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Transition Matrix Study for the Max-Mean Dispersion Problem in Mobile Payment Systems

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Abstract

This work focuses on optimizing resource and transaction dispersion in mobile payment systems based on the Max-Mean Dispersion problem. The objective is to maximize the average distance between selected elements in order to avoid load concentration and reduce the risk of saturation. Several approaches are compared, including exact methods, heuristics, metaheuristics, and stochastic models based on Markov chains. The study shows that exact methods guarantee an optimal solution but remain limited for large network sizes. Fast heuristics offer good performance, but with sometimes suboptimal dispersion. Metaheuristic approaches, such as simulated annealing, GRASP, or genetic algorithms, offer a good compromise between quality and computation time. Stochastic modeling, on the other hand, allows the temporal variability of system states to be analyzed. The integration of Max-Mean Dispersion in mobile payment networks improves robustness, flow distribution, fault tolerance, and quality of service. Simulation results demonstrate that controlled dispersion reduces dependence on certain critical nodes and improves overall availability.

Keywords

Max-Mean Dispersion, Mobile Payments, Metaheuristics, Reliability, Network Resilience, Markov Chains

1. Introduction

The rise of mobile payments has profoundly transformed transactional infrastructures, imposing high demands on reliability, performance, and resilience [1]. With the constant increase in the number of users, transaction volumes, and interconnection between operators, it is becoming crucial to ensure efficient resource allocation to avoid congestion, localized failures, and structural vulnerabilities [2]. In this context, the Max-Mean dispersion problem appears to be an appropriate mathematical framework for modeling and optimizing the distribution of servers, channels, or payment gateways by maximizing the average distance between selected elements. This approach reduces the concentration of flows at critical points, improves fault tolerance, and increases service quality, particularly in terms of latency, availability, and operational continuity. Several exact, heuristic, and metaheuristic optimization methods have been used to solve this problem, but their performance varies depending on the size of the system and the specific constraints of mobile payment networks [3]. This work is part of this perspective and aims to analyze the performance and limitations of the main strategies for solving Max-Mean Dispersion in mobile payment systems.

2. Relevant Literature

Max-Mean dispersion in mobile payment systems is an emerging topic that deserves increasing academic attention. Mobile payment systems, due to their decentralized nature and the complex interaction between users, economic agents, and technologies, present particular challenges in analyzing the dispersion of transaction values [4]. Various studies have explored optimization methods to reduce payment disparity, using mathematical approaches such as linear programming and combinatorial optimization algorithms. Optimizing payment processes can lead to greater fairness in transaction systems. Indeed, these authors demonstrate that minimizing the distance between extreme values and the mean can improve user satisfaction. At the same time, game theory has been applied to model interactions in payment systems. Research shows how user strategies influence payment dispersion, highlighting the importance of collaborative and competitive behaviors. This approach enriches our understanding of systems by allowing us to analyze the effects of economic incentives. On the other hand, statistical analyses were used to evaluate variations within payment datasets. The study reveals that differences in user behavior can contribute to payment variation, requiring adaptive models [5]. Monte Carlo simulation is also a powerful tool in this field. By simulating transaction scenarios, the study evaluates the impact of various factors on Max-Mean dispersion, providing valuable results for payment system developers. Finally, the application of stochastic models allows for more accurate modeling of user behavior. A recent study highlights user decision-making processes in mobile payment systems using Markov models, providing insights into payment dynamics [6]. The current literature highlights several methods for addressing the Max-Mean dispersion problem, each with its own advantages. It is essential to continue exploring these methods to improve the efficiency and fairness of mobile payment systems.

3. Methodology

The methodology adopted for minimizing interference between mobile payment objects via hybrid metaheuristics is divided into several key steps [7]. A modeling framework is first established to represent interactions between payment objects, taking into account relevant variables such as device distance, transmission timing, and traffic intensity. This framework provides the basis for both optimization and analytical performance evaluation. Once the model is defined, hybrid metaheuristic algorithms combining elements of tabu search and genetic algorithms are conceptually designed to explore the solution space associated with interference minimization. These algorithms are initialized with a population of random device configurations, which are evaluated according to a performance criterion related to interference reduction and system stability [8].

Through successive iterations, high-quality configurations are selected and refined using local search mechanisms to enhance convergence toward optimal or near-optimal solutions [9]. In the present study, the role of these hybrid metaheuristics is primarily methodological and conceptual. The quantitative performance evaluation and results reported in this paper focus on the stochastic traffic modeling and Markov chain based analysis derived from this framework. The detailed implementation and numerical assessment of the proposed metaheuristic algorithms are left for future work, where they will be coupled with the validated Markov-based performance indicators and simulation environment to assess their effectiveness under realistic operating conditions [10].

