation value can be written as $W_{\tau^l} + lK_{\tau^l}$. Upon liquidation, the agent sells the firm for $W + lK$ and becomes a Merton consumer (Merton [17]) where her value function takes the form of

$$V(W + lK) = \frac{\left[b(W + lK)\right]^{1-\gamma}}{1-\gamma}, \tag{3.18}$$

where $V(W) = \frac{(bW)^{1-\gamma}}{1-\gamma}$ which denotes the value function for a Merton consumer with the financial wealth W; b is given by (3.28). Our quantitative analyses show that $W_t + lK_t > 0$ always holds so that the entrepreneur as a Merton consumer always starts with a positive wealth.[10]

Let $\underline{W}$ denote the firm's liquidation boundary with $\underline{w} \equiv \frac{\underline{W}}{K}$. Since $\underline{W}$ is optimally chosen by the agent of the firm, we have the following value matching and smooth-pasting conditions:

$$J(K, \underline{W}) = V(\underline{W} + lK), \tag{3.19}$$

$$\left.\frac{\partial J(K,W)}{\partial W}\right|_{W=\underline{W}} = \left.\frac{\partial V(W + lK)}{\partial W}\right|_{W=\underline{W}}. \tag{3.20}$$

where $J(K,W)$ and $V(W + lK)$ are defined by (3.7) and (3.18), respectively. Simplifying (3.19)-(3.20) by making use of (3.7), (3.18), and the scaled variables, we obtain

$$p(\underline{w}) = \underline{w} + l, \tag{3.21}$$

$$p'(\underline{w}) = 1. \tag{3.22}$$

3.5. Boundary Condition at the Higher End

Since (3.17) is a second-order ODE with the free boundary $\underline{w}$, we need an additional boundary condition at the upper end to fully specify its solution. A natural guess is when w approaches infinity, we have

$$\lim_{w\to\infty} p(w) = p^{FB} = w + q^{FB} \tag{3.23}$$

where q^{FB} is a constant which denotes the valuation of capital under first-best (FB). Intuitively, the liquidation risk vanishes as $w \to \infty$. Without the concern about the potential liquidation, the firm achieves its first-best under which the value of capital q no longer varies with the financial slack.

To identify q^{FB}, we pursue both the firm's investment policy and the ODE for

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[10] It also verifies that firm-level borrowing before its liquidation is always risk-free.

$p(w)$ under the limiting case when $w \to \infty$. Firstly, substituting (3.14) into (3.15) to obtain

$$i^* = \frac{p-(w+1)p' - \rho_I \epsilon p' \frac{\gamma p - wh}{p}\left[\eta \frac{p}{h} - \rho\sigma_K\left(\frac{\gamma p}{h} - w\right)\right]}{\theta p' + \left(1-\rho_I^2\right)\epsilon^2 \frac{p'}{p}\frac{\left(\gamma p - wh\right)^2}{h} - \gamma\epsilon^2 \frac{p^2 p''}{hp'}} \tag{3.24}$$

which no longer loads on *x*. Secondly, rewrite (3.17) by

$$\begin{aligned} 0 = \max_i & \frac{\zeta^\psi b^{1-\psi}\left(p'\right)^{1-\psi} - \zeta\psi}{\psi - 1} p + \left[rw + A - i - \frac{1}{2}\theta i^2\right]p' + \frac{1}{2}\frac{\eta^2 pp'}{h} \\ & + \left(i-\delta_K\right)\left(p - wp'\right) - \frac{1}{2}\left[\left(1-\rho_I^2\right)\epsilon^2 i^2 + \left(1-\rho^2\right)\sigma_K^2\right]\frac{p'p}{h}\left(\gamma - \frac{wh}{p}\right)^2 \\ & - \left(\rho_I \epsilon i + \rho\sigma_K\right)\eta p'\left(\frac{\gamma p}{h} - w\right) + \rho_I \epsilon i \rho\sigma_K \frac{p'p}{h}\left(\gamma - \frac{wh}{p}\right)^2 \\ & + \frac{1}{2}\gamma\left(\epsilon^2 i^2 + \sigma_K^2\right)p^2 \frac{p''}{hp'}, \end{aligned} \tag{3.25}$$

where the optimal choice of *i* is given by (3.24). Substituting (3.23) into (3.24)-(3.25), taking the limit $w \to \infty$, and simplying,[11] we obtain

$$i^* = i^{FB} = \frac{q^{FB} - 1 - \rho_I \epsilon \eta q^{FB}}{\theta} \tag{3.26}$$

and

$$\begin{aligned} 0 = \max_i & \left(\frac{\zeta^\psi b^{1-\psi}\gamma - \zeta}{1-\gamma} + r + \frac{\eta^2}{2\gamma}\right)\left(w + q^{FB}\right) \\ & + A - i - \frac{1}{2}\theta i^2 - \left(r + \delta_K - i\right)q^{FB} - \left(\rho_I \epsilon i + \rho\sigma_K\right)\eta q^{FB}. \end{aligned} \tag{3.27}$$

Since (3.27) has to hold for all *w*, we set *b* in a way such that $\frac{\zeta^\psi b^{1-\psi}\gamma - \zeta}{1-\gamma} + r + \frac{\eta^2}{2\gamma} = 0.$ This gives

$$b = \zeta\left[1 + \frac{1-\psi}{\zeta}\left(r - \zeta + \frac{\eta^2}{2\gamma}\right)\right]^{\frac{1}{1-\psi}}. \tag{3.28}$$

(3.27) now degenerates to

$$0 = \max_i A - i - \frac{1}{2}\theta i^2 - \left(r + \delta_K - i\right)q^{FB} - \left(\rho_I \epsilon i + \rho\sigma_K\right)\eta q^{FB}, \tag{3.29}$$

and a substitution of (3.26) into (3.29) gives the valuation of capital under FB as follows:

$$q^{FB} = \frac{1 + \theta i^{FB}}{1 - \rho_I \epsilon \eta}, \tag{3.30}$$

where[12]

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[11] Under $p(w) = w + q^{FB}$, *h* as defined by (3.16) degenerates to γ.

$$i^{FB} = \frac{r+\rho\sigma_K\eta+\delta_K}{1-\rho_I\epsilon\eta} - \sqrt{\left[\frac{r+\rho\sigma_K\eta+\delta_K}{1-\rho_I\epsilon\eta}\right]^2 - \frac{2}{\theta}\left[A - \frac{r+\rho\sigma_K\eta+\delta_K}{1-\rho_I\epsilon\eta}\right]}. \quad (3.31)$$

4. Numerical Solution to the Model

Taking into account the boundary conditions of (3.21)-(3.23) and (3.23), we solve (3.17) numerically for a plausible set of parameterization which is summarized in Table 1.

