

A Solution of the Cosmological Constant, Using Multiverse Version of Penrose CCC Cosmology, and Klauder Enhanced Quantization for Obtaining a Cosmological Constant Compared

Andrew Beckwith

Physics Department, College of Physics, Chongqing University, Chongqing, China
Email: rwill9955b@gmail.com

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Abstract

This study compares a multiverse generalization of CCC Penrose cosmology with my work on Klauder enhanced quantization and the cosmological constant problem. While it is not linked to tokamaks, it is still interesting to contemplate.

Keywords

Minimum Scale Factor, Cosmological Constant, Space-Time Bubble, Bouncing Cosmologies

1. Introduction

We bring up this study due to the general failure of field theory methods to obtain a working solution to the Cosmological Constant problem. One of the reasons for this work is that Quintessence studies involving evolution of the Cosmological Constant over time have routinely failed in match ups with Early supernova data. The working hypothesis is that if Dark Energy is commensurate with the Cosmological Constant, then the Cosmological Constant, and Dark energy would be the same from the start of inflation. Due to the Penrose Singularity theorems, the traditional model of inflation also involves [1] a singular point for initial expansion of the Universe. Our study clearly initiates a Cosmological constant which is invariant over time. We digress from using the Penrose Singularity theorem [1] in terms of a Cosmic bounce [2] style nonsingular initial bubble of space-time for reason we state in the manuscript. The importance we have to our work is that we are convinced that the Cosmological Constant [3] [4] was set initially for conditions in the interior of a

nucleated space-time bubble, with a wavefunction style calculation for collapsing infill into the space-time bubble with that wavefunction of Dark energy [5] [6] to be at a minimum 10 to the 30 power times Planck length. We conclude that Polarization states for obtaining early universe high Frequency Gravitational waves detected and analyzed may be more favorable for higher dimensional generalizations of our Dark Energy generating procedures, with Klauder Enhanced Quantization [7] [8] limited to 3 + 1 Geometry whereas our Vacuum energy [9] [10] calculation may be amendable to higher frequency calculation incantations. While considering [11]-[19] issues. In particular, [19] gives us future goals to consider. As an example, could multiverse CCC allow incorporating [19] with [13]?

2. Methods

We will first start off with the redone calculation as to the Vacuum energy as given in [1] and how we rescale them to be in sync as to the observed experimental value for vacuum energy which is of the present era. This methodology is consistent with the Zero-point energy calculation. We start off with the following as given by [11]

$$\frac{1}{2} \cdot \sum_i \omega_i \equiv V (\text{volume}) \cdot \int_0^{\hat{\lambda}} \sqrt{k^2 + m^2} \frac{k^2 dk}{4\pi^2} \approx \frac{\hat{\lambda}^4}{16\pi^2} \quad (1)$$

$$\xrightarrow{\hat{\lambda}=M_{\text{Planck}}} \rho_{\text{boson}} \approx 2 \times 10^{71} \text{ GeV}^4 \approx 10^{119} \cdot \left(\rho_{DE} = \frac{\Lambda}{8\pi G} \right)$$

In stating this we have to consider that $\rho_{DE} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}^4}$, so then that the

equation we have to consider is a wavelength $\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}$ which is about 10^{30} times a Planck length radius of a space-time bubble [13] as a nonsingular expansion point for Cosmology, at the start of inflation with the space-time bubble of about a Planck length radius in size. Having said that, how do we get having the Penrose multiverse condition in this problem, for $\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}$ just before the near singularity?

2.1. Looking at Modifications of Penrose CCC Cosmology

We now outline the generalization for Penrose CCC(Cosmology) before inflation which we state we are extending Penrose's suggestion of cyclic universes, black hole evaporation, and the embedding structure our universe is contained within, This multiverse has BHs and may resolve what appears to be an impossible dichotomy. The following is largely from [13] and has serious relevance to the final part of the conclusion. That there are N universes undergoing Penrose 'infinite expansion' (Penrose) [18] contained in a mega universe structure. Furthermore, each of the N universes has black hole evaporation, which is with Hawking radiation from decaying black holes. If each of the N universes is defined by a partition function, called $\{\Xi_i\}_{i=1}^{i=N}$, then there exists an information ensemble of mixed minimum information correlated about $10^7 - 10^8$ bits of information per partition function in the set $\left. \{\Xi_i\}_{i=1}^{i=N} \right|_{\text{before}}$, so minimum information is conserved between

a set of partition functions per universe [13]

$$\{\Xi_i\}_{i=1}^{i=N} \Big|_{\text{before}} \equiv \{\Xi_i\}_{i=1}^{i=N} \Big|_{\text{after}} \tag{2}$$

However, there is non-uniqueness of information put into individual partition function $\{\Xi_i\}_{i=1}^{i=N}$. Also Hawking radiation from black holes is collated via a strange attractor collection in the mega universe structure to form a new inflationary regime for each of the N universes represented.