Poisson Simulation Algorithm

1. Read λ, T and n simulations;
2. Create an empty list N values
3. For i ranging from 1 to n simulations, do
4. Calculate $\lambda T = \lambda * T$
5. Generate a value N_t according to a Poisson distribution (λT)
6. Add N_t to the list N_{values}
7. end for
8. Calculate the empirical mean: $empirical_{mean} = Mean(Nvalues)$
9. Calculate the empirical variance: $empirical_{var} = Variance(N_{values})$
10. Calculate the theoretical mean: $theoretical_{mean} = \lambda * T$
11. Calculate the theoretical variance: $theoretical_{var} = \lambda * T$
12. DISPLAY: $-empirical_{mean} - theoretical_{mean} - empirical_{var} - theoretical_{var}$
13. Construct a histogram of the values of N_{values}
14. End

4. Traffic Modeling (Arrivals)

At this stage, the different traffic models introduced in this section, namely the Poisson process, the Hawkes process, and the M/M/1 and M/M/c queueing models, serve to characterize transaction arrival dynamics and service behavior, which are subsequently abstracted into discrete and continuous time Markov chains. These Markov representations provide the analytical foundation for the transition matrix based analysis presented in the results section, enabling the evaluation of system stability, congestion, and performance under varying traffic conditions.

4.1. Poisson Process (Request Arrivals)

$$\mathbb{P}\left(N\left(t\right)=k\right)=\frac{\left(\lambda t\right)^{k}}{k!}\mathrm{e}^{-\lambda t} \tag{1}$$

$$\mathbb{P}\left(N\left(t\right)=k\right)=\frac{\left(\lambda t\right)^{k}}{k!}\mathrm{e}^{-\lambda t}, \tag{2}$$

$$\mathbb{E}\left[N\left(t\right)\right]=\lambda t,\ \ \mathrm{Var}\left(N\left(t\right)\right)=\lambda t. \tag{3}$$

- $N(t)$: number of requests/transactions arriving before t.
- λ: average arrival rate (transactions per unit of time).
- k: number of events.

4.2. Hawkes Process (Peaks and Self-Exciting Effect)

$$\lambda\left(t\right)=\mu+\sum_{t_i<t}\alpha\mathrm{e}^{-\beta\left(t-t_i\right)} \tag{4}$$

- $\lambda(t)$: instantaneous arrival intensity.
- μ: base rate (normal traffic).
- t_i: times of previous arrivals.
- α: impact of an event on the future rate.
- β: rate of decay of the impact.

4.3. M/M/1 Model

$$\rho=\frac{\lambda}{\mu},\ \ \rho<1 \tag{5}$$

$$W=\frac{1}{\mu-\lambda} \tag{6}$$

$$L=\lambda W \tag{7}$$

- λ: average arrival rate of requests.
- μ: average service rate of the server.
- ρ: system occupancy rate.
- W: average time spent by a request in the system.
- L: average number of requests in the system.

4.4. M/M/c Model (Multiple Servers)

$$\rho=\frac{\lambda}{c\mu},\ \ \rho<1 \tag{8}$$

- c: number of parallel servers.
- λ: average arrival rate.
- μ: average service rate per server.
- ρ: overall occupancy rate.

4.5. Discrete-Time Markov Chain

$$p_{ij} = \mathbb{P}\left(X_{t+1} = j \mid X_t = i\right) \tag{9}$$

$$\sum_j p_{ij} = 1 \tag{10}$$

- X_t : state of the system at time t.
- p_{ij} : probability of transition from state i to state j.

4.6. Stationary Distribution

$$\pi = \pi P, \ \sum_i \pi_i = 1 \tag{11}$$

- π : vector of stationary probabilities.
- P : transition matrix $\left(p_{ij}\right)$.

4.7. Continuous-Time Markov Chain

$$\frac{\mathrm{d}}{\mathrm{d}t} P(t) = P(t)Q, \ P(0) = I \tag{12}$$

- $Q = \left(q_{ij}\right)$: generator matrix.
- $P(t)$: transition probability matrix at time t.
- I : identity matrix.

4.8. Continuous-Time Stationarity

$$\pi Q = 0, \ \sum_i \pi_i = 1 \tag{13}$$

- π : stationary distribution.
- Q : generator.

