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The Zitterbewegung, Such as It Is, and Etiology of the Corpuscular Speed

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Abstract

In this paper, we study the Zitterbewegung, as the most anomalous and controversial effect of the Dirac equation. For this purpose, we do not establish a study framework, but we carry out a dissection of the Dirac solution for the velocity operator from a wave optics perspective, which will provide us (such as it is) with all the information about the Zitterbewegung, about the true sense and legality of the solution, and of the Dirac equation itself. In particular, we will obtain that said solution of the Dirac equation is composed of two wave packets coupled through the phase functions, and that it is the destructive and constructive confrontation of their group velocities that will give rise to the classical speed of the particle and the quantum anomalous term, respectively. This coupling is the kinetic aspect of a more general energetic development and of a corpuscular etiology, from which we derive a geometry that will then be an active part of any related phenomenology. A geometry will allow us to uncover some primary questions about physical principles, kinetic energy, spin and velocities, as well as differentiate between wave packets, photons and particles.

Keywords

Standard Model, Wave Packet, Zitterbewegung, Dirac Equation

1. Introduction

The Zitterbewegung (from the German, Bewegung, "movement" and zitter "tremulous, trembling") happens to be one of the most unknown, little studied or ignored effect or theoretical peculiarities, and lacking in certainties, of physics, since its appearance.

In a very preliminary way, we can say that the Zitterbewegung (zitter) is an ultrafast vibrational motion along the classical trajectory of a quantum particle, specifically of electrons and other 1/2 spin particles. More specifically, or originally, it is a velocity term appearing in the solution of the Dirac equation for the velocity operator of a free particle, which departs from the classical result by appearing as an extra term, and apparently anomalous, since it is developed over reference speed c, indicially violating relativistic principles, which is why its physical meaning is debated or even its existence or primordial character is questioned.

As a consequence of this anomaly, the zitter cannot be easily understood in the relativistic corpuscular environment of Dirac's equation itself, for which, ultimately (as a compromise solution), this is due to an energy fluctuation in the Dirac sea, associated with the creation and annihilation of pairs, and not to a real oscillation. While other theoretical models discard this interpretation and/or find another alternative. Either because, regardless of its spatial conception, it is not the mass but the charge of the electron that participates in the rotational movement (overcoming the relativistic impediment). Either because, starting from a point or non-dimensional conception of the electron, they consider the phenomenon to be non-existent or amortized within the framework of their own theories (QFT/QED), in which the zero-point energy (zpe) of the quantum vacuum is studied alternatively, which, being associated with the same energy fluctuation, does not require this corpuscular participation.

In conclusion, we have a series of models, or rather positions, which represent a range of possibilities for each of the questions that arise, so we do not know whether when we speak of zitter and zpe we are talking about the same thing or whether one gives rise to the other, whether we are talking about a fluctuation or a circular motion, whether it is associated with the particle or with the quantum vacuum, whether it is the result of the internal structure of the particle or whether, on the contrary, this structure (if we contemplate it) arises from the quantum field itself. This gives rise, consequently, to a series of disquisitions concerning the zitter, which bring us face to face with the dichotomy, cause or effect, of the origin of the internal structure of the particles, or models of the electron (electron models), in addition to the more radical one regarding the inherent nature of its existence. Disquisitions of which we can have a sufficient idea through an updated review [1], in which, in addition to gathering all the perspectives to date on the zitter phenomenon that provide data on this structure, it gives us an indication in the same title, "The zitterbewegung electron puzzle", of the collage or unfinished state of this configuration.

It is not that we are unable to put the puzzle together, it is that there are missing pieces, important pieces that provide the keys to the itinerary to be followed, with the result that everything that has been said is only part of a picture that lacks support. It is for this reason, and because we realize that there is no truly predominant or reference model, that we, taking another route, are not going to drag the different proposals into our development, nor counter-argue them more than necessary. That is to say, the different models promote geometric configurations that are expected to achieve legitimacy (which is not achieved) through the results obtained with regard to certain known variables or the explanation of some physical quantities, without paying sufficient attention to the solution of Dirac's equation itself, that is, without addressing the zitter from the only perspective that makes its existence feasible (which proves the reality of this phenomenon as part of a phenomenology).

This unique perspective is nothing more than the necessary coexistence and intervention of the two aspects of a dual nature and the presence in the process, therefore, of an underlying and at the same time separate wave nature, which, as such, makes it possible to separate or decouple velocity into two physical species, that is, to externalize the fluctuations or dissociate them from the mass, regardless of whether they come from its internal structure or are simply supported by it, which is precisely what the aforementioned solution expresses.

The dual nature of the velocity terms is so evident through the present phase function $\exp(-2iHt/\hbar)$ that it can only be understood that the interpretation of the zitter has not opted for this wavelike direction due to scientific pragmatism and the inexistence of a model that conveniently represents this dual nature or, in other words, the internal structure of particles (whether material or not). Once we have this, as we will develop, the argumentative and causal connection is natural and almost immediate and congruent from other substantial points of view, which go beyond the phenomenon and speak of the material formative aspects and of its etiology, which is, in the last analysis, what matters. Indeed, a puzzle cannot be put together if the main piece of the puzzle is missing, around which everything revolves, and everything makes sense. More specifically, if we do not know what the relationship $\omega = H/\hbar$ means, because we do not know how to interpret it as an argument of a phase or recognize that phase (because scientific pragmatism has dissociated it from physical reality) and, as a result, we strive to find that relationship (which is at hand) through a ratio between geometric radii, we will find it difficult to know what zitter is.

Succinctly expressed, we will present the zitter phenomenon and then show that the resulting equation obeys a process involving wave packets (WPs). More in detail, we will show that the zitter phenomenon is a consequence of the combination of two WPs, as a consequence, in turn, that the particle describing the Dirac equation through the Hamiltonian, and matter itself, is composed, in effect, of these two WPs, which, in a particular and necessary way, corresponds to a symmetrized wave packet (SWP), or symmetric composition of these two WPs, and the whole energy balance of the symmetrization process carried out [2], which we will address here in a sufficient way.

We will develop this from several perspectives that will give us a more and more complete view of this reality. First, we will analyze the solution for the velocity of the Dirac equation, *i.e.*, the term itself that describes the zitter, to show that the velocity, which it refers to, has the form of a WP. Second, we will show that the zitter actually corresponds to two WPs with a sort of coupling or combination. Third, we will develop decoupling alternatives, which will allow us to achieve the form that decoupled WPs must have, regardless of whether we can obtain them effectively. Fourth, we will investigate whether there is in optical physics a WP whose shape corresponds to the previous ones. Fifth, we will develop the final solution, *i.e.*, what these WPs are like and how they combine. The development of the last point will imply realizing that these WPs will be the solutions of the problem thanks to a geometry that puts them in communication, that is to say, that serves as support for their necessary combination. A geometry that in this case is toroidal and that, obviously, the WPs themselves have to generate in the symmetrization process [3], hence the importance of this framework in this process.

We see that in this development we are not taking the same path as other theoretical models regarding the topology of the system, in that we are not establishing an ad hoc one, but rather we already have one in place that satisfies structural or material configuration requirements, and which also derives from the envelopes of two WPs (from the physics of the system). Consequently, starting from the SWP and its geometric shape, we will only have to apply the development achieved for the zitter and check its response, *i.e.*, check whether this toroidal geometry (regardless of their size, because that's not what matters about it) is capable of coupling the individual WPs in a form from which the zitter is derived. A toroidal geometry that would be the one that would allow there to be, associated with the zitter, an angular velocity and, consequently, a spin angular momentum, while other physical parameters such as mass, size, or particle charge are already defined in SWP's own starting function for each particle family, without prejudice to the possibility of a more instrumental study of certain aspects, such as that carried out [4] for, precisely, a photon confined in a toroid, which can give us a complementary idea of this scheme SWP from a QED perspective.

The SWP constitutes the physical environment of the zitter, and the zitter provides a new proof of the SWP as the foundation of the energetic development of matter, that is, one more evidence of the wave-corpuscle duality at a structural level. We will not pretend here to make a foundation of the SWP, but we will describe it in essence to be able to verify sufficiently the analogy or connection of both processes, that is, the one that gives rise to the particles, utilizing an energetic treatment, and the one that gives rise to the fluctuating velocity zitter in the said process of corpuscular generation, as well as to the classical one, since the classical speed, by means of this binary scheme, arises (and that is its origin) as a difference of wave velocities. This description implies delving into the etiological foundations of particle formation through the combination of two WPs as the only understandable way to preserve the conservation principles, and, in particular, the momentum conservation.

With all this, given the formal connection established between the Dirac spinor and the SWP, which for this reason we alternatively call the intrinsic spinor [5], and given the functional connection between both types of spinors developed here, we can say that the foundation we will carry out of the zitter not only validates the Dirac equation, by validating that initially anomalous term, but corrects it through the specified spinors, which, unlike the Dirac spinors, conserve the functionality of the carrier of the wave function. With this we are saying that the solutions of the Dirac equation are not and cannot be the individual wave functions taken as mere mathematical supports but the real and double wave functions that describe the WP in its two parts, the envelope and the carrier. From this consideration, that is, from the consideration of the carrier, arises the interpretation of the zitter and the interpretation of a myriad of questions in which it participates, such as the spin, as a consequence or part of the process of corpuscular generation, in which the carriers are, in effect, the phase functions resulting from the conformation of both WPs of the WPS.