Table 1. Summary of model parameterization.

Panel A: Market environment					
$r = 0.046$	$\mu_R - r = 0.06$	$\sigma_R = 0.2$	$\eta = 0.3$		
Panel B: Preferences					
$\zeta = 0.046$	$\gamma = 2$	$\psi = 2.2$			
Panel C: Firm-related parameters excluding volatility parameters					
$\theta = 2$	$l = 0.9$	$\delta_K = 0.125$	$A = 0.2$	$\rho = 0$	$\rho_I = 0.3$
Panel D: Volatility parameters					
$\sigma_K = 0.1$	ϵ: between 0 and 1				

This table summarizes the baseline parameterization to our model. Panel A describes the market-related parameters, where $r, \mu_R - r$ and σ_R denote, respectively, the risk-free rate, the market equity risk premium, and the volatility of the market portfolio. Panel B reports preference parameters,. where ζ, γ and ψ denote the subjective discount rate, the degree of risk aversion, and the elasticity of intertemporal substitution (EIS), respectively. Panel C calibrates firm-related parameters excluding volatility parameters, where $\theta, l, \delta_K, A, \rho$ and ρ_I denote, respectively, the adjustment cost parameter, the capital liquidation price, the rate of capital depreciation, the productivity of capital, the correlation between the market portfolio returns and capital shocks, and the correlation between the market portfolio returns and investment-specific shocks. Panel D reports the volatility parameters, where σ_K denotes the volatility of capital depreciation shocks while ϵ governs the volatility of investment-specific shocks. All parameters are annualized.

More specifically, the baseline values of risk-free interest rate *r*, the market equity risk premium $\mu_R - r$, the volatility of the market portfolio return σ_R, the risk aversion coefficient γ, the rate of time preference ζ, the capital depreciation rate δ_K, and the productivity of capital A are taken from Wang, Wang, and Yang [9][13]: Their parameterization implies a sizable equity risk premium of 0.06 and a reasonably low risk aversion of 2. Motivated by its recent estimate by Kapoor and Ravi [18], we set the EIS parameter ψ to 2.2. Consistent with the estimate by Whited [19], we take the adjustment cost parameter θ to be 2. As suggested by Hennessy and Whited [20], we choose the capital liquidation price l to be 0.9. We set $\rho = 0$ and $\rho_I = 0.3$ so as to stay focused on the impact of investment risk when it is positively correlated with the stock market. We choose the volatility of capital shocks σ_K to be half of the level used in Bolton, Wang,

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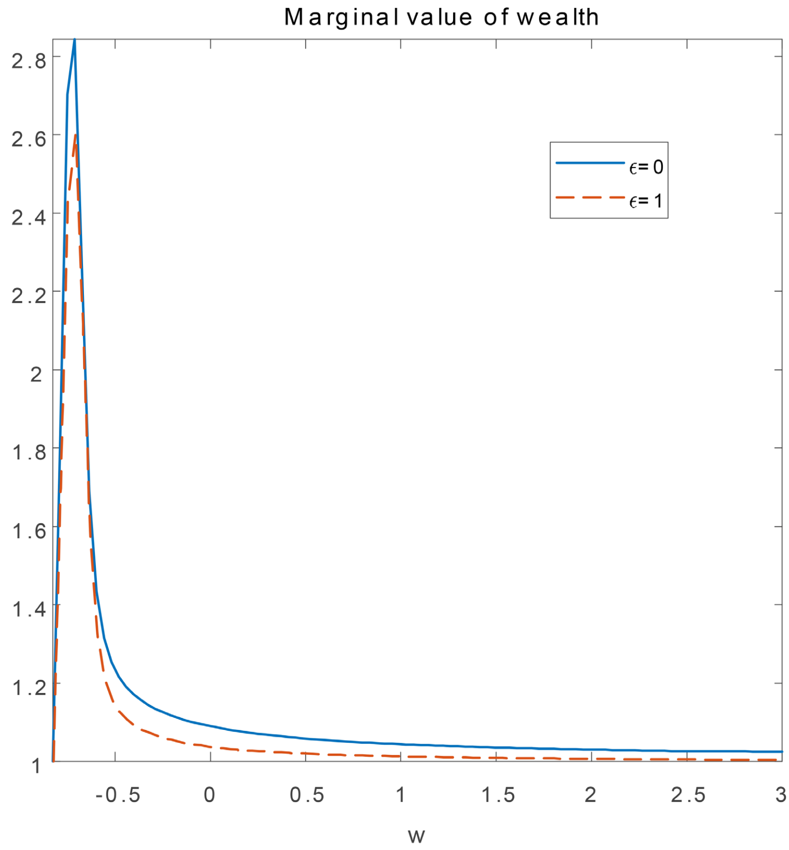
[12] It is easy to see that a substitution of (3.30) into (3.29) gives a quadratic equation on *i*. Of its too roots, we pick the one given by (3.31) as the solution to i^{FB} so that the resulting i^{FB} is increasing in the productivity of capital, A.

[13] Wang, Wang, and Yang [9] allow A to be stochastic, so our A corresponds to their μ_A denoting the average productivity of capital.

and Yang [10] who do not consider investment-specific shocks. Finally, For the key parameter ϵ which control the volatility induced by investment risk, we allow it to vary between 0 and 1 and study the implied impact on investment and the valuation of capital. All parameters are annualized.