We use what is known as CCC cosmology [13] [18], which can be thought of as the following. First. Have a big bang (initial expansion) for the universe which is represented by $\{\Xi_i\}_{i=1}^{i=N}$. Verification of this mega structure compression and expansion of information with stated non-uniqueness of information placed in each of the N universes favors ergodic mixing of initial values for each of N universes expanding from a singularity beginning. The n_f stated value, will be using (Ng, 2008) $S_{\text{entropy}} \sim n_f$ [20]. How to tie in this energy expression, as in Equation (12) will be to look at the formation of a nontrivial gravitational measure as a new big bang for each of the N universes as by $n(E_i)$ the density of states at energy E_i for partition function [13] [14].

$$\{\Xi_i\}_{i=1}^{i=N} \propto \left\{ \int_0^\infty dE_i \cdot n(E_i) \cdot e^{-E_i} \right\}_{i=1}^{i=N} \tag{3}$$

Each of E identified with Equation (3) above, are with the iteration for N universes [13], and [18] (Penrose, 2006) Then the following holds, by asserting the following claim to the universe, as a mixed state, with black holes playing a major part, *i.e.*

Claim 1

See the below [13] representation of mixing for assorted N partition functions per CCC cycle

$$\frac{1}{N} \cdot \sum_{j=1}^N \Xi_j \Big|_{j \text{ before nucleation regime}} \xrightarrow{\text{vacuum nucleation transfer}} \Xi_i \Big|_{i \text{ fixed after nucleation regime}} \tag{4}$$

For N number of universes, with each $\Xi_j \Big|_{j \text{ before nucleation regime}}$ for $j = 1$ to N being the partition function of each universe just before the blend into the RHS of Equation (4) above for our present universe. Also, each independent universe as given by $\Xi_j \Big|_{j \text{ before nucleation regime}}$ is constructed *by the absorption of one to ten million black holes taking in energy, i.e. (Penrose).*

Furthermore, the main point is done in [13] in terms of general ergodic mixing [21]-[23].

Claim 2

$$\Xi_j \Big|_{j \text{ before nucleation regime}} \approx \sum_{k=1}^{Max} \tilde{\Xi}_k \Big|_{\text{black holes } j\text{th universe}} \tag{5}$$

What is done in **Claims 1 and 2** [13] is to come up as to how a multi dimensional representation of black hole physics enables continual mixing of spacetime [13] [18] [22] [24] largely as a way to avoid the Anthropic principle [13] [25], as to a

preferred set of initial conditions

2.2. Looking at the Modification of the Penrose CCC (Cosmology)

We argue this modification is mandated by having the initial DE wavefunction set as

$$\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}} \quad (6)$$

After having set this in place we will be comparing this method to the Klauder argument.

2.3. Looking at Klauder Enhanced Quantization for Cosmological Constant

We use the Padmanabhan 1st integral [26] of the form, with the third entry of Equation (1) having a Ricci scalar defined via [27] [28] and usually the curvature \aleph [13] set as extremely small, with the general relativity [13] [28] action of

$$\begin{aligned} S_1 &= \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\aleph - 2\Lambda) \\ &\& -g = -\det g_{\mu\nu} \\ &\& \aleph = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right) \end{aligned} \quad (7)$$

2.4. Next for the Idea from Klauder

We are going to go to page 78 by Klauder [8] [13] of what he calls on page 78 a restricted Quantum action principle which he writes as: where we write a 1-1 equivalence as in [8] [13], which is

$$S_2 = \int_0^T dt \cdot [p(t)\dot{q}(t) - H_N(p(t), q(t))] \approx S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\aleph - 2\Lambda) \quad (8)$$