2. Zitterbewegung

When we try to obtain the particle velocity from the Dirac equation [6]:

$$H_D = c\alpha \cdot p + \beta mc^2, \tag{1}$$

applying Ehrenfest's theorem, we obtain:

$$\dot{x}_k = (i/\hbar)[H, x_i] = c\alpha_k, \tag{2}$$

which indicates that α_k is the velocity in units of c, which can also be interpreted as the k-th component of the velocity operator, of which we can know its derivative:

$$\begin{aligned}\dot{\alpha}_k &= (i/\hbar)[H, \alpha_i] = (i/\hbar)(-2\alpha_k H + \{H, \alpha_k\}) \\ &= (i/\hbar)(-2\alpha_k H + 2cp_k).\end{aligned} \tag{3}$$

It can be seen that $\dot{\alpha}_k \neq 0$ and that, consequently, $\dot{x}_k = c\alpha_k$ is not a constant of motion, which allows us to consider Equation (3) as a differential equation for $\alpha_k(t)$, with the immediate solution:

$$\alpha_k = cp_k H^{-1} + \left(\alpha_k(0) - cp_k H^{-1}\right) e^{\frac{-2iHt}{\hbar}}, \tag{4}$$

which applied to Equation (2), results:

$$\dot{x}_k = c\alpha_k = c^2 p_k H^{-1} + c\left(\alpha_k(0) - cp_k H^{-1}\right) e^{\frac{-2iHt}{\hbar}} = \upsilon + \upsilon_\omega e^{-2i\omega_0 t}, \tag{5}$$

where we see that there appears a first term υ that accounts for the classical speed of the particle plus another unexpected term that shows through the exponential a fluctuating velocity (that may even be c), superposed to the expected value, which gives rise to an ultrafast vibrational motion, which is precisely the zitter. A fluctuation that we have initially associated to the most elementary frequency, corresponding to the fundamental state of the particle, $H = E_0 = \hbar\omega_0 = m_0 c^2$, on which we can define for Equation (5) the Dirac frequency $\omega_D \equiv 2\omega_0 = 2m_0 c^2/\hbar$, which we can generalize to any non-fundamental state ω_i.

Not understanding what the zitter consists of lies in not knowing what this exponential obeys, but also, and very principally, in not knowing what the factor υ_ω in Equation (5) is, since what we can say about the exponential we can say through υ_ω and, very principally, of its nature, which can in no way be corpuscular, since, in case of having such a corpuscular nature, affine to the particle, it would imply that the particle performs a motion at high frequency and reaches the speed $\pm c$ [7], according to its eigenvalues ($\dot{x}_k/c = \pm 1$), violating the relativistic principles.

This is what was historically noticed in the first analysis and is what has been tried to be overcome through the different models, which, however, have always had to suppress a part of reality or of their pretensions. Thus, if the zitter is considered as a fluctuation between energy states of the Dirac sea, or an interference between positive and negative states, it is able to explain the spin characteristic of the particle, as Schrödinger did [8], but only as a quantum mechanical element, without a classical correlation. And if this correlation is sought through the model, giving the motion of the electron a real physical character through an undulatory reformulation [9], one can only say, according to this relativistic conditioning, that the particle has no mass, or that it does not have mass at the level at which these fluctuations occur and that the mass is therefore present in a frame different from that of the fluctuations. Which may be fine if both frames are connected, being otherwise an insufficient wave formulation and a failed model. Connecting the two frames is what a dual conception of particles can and does do, that is, an underlying wave structure of these particles, the feasibility of which we have already discussed. To dispense with this ambivalent nature is to dispense with all argumentative and embedding possibilities.

In our case, we will not only fit both requirements, but they will fit themselves as a consequence of a thorough explanation of the behavior of the Dirac equation, which is and has always been, by the way, the final pretension of any explanation of zitter (and of other initiatives on the margin), that is, to show if in fact the equation is consistent, and why. That is to say, here we are not dealing with a proposal or a model but with a dissection of the solution offered by the Dirac equation, represented in Equation (5), that will be supported circumstantially and then essentially in a geometry (SWP), which gives foundation to the equation from its own raison of being, and not in one, as we said, that we have to conceive for the purpose of zitter.

In fact, part of the result of that dissection, part of this dissection, concerning the nature of the exponential function present (as the first anomalous element), would already be in sight, that is, it would be clarified prior to the study if we take into account that the solution of the Dirac equation is not based, as we have already mentioned, on the Dirac spinors but on the intrinsic spinors that we formulated [5], which already contain this exponential function with a clear meaning, and for which the former are only a simplified form and incapable of explaining the phenomenology. This is an enlightening perspective that we will leave aside for now in order to approach the analysis from a more basic position, one that is as concrete and neutral (free of additional theoretical elements) as the breakdown and study of the parts of an equation can be.

2.1. Undulatory Nature of the Zitter

In our case, and as we have already mentioned, the relativistic conditioning leads us to attribute to the exponential (as the only kind of reformulation or starting point) the same wavelike character as to υ_ω, that is, to think that the fluctuating velocity they express is initially wavelike and not related to the motion or oscillation of a particle nor to the interference between energetic states, without prejudice that it may finally have a repercussion of an oscillatory nature in that particle.

What we are saying is that the second term of Equation (5) is genuinely undulatory and that it is ascribed to the undulatory nature of the particle itself, brought into play during its formation, by which the zitter ceases to be a strange phenomenon to be something consubstantial to the material states, being for this reason, as we shall see, that these material states are affected.

To confirm this undulatory nature, we are going to compare the group velocity of a WP, as simple and generic as the one formed by two almost identical individual waves, with the speed of a particle, to which we can associate this same undulatory nature through the group velocity of de Blogie matter-wave.

We can concretize the above for the first case [10] if, taking into account the expression υ_p for the phase velocity,

$$\upsilon_p = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda \quad \Rightarrow \quad \frac{\mathrm{d}\nu}{\mathrm{d}\lambda} = \frac{\upsilon_p}{\lambda^2} + \frac{\mathrm{d}\upsilon_p}{\lambda \mathrm{d}\lambda}, \tag{6}$$

we calculate the group velocity of the WP,

$$\upsilon_g^c \equiv \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\mathrm{d}\nu}{\mathrm{d}(1/\lambda)} = -\lambda^2 \frac{\mathrm{d}\nu}{\mathrm{d}\lambda} = \upsilon_p - \lambda\left(\mathrm{d}\upsilon_p/\mathrm{d}\lambda\right), \tag{7}$$

where the relation between υ_g^c and υ_p clearly becomes evident, so that the former represents a variation (decrease) of the latter, on the reference $\upsilon_p = c$, as the initial state, which we have reflected as a superscript in υ_g^c.

And if we apply, on the other hand, for the second case, de Blogie-Einstein relations $p = h/\lambda$ and $E = h\nu$ [11]:

$$\upsilon_g^0 \equiv \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\mathrm{d}\nu}{\mathrm{d}(1/\lambda)} = \frac{\mathrm{d}E}{\mathrm{d}p} = \frac{m\upsilon\mathrm{d}\upsilon}{m\mathrm{d}\upsilon} = \upsilon. \tag{8}$$

it follows that the group velocity $\upsilon_g^0 = \upsilon$ (corresponding with the particle speed) is for this case a velocity differential (increment) over the reference $\upsilon = 0$, as initial state, which in the same way we have reflected as superscript in υ_g^0.

Note that in this proposition we are not identifying one species with the other through a wave-corpuscle equivalence but presenting what we know about the group velocity in both physical species. Note also that to carry out this wave-corpuscle identification would imply taking the form of the wave group velocity to the corpuscular one, that is, from one whose reference is $\upsilon = c$ to another whose reference is $\upsilon = 0$, which could have many consequences concerning this energetic transmutation and the principles of conservation, and consequences concerning the physical systematics that nature would use to make it possible (that's where we are), which would entail having to manage this apparent collapse of the velocity and camouflage or bring to the background the undulatory nature, of which, in this case, zitter could be the proof.

Having carried out this comparison, and differentiated the wave and corpuscular forms of the group velocities, one that is effectively of a WP and the other that is of a particle, we can observe that, given $\upsilon_p = c\alpha_k(0) = \pm c$ for the eigenvalues $\alpha_k(0) = \pm 1$, there is a correspondence between υ_g^c of Equation (7) and υ_ω of Equation (5). That is:

$$\upsilon_g^c = \upsilon_p - \lambda\left(\mathrm{d}\upsilon_p/\mathrm{d}\lambda\right) = \left(c\alpha_k(0) - c^2 p_k H^{-1}\right) = -\left(c\alpha_{-k}(0) + c^2 p_k H^{-1}\right) = \upsilon_\omega, \quad (9)$$

where we have considered and differentiated by means of $\pm k$ the two possible values $\upsilon_p = c\alpha_{\pm k}(0) = \pm c$ (in phase opposition) of the second equation and its relation with the classical speed of the particle so that, regardless of the sign, it is always $\upsilon_\omega \le c$.