Figure 1 plots the numerical solution to the model in terms of $p'(w)$ under two scenarios of ϵ. Financially, $p'(w)=P_W$ denotes the firm's marginal value of wealth and **Figure 1** shows that it generally stays above one. Exceptions are i) when w approaches the liquidation boundary $\underline{w}$ under which $p(\underline{w})=1$ (see Equation (3.22)); ii) when w approaches infinity under which $\lim_{w\to\infty} p'(w)=w+q^{FB}$ so that $\lim_{w\to\infty} p'(w)=1$. Furthermore, **Figure 1** shows that raising ϵ depresses $p'(w)$ for all w which implies a higher liquidation boundary. Intuitively, the higher investment risk prompts the risk-averse agent to abandon the firm earlier. In addition, it depresses the marginal value of wealth for the given level of w which prompts the agent to accumulate more of the financial reserves working as the buffer against the costly liquidation.



$w\equiv W/K$ which measures the financial slack of the firm. The marginal value of the wealth approaches one either when w approaches its liquidation boundary $\underline{w}$ (-0.8278 for $\epsilon=0$ and -0.8230 for $\epsilon=1$) or when $w\to\infty$.

Figure 1. Marginal value of wealth P_W for the firm under two scenarios of ϵ, where ϵ controls the volatility of investment-specific shocks.

5. Optimal Investment for a Firm under Autarky

For our financial analyses, we also need to solve the problem for a firm operated under autarky. Such a firm does not accumulate liquid wealth nor take positons in the stock market. Therefore, (3.1) degenerates to

$$0=\max_{I} f\left(C,J\right)+\left(I-\delta_K K\right)J_K+\frac{\epsilon^2 I^2+\sigma_K^2 K^2}{2}J_{KK}, \tag{5.1}$$

which is subject to the autarky condition of

$$C_t=AK_t-I_t-G\left(I_t,K_t\right). \tag{5.2}$$

In (5.1), *J* denotes the recursive preference that is characterized by (2.4)-(2.6).[14] Under autarky, the only control by the firm is its investment policy, and the FOC of *I* gives

$$\left[1+G_I\left(I,K\right)\right]f_C=J_K+\epsilon^2 IJ_{KK}. \tag{5.3}$$

where we have used $\frac{\partial C}{\partial I}=1+G_I$ from the autarky condition.

Conjecture (and verify later) that the implied value function *J* takes the following form

$$J\left(K\right)=\frac{\left(b^A\right)^{1-\gamma}}{1-\gamma}\left(Kq^A\right)^{1-\gamma}, \tag{5.4}$$

where q^A denotes the valuation of capital under autarky and we allow its coefficient b^A to differ from the coefficient *b* in the value function of (3.7). Under (5.4), we have the following relations:

$$J_K=\frac{(1-\gamma)J}{K};J_{KK}=\frac{-\gamma(1-\gamma)J}{K^2} \tag{5.5}$$

Using (5.5) and the definition of *G* in (2.3), (5.3) simplifies to

$$\left(1+\theta i\right)f_C=J_K\left(1-\gamma\epsilon^2 i\right). \tag{5.6}$$

From the definition of *f* given by (2.5),

$$f=\frac{Cf_C}{1-1/\psi}-\frac{\zeta(1-\gamma)}{1-1/\psi}. \tag{5.7}$$

Substituting *f* from (5.6) and (5.7) into (5.1), we obtain

$$0=\frac{1-\gamma\epsilon^2 i}{1+\theta i}\frac{CJ_K}{1-1/\psi}-\frac{\zeta(1-\gamma)}{1-1/\psi}J+\left(I-\delta_K K\right)J_K+\frac{\epsilon^2 I^2+\sigma_K^2 K^2}{2}J_{KK}, \tag{5.8}$$

Applying (5.5), (5.8) simplifies to

$$0=c\frac{1-\gamma}{1-1/\psi}\frac{1-\gamma\epsilon^2 i}{1+\theta i}-\frac{\zeta(1-\gamma)}{1-1/\psi}+(1-\gamma)\left(i-\delta_K\right)-\frac{\gamma(1-\gamma)}{2}\left(\epsilon^2 i^2+\sigma_K^2\right) \tag{5.9}$$

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[14] With a slight abuse of notation, we still use *J* to denote the firm agent's value function when the firm is operated under autarky.

Substituting c from the autarky condition of (5.2), (5.9) can be equivalently written as

$$\frac{A - i - \frac{1}{2}\theta i^2}{\zeta + \left(\frac{1}{\psi}-1\right)\left[\left(i-\delta_K\right) - \frac{1}{2}\gamma\left(\epsilon^2 i^2+\sigma_K^2\right)\right]} = \frac{1+\theta i}{1-\gamma\epsilon^2 i} \tag{5.10}$$

which implies the following cubic equation on *i*:

$$ai^3 + bi^2 + ci + d = 0 \tag{5.11}$$

with

$$a = \frac{1}{2}\frac{\gamma}{\psi}\theta\epsilon^2;\, b = \frac{1}{2}\theta + \frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\epsilon^2 - \frac{1}{\psi}\theta \tag{5.12}$$

$$c = -\gamma A\epsilon^2 - \frac{1}{\psi} - \zeta\theta + \left(\frac{1}{\psi}-1\right)\theta\delta_K + \frac{1}{2}\left(\frac{1}{\psi}-1\right)\theta\gamma\sigma_K^2$$

$$d = A - \zeta + \left(\frac{1}{\psi}-1\right)\delta_K + \frac{1}{2}\left(\frac{1}{\psi}-1\right)\gamma\sigma_K^2$$