Our assumption is that Λ is a constant [13], hence we assume then use the approximation, from [8], and [13]

$$\begin{aligned} \frac{p_0^2}{2} &= \frac{p_0^2(N)}{2} + N; \text{ for } 0 < N \leq \infty \text{ and } q = q_0 \pm p_0 t \\ V_N(x) &= 0; \text{ for } 0 < x < 1 \\ V_N(x) &= N; \text{ otherwise} \end{aligned} \quad (9)$$

$$H_N(p(t), q(t)) = \frac{p_0^2}{2} + \frac{(\hbar \cdot \pi)^2}{2} + N; \text{ for } 0 < N \leq \infty$$

Our innovation is to then equate $q = q_0 \pm p_0 t \sim \phi$ and to assume small time step values. Then [13]

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2N + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \bar{t}^2} \right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} + \left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right) \right) \Bigg|_{t=\bar{t}} \quad (10)$$

If we assume that Equation (10) was formed within a cosmic bubble of space-time [12]

$$\left(\frac{\dot{a}}{a}\right)^2 = H_{\text{bounce-cos}}^2 = \frac{8\pi}{3M_p^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{11}$$

Here, we have that ρ is a space-time density function, whereas σ is the tension of a space-time bubble presumably of the order of a Planck radius. Also, within the bubble of space-time $\frac{\ddot{a}}{a} \approx \varepsilon^+ = 0$ and $a(t) = a_{\text{min}} t^\gamma$ at the surface of the space-time bubble. Hence

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2\tilde{N} + \frac{\gamma(3\gamma-1)}{8\pi G t_{\text{Planck}}}\right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_p^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{12}$$

c_1 is to be determined, whereas the inflaton [13] [26].

$$\phi = \sqrt{\frac{\gamma}{4\pi G}} \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\} \tag{13}$$

And what we will use the “inflaton potential” we write as [13] [26]

$$V = V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{14}$$

Also

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{15}$$

3. Results: Comparing Equation (1) Implications with Equation (12)

The significance of the procedure is that with tweaking we may be seeing the actual realization of classical gravity as an Eikonal Approximation to quantum theory [29] [30] as what was brought up by Horowitz and Oron in a highly nonstandard way as seen in reference [30] which gives a different interpretation to Equation (15) above. The term we refer to as Eikonal approximation to a quantum state has the form of the following decomposition. From Powell and Crasemann, [31] we have the decomposition of Geometric style decomposition of the optical wave equation of

$$\nabla^2 \psi - \frac{1}{v_{\text{Velocity}}^2} \cdot \frac{d^2 \psi}{dt^2} = 0 \tag{16}$$

Whereas we use the common $\lambda \cdot \omega = 2\pi v_{\text{Velocity}}$, and if we substitute in an angular frequency dependence of $e^{-i\omega t}$ in ψ , we get from (16) that if $\lambda = h/p$ and $p = \sqrt{2m \cdot (E - V(\text{potential}))}$, Then we have a transformation of

Equation (17) to Equation(18)

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2} \cdot \psi = 0 \quad (17)$$

Leads to a Schrodinger equation of the form given by

$$\nabla^2\psi + \frac{8\pi^2}{h^2} \cdot (E - V(\text{potential})) \cdot \psi = 0 \quad (18)$$

Equation (17) has [32]

$$\psi = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (a_{\ell m} j_{\ell}(kr) + b_{\ell m} y_{\ell}(kr)) \cdot Y_{\ell}^m(\theta, \phi) \quad (19)$$

Where we could get away with making a substitution of [31]

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} \cdot (E - V(\text{potential}))^{1/2} \quad (20)$$

Where the first and second Bessel Equation solutions [32] are of the form

$$j_{\ell}(kr = x) = (-x)^{\ell} \left(\frac{1}{x} \frac{d}{dx} \right)^{\ell} \frac{\sin x}{x} \quad (21)$$

$$y_{\ell}(kr = x) = -(-x)^{\ell} \left(\frac{1}{x} \frac{d}{dx} \right)^{\ell} \frac{\cos x}{x} \quad (22)$$

Then we have via making use of Kieffer's result of variance of the energy [33] is

$$\Delta E = \frac{\sqrt{2\tilde{n}+1}}{\lambda} \approx \frac{h\sqrt{2\tilde{n}+1} \cdot (E - V(\text{potential}))^{1/2}}{2\pi} \equiv \hbar\sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot t \quad (23)$$

We get via [33] on page 239 of this reference. (23). after we use (23), then

$$\Delta E \Delta t \approx \sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot t \cdot \Delta t \cdot \hbar \approx \hbar \quad (24)$$