That is to say, ignoring this duplicity or symmetry of states, and taking $+c$ as the default, we can observe that in both cases we are expressing, according to the above correspondence, a velocity that is the result of a decrease over the phase maximum velocity of a wave $\upsilon_p = c$ or, what is the same, in both cases we are talking about the group velocity of a WP. A decrease that we can write in a unique alternative way for both expressions:

$$\lambda\left(\mathrm{d}\upsilon_p/\mathrm{d}\lambda\right) = c^2 p_k H^{-1} = \upsilon_g^0, \quad (10)$$

which allows us to write Equation (5) in the form:

$$\dot{x}_k = c\alpha_k = \upsilon + \upsilon_\omega \mathrm{e}^{-2i\omega_0 t} = \upsilon_g^0 + \left(c - \upsilon_g^0\right)\mathrm{e}^{-2i\omega_0 t}. \quad (11)$$

in which, for $t = 0$, it becomes clear why $\dot{x}_k = c = cte$, which logically holds for $t \ne 0$ in an averaged form, although $\dot{x}_k$ is not constant of the motion. That is, it becomes clear how in the process described by H_D, which may involve the constitution or even the creation of a particle, there is a partition of the speed c with two different characteristics, that which is or will be corpuscular and that which will be (or remain) undulatory.

Equation (11) tells us that the factor υ_g^0 is the same in both terms and that the magnitude increased in one over zero within the interval $[0, c]$, according to Equation (5), is the same as that decremented in the other over c in said interval, already outlined in Equation (7), from which we reach that υ_g^0 expresses the same phase shift over both systems (corpuscular and undulatory) and that, consequently, υ_g^0 is the phase shift or interval of velocities complementary to υ_g^0 with respect to c. This was already stated in Equation (5), but now we have given it physical meaning through the variables introduced.

What we are saying, consequently, is that H_D is expressing through Equation (5) both the velocity υ of the particle and a velocity υ_ω that could be associated with the modulation of a WP, which would evidence not only the wave character of the term but also the wave nature, and therefore dual, of H_D. A velocity that also has a related wave factor associated with it (which we will discuss later), that is what differentiates the two species, and which is what makes υ_ω not become a detectable linear displacement (like υ) but a periodic one the amplitude λ. A question that we will see more clearly if we integrate in time the second term of Equation (5), from which it results:

$$x_k^\omega \equiv \frac{i\hbar c}{2H}\left(\alpha_k(0) - cp_k H^{-1}\right)\left[e^{\frac{-2iHt}{\hbar}} - 1\right] = i\lambda\left(\alpha_k(0) - cp_k H^{-1}\right)\left[e^{\frac{-2iHt}{\hbar}} - 1\right]. \quad (12)$$

We can easily see that Equation (5) is not only the partition of velocities expressed in Equation (11), but implies through its two terms the separation of two distinct species with their respective group velocities, which we can even put in a more homogeneous form through the already established identities $\upsilon_g^0 = \upsilon$ and $\upsilon_g^c = \left(c - \upsilon_g^c\right) = \upsilon_\omega$,

$$\dot{x}_k = \upsilon_g^0 + \upsilon_g^c e^{-2i\omega_0 t} \equiv \upsilon_D, \quad (13)$$

and more significant, since each one of them indicates in the superscript its reference, on which we can reiterate the complementary nature or dependence of its terms, as made explicit for $t = 0$, respect to c.

$$\dot{x}_k(0) = \upsilon_g^0 + \upsilon_g^c = c. \quad (14)$$

Once we know the meaning of $\upsilon_g^c = \upsilon_\omega$ (coinciding with a group velocity of a WP), and we have reached the previous equation, we are in a position to ask ourselves what the second term of Equation (13) means, and, as a consequence of this, to ask ourselves if it can have a real representation, since it is reached through H_D.

Regarding the first question, what Equation (13) is saying is that, in addition to finding a velocity υ_g^0 that we associate with the particle, there is another superposed group velocity υ_g^c, associated with a WP and modulated by a function that has the form of a phase function, which could be its own phase function. Since the group velocity is associated with the wave equation of the WP envelope, what establishes the Equation (13) so far, consequently, is the relationship between a particle and the envelope of a WP, of which curiously we do not appreciate a main, explicit, genuine and differentiated velocity, energy, or presence, or other properties, but only (as proof of its existence) a perturbed (and, consequently, secondary) speed, through a phase function. This suggests that the main part has been neutralized in some way, such as through a process of absorption or conversion between entities (materialization). This does not seem strange if we consider the SWP. A materialization process by which the particle, according to the equation, would escape from almost all the wave influx, but not all, being forced to vibrate to the rhythm of the guiding wave (which, as we shall see, is the spin) to which the particle (the envelope) is still attached.

Regarding the second question, that is, the reliability of H_D, it is evident that the certainty of an equation is given by experimental proof but, on the contrary, it is often the equations that discover a reality that is not observable or that does not have a description by other poorer equations, or other more sophisticated ones, but incapable nevertheless of giving an account of the phenomenology. It may be that H_D has this potential, similar to the spontaneous manifestation of the Coriolis force when we deal with the system using analytical mechanics, with respect to the result achieved with Newton's laws, that is, with respect to their inability to present hidden relations. In this sense, it is different to deal with E than to deal with H, on the other hand, it is different to place the reference in $\dot{x}_k = 0$ than to do it in $\dot{x}_k = c$. In the first case, we only have to pay attention to what happens with $\dot{x}_k = \upsilon$, in the second case we have to do it with the interval $\dot{x}_k = \upsilon$ as well.

2.2. Coupled Wave Velocity of Material Particles

In Equation (13) we have seen that associated with the particle velocity we have a secondary term consisting of a phase function that accompanies a factor that resembles the group velocity of a WP. This puts us before the idea that, in effect, the particle is partially or essentially a WP, which would also imply, as a more general expression of the above, that it has an internal construction, responsible for much of the quantum phenomenology that is currently forcibly explained as a consequence of this theoretical deficiency.

In the case of a WP (with WP group velocity), the normal thing is that this phase function corresponds to the phase function of the same, that is to say with the unmodulated part, which is the only one that can accounts for an oscillation in the form seen, which responds to a more general form of the type:

$$\mathrm{e}^{\frac{-iHt}{\hbar}} = \mathrm{e}^{i(\omega_0 t - k_0 x)} = \mathrm{e}^{-i\omega_0 t} \quad (x = 0), \tag{15}$$

which serves us to emphasize that in the oscillation of the zitter, in front of that general type, appears a frequency that doubles the expected frequency, that is, the one that we can associate or is energetically equivalent to the fundamental particle that we are dealing with.

The most recurrent explanation to this duplication in the environment of the Dirac equation and its sea of (anti)particles has been to establish a correspondence between this oscillation and the energy $E = 2m_0c^2$, that is, the one that would exist in the interval $\left[m_0c^2, -m_0c^2\right]$, in correspondence with the energy differential existing between a particle and its antiparticle, proposing that it is the interference of these positive and negative states that gives rise to the zitter phenomenon, since the phenomenon disappears in WPs composed of waves with only positive or negative energies, which also implies that it does not apply to a single-particle system [12].

Given that Dirac's equation makes no mention of the number of particles and is valid for a single particle, the promotion of these claims is a consequence of: (1) not knowing what the phase $-i\omega_D = -i2m_0c^2/\hbar$ responds to, (2) resorting to the only thing that resembles it quantitatively, that is, the energy differential between two masses, which, on the other hand, resides in the factor that accompanies the phase, that is, in the amplitude or envelope, even though for the rest of the (classical) treatment only one appears, (3) and indeed present two components for the zitter (because matter itself has them, as we will see as we progress in our study).

In conclusion, the duplication of the phase highlights that we have a coupling of two things that need to be identified and decoupled, for which we will make the following preliminary considerations:

If we stick to H_D, we see that in its solution there is a duplication of terms (derived from our mathematical handling, obviously), which then, except for the exponential, disappears throughout the development. That is to say, the own (intermediate) solution given in Equation (3) tells us that two separable energy elements intervene in the process (regardless of whether they are ultimately presented, in accordance with the physical reality they represent, as a whole):

$$\begin{aligned}\dot{\alpha}_k &= (i/\hbar)(-2\alpha_k H + 2cp_k) = (i/\hbar)\left[(-\alpha_k H + cp_k)_a + (-\alpha_k H + cp_k)_b\right] \\ &= \dot{\alpha}_{ka} + \dot{\alpha}_{kb},\end{aligned} \tag{16}$$

of those we would achieve the result:

$$\begin{aligned}\dot{x}_k \neq \dot{x}_{k1} + \dot{x}_{k2} &= \left[c^2 p_k H^{-1} + c\left(\alpha_k(0) - cp_k H^{-1}\right)\mathrm{e}^{\frac{-iHt}{\hbar}}\right]_a \\ &\quad + \left[c^2 p_k H^{-1} + c\left(\alpha_k(0) - cp_k H^{-1}\right)\mathrm{e}^{\frac{-iHt}{\hbar}}\right]_b \\ &= \left(\upsilon_{ka} + \upsilon_\omega \mathrm{e}^{-i\omega_0 t}\right) + \left(\upsilon_{kb} + \upsilon_\omega \mathrm{e}^{-i\omega_0 t}\right) = f(a) + f(b),\end{aligned} \tag{17}$$

which is clearly not legitimate or consistent with physical reality, since the operation associated with the differential equation is not linear:

$$\dot{x}_k = c\alpha_k = \left(\upsilon + \upsilon_\omega \mathrm{e}^{-2i\omega_0 t}\right) = f(a+b) \neq \dot{x}_{ka} + \dot{x}_{kb} = f(a) + f(b). \tag{18}$$