When there is no adjustment cost as assumed in AW [1] so that $\theta = 0$, (5.11) degenerates to the following quadratic equation:

$$0 = \frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\epsilon^2 i^2 - \left(\gamma A\epsilon^2 + \frac{1}{\psi}\right)i + A - \zeta + \left(\frac{1}{\psi}-1\right)\delta_K + \frac{1}{2}\left(\frac{1}{\psi}-1\right)\gamma\sigma_K^2$$

whose solution is given by

$$i_q^A = \frac{\gamma A\epsilon^2 + \frac{1}{\psi} - \sqrt{\Delta}}{\gamma\left(1+\frac{1}{\psi}\right)\epsilon^2}$$ [15]

where

$$\Delta \equiv \left(\gamma A\epsilon^2 + \frac{1}{\psi}\right)^2 - 2\gamma\left(1+\frac{1}{\psi}\right)\epsilon^2\left(A - \zeta + \left(\frac{1}{\psi}-1\right)\delta_K + \frac{1}{2}\left(\frac{1}{\psi}-1\right)\gamma\sigma_K^2\right).$$

Back to the cubic equation of (5.11) and (5.12): Among its three roots, we verify numerically that only one root, taking the form of

$$i^A = -\frac{b}{3a} + \omega^2\left(-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\right)^{\frac{1}{3}} + \omega\left(-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\right)^{\frac{1}{3}}, \tag{5.13}$$

converges to i_q^A when $\theta \to 0$, where

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[15] We take the root with “$-\sqrt{\Delta}$” so that as ϵ approaches zero, i_q^A approaches the special solution of $i_l^A = \dfrac{A - \zeta + \left(\frac{1}{\psi}-1\right)\delta_K + \frac{1}{2}\left(\frac{1}{\psi}-1\right)\gamma\sigma_K^2}{\gamma A\epsilon^2 + \frac{1}{\psi}}$ for $\epsilon = 0$.

$$p=\frac{3ac-b^2}{3a^2};q=\frac{27a^2d-9abc+2b^3}{27a^3};\omega=\frac{-1+\sqrt{3}\tilde{i}}{2}, \tag{5.14}$$

where $\tilde{i}$ denotes imaginary unit. The optimal investment for an autarky firm is thus given by (5.13) and (5.14). Unlike i^{FB} given in (3.31), i_q^A does not load on market-related parameters such as r or η because the firm is now isolated from financial markets.

6. Financial Implications of the Model

Based on the numerical solution to our model and the characterization of the optimal investment under autarky, we now discuss the financial implications of investment risk and the switch-on effect when a firm under autarky has suddenly gained the full access to financial markets.

6.1. Financial Analyses from Formulas

Rewrite (3.15) by

$$1+\theta i=\frac{p-wp'}{p'}-\epsilon^2 i\frac{\gamma p\left(p-2wp'\right)+w^2p'h}{pp'}-\rho_I\epsilon\sigma_R x\left(\gamma-\frac{wh}{p}\right). \tag{6.1}$$

The left-hand side (LHS) of (6.1) denotes the marginal pecuniary cost of investment (*i.e.*, I plus the adjustment cost) while its RHS denotes the discounted marginal benefit of investment. The discount is attributed to investment risk and it takes the form of i) the disutility effect to a risk-averse agent as captured by the second term; and ii) the market correlation effect as captured by the third term which we will discuss shortly.

Under autarky, the (scaled) CE wealth of the firm, p^A, and the value of capital, q^A, is the same since there is no liquid wealth. To make the full use of (6.1), we equivalently rewrite the relation between p^A and q^A by

$$p^A=\lim_{w\to 0}p^A(w)\equiv\lim_{w\to 0}\left(w+q^A\right) \tag{6.2}$$

The expression of (6.2) implies $\left(p^A\right)'=1$[16] so that $\left(p^A\right)''=0$ and h defined in (3.16), with p replaced with p^A, again degenerates to γ. Substituting these simplifications and (6.2) into (6.1) and simplifying, we obtain

$$1+\theta i^A=\lim_{w\to 0}\left(q^A-\gamma\epsilon^2 i^A\frac{\left(q^A\right)^2}{w+q^A}-\rho_I\epsilon\sigma_R x^A\frac{\gamma q^A}{w+q^A}\right). \tag{6.3}$$

Under autarky, the firm does not take position in the stock market so the term $-\rho_I\epsilon\sigma_R x^A\frac{\gamma q^A}{w+q^A}$ is simply zero. (6.3) then degenerates to

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[16] To obtain it, we assume that taking derivative and taking limit can change orders. This result also makes financial sense: While the marginal value of wealth p' in the full model generally stays above one as plotted in **Figure 1**, $\left(p^A\right)'$ under autarky is always one. This is because in the former case, raising liquid wealth has the extra benefit of providing the buffer against the costly liquidation. In contast, firm never gets liquidated under autarky (since the firm never borrows) and as a result the marginal benefit of (hypothetically) acquiring one unit of liquid wealth is exactly one.

$$1+\theta i^{A}=q^{A}-\gamma\epsilon^{2}i^{A}q^{A} \tag{6.4}$$

after taking the limit. When there is no adjustment cost so that $\theta=0$, (6.4) is equivalent to Equation (24) of AW [1].[17] Specifically, it shows the disutility effect to the risk-averse agent, as captured by $-\gamma\epsilon^{2}i^{A}$, depresses the marginal benefit of investment leading to a lower i^{A}. Mathematically,

$$i^{A}=\frac{q^{A}-1}{\theta+\gamma\epsilon^{2}q^{A}}<\frac{q^{A}-1}{\theta}\quad\text{when } \epsilon=0. \tag{6.5}$$