If so, then we have that if so by use of [34] since this is also applicable to black holes.

$$\begin{aligned} t \propto t_{\text{Planck}} \propto \Delta t \Rightarrow \Delta E \Delta t \approx \sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot (\Delta t)^2 \cdot \hbar \propto \hbar \\ \Rightarrow (\Delta t)^2 \propto \frac{1}{\sqrt{2\tilde{n}+1}} \cdot \sqrt{\frac{8\pi G}{V_{\text{volume}}}} \end{aligned} \quad (25)$$

Using Planck units, what this means is that if $G = \hbar = \ell_{\text{Planck}} = k_B = 1 =$ Planck time

$$(\Delta t)^2 \propto \frac{1}{\sqrt{2\tilde{n}+1}} \cdot \sqrt{\frac{8\pi G}{V_{\text{volume}}}} \equiv \sqrt{\frac{8\pi}{2\tilde{n}+1}} \cdot (t_{\text{Planck}})^2 \quad (26)$$

If \tilde{n} = quantum number , then for Δt being proportional to $t_{\text{Planck}} \propto \Delta t$, we have definite restrictions on \tilde{n} = quantum number , i.e. of the type given by

$$\sqrt{\frac{8\pi}{2\tilde{n}+1}} \leq 1 \quad (27)$$

Whereas the Kieffer Wavefunction commensurate as to Equation (23) for a quantum Dust universe initially is given as [33] on page239 of this reference

$$\psi_{\tilde{n},\lambda}(t,r) \equiv \frac{1}{\sqrt{2\pi}} \cdot \frac{\tilde{n}! \cdot (2\lambda)^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}} \cdot \left[\frac{1}{(\lambda+i \cdot t+i \cdot r)^{\tilde{n}+1}} - \frac{1}{(\lambda+i \cdot t-i \cdot r)^{\tilde{n}+1}} \right] \quad (28)$$

Here the time t would be proportional to Planck time, and r would be proportional to Planck length, whereas we set

$$\lambda \approx \sqrt{\frac{8\pi G}{V_{\text{volume}} \hbar^2 t^2}} \xrightarrow{G=\hbar=\ell_{\text{Planck}}=k_B=1} \sqrt{\frac{8\pi}{t^2}} \equiv \frac{\sqrt{8\pi}}{t} \quad (29)$$

Then a preliminary emergent space-time wavefunction would take the form of

$$\psi_{\tilde{n},\lambda}(\Delta t,r) \equiv \frac{1}{\sqrt{2\pi}} \cdot \frac{\tilde{n}! \cdot (2 \cdot \sqrt{8\pi} \cdot (\Delta t)^{-1})^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}} \cdot \left[\frac{1}{(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t + i \cdot r)^{\tilde{n}+1}} - \frac{1}{(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t - i \cdot r)^{\tilde{n}+1}} \right] \quad (30)$$

Just at the surface of the bubble of space-time, with $t_{\text{Planck}} \propto \Delta t$, and $r \propto \ell_{\text{Planck}}$.

This is from a section, page 239 of the 3rd edition of Kieffer’s book, as to a quantum theory of collapsing dust shells, and is a way to use what is referred to as what Isham in 1984 referred to as “group quantization”, as we need to remember that we are assuming. Having said that, let us look at Equation (1) and Equation (12) again $G = \hbar = \ell_{\text{Planck}} = k_B = t_{\text{Planck}} = 1$.

3.1. We Compare Equation (1) and Equation (12) with the Above New Given Information

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2\tilde{N} + \frac{\gamma(3\gamma-1)}{8\pi G t_{\text{Planck}}} \right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_P^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|} \right) \quad (31)$$

$$\Leftrightarrow \rho_{DE} = \frac{\Lambda}{8\pi G} \Leftrightarrow \Lambda \approx \hbar \cdot 8\pi G \cdot \left. \frac{(2\pi)^4}{\lambda_{DE}^4} \right|_{\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}} \equiv 8\pi \cdot \left. \frac{(2\pi)^4}{\lambda_{DE}^4} \right|_{\lambda_{DE} \approx 10^{30}}$$

To compare these two values we can state that within the bubble, that just before the bubble boundary, we have $\tilde{N} = 0$, hence we look at, initially

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + \frac{\gamma(3\gamma-1)}{8\pi G t_{\text{Planck}}} \right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_P^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|} \right) \quad (32)$$

$$\Leftrightarrow \Lambda \approx \hbar \cdot 8\pi G \cdot \left. \frac{(2\pi)^4}{\lambda_{DE}^4} \right|_{\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}} \equiv 8\pi \cdot \left. \frac{(2\pi)^4}{\lambda_{DE}^4} \right|_{\lambda_{DE} \approx 10^{30}}$$

i.e. we pick c_1 so that the two are equivalent in value.