Despite that, any two velocities in the final system, that is, two solutions of H_D, have that linearity, as evidenced by making use of Equation (11):

$$\begin{aligned}\dot{x} = c\alpha_k &= \upsilon_{g1}^0 + \left(c - \upsilon_{g1}^0\right)\mathrm{e}^{-2i\omega_0 t} \pm \upsilon_{g2}^0 + \left(c - \upsilon_{g2}^0\right)\mathrm{e}^{-2i\omega_0 t} \\ &= \left[\upsilon_1 + (c - \upsilon_1)\mathrm{e}^{-2i\omega_0 t}\right] \pm \left[\upsilon_2 + (c - \upsilon_2)\mathrm{e}^{-2i\omega_0 t}\right] \\ &= (\upsilon_1 \pm \upsilon_2) + \left[(c - \upsilon_1) \pm (c - \upsilon_2)\right]\mathrm{e}^{-2i\omega_0 t} \\ &= (\upsilon_1 \pm \upsilon_2) + \left[c - (\upsilon_1 \pm \upsilon_2)\right]\mathrm{e}^{-2i\omega_0 t} \\ &= \upsilon_g^0 + \left(c - \upsilon_g^0\right)\mathrm{e}^{-2i\omega_0 t},\end{aligned} \tag{19}$$

It becomes clear that the sum of the classical term (disregarding the relativistic treatment) is as expected, and that the decrements υ_1 and υ_2 on each c, corresponding to υ_{g1}^c and υ_{g2}^c, refer jointly in υ_g^c to a single c, since it is universal and unique. It also shows that the negative relationship between $\upsilon_{g1}^c = (c - \upsilon_1)$ and $\upsilon_{g2}^c = (c - \upsilon_2)$ represents a state of phase opposition between both phase functions. A sign relation that the velocities $(\upsilon_1 - \upsilon_2)$ assume when we bring those phases facing each other to a single one by using Equation (9), as we shall see explicitly later.

That the function f is not linear means that the terms of Equation (17) are not exactly like that or that those terms can continue their evolution from there in a non-independent way, or that it reaches separability in another way, which, far from disregarding them, leads us to reconsider on that basis how that separation can be reached or in what it consists, since it is desirable and, as we have seen in Equation (16), pre-existing or connatural.

The previous unsuccessful development makes it clear that H_D treats individual particles, not half particles, but we know ($\left[H,\alpha\right]\neq 0$) that it does not deal with WPs that do not contain any particles either, which will force us to look for that wave character through a dissection of the solution that does not spoil the corpuscular component of the solution, that is, that of the first term of Equation (5).

Accordingly, although it is obvious that there is no correspondence (linearity) between $\dot{x}_k$ and $\dot{x}_{ki}$ in Equation (17), it is also obvious that there is such a correspondence concerning the first term, since we can express every velocity in the separable form $\upsilon=\upsilon_1-\upsilon_2$, that is, by a difference of two velocities υ_{gi}^{0}, with physical meaning (premise). A difference that is susceptible to being considered as the result of a difference of two group velocity υ_{gi}^{c}, which implies the existence and definition of these two group speeds. While the second term, carrying out an inverse process to that carried out in Equation (19) which also contemplates the universality and uniqueness of $c\alpha_k\left(0\right)$, is equally separated into two velocities υ_{gi}^{c}. That is:

$$\begin{aligned}\dot{x}_k &= c\alpha_k=\upsilon+\upsilon_\omega \mathrm{e}^{-2i\omega_0 t}=\left(\upsilon_1-\upsilon_2\right)+\left[c\alpha_k\left(0\right)-\left(\upsilon_1-\upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t}\\ &=-\left(\upsilon_2-\upsilon_1\right)+\left[c\alpha_k\left(0\right)-c\alpha_k\left(0\right)\right]+\left[c\alpha_k\left(0\right)-\left(\upsilon_1-\upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t}\\ &=\left[c\alpha_k\left(0\right)-\left(\upsilon_2-\upsilon_1\right)-c\alpha_k\left(0\right)\right]+\left[\left(c\alpha_k\left(0\right)-\upsilon_1\right)+\left(c\alpha_{-k}\left(0\right)+\upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t}\\ &=\left[\left(c\alpha_k\left(0\right)-\upsilon_2\right)-\left(c\alpha_k\left(0\right)-\upsilon_1\right)\right]+\left[\left(c\alpha_k\left(0\right)-\upsilon_1\right)-\left(c\alpha_k\left(0\right)-\upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t}\\ &=\left(\upsilon_{\omega 2}-\upsilon_{\omega 1}\right)+\left[\upsilon_{\omega 1}-\upsilon_{\omega 2}\right]\mathrm{e}^{-i\omega_0 t}\times\mathrm{e}^{-i\omega_0 t}\\ &=\left(\upsilon_{\omega 2}-\upsilon_{\omega 1}\right)-\left[\upsilon_{\omega 2}-\upsilon_{\omega 1}\right]\mathrm{e}^{-i\omega_0 t}\times\mathrm{e}^{-i\omega_0 t},\end{aligned} \tag{20}$$

where we have not only finally separated the carriers, but we have done so in a dimensional way, that is, for the most general case, and where it is shown that both $\upsilon_1>0$ and $\upsilon_1-\upsilon_2>0$ operate on phase c, but that $\left(-\upsilon_2\right)<0$ operates on phase $-c$ initially, *i.e.*, $\left(c\alpha_{-k}\left(0\right)+\upsilon_2\right)$, being then extrapolated to c so that all elements remain on a phase in the same way, as we did in Equation (19) more directly.

With this, we see that both the first term and the factor of the second one transit to the coupled form of (two) WPs group velocities in a natural way by a simple composition and disintegration of velocities. Going in one case from the reference $\upsilon=0$ (characteristic of the particles) to the reference $\upsilon=c$ (undoing the supposed coupling of the phases in the corpuscular term), and doing in the other a simple disintegration on the reference $\upsilon=c$ (characteristic of the WP), being the exponential the one that breaks the linearity of that factor, that prevents it from canceling its phases, that maintains the coupling, that makes both terms follow different developments, despite being identical, $\left(\upsilon_{\omega 2}-\upsilon_{\omega 1}\right)=\left[\upsilon_{\omega 2}-\upsilon_{\omega 1}\right]$).

A different development of the terms, that is, a different and apparently arbitrary way of relating to speed (essential for reaching the two initial terms), which will necessarily have to be motivated by something of a primordial, prior, and common nature that sets in motion, differentiates, and allows for the coexistence of the two types of relationships, and the coexistence of the two associated natures (classical/quantum versus corpuscular/non-corpuscular). That is, Equation (20) reveals the wave-particle duality, for dynamic purposes (zitter), and reduces this duality (we are no longer dealing with a particle velocity and a wave velocity but with two wave velocities) to a single thing or previous (unified) nature, which will have to correspond to a physical support (entity) capable of this and with a certain arrangement, as we will see later.

For now, in terms of what matters to us, this reduction or unification will allow us to undertake, based on Equation (20), a separation of terms that is much more promising than that carried out in Equation (17) or that existing in Equation (5) itself, in which, through the conversion of corpuscular velocity into wave velocities, it will become clear that although H_D deals with individual particles, it could also deal with the combination of (two) WP that give rise to them, which is nothing more than an etiological connection between the corpuscular term (velocity and properties) and characteristically primary wave forms (not the secondary ones in the second term of the initial equation), *i.e.*, ones in which the velocities of the envelopes (which are the ones carrying the energy) appear in the open without having been subjected to perturbation or neutralization.

The velocity of material particles has, obviously, as part of that connection (which we will develop later). a wave nature through the WP because the material particles themselves must have the same nature, *i.e.*, Equation (20), as a solution of H_D, would show that all particles necessarily derive from the confrontation of two WPs, which at some point in their construction are partially neutralized in the corpuscular part, and that of two phase functions $\mathrm{e}^{-i\omega_0 t}$, coupled in the form $\mathrm{e}^{-i2\omega_0 t}$, as presented in Equation (13), although we are not yet working, as we will do later, with the wave equations of the WPs but with the velocities associated with their envelopes.

Concerning what we have just proposed, it is mandatory to comment that the already mentioned interference of positive and negative states [12], and the necessary fluctuation between energetic material entities of different signs for the zitter to occur, is not such, or it is not associated to the particles, but it is the concurrence of two WPs, from which an interference or combined form of them is derived, which at a zero (or compensated) energy level takes the coupled form $\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t}$. It is not only that two WPs (particles) are necessary for the zitter to occur, it is that two WPs (as pre-corpuscular states) are necessary for the classical part of the event to occur, linked to the formation of the particle or energetic coupling of the WP, highlighted through υ. In this case, the positive or negative values of the WP do not refer, as we will see later, to the energetic states of the (anti)particles of the Dirac sea, but to the fact that the phase functions of the WPs are initially in phase opposition for this energetic coupling, even though for the kinetic coupling they have the same sign.

2.3. Decoupled Forms of the Wave Velocity of Material Particles

Equation (20) presents a coupling of solutions (presumably associated with WPs) to be decoupled, and also a coupling of phase functions to be treated, which together will offer us different possibilities or configurations more or less intuitive, and capable of satisfying this equation, which for now we will simply present, as well as justify its validity and the possible feasibility of being achieved through a theoretical development.