In contrast, the disutility effect plays no role to a firm which has already built up a high degree of financial slack. To see the point, let $w\to\infty$ so that $p(w)$ is given by (3.23). Substituting (3.23) into (6.1) simplifying, we have

$$1+\theta i^{FB}=\lim_{w\to\infty}\left(q^{FB}-\gamma\epsilon^{2}i^{FB}\frac{\left(q^{FB}\right)^{2}}{w+q^{FB}}-\rho_{I}\epsilon\sigma_{R}x^{FB}\frac{\gamma q^{FB}}{w+q^{FB}}\right), \tag{6.6}$$

where

$$x^{FB}=\frac{\eta}{\sigma_{R}}\frac{w+q^{FB}}{\gamma}-\frac{\rho_{I}\epsilon i^{FB}+\rho\sigma_{K}}{\sigma_{R}}q^{FB} \tag{6.7}$$

which is obtained by substituting (3.23) into (3.14). (6.6) and (6.7) is the FB-counterpart of (6.4) with several fundamental differences. First of all, disutility effect as captured by $-\gamma\epsilon^{2}i$ no longer affects the firm's investment policy because the whole term of $-\gamma\epsilon^{2}i^{FB}\frac{\left(q^{FB}\right)^{2}}{w+q^{FB}}$ vanishes as $w\to\infty$. Intuitively, the risk-averse agent no longer cares about the extra volatility induced by investment because the firm's valuation now comes mainly from its financial reserve. Secondly, the market correlation effect, which is not a concern for a firm under autarky, now drives the impact of ϵ on firm's investment decisions. More specifically, by substituting (6.7) into (6.6) and taking the limit, we obtain

$$1+\theta i^{FB}=q^{FB}-\rho_{I}\epsilon\eta q^{FB}. \tag{6.8}$$

When $\rho_{I}>0$ as what we have parameterized in **Table 1**, a higher investment exposes the firm more to the stock market. Consequently, it adds to the riskiness of investment which prompts a more conservative investment policy. Mathematically,

$$i^{FB}=\frac{q^{FB}-1-\rho_{I}\epsilon\eta q^{FB}}{\theta}<\frac{q^{FB}-1}{\theta}\quad\text{when } \epsilon=0. \tag{6.9}$$

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[17] (6.4) also helps identify b^{A} in the valuation function of (5.4). Firstly, a direct simplification of (5.6) by making use of (5.4) and (2.5) gives the FOC in the form of $\zeta c^{-\frac{1}{\psi}}\left(b^{A}q^{A}\right)^{\frac{1}{\psi}-1}q^{A}\left(1+\theta i^{A}\right)=q^{A}\left(1-\gamma\epsilon^{2}i^{A}\right)$. This condition is consistent with (6.4) if we set b^{A} in a way such that $\zeta c^{-\frac{1}{\psi}}\left(b^{A}q^{A}\right)^{\frac{1}{\psi}-1}q^{A}=1$. This equation identifies b^{A} as the function of q^{A} and *c*, where both q^{A} and *c* are functions of i^{A} as indicated by (6.4) and the autarky condition of $c=c^{A}=A-i^{A}-\frac{1}{2}\theta\left(i^{A}\right)^{2}$, where i^{A} is obtained in closed-form by (5.13) and (5.14).

Thirdly, while the magnitude of disutility effect with a self-sufficient firm is determined by the agent's idiosycratic taste for risk, *i.e.*, γ, the magnitude of the market correlation effect is determined by the market Sharpe Ratio η, which can be interpreted as the price of the market risk. Finally, a firm under autarky does not take positions in the stock market while a firm under FB chooses its stock market allocation to be linearly decreasing in i^{FB} as indicated by (6.7), provided that $\rho_I > 0$.

6.2. Quantitative Results

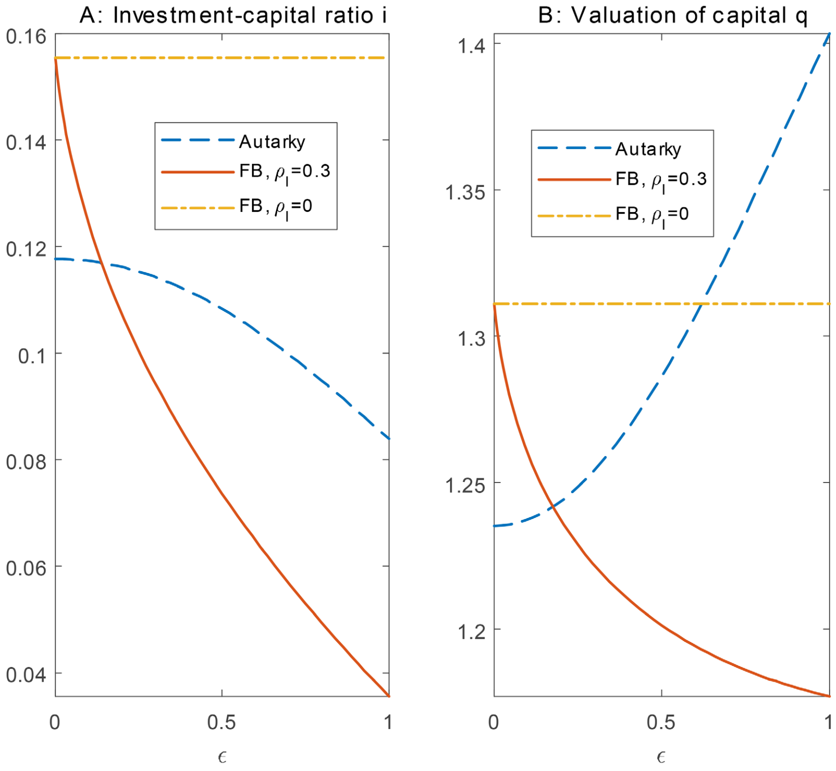
6.2.1. Impact of Investment Risk

Panel A of Figure 2 plots i^A and i^{FB} as the function of ϵ which serves as the measure of investment risk. With a positive ρ_I, both i^A and i^{FB} decreases with ϵ but for different reasons. For i^A (dashed line), it is due to the disutility effect on the risk-averse agent and, as suggested by (6.5), it vanishes when the agent is risk-neutral with $\gamma = 0$. For i^{FB} (solid line), the decreasing pattern is attributed to the correlation effect at $\rho_I > 0$ which links ϵ positively to the market risk. Quantitatively, the correlation effect is more pronounced than the disutility effect over the range of ϵ that we plot. This is because the correlation effect rises linearly with ϵ (see Equation (6.9)) while the disutility effect rises quadratically in ϵ (see Equation (6.5)) which implies a higher-order effect when ϵ is less than one. The correlation effect is shut down once ρ_I is set to zero indicating that the investment-specific shocks are now completely idiosyncratic. Consequently, the implied i^{FB} (dash dotted line) stays at the highest level throughout. In contrast, varying ρ_I has no impact on i^A (not plotted) because the underlying firm is isolated from the stock market.