Note this is just before a very large value set to \tilde{N} at the boundary of the initial space-time bubble. Hence, we also would be looking at

$$\Lambda \approx \gamma \cdot t^2 + 8\pi V_0 \cdot \left\{ \sqrt{\frac{8\pi V_0}{\gamma(3\gamma-1)}} \cdot t \right\} \Bigg|_{t \approx \ell_{\text{Planck}}=1}^{\sqrt{\frac{\gamma}{4\pi}} \cdot \sqrt{\frac{8\pi}{\gamma}}}$$

$$\approx \frac{-\left[\frac{V_0}{3\gamma-1} + \frac{\gamma(3\gamma-1)}{8\pi} \right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_p^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|} \right) \tag{33}$$

$$\approx \hbar \cdot 8\pi G \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Bigg|_{\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}} = 8\pi \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Bigg|_{\lambda_{DE} \approx 10^{30}}$$

In Equation (31) we will also make the following identification

$$\rho \approx \rho_{DE} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Bigg|_{\lambda_{DE} \approx 10^{30}} .$$

I.e. just before the bubble, we will factor in a very large bubble tension, σ .

Doing so would be to have an optimal c_1 value, and that this choice of c_1 would allow us to make the identification of the choice of wavefunction for our model which would according to the Equation(28) model have a quantum flavor, while at the same time in adherence be similar to the Equation(31) values of Dark Energy density.

3.2. What We Obtain If We Model the Cosmological Constant This Way?

First of all, as noted by Kieffer, the wavefunction in Equation (28) vanishes if $r = 0$. We avoid having a vanishing wavefunction by asserting that we will be making an evaluation of it in the neighborhood of a Planck Length. Secondly in making this assumption we are at least proceeding in a direction where we can ditch the Anthropic principle, which I consider a MAJOR blight on cosmology. I also too though this this procedure as outline is experimentally verifiable, once we understand what may constitute embedding of Quantum Mechanics within a deterministic structure. The entire referral of this problem can be contrasted with the solution given in [19] by Rosen, which is in the mode of Quantum mechanics for a single particle in an isotropic universe.

The primary difference between the Rosen approach, [19] and what is done in this document is that the mass of a graviton, is proportional to the square of the Cosmological constant, an assumption which Rosen does not make. In addition the Rosen document assumes that “The situation is somewhat analogous to the quantum mechanics of a single particle as compared to quantum field theory” and assumes an isotropic background.

We do not confine ourselves to Isotropic backgrounds. We also do share

with Rosen [19] a procedure as to rendition a linkage between classical to quantum mechanical treatment of our problem, but our work is more in tune with using what Powell and Craseman in their book on quantum mechanics used, which is similar to a limiting case of geometric optics.

I wish to thank Christian Corda as to alerting me to the Rosen article which is also applicable to a Dust Cosmology. I also wish to thank Fangyu Li, Wen Hao, and others for alerting me as to the crucial role of polarization. As it is, I presented part of this idea in Zeldovich 4, in the ICRANET on line conferences (a small part of it) and found that the audience and the reviewers did not understand the Klauder treatment of the cosmological constant via Klauder Enhanced quantization.

Hence this expanded treatment whereas the Rosen treatment as Corda outlined is most effective for quantum black holes.

3.3. Final Reference to High Frequency Gravitational Waves

$$(1 + z_{\text{initial era}}) \equiv \frac{a_{\text{today}}}{a_{\text{initial era}}} \approx \left(\frac{\omega_{\text{Earth orbit}}}{\omega_{\text{initial era}}} \right)^{-1} \tag{34}$$

$$\Rightarrow (1 + z_{\text{initial era}}) \omega_{\text{Earth orbit}} \approx 10^{25} \omega_{\text{Earth orbit}} \approx \omega_{\text{initial era}}$$

Equation (34) can be seen in [35] as well as [36].