2.3.1. First Form

The simplest option is to separate the solutions without separating the phase functions, but it implies a greater interpretative difficulty of the solutions, that is, to know what these solutions are in the wave frame. Indeed, without separating the phase functions the solution of Equation (20) is $\dot{x}_k = \dot{x}_{k2} - \dot{x}_{k1}$, and its two terms have the form:

$$\begin{aligned} \dot{x}_{k1} &= \upsilon_{\omega 1} - \upsilon_{\omega 1}\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t} = \upsilon_{g1}^c - \upsilon_{g1}^c \mathrm{e}^{-i2\omega_0 t}, \\ \dot{x}_{k2} &= \upsilon_{\omega 2} - \upsilon_{\omega 2}\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t} = \upsilon_{g2}^c - \upsilon_{g2}^c \mathrm{e}^{-i2\omega_0 t}, \end{aligned} \tag{21}$$

in which we would have two WPs that join term by term and give the result without having to be attentive to the geometry (although it is involved). Being also the paradigmatic case of the required linear application,

$$\dot{x}_k = c\alpha_k = g(a-b) = g(a) - g(b) = \dot{x}_{k1} - \dot{x}_{k2}, \tag{22}$$

in which, attending exclusively to the terms of the phase, for $A = \upsilon_{\omega 1}\mathrm{e}^{-i2\omega_0 t}$ and $B = \upsilon_{\omega 2}\mathrm{e}^{-i2\omega_0 t}$, $A - B = [\upsilon_{\omega 1} - \upsilon_{\omega 2}]\mathrm{e}^{-i2\omega_0 t}$ is fulfilled.

In contrast to the presented facility, we would have, on the other hand, two WPs with corrected group velocities υ_C^1 (for this first case), on initial group velocities υ_{gi}^c:

$$\upsilon_{Ci}^1 \equiv \dot{x}_{ki} = \upsilon_{gi}^c - \upsilon_{gi}^c \mathrm{e}^{-i2\omega_0 t}, \tag{23}$$

which not only presents an additional term with a phase function, with respect to that defined in Equation (7), but also involves a phase that does not correspond to the fundamental phase $\omega_0 t$ that presumably a WP would have to have, but $\omega_D t$, in a way that is initially as inexplicable as in the starting Equation (5). This discrepancy would make us consider υ_C^1 not so much a real correction of the WP group velocity, which would have to be demonstrated in a strictly wave scenario, as a correction associated with the geometry in the pro-corpuscular configuration we are dealing with.

2.3.2. Second Form

As a second option, as opposed to all of the above, we can understand another one that involves a separation of the phase factors of $\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t}$. In this case, since there is a correspondence between this product of factors and the product of the phase factors themselves, a shared use of these factors by both solutions is evident, from which it would be necessary to extract what corresponds to each WP by means of the aforementioned separation.

Continuing with this idea, we can label for greater ease each of these fundamental frequencies with the subscript of the WP, that is, differentiate them. Without losing sight of the fact that $\mathrm{e}^{-i2\omega_0 t}$ itself could be a simplification, or reduction, of the product $\mathrm{e}^{-i\omega_2 t} \times \mathrm{e}^{-i\omega_1 t}$ for $\omega_1 = \omega_2 = \omega_0$, that is, of the real initial wave state of two independent WPs that can report small differences with respect to their fundamental states. This, either for real or didactic motivations, which we can reverse at will, would lead us to rewrite the result in the form:

$$\dot{x}_k = \left(\upsilon_{\omega 2} - \upsilon_{\omega 1}\right) + \left[\upsilon_{\omega 1} - \upsilon_{\omega 2}\right] \mathrm{e}^{-i\omega_2 t} \times \mathrm{e}^{-i\omega_1 t}, \tag{24}$$

which differentiates in a clearer way that there are two elements that combine in a different way in each of their parts or natures to arrive at the final result $\dot{x}_k$. That is to say, apart from the particular concomitance of the wave terms and the underlying phenomenology that derives in Equation (24), everything would seem to indicate that we would have two strictly wave separable elements that fulfill:

$$\begin{aligned} \cancel{\dot{x}}_{k1} &= \upsilon_{\omega 1} - \upsilon_{\omega 1} \mathrm{e}^{-i\omega_1 t} = \upsilon_{g1}^{c} - \upsilon_{g1}^{c} \mathrm{e}^{-i\omega_1 t}, \\ \cancel{\dot{x}}_{k2} &= \upsilon_{\omega 2} - \upsilon_{\omega 2} \mathrm{e}^{-i\omega_2 t} = \upsilon_{g2}^{c} - \upsilon_{g2}^{c} \mathrm{e}^{-i\omega_2 t}, \end{aligned} \tag{25}$$

which, obviously, with $\dot{x}_{ki} \neq \cancel{\dot{x}}_{ki}$, require further development to satisfy Equation (20).

From there on, and following Equation (24), the first terms of these two elements tend to cancel each other because they are velocities of progression (or increase from zero), in opposition of movement, while the second ones are unified in a sort of wave interference, of integration or inclusion, being subjected to their own oscillation and to that of others. That is to say, $\upsilon_{\varpi 1} \mathrm{e}^{-i\omega_1 t}$ is subjected to phase $\mathrm{e}^{-i\omega_2 t}$, or supported by it (since it is a carrier), resulting in $A = \upsilon_{\omega 1} \mathrm{e}^{-i\omega_1 t} \times \mathrm{e}^{-i\omega_2 t}$, and $\upsilon_{\varpi 2} \mathrm{e}^{-i\omega_2 t}$ is supported by phase $\mathrm{e}^{-i\omega_1 t}$, resulting in $B = \upsilon_{\omega 2} \mathrm{e}^{-i\omega_2 t} \times \mathrm{e}^{-i\omega_1 t}$, and finally in $A - B = \left[\upsilon_{\omega 1} - \upsilon_{\omega 2}\right] \mathrm{e}^{-i\omega_2 t} \times \mathrm{e}^{-i\omega_1 t}$.

A process of inclusion that has much to do here with the geometry of the system, that is, of the particle, which will allow the second term of each equation of Equation (25) to be an unsuspected traveler of the carrier of the other equation, being able to say that the possibility of forming a particle from two WPs lies in this interlock, without going into the energetic mechanisms that have to be put in place, and we will deal with later.

For this case, as derived from Equation (25), the WP would have the corrected group velocity υ_C^2:

$$\upsilon_{Ci}^2 \equiv \cancel{\dot{x}}_{ki} = \upsilon_{gi}^{c} - \upsilon_{gi}^{c} \mathrm{e}^{-i\omega_i t}, \tag{26}$$

A configuration susceptible of being, although it will not be the case, a real correction of the group velocity expressed in Equation (7), in that it only presents its single own phase $\omega_i t$, and is a priori independent of the geometry, a geometry that nevertheless would have to be applied with the described process to reach

Equation (23), and thus satisfy Equation (22) and the starting equation.

Notwithstanding this, and despite its apparent simplicity, we do not know the geometry that makes this development possible, nor the one that originates this simplified phase, which makes it in practice an unpromising version of the previous one. In other words, in this configuration, there is a lack of knowledge of the geometry that would establish the connection between the solutions of Equation (21) and the intrinsic ones of Equation (25), of which we would not know their origin either, since we start from Equation (7) which is analogous, but different and contrasted.

2.3.3. Third Form

For a third configuration option, we can realize that the separation of the elements carried out in Equation (21) on Equation (20), which we can extend to that carried out in Equation (25) on Equation (24), is not unique, since we can do the same:

$$\begin{aligned} \dot{x}_{k1} &= \upsilon_{\omega 1} + \upsilon_{\omega 2}\mathrm{e}^{-i2\omega_0 t} = \upsilon_{g1}^{c} + \upsilon_{g2}^{c}\mathrm{e}^{-i2\omega_0 t}, \\ \dot{x}_{k2} &= \upsilon_{\omega 2} + \upsilon_{\omega 1}\mathrm{e}^{-i2\omega_0 t} = \upsilon_{g2}^{c} + \upsilon_{g1}^{c}\mathrm{e}^{-i2\omega_0 t}. \end{aligned} \tag{27}$$

This leads us to a different concept in the conformation of $\dot{x}_{ki}$, in which we will not be able to speak of WP of homogeneous terms, as those presented in Equation (23) and Equation (26), but of a single proper term, in the usual form given in Equation (7), and a subsequent process on such an external element as is the WP paired to the reference one. It follows that the additional term must necessarily be unrelated to the group velocity and that an assumed corrected group velocity υ_C^3, in this third case, is coincident with $\upsilon_{\omega i}$ of Equation (7). Resulting, consequently, that the whole weight of the solution, *i.e.*, the one reached through the geometry from $\upsilon_{\omega i}$, rests on the interdependence of the two WPs, nonexistent in the other cases.