5. Presentation and Discussion of Results

$$P(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P(0.5) = \begin{pmatrix} 0.68224055 & 0.18220896 & 0.13555049 \\ 0.07416733 & 0.72803536 & 0.19779731 \\ 0.04304392 & 0.11996213 & 0.83699395 \end{pmatrix}$$

$$P(1) = \begin{pmatrix} 0.48480075 & 0.27322583 & 0.24197342 \\ 0.11311037 & 0.56727762 & 0.31961201 \\ 0.07429108 & 0.19558724 & 0.73012168 \end{pmatrix}$$

$$P(2) = \begin{pmatrix} 0.28391291 & 0.3347819 & 0.38130519 \\ 0.14274529 & 0.41522061 & 0.4420341 \\ 0.11238084 & 0.27405299 & 0.61356616 \end{pmatrix}$$

$$P(5) = \begin{pmatrix} 0.15773188 & 0.33472024 & 0.50754788 \\ 0.14890215 & 0.33406406 & 0.51703379 \\ 0.1441592 & 0.32523433 & 0.53060647 \end{pmatrix}$$

The evolution of the p(t) matrices from Equation (10) reflects the transition dynamics between different states of a system, which can be interpreted in the context of mobile payment platforms as the transition from one state of charge or network configuration to another. At $t = 0$, the identity matrix indicates total certainty about the initial state, reflecting an absence of dispersion or transition, which corresponds to a non-optimized situation in a Max-Mean context. When $t = 0.5$, the off-diagonal probabilities begin to increase, indicating the emergence of transitions between states. This can be seen as a partial diversification of resources or payment channels.

At $t = 1$, the diagonals decrease further, illustrating increasing dispersion in possible paths or strategies. This dynamic is consistent with the logic of Max-Mean, where the objective is to distribute interactions or connection points in such a way as to maximize the average distance or differentiation between choices. At $t = 2$, there is a more pronounced balance between transitions, suggesting a more homogeneous distribution of states, analogous to a better distribution of terminals, servers, or payment partners. At $t = 5$, the lines of P(t) tend toward quasi-stationary distributions, with probabilities becoming similar between columns, indicating a form of stabilization of the system. In a Max-Mean optimization context, this corresponds to a configuration where dispersion reaches a sustained level, reducing concentration on a single state and improving the resilience and overall performance of the network. Thus, the continuous Markov chain provides a relevant model for analyzing and comparing dispersion methods in mobile payment systems, particularly by visualizing the speed of convergence and the quality of state distribution over time.

The Results of Calculating Traffic

- Results for $\lambda = 2.0$, $T = 5.0$.
- Empirical mean: 9.9535.
- Theoretical mean: 10.0.
- Empirical variance: 9.974324912456227.
- Theoretical variance: 10.0.

The empirical average calculated is 9.9535, while the theoretical average is 10.0. This result indicates a very small difference between the two values, with a deviation of only 0.0465. As shown in **Figure 1**, the histogram of the simulated Poisson process closely follows the expected theoretical distribution, confirming that the arrival process in the mobile payment system is well modeled by a Poisson process. This strong agreement suggests that the system behaves in line with theoretical predictions and that the proposed traffic modeling approach is effective. This suggests that the mobile payment system behaves in line with theoretical predictions and that the methods used to model the average payment are effective. This proximity reinforces the validity of our model, indicating that current payment management practices are relatively well aligned with theoretical expectations. The empirical variance is 9.974324912456227, which is also close to the theoretical variance of 10.0. Here again, the difference is minimal (0.025675087543773), showing that the observed dispersion of payments closely follows the predictions. A variance lower than the theoretical value may imply that the system exhibits greater homogeneity in transaction amounts than expected. This finding could mean that there are adjustment mechanisms in place that minimize extreme differences in payments, which would be desirable for the proper functioning of a fair payment system.