Whereas we postulate that we specify an initial era frequency via use of simple dimensional analysis which is slightly modified by Maggiore for the speed of a graviton [35] whereas

$$c(\text{light speed}) \approx \omega_{\text{initial era}} \cdot (\lambda_{\text{initial post bubble}} = \ell_{\text{Planck}}) \tag{35}$$

and that dimensional comparison with initially having a temperature built up so as

$$\Delta E \approx \hbar \omega_{\text{initial era}} \tag{36}$$

where $T_{\text{universe}} \approx T_{\text{Planck temperature}} = 1.22 \times 10^{19} \text{ GeV}$. If so then the Planck era we would have temperature would be extremely high leading to a change in temperature from the Pre Planckian conditions to Planck era leading to

$$\Delta E = \frac{d(\text{dim})}{2} \cdot k_B \cdot T_{\text{universe}} \tag{37}$$

In doing so, be assuming

$$\omega_{\text{initial era}} \approx \frac{c}{\ell_{\text{planck}}} \leq 1.8549 \times 10^{43} \text{ Hz} \tag{38}$$

Where we would be assuming $\omega_{\text{initial era}} \approx \frac{c}{\ell_{\text{planck}}} \leq 1.8549 \times 10^{43} \text{ Hz}$ so then we would be looking at frequencies on Earth from gravitons of mass $m(\text{graviton})$ less than of equal to

$$\omega_{\text{Earth orbit}} \leq 10^{-25} \omega_{\text{initial era}} \tag{39}$$

And this partly due to the transference of cosmological ‘information’ as given

in [13] for a phantom bounce type of construction .Further point that since we have that gravitons travel at nearly the speed of light [36], that gravitons are formed from the surface of a bubble of space-time up to the electroweak era that mass values of the order of 10^{-65} grams (rest mass of relic gravitons) would increase due to extremely high velocity would lead to enormous $\Delta E \approx \hbar\omega_{\text{initial era}}$ values per graviton, which would make the conflation of ultrahigh temperatures with gravitons traveling at nearly the speed of light as given in Equation(39) as compared with $\Delta E \approx \hbar\omega_{\text{initial era}}$. We can in future work compare this with the Rosen [19] value of energy for a mini universe of (from a Schrodinger equation) with ground state mass of $m = \sqrt{\pi}M_{\text{Planck}}$ and an energy of

$$E_{\tilde{n}} = \frac{-Gm^5}{2\pi^2\hbar^2\hat{n}^2} \quad (40)$$

Our preliminary supposition is that Equation (40) could represent the initial energy of a Pre Planckian Universe and that Equation(37) be the thermal energy dumped in due to the use of Cyclic Conformal cosmology (maybe in multiverse form) so that if there is a buildup of energy greater than Equation(40) due to thermal buildup of temperature due to in fall of matter-energy, we have a release of Gravitons in great number which would commence as a domain wall broke down about in the Planckian era with a temperature of the magnitude of Planck Energy for a volume of radius of the order of Planck Length. This will be investigated in detailed future calculations. All this should be in fidelity, in experimental limits to [37], as well as looking at ideas about Quantum tunneling we may gain from [38] as to understand the transition from Pre Planckian to Planckian physics [39] [40].

4. Another Treatment of Entanglement Entropy to Consider *i.e.* the Universe as a Giant Black Hole

This involves using Muller and Lousto Early universe entanglement entropy, and its implications. Solving the spatial length issue, provided a minimum time step is preserved in the cosmos. We look at [41]

Here we have from [41] *i.e.*

Then consider how Entropy is created by treating the Universe initially as a giant black hole.

$$S_{\text{Entropy}} = 0.3r_H^2/a^2 \quad \text{for a time dependent horizon radius } r_H \text{ in cosmology} \quad (41)$$

Equation (16) above was shown by the author to be fully equivalent to

$$S_{\text{Entropy}} = 0.3r_H^2/a^2 \sim \frac{0.3}{a^2} \exp \left[-t \cdot \sqrt{\frac{\Lambda}{3}} \right] \quad (42)$$

i.e.

$$\left[-t \sqrt{\frac{\Lambda}{3}} \right] \sim \ln_e \left(\frac{a^2}{0.3} \cdot S_{\text{entropy}} \right) \quad (43)$$

So, then one has

$$\Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{0.3} \cdot S_{\text{entropy}} \right) \right]^2 \tag{44}$$

No matter how small the length gets, S_{entropy} if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t , re scaled does not go to zero. *This preserves a minimum non zero Λ vacuum energy, and in doing so keep the non zero initial bits, for computational bits contributions to evolving space time behavior even if $R_{\text{initial}} \rightarrow 0$.*

Comment 1. I.e. the Klauder enhanced quantization will not cohere if the initial radii goes to zero. But it appears to be in fidelity as to what to expect if there is a Planck sized minimum length.