2.3.4. Summary

We have separated the decoupling possibilities into three different configurations which are only, as we shall see, apparently different, since, if we carry out the inclusion or interlock referred to in Equation (25) on the carrier of the other WP, what we are doing actually is a transfer of the term (which may include the sign that accompanies it) to the wave equation of that carrier, that is, putting that term in the other equation, and arriving at:

$$\begin{aligned} \dot{x}_{k1} &= \upsilon_{\omega 1} + \upsilon_{\omega 2}\mathrm{e}^{-i\omega_1 t} \times \mathrm{e}^{-i\omega_2 t}, \\ \dot{x}_{k2} &= \upsilon_{\omega 2} + \upsilon_{\omega 1}\mathrm{e}^{-i\omega_1 t} \times \mathrm{e}^{-i\omega_2 t}, \end{aligned} \tag{28}$$

which is analogous or indistinguishable to Equation (27) for $\omega_1 = \omega_2 = \omega_0$. Whereas Equation (27) and Equation (21) are the same two configurations from different perspectives, which we will later uncover. It follows that only these two configurations exist and that the good configuration will be the one that is accessible from any physical or geometrical perspective that we can devise.

That said, it remains to investigate whether the equations are feasible and the geometry that would allow or justify the overlapping of the WPs in any of these forms, as well as to elucidate by other methods whether any of the assumptions are accessible, such as the purely wave analysis that we are going to perform next.

3. Undulatory Treatment

All the development seen makes evident the existence of an additional velocity to the classical velocity, which we can revalidate by the composition of WP, whether it results in a change in the generic interpretation or correction υ_C of the group velocity, or not. Once we are no stranger to this additional velocity we may be in a position to understand and reach it through a purely undulatory treatment, and in a position, therefore, to formulate υ_C from this perspective, which will undoubtedly help to ground υ_D.

Through this undulatory treatment, we know that the equation of a WP, derived from the superposition of two waves, obeys an expression of the type [10]:

$$\Psi = 2A\cos 2\pi\left(t\mathrm{d}\nu - x\frac{\mathrm{d}\lambda}{\lambda^2}\right)\cos 2\pi\left(\nu t - \frac{x}{\lambda}\right) = B\cos\alpha\cos\beta, \tag{29}$$

in which both the base wave function $\cos\beta$ and the envelope $\cos\alpha$ have a propagation velocity, determined by their respective arguments, in the x direction. Through the former we obtain the phase velocity, while by means of $\cos\alpha$ we obtain that the WP develops a velocity υ_g^c which is affine to the WP motion itself, given by the endogenous condition $\alpha = cte$, which is the one that gives rise to Equation (7).

However, Equation (29), as a type equation for a WP consisting of two waves with slightly different frequencies with their respective wavelengths $\lambda + \mathrm{d}\lambda$ and $\lambda - \mathrm{d}\lambda$, is not exact, since in the crossing of terms the $(\mathrm{d}\lambda)^2$ factors are discarded, with the result that Equation (7) for the group velocity is not completely correct either, and that the correct one, which we previously called υ_C for our purpose, is:

$$\begin{aligned}\upsilon_C &= \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\mathrm{d}\nu}{\mathrm{d}(1/\lambda)} = -\left[\lambda^2 - (\mathrm{d}\lambda)^2\right]\frac{\mathrm{d}\nu}{\mathrm{d}\lambda} \\ &= \upsilon_p\left[1 - \frac{(\mathrm{d}\lambda)^2}{\lambda^2}\right] - \frac{\lambda^2 - (\mathrm{d}\lambda)^2}{\lambda}\left(\mathrm{d}\upsilon_p/\mathrm{d}\lambda\right) \\ &= \upsilon_p\left[\frac{\lambda^2 - (\mathrm{d}\lambda)^2}{\lambda^2}\right] - \frac{\lambda^2 - (\mathrm{d}\lambda)^2}{\lambda^2}\lambda\left(\mathrm{d}\upsilon_p/\mathrm{d}\lambda\right) \\ &= \left(\upsilon_p - \upsilon_g^0\right)\left[\frac{\lambda^2 - (\mathrm{d}\lambda)^2}{\lambda^2}\right] = \upsilon_g^c\left[1 - \frac{(\mathrm{d}\lambda)^2}{\lambda^2}\right],\end{aligned} \tag{30}$$

which presents a variation with respect to Equation (7) in each of its terms, and consequently in $\upsilon_g^c = \upsilon_\omega$, with respect to the expected and fixed value. That is:

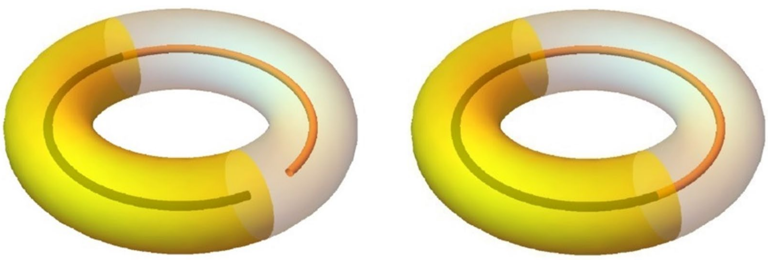
$$\upsilon_C = \upsilon_g^c - \upsilon_g^c\left(\frac{\mathrm{d}\lambda}{\lambda}\right)^2, \tag{31}$$

in which appears a squared factor that reminds us of the squared factor $\mathrm{e}^{-i2\omega_0 t}$ and, consequently, a greater correspondence or similarity with υ_C^1 of Equation (23) than with any other variant, which could be greater if we consider the possibility that there is a temporal evolution of the factor $(\mathrm{d}\lambda/\lambda)$.

We have no reason to think that this time evolution occurs in a WP, however, we might think, attending once again to the geometry and the existence of two WPs, that the time evolution of a wavelength of one of the WP constitutes the displacement margin of the wavelength of the other WP, and that, consequently, for WP of equal size, it is fulfilled:

$$\upsilon_C = \upsilon_g^c - \upsilon_g^c \left(\frac{\mathrm{d}\lambda_1}{\lambda_2}\right)^2 = \upsilon_g^c - \upsilon_g^c \left(\frac{\lambda_1 \mathrm{e}^{-i\omega_0 t}}{\lambda_2}\right)^2 = \upsilon_g^c - \upsilon_g^c \mathrm{e}^{-i2\omega_0 t}, \quad (\lambda_1 = \lambda_2). \tag{32}$$

If we observe, the proposed configuration is nothing more than the connection of the two halves of a toroid, or a sphere, or a similar symmetrical topology, in which the only accessible route for one half is the other, hence the aforementioned margin of displacement. In this case, using a toroid as a model, we can see that evolution $\mathrm{d}\lambda$ in a WP-I (yellow) can progress through the other WP-II (green) clockwise, as shown in **Figure 1(a)**, or counterclockwise, reaching the origin of λ_1 for $\mathrm{d}\lambda = \lambda_1$ and establishing a closed circuit, as shown in **Figure 1(b)**. Note that we can say the same about λ_2 and $\mathrm{d}\lambda$ of WP-II with respect to WP-I, which would result in a second circuit.



(a) (b)

Figure 1. (a) Partial evolution $\mathrm{d}\lambda$, relative to λ, in a WP on the topology of a toroid; (b) Maximum or total evolution $\mathrm{d}\lambda = \lambda$, relative to λ, in a WP on the topology of a toroid.

With this we are saying, on the one hand, that by simply admitting the time evolution $\mathrm{d}\lambda/\lambda = \mathrm{e}^{-i\omega_0 t}$, and venturing a topology, we are able to explain what happens with respect to the zitter in a particle, that is, capable of achieving the Equation (13) and to reproduce, consequently, by means of a classical wave treatment, what happens in that particle.

On the other hand, we are saying that the zitter could be showing precisely the existence of this temporal evolution at least in the WPs that involve particles, that is, in the processes in which these WPs are paired. We are also saying that in this assumption the zitter would be the expression or result of the correction of the group υ_g^c velocity suffered by each WP as a consequence of having a pair WP.

4. The Symmetrized Wave Packets (SWP)

Equation (20) already expresses by itself a certainty about the material configuration on the basis of two WPs with their respective group velocities, regardless of whether we are then able to decouple them convincingly. If, despite the above, we did not have a theoretical framework that postulated the creation of particles from some sort of combination of two WPs (which involves a topology), we would have to try out different configurations or leave the above statements in abeyance, waiting for some kind of experimental confirmation or some kind of theoretical breakthrough that would overcome them definitively, but this is not the case.

It is not the case because we have that theoretical framework capable of justifying such statements, while reaffirming itself as a stage [2]. A theoretical framework with which other phenomena have already been justified, which provides us, for our purposes, with the secondary wave functions that give rise to zitter and their transit space, which also opens the door to an etiological understanding of the different dynamics, because it not only says how matter is constructed from two WPs but, as we shall see in the next section, why it has to be constructed in this way.