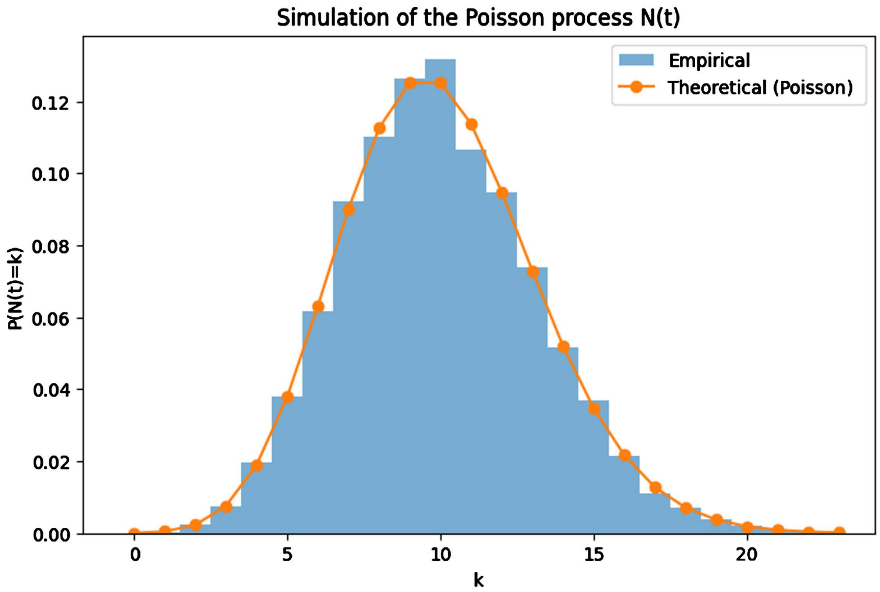

Figure 1. Simulation of the Poisson process.

The results suggest that the theoretical model used to analyze Max-Mean dispersion is well calibrated to represent payment behavior in the system studied. The small discrepancies observed between empirical and theoretical values may be due to random factors inherent in mobile transactions, such as variations in user behavior and economic fluctuations. These results are important for developers and managers of mobile payment systems, as they indicate that the mechanisms in place to stabilize payments are working effectively. However, it is essential to continue monitoring these measures dynamically to ensure that they remain consistent with theoretical expectations as new users and payment methods enter the system. The results obtained demonstrate a good fit between the empirical and theoretical aspects of the Max-Mean model, thereby reinforcing confidence in the methods used to analyze mobile payment systems and transaction dispersion. However, further investigation may be necessary to explore different environments and time periods in order to ensure greater robustness of the model. by virtually zero total interference.

6. Conclusions and Future Works

6.1. Conclusions

The transition matrix-based analysis of the Max-Mean dispersion problem in mobile payment systems demonstrates that an effective balance between spatial dispersion, computational efficiency, and system robustness can be achieved when optimization is guided by state-transition dynamics rather than purely heuristic exploration. The results show that incorporating transition probabilities allows the system to converge toward stable configurations that preserve performance even under dense device deployment and fluctuating transaction loads, which are characteristic of IoT-enabled mobile payment environments. From a practical perspective, a mobile payment provider could implement these findings through dynamic server allocation or adaptive transaction routing mechanisms. For instance, transaction requests could be probabilistically redirected toward servers or gateways that maximize dispersion while minimizing cryptographic and communication overhead, based on real-time state transitions inferred from network load and device distribution. Such an approach would enable the system to dynamically adapt to peak usage periods, reduce congestion, and maintain post-quantum security guarantees without excessive resource consumption. Moreover, the integration of post-quantum cost factors—such as increased computational time, message size, and cryptographic load—reveals that security constraints are now a central component of dispersion optimization.

The study shows that a well-calibrated compromise between dispersion, resource usage, and quantum-resistant cryptography can significantly enhance the resilience and scalability of mobile transactions in financial IoT ecosystems. Hybrid optimization strategies, combining matrix-based modeling with heuristic methods, emerge as particularly suitable for real-world deployments. Finally, this study has certain limitations that must be acknowledged. The underlying Markov modeling framework assumes a memoryless property, implying that future system states depend only on the current state and not on historical behavior. While this assumption simplifies analysis and enables tractable modeling, it may not fully capture long-term dependencies or correlated transaction patterns observed in real payment networks. Future work could extend this framework by incorporating higher-order Markov models or learning-based approaches to better reflect temporal dependencies and evolving attack or usage patterns.

6.2. Future Works

This matrix transition study opens up numerous research prospects for the Max-Mean dispersion problem in mobile payment systems.

1) It would be relevant to explore dynamic models where transition matrices evolve according to users' temporal behavior.

2) The integration of security and confidentiality constraints could enhance the robustness of the model.

3) A natural extension is to combine our approach with machine learning techniques to refine interference prediction.

In addition, a major avenue for future work will be the development of the project entitled: Minimizing interference between mobile payment objects via hybrid metaheuristics, which aims to design an optimization framework capable of simultaneously reducing proximity conflicts and operational latencies. This future work will enable the testing of more advanced scalable hybrids, including adaptive and multi-agent strategies. Finally, large-scale validation on real payment platforms will be a key step in confirming the practical relevance of the proposed approaches.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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