Panel B of Figure 2 plots the corresponding valuations of capital, q^A and q^{FB}, as the function of ϵ. As firstly shown by AW [1], investment risk induces the disutility effect which makes the newly invested capital less desirable than the installed capital in the eyes of a risk-averse agent. As a result, the installed capital is valued at a higher level than its replacement cost leading to $q^A > 1$ even when there is no adjustment cost. Naturally, a higher ϵ implies a more severe disutility effect which further drives up q^A (dashed line). In contrast, market correlation effect at a positive ρ_I implies that a higher ϵ depresses q^{FB} instead of raising it (solid line). The reasoning lies in the basic asset pricing logic in that the price of an asset (e.g., firm-held capital) depresses more when it is more exposed to the market and hence commands a higher premium, and a larger ϵ at $\rho_I > 0$ exactly does that because it serves to raise the firm's market exposure. Once ρ_I is set to zero, q^{FB} stays flat irrespective of ϵ (dash dotted line) implying that there is no more discount to the valuation of capital when investment risk is completely idiosyncratic.

6.2.2. The Switch-On Effect

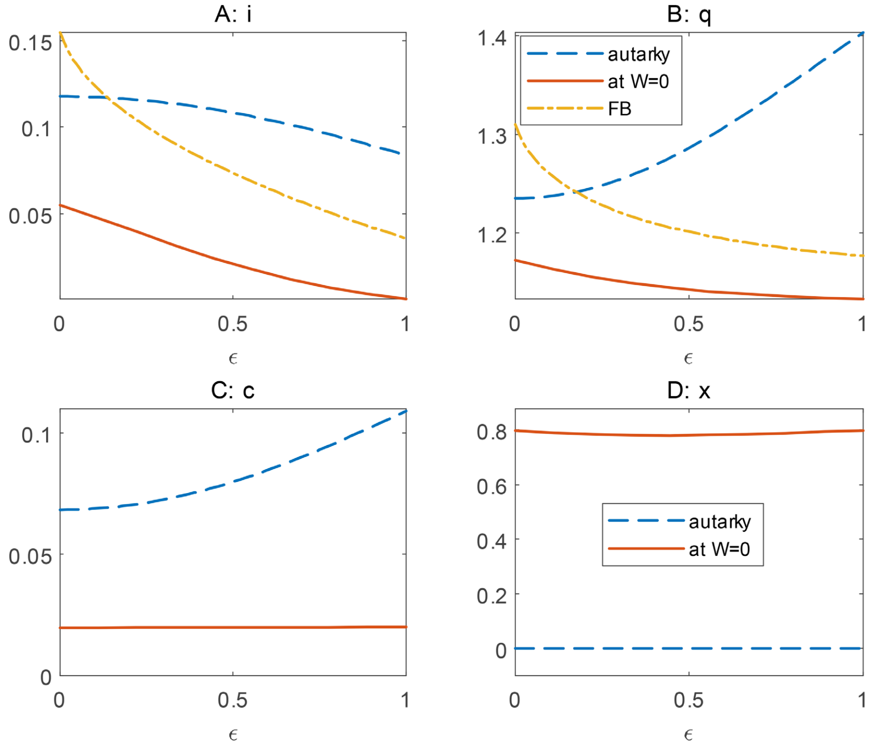
Plots in Figure 2 show that a firm with abundant liquid wealth may "under-invest" and may have a lower valuation of its capital when compared to a self-sufficient firm. For a more systematic examination of the underlying reasoning, we calculate relevant variables for a firm with temporary zero degree of financial slack at $W = 0$, but with the full access to financial trading which enables it to accumulate financial reserves in the future. We compare the implied values of these variables to their autarky-counterparts and plot the results in **Figure 3**. For illustration, we also plot some of variables under FB. In all plots, we allow ϵ to vary between 0 and 1 and all parameters other than ϵ take their baseline values reported in **Table 1**.



In Panel A, we plot investment under autarky and under first-best (FB, achieved when firm with the full access to financial markets has accumulated abundant liquid wealth with $w \to \infty$), *i.e.*, i^A and i^{FB} as the function of ϵ. In Panel B, we plot the valuation of capital under autarky and under FB, *i.e.*, q^A and q^{FB}, as the function of ϵ. For FB, we plot for two scenarios of ρ_I: $\rho_I = 0.3$ as its baseline value and $\rho_I = 0$, where ρ_I denotes the correlation coefficient between investment risk and the market risk. All parameters other than ρ_I and ϵ are at their baseline levels reported in **Table 1**.

Figure 2. Impact of investment risk as measured by ϵ.

Let's first examine dashed lines (for a firm under autarky) vs. solid lines (for a firm with the temporary $W = 0$) plotted in Panel A&B of **Figure 3**. In combination, they indicate that a firm uniformly lowers its capital investment and the valuation of its capital once it has broken out of autarky by gaining the access to financial trading. Quantitatively, the magnitude of this switch-on effect is quite large. Indeed, when the degree of slack goes from 0 to infinity (dash dotted lines), the resulting i^{FB} and q^{FB}, which is naturally much higher than $i(w=0)$ and $q(w=0)$, still lie below the corresponding i^A and q^A for most of the ϵ-realizations.



The dashed lines and solid lines plot variables as the function of ϵ for a firm operated under autarky and for an otherwise identical firm (e.g., liquid wealth W is zero) which has the full access to financial markets. The plotted variables in Panel A, B, C, D are investment-capital ratio i, the valuation of capital q, consumption-capital ratio c, and market asset allocation-capital ratio x, respectively. For Panel A&B, we also plot values of corresponding variables under first-best.