To find a reason that justifies Equation (13), and with it Equation (5), we have to put ourselves, therefore, in the SWP environment, in which starting from a WP type:

$$\Psi_k(x,t) = B(i)\frac{\mathrm{e}^{-i[\Delta k/2(\upsilon t - x)]}}{\upsilon t - x}\mathrm{e}^{i(\omega_0 t - k_0 x)}, \tag{33}$$

we obtain two symmetrized WPs, in the form:

$$\Psi(x,t) = \frac{\left(\Psi_1(x,t) + \Psi_2(x,t)\right)}{\sqrt{2}} = B(i)\frac{\mathrm{e}^{-i(M/2)}\mathrm{e}^{-i(N)} - \mathrm{e}^{i(M/2)}\mathrm{e}^{i(N)}}{\sqrt{2}(\upsilon t - x)}, \tag{34}$$

which reaches an energetic form with a geometrical shape, which we call a particle, and a certain corpuscular speed and, consequently, kinetic energy, as part of a more general energy balance expressed through the energy transfer equation (ETE):

$$\begin{aligned}
\bar{E} &= A^2\int \zeta^m(\upsilon)\mathrm{d}\upsilon + \zeta^\omega(\upsilon)\mathrm{d}\upsilon \\
&= A^2\int\left(\zeta_k(\upsilon) + \zeta_f(\upsilon) + \zeta_\omega(\upsilon)\right)\mathrm{d}\upsilon = \bar{E}_k + \bar{E}_f + \bar{E}_\omega \\
&= \left(\frac{\hbar \not{b}}{\pi a^3}\right)\sin[\Phi]\int_{\upsilon_o}^{\upsilon_f}\frac{\upsilon}{\gamma^{-3}}\mathrm{d}\upsilon \qquad \text{(a)} \\
&\quad -\left(\frac{\hbar\Delta k \not{b}}{2\pi a^2}\right)c^2\int\frac{\cos[\Phi_\nu]}{\nu}\mathrm{d}\nu \qquad \text{(b)} \\
&\quad +\left(\frac{\hbar\omega \not{b}}{\pi a^2}\right)c\int\frac{\cos[\Phi_\nu]}{\left(1-\nu^2\right)^{1/2}\nu}\mathrm{d}\nu, \qquad \text{(c)}
\end{aligned} \tag{35}$$

which includes everything that energetically happens in the process of symmetrization and consequent materialization of the two WPs which, for dynamic purposes, should give rise to a process inverse to that expressed by Equation (20), that is:

$$\begin{aligned}\dot{x}_k &= c\alpha_k = \left(\upsilon_{\omega 2} - \upsilon_{\omega 1}\right) + \left[\upsilon_{\omega 1} - \upsilon_{\omega 2}\right]\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t} \\ &= \left(\upsilon_1 - \upsilon_2\right) + \left[c\alpha_k\left(0\right) - c\alpha_k\left(0\right)\right] + \left[\left(c\alpha_k\left(0\right) - \upsilon_1\right) - \left(c\alpha_k\left(0\right) - \upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t} \\ &= \left(\upsilon_1 - \upsilon_2\right) + \left[c\alpha_k\left(0\right) - \left(\upsilon_1 - \upsilon_2\right)\right]\mathrm{e}^{-2i\omega_0 t} = \upsilon + \upsilon_\omega \mathrm{e}^{-2i\omega_0 t}.\end{aligned} \quad (36)$$

in which we see that there is a final term ($\upsilon_\omega \mathrm{e}^{-2i\omega_0 t}$) that does not correspond to $\overline{E}_k$, that is, that they do not transport energy or that the energy balance of that transport is zero, in the same way that it is zero in Equation (35) the final energy balance of $\overline{E}_\omega$ as a result of the energy transfer between $\overline{E}_\omega$ and $\overline{E}_f + \overline{E}_k$, attributed to the formation of the particle or construction of its mass from the formants [2]. In other words, since all wave energy is converted during the formation process, only this wave form remains in its elementary form (the carrier), which is the same as that which becomes apparent in the zitter after said process.

A process in which, as already said, the reference speed falls from the light c to the corpuscular υ , as expressed by the first term of both Equation (35) and Equation (36). In fact, it is the appearance of this speed of the first term, on a new reference $\upsilon = 0$, that characterizes and reveals a change to a physical system that we perceive, as opposed to the second term, as material or dense, and different from the initial undulatory one, which is thus perceived as unreachable or alien to the relative space created. Here it would be necessary to speak of material phases or material states, and to introduce all the repercussions of phase changes, of the coexistence of two phases [2], of the generation of three-dimensional space from two one-dimensional spaces, and of the movement through this space [3].

Apart from this de-individualization (densification or materialization) we have and continue to have in an underlying form two waves, with a packaged part, associated to the factors $\mathrm{e}^{\pm i(M)}$ of the Equation (34), with the same initial group velocity for $\upsilon = 0$, on two carrier waves $\mathrm{e}^{\pm i(N)}$, which have no energetic repercussion, but which are still there as a reference, being for this reason that, according to the ETE terms, any inverse process (annihilation versus creation) finds the adequate wave support, being feasible the energetic transfer between said terms.

Regarding the corpuscular speed, the envelopes of the WPs give rise to a particle with a classical speed represented by the first term of Equation (5), without our being able to notice any oscillating speed in the particle through this energetic conformation as a consequence of the cancellation of the phases with their conjugates in the energy balance and the energetic irrelevance of plane waves after it [2]. This cancellation of the phases in the energy balance does not imply, as we said, their suppression, but rather the cancellation for those purposes, not for dynamic purposes such as those of the zitter we are dealing with. The non-suppression of the phases implies, on the other hand, the total non-suppression of the wave equations.

Regarding the geometrical shape, continuing with the description, the envelopes form a particle with a toroidal shape [3], as shown in **Figure 2(a)**, *i.e.* we are dealing with two WPs, represented by two colors in **Figure 2(b)**, which form a kind of tubular circuit between them. We cannot guarantee that the particle (electron) coincides with this shape (occupying the entire volume), only that this is (for its mass) its area of influence about the WPs that form it, according to Equation (35a).

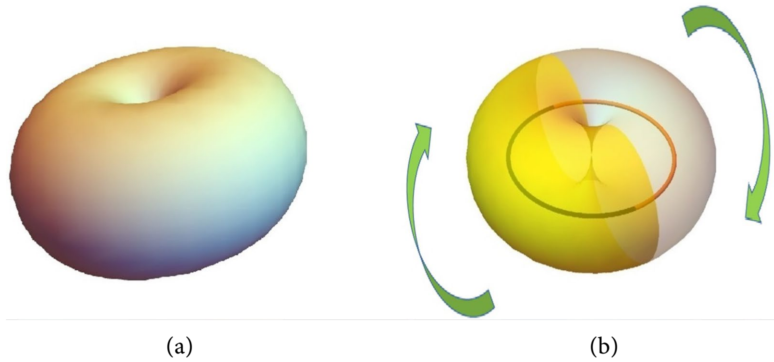


Figure 2. (a) Side view of the toroid for $r^2 \times R = a^3 (R = r)$, according to Equation (35a); (b) View of the toroid highlighting the two semitoroids in yellow and green, and the circulation of the inner phase.

Before symmetrization, we will have two independent and opposite carriers $e^{-i(N)}$ and $e^{+i(N)}$, which are the ones that face each other (but do not disappear) in the conformation of the corpuscle and its corpuscular speed, as detailed in Figure 2(b) itself. From there, we may award to phase $e^{-i(N)}$ a right-handed progression in the semitoroid green and to the phase $e^{+i(N)}$, without solution of continuity, a left-handed progression in the symmetrized (inverted) semi-toroid yellow, which in practice supposes, as indicated in Figure 2(b), constituting a clockwise endless movement on the circuit already established by the envelopes, which implicitly involves the virtual inversion of the phase $e^{+i(N)} \leftrightarrow e^{-i(N)}$ and its synchronization or effective superposition, $e^{-i2(N)} = e^{-i(N)} \times e^{-i(N)}$, with the other phase.

4.1. The SWP as Quasi-Stationary System

To understand this (at least, and to begin with, its basic logic), we need to realize that, generally speaking, when we tie two strings together at their ends, analogous to the process of symmetrization of two elements carried out in Equation (34), we no longer have two strings but one with a circular arrangement. This circular arrangement manifests itself in the form of a toroid for the envelopes, and is adopted by the carriers, even if there is no energetic representation of them, due to the knotting and the toroidal space created.

When instead of two strings we have a single circular one, the displacement of one point is transmitted in the same way to all points, even though we see that half of the string moves in the opposite direction to the impulse imparted if we bend it at that point. Without the bend, a total reflection is a transmission. In our case, phase $e^{-i(N)}$ of the green semi-toroid, taken as a reference, will be followed by phase $e^{+i(N)}$, but not in opposition to the movement, since they are not on the same straight trajectory, but rather adding their effect to $e^{-i(N)}$ (and hence that $e^{+i(N)} \to e^{-i(N)}$) by the accommodation of the straight line movement (which each would have outside that contour) to the curved one. As a result, in the green semi-toroid we would have the form $e^{-i(N)} \times e^{-i(N)}$.

That is, carrier $e^{+i(N)}$ catches up with $e^{-i(N)}$ and pushes it (overlaps it) in the green part, and similarly carrier $e^{-i(N)}$ catches up with $e^{+i(N)}$ and pushes it in the yellow part, resulting in the effects adding up. We might think that in the yellow semi-toroid it is $e^{+i(N)} \times e^{+i(N)}$, but it is not so, but $e^{-i(N)} \times e^{-i(N)}$, because what marks the sign is the direction of rotation, and in both it is the same (arbitrary or conditioned), that is, there is no longer movement to the right and left but one of rotation that they share, and because (as a consequence) at every point of the circular trajectory there is now only one wave vector tangent to it. The same wave vector, distinguishable only by the coefficient (ϑ_1 or ϑ_2) that accompanies the native carrier in each section (the thickness of each string or the physical characteristics of each envelope).