Comment 2. On the other hand, there is nothing explicitly quantum mechanical in Equation (44). I.e. a super massive black hole treatment of entropy as a starting point has no input parameters which are quantum mechanical.

Equation (44) has no need as to quantum processes. The Klauder enhanced quantization uses a comparison to two first integrals with one of the first integrals explicitly quantum mechanical.

5. Conclusions

We assume then due to Equation (44) having no need for explicit quantization to be incommensurate as to quantum gravity whereas the Klauder Quantization used to obtain a cosmological constant IS indeed invoking a procedure in tune with quantum gravity.

We further postulate that a further analysis may show overlap via the following bridge between these different pathways.

Our variant of eternal inflation would be using the approximation that we look at [42] where we have

$$V(\varphi) \approx \frac{k^2}{a^2} \cdot \varphi^2 \tag{45}$$

Use the approximation that the time derivative is $d/d\tau$, and $\varphi \equiv \varphi_k$, and if so, then if the scalar field $\varphi \equiv \varphi_k$ is equivalent to the beginning of our article, we can have the result we want if a prior universe creates an effective B field so that

$$\begin{aligned} \frac{\dot{\varphi}_k^2}{2} &= \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \rho \right) \\ &\& \\ \frac{\dot{\varphi}_k^2}{2} &= \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \end{aligned} \tag{46}$$

The last line of Equation (46) states that, if we apply it to the Pre Planckian to Planckian regime, that there will be a change in the energy, which we will call

$$\Delta E \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \tag{47}$$

We then will call this shift in energy, as equivalent to a change in KINETIC energy, and then reference the Virial theorem which in a general form, will be interpreted as

$$\langle \psi | \text{Kinetic Energy} | \psi \rangle = \langle \psi | r \cdot \nabla V (\text{Potential energy}) | \psi \rangle \quad (48)$$

Leading to

$$\begin{aligned} \langle \psi | \left[\text{Kinetic Energy} \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \right] | \psi \rangle \\ \approx \langle \psi | \left(r \cdot \nabla [V (\text{Potential energy}) \approx c_2 / r^\alpha] \right) | \psi \rangle \end{aligned} \quad (49)$$

The wave functions assumed are similar to what we use in our paper prior to this section.

In the Pre Planckian to Planckian space time, we will approximate, in the instant before time is initialized, formally, the mean value theorem as to the computed values of both the Left and right hand sides of Equation (48) and Equation (49) with the results that we obtain

$$\begin{aligned} \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \approx -\alpha / r^\alpha \equiv -\alpha / (\text{Planck length})^\alpha \\ \Leftrightarrow \left[\alpha / (\text{Planck length})^\alpha \right] \approx \frac{16}{3} \cdot c_1 \cdot B^4 - \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 \right) \end{aligned} \quad (50)$$

THIS VARIANT OF DARK ENERGY should be compared with the main scheme which has re acceleration of the Universe as its main constituent point. *I.e* what is equivalent to DE is given with Equation (49) in the Planckian regime.

This shows in part that α is no longer strictly real valued but is strongly influence by the input from φ_k , i.e. which has real and imaginary components.

Finding an effective B field, from a prior universe, and having the scalar field appropriately constrained, we postulate may allow an overlap between the Klauder approach and Equation (44) above.

Also if that happens

$$\omega_{\text{equation of state DE}} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \approx \frac{\dot{\phi}^2 - 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}} \frac{\phi}{m_{\text{Plank}}}\right)}{\dot{\phi}^2 + 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}} \frac{\phi}{m_{\text{Plank}}}\right)}$$

for acceleration of universe $\tilde{m} > 1$

reconcile above with

$$\begin{aligned} \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \approx -\alpha / r^\alpha \equiv -\alpha / (\text{Planck length})^\alpha \\ \Leftrightarrow \left[\alpha / (\text{Planck length})^\alpha \right] \approx \frac{16}{3} \cdot c_1 \cdot B^4 - \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 \right) \end{aligned} \quad (51)$$

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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