Figure 3. Impact of financial trading.

Panel C of **Figure 3** shows that a firm which has just gained the access to financial markets not only cuts its investment but also significantly cuts its consumption. Simultaneously and as plotted in Panel D, the firm substantially raises its market asset allocation from zero to a level equivalent to 80% of its capital stock for the purpose of harvesting the equity risk premium.[18] Since the firm plotted in solid lines hasn't yet accumulated any liquid wealth, it essentially uses its capital as the collateral to take position in stock market which is further facilitated by a cut in its consumption and investment. How to interpret these results financially? To answer it, we plot in Panel A–F of **Figure 4** the simulated averages of wealth-capital ratio (w), market asset allocation-capital ratio (x),

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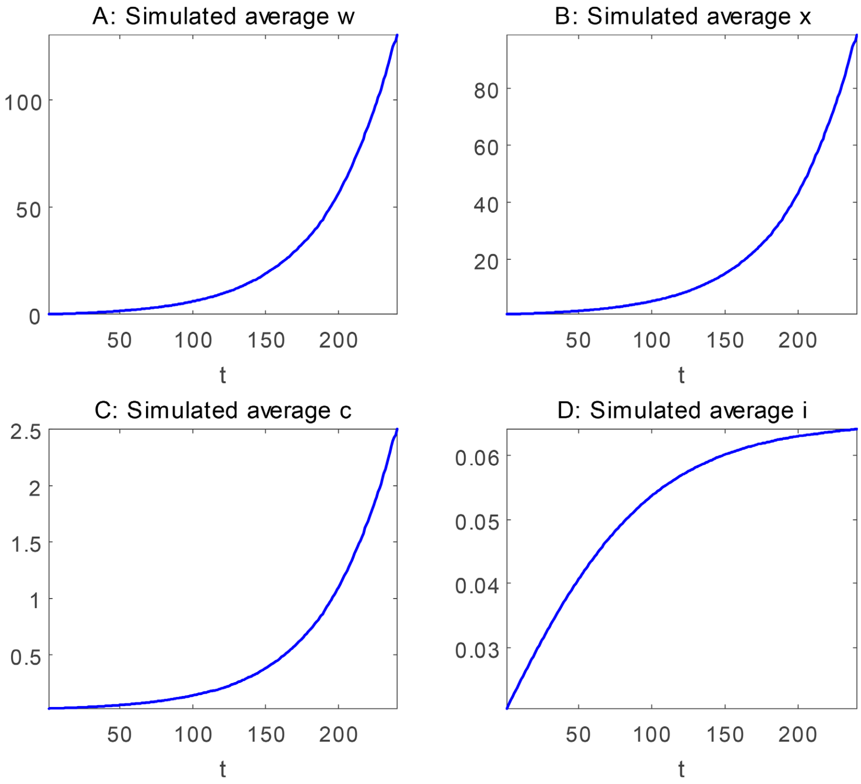
[18] Recall the plotted x in Panel D denotes the allocation-capital ratio. Numerically, x is mainly driven by the agent's myopic demand, which is determined by the market risk premium, instead of by the hedging demand.

consumption-capital ratio (*c*), and investment-capital ratio (*i*) for a time span of 240 months. The averages are based on 10,000 simulated firms with the starting levels of financial slack w_0 all set to zero. To conduct the simulation, we set $\epsilon = 0.5$. All other parameters are set at their baseline values that are reported in **Table 1**.



Panel A-D plot the time series for the simulated averages of wealth-capital ratio (*w*), market asset allocation-capital ratio (*x*), consumption-capital ratio (*c*), and investment-capital ratio (*i*). We simulate 10,000 firms with the starting level of financial slack, w_0, set at zero for all firms. The time span is 240 months. Liquidated firms are removed from simulation so that the simulated averages are only based on alive firms. We set $\epsilon = 0.5$ and all other parameters take their baseline values that are reported in **Table 1**.

Figure 4. The implied model dynamics by simulation.

With the full access to financial markets, the firm raises its stock market position (Panel B) almost linearly with w (Panel A). Since $\eta > 0$, this gives rise to the value-creation effect that allows the liquid wealth to grow without bound. Consequently, the implied $c(w)$ (Panel C) also rise unboundedly which points to a large increase of utility to the firm's agent once the firm has gained access to the stock market. This is in contrast to a self-sufficient firm under which the consumption-capital ratio c^A is a constant (see footnote 17 for a justification). In contrast, the rise of $i(w)$ (Panel D) is clearly capped because it can never exceed i^{FB} which is 0.0736 at $\epsilon = 0.5$. The reasoning for the switch-on effect on investment and valuation of capital is now clear. With the new possibility to gain infinite amount of liquid wealth relative to firm's capital, the agent of the firm naturally turns her attention to building up the financial slack which (on average) would substantially raise her consumption level in the future. Such a switch of emphasis implies i) a permanent cut in i as well as temporary cut in c so as free up more resources to be put into the stock market; ii) a permanent downward adjustment in her valuation of capital whose contribution to the firm is now downplayed by the value-creation effect from the stock market.

7. Conclusion and Suggestions

Capital investment not only requires adjustment cost but also involves risk, which is the concern of a risk-averse agent. To study the financial impact of investment risk within a corporate environment, we develop and solve a dynamic stochastic model for a firm featuring intertemporal asset allocation, consumption, costly business liquidation, and investment-specific shocks. We further extend AW's [1] model on a self-sufficient firm to the case with recursive preference and adjustment cost, and we solve its problem in closed form. Our financial analyses show that investment risk has a fundamentally different impact on a firm with a high degree of financial slack when compared to that on a firm operated under autarky. We further show that gaining access to financial markets implies a switch of emphasis from capital investment to financial trading which is intended for the accumulation of liquid wealth. As a result, the firm temporarily cuts its consumption but permanently cuts its investment and the valuation of its capital. Our results are broadly consistent with the literature that emphasizes the importance of financial development and investor protection for creating a healthy stock market. In addition, they also support Warren Buffet's long-term advocate of using the financial markets for creating and preserving wealth (after acquiring the initial capital to start with).