In conclusion, with regard to rectilinear motion, we can distinguish between $e^{-i(N)}$ and $e^{+i(N)}$, whereas with regard to curved motion, they are indistinguishable (both can be considered $e^{-i(N)}$) because at all times one occupies the position of the other (they are no longer distinguished by origin and direction) and they have the same effect (circular motion) with regard to a given axis, to which both carriers contribute (as a pair of forces) in the form $e^{-i(N)} \times e^{-i(N)}$. Carriers that we can understand for each semi-toroid as a single carrier $e^{-i2(N)}$, with double frequency [and this is how we have represented it in Figure 3]. Obviously, there is no force or impulse here that causes rotation as such; rather, the action is simply duplicated by superposition, that is, the frequency of the wave, which already has a circular motion imposed by its topology and its own wave dynamic.

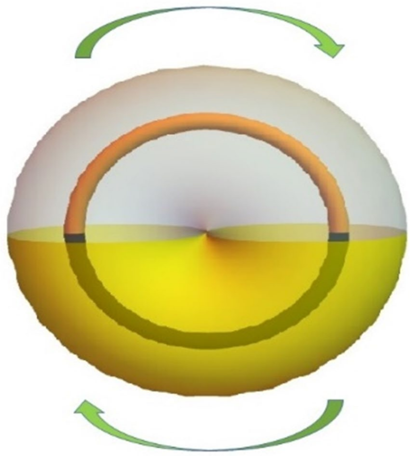


Figure 3. Zenithal view of the toroid for phase-coupled circulation $e^{-i2(N)}$.

That said, it is worth raising a few questions, some of which are merely reminders of what we know, in order to set the scene correctly. We are not intent on analytically demonstrating that the final relationship between phases for the SWP is $\mathrm{e}^{-i(N)}\times\mathrm{e}^{-i(N)}$, which we already take for certain through the zitter factor $\mathrm{e}^{-i2\omega t}$, but rather to demonstrate that this factor corresponds to that final relationship between phases or is achieved from it, trying to explain, in accordance with known phenomenology or theoretical treatments, why this is so, in addition to being able to justify it from the SWP itself and being key, as we shall see, to achieving the zitter. Among these frameworks are those relating to resonant cavities ("stationary waves"), which bear a certain resemblance to SWP, although they differ in other aspects.

Stationary waves are formed by wave $E_1 = E_0\mathrm{e}^{i(\omega t+kx)}$ and the reflected wave $E_2 = E_0\mathrm{e}^{i(\omega t-kx)}$ on the interface between two media (see Chapter 21 of [10]), giving rise to spatial modulation in amplitude, that is, a WP with a modulated wave or stationary pattern $2E_0\cos(kx)$ and a phase $\mathrm{e}^{i\omega t}$ (common to the two initial waves and, consequently, common to the two orientations of space), which is what truly gives the aforementioned pattern its stationary character because, since it does not propagate through x (it is not a progressive wave), all points have the same phase at a given instant t. In the SWP, it is not two waves that undergo a combination but two WPs (both with a stationary pattern), being the symmetrization achieved in Equation (34) the most compact possible expression of this. It is not possible to make it any more compact because the formants of the SWP do not join together, except at the ends (hence the toroidal shape), and it is these joints that act as reflection points between one WP and another, that is, they emulate a surface of separation between media, which is nothing more than emulating the external medium and differentiating itself de facto from the external medium, that is, creating space or space outside to the interior. It is when we take stock of the energy balance that we are able to observe the final result of the process and the total stationary pattern (the particle), and it is through the zitter that we are able to observe what happens to the carriers, which is different for each of the formants, but which are also associated with this pattern in a specific way. All of this makes up the difference between a symmetrization and a stationary wave.

As we said, $\mathrm{e}^{i\omega t}$ in the stationary wave is unrelated to spatial orientation (incident-reflected), so that differentiating $\mathrm{e}^{-i(N)}$ from $\mathrm{e}^{+i(N)}$ in the SWP based on orientation is more than inappropriate, or less appropriate than doing so based on a different temporal evolution of the formants or the energetic sign (matter-antimatter), as we have already pointed out [5] and will develop in other works. In other words, we could say that the SWP is made up of a function with positive energy, according to its quadri-momentum, and another with negative energy, which would explain more clearly the difference between Dirac spinors and intrinsic spinors, as well as giving us an idea of why the zitter has been associated with energy fluctuations (between pairs). Not only that, but the existence of two different energy signs combined is necessary for spin configuration, that is, it is necessary because spin itself is not something intangible but the result of the combination of these signs or circulation of the two carriers in the toroid [5], which otherwise breaks the degeneracy of the states (which the Dirac equation accomplishes by means of a label for the same energy sign), since it is possible to create two different states of negative energy by extrapolating the phases in functions Ψ_1 and Ψ_2 (with combined energy sign), as well as two states of positive energy, one of which is achieved through Equation (34).

What does seem certain, or is evidenced by the zitter, and from the mentioned Equation (34), is that whatever the distinction may be between phases, it is nullified by the SWP, that is, the SWP does not allow the two to coexist and forces us to choose one form of progress or the other, so that the final balance for both spin configurations is positive for $(\Psi_1 + \Psi_2)$ and negative for $-(\Psi_1 + \Psi_2)$. If we refer to the aforementioned temporal evolution, it would seem logical that two identical opposing evolutions cannot coexist (in the best case scenario, everything would be canceled out), as matter-antimatter cannot, only their proper fit. In this sense, everything said about the convergence of the sign in a circular domain is valid for this temporal factor, that is, it seems logical that two opposing temporal progressions will unify, overlap, or synchronize if we integrate them into a single circular path, and it is logical that this is a sine qua non condition for that unification, in order to achieve a compatible and stable state.

4.2. The Zitter from SWP

We can, on that argumentative basis, expand and structure the scenario described from the wave perspective developed at the beginning, to try to explain, going further, how the second terms are created or incorporated into the zitter, cross their factors between the two WPs, and the sign of the phase is inverted. To this effect, we can take the two wave equations of Equation (34) and write them in the following simplified form.

$$\begin{aligned}\Psi_1 &= \vartheta_1 \mathrm{e}^{-i\omega_0 t},\\ \Psi_2 &= \vartheta_2 \mathrm{e}^{+i\omega_0 t}.\end{aligned} \tag{37}$$

It should be noted that here, as in Equation (34), we are referring to the WPs through their wave equations and not through the group velocities to which they give rise, as in Equation (36). Wave equations with positive and negative values that, now yes, are susceptible to give rise to a process of interference or combination of states (not ascribed to a particle and an antiparticle) from which are derived implications for the velocities such as those that we study and express in Equation (36).

These functions Ψ_1 and Ψ_2 would have their natural development in their respective envelopes, we can say in their respective semitoroids or application channels A and B, giving rise to the respective group velocities, that is, $\upsilon_{g1}^{c} = \upsilon_{\omega 1}$ of the envelope function ϑ_1 and $\upsilon_{g2}^{c} = \upsilon_{\omega 2}$ of the envelope function ϑ_2, and to the consequent classical corpuscular velocity υ_g^0, according to what is proposed by the SWP and the first term of Equation (36).

On the other hand, since there is neither a physical barrier between the junction ends of the envelopes nor a potential that prevents propagation, wave equation $\Psi_2 = \vartheta_2 e^{+i\omega_0 t}$ will be introduced or progress in channel A, and equation $\Psi_1 = \vartheta_1 e^{-i\omega_0 t}$ will progress in channel B, giving rise to additional superposed wave equations, unrelated to the materialization or wave neutralization carried out in the aforementioned first term of Equation (36).

An additional wave equation unrelated to the materialization, although subordinated to it, since from the moment in which both WPs are joined by the symmetrization process there will no longer exist, as we said in the previous description, two channels and two carriers, but a single propagation circuit and a single carrier, either $e^{-i\omega_0 t}$ or $e^{+i\omega_0 t}$. We will not have, therefore, two phases in opposition of movement but a closed circuit and a single circular path in which both movements, initially opposite, become contributory with respect to the changing direction of that circular path.

In this case, taking as reference the first carrier, $\Psi_2 = \vartheta_2 e^{+i\omega_0 t}$ will be presented as $\Psi_2^* = \vartheta_2 e^{-i\omega_0 t}$ and will be carried by that first carrier $e^{-i\omega_0 t}$ in channel A, while $\Psi_1 = \vartheta_1 e^{-i\omega_0 t}$ will not be carried in channel B by $e^{+i\omega_0 t}$ of Ψ_2, but by $\left(e^{+i\omega_0 t}\right)^* = e^{-i\omega_0 t}$ attached to $\Psi_2^* = \vartheta_2 e^{-i\omega_0 t}$, which is how it is presented when it reaches it. More specifically, the real and effective additional wave equations will be:

$$\begin{aligned} \bar{\Psi}_2 &= \Psi_2^* \times e^{-i\omega_0 t} = \vartheta_2 e^{-i\omega_0 t} e^{-i\omega_0 t} = \vartheta_2 e^{-i2\omega_0 t} \equiv \psi_2 e^{-i\omega_0 t}, \\ \bar{\Psi}_1 &= \Psi_1 \times \left(e^{+i\omega_0 t}\right)^* = \vartheta_1 e^{-i\omega_0 t} e^{-i\omega_0 t} = \vartheta_1 e^{-i2\omega_0 t} \equiv \psi_1 e^{-i\omega_0 t}, \end{aligned} \quad (38)$$

which will be added to the starting ones in the form:

$$\begin{aligned} \Psi_A &= \Psi_1 + \bar{\Psi}_