Our model makes several predictions that are empirically testable, which we summarize in Panel B of **Table 2**. While a formal empirical test is beyond the scope of this paper, we outline in the following a suggested procedure for one possible test on the correlation effect, namely that both capital investment and valuation of capital should go down when investment risk is positively correlated with the stock market. First, collect the industry-level data and confirm the existence of investment-specific shock in that higher investment is associated with higher volatility of outputs in that industry. Second, identify a wide array of non-industry factors that are related to investment and valuation of capital as the controls, and use a dummy variable to denote the different industries. Third, run time-series regressions industry by industry with (the appropriate empirical measure of) investment and capital valuation as the dependent variables and the industry dummy and all the controls as the independent variables. The model prediction on correlation effect will be validated if regression coefficient on the dummy variable is statistically higher with industries that are negatively correlated with the stock market (e.g., the insurance industry) than with industries that are positively correlated with the stock market.

Table 2. Summary of theoretical findings and empirical predictions.

Panel A: Theoretical contributions:
1) Numerically solve a dynamic stochastic model for entrepeneurial firm that incorporates intertemporal asset allocation, consumption decisions, and investment-specific shocks.
2) Extend the model of AW [1] on firms under autarky to the case with recursive prefernece and adjustment, and solve the extended model in closed-form.
Panel B: Model implications and predictions
1) Under autarky, investment risk depresses investment but raises the value of capital. (This implication confirms the finding of AW [1] in a more general setup)
2) With a high degree of financial slack, investment risk no longer affects investment or capital valuation if the risk is purely idiosyncratic.
3) Correlation effect: When investment risk is positively correlated with the stock market, it depresses both investment and capital valuation.
4) Switch-on effect: Upon breaking out of autarky, a firm cuts consumption and investment, assigns a lower value to its capital, and takes a large position in the stock market.

This table summarizes the key findings of the paper. Panel A describes its theoretical contributions. Panel B summarizes the model implications and predictions which are all empirically testable.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Albuquerue, R. and Wang, N. (2008) Agency Conflicts, Investment, and Asset Pricing. *The Journal of Finance*, **63**, 1-40. https://doi.org/10.1111/j.1540-6261.2008.01309.x

[2] Keynes, J.M. (1936) The General Theory of Employment, Interest and Money. Macmillan.

[3] Greenwood, J., Hercowitz, Z. and Huffman, G.W. (1988) Investment, Capacity Utilization and the Real Business Cycle. *American Economic Review*, **78**, 402-417.

[4] Greenwood, J., Hercowitz, Z. and Krusell, P. (2000) The Role of Investment-Specific Technological Change in the Business Cycle. *European Economic Review*, **44**, 91-115. https://doi.org/10.1016/s0014-2921(98)00058-0

[5] Fisher, J.D.M. (2006) The Dynamic Effects of Neutral and Investment-specific Technology Shocks. *Journal of Political Economy*, **114**, 413-451. https://doi.org/10.1086/505048

[6] Brainard, W.C. and Tobin, J. (1968) Pitfalls in Financial Model-Building. *American Economic Review*, **58**, 99-122.

[7] Tobin, J. (1969) A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, **1**, 15-29. https://doi.org/10.2307/1991374

[8] Ai, H. and Li, R. (2015) Investment and CEO Compensation under Limited Commitment. *Journal of Financial Economics*, **116**, 452-472. https://doi.org/10.1016/j.jfineco.2015.04.002

[9] Wang, C., Wang, N. and Yang, J. (2012) A Unified Model of Entrepreneurship Dynamics. *Journal of Financial Economics*, **106**, 1-23.

https://doi.org/10.1016/j.jfineco.2012.05.002

[10] Bolton, P., Wang, N. and Yang, J. (2019) Optimal Contracting, Corporate Finance, and Valuation with Inalienable Human Capital. *The Journal of Finance*, **74**, 1363-1429. https://doi.org/10.1111/jofi.12761

[11] Rajan, R. and Zingales, L. (1998) Financial Dependence and Growth. *American Economic Review*, **88**, 559-586.

[12] Shleifer, A. and Wolfenzon, D. (2002) Investor Protection and Equity Markets. *Journal of Financial Economics*, **66**, 3-27. https://doi.org/10.1016/s0304-405x(02)00149-6

[13] Valickova, P., Havranek, T. and Horvath, R. (2014) Financial Development and Economic Growth: A Meta-Analysis. *Journal of Economic Surveys*, **29**, 506-526. https://doi.org/10.1111/joes.12068

[14] Hayashi, F. (1982) Tobin's Marginal Q and Average Q: A Neoclassical Interpretation. *Econometrica*, **50**, 213-224. https://doi.org/10.2307/1912538

[15] Epstein, L.G. and Zin, S.E. (1989) Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica*, **57**, 937-696. https://doi.org/10.2307/1913778

[16] Duffie, D. and Epstein, L.G. (1992) Stochastic Differential Utility. *Econometrica*, **60**, 353-394. https://doi.org/10.2307/2951600

[17] Merton, R.C. (1971) Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economic Theory*, **3**, 373-413. https://doi.org/10.1016/0022-0531(71)90038-x

[18] Kapoor, M. and Ravi, S. (2017) Elasticity of Intertemporal Substitution in Consumption in the Presence of Inertia: Empirical Evidence from a Natural Experiment. *Management Science*, **63**, 4188-4200. https://doi.org/10.1287/mnsc.2016.2564

[19] Whited, T.M. (1992) Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data. *The Journal of Finance*, **47**, 1425-1460. https://doi.org/10.1111/j.1540-6261.1992.tb04664.x

[20] Hennessy, C.A. and Whited, T.M. (2007) How Costly Is External Financing? Evidence from a Structural Estimation. *The Journal of Finance*, **62**, 1705-1745. https://doi.org/10.1111/j.1540-6261.2007.01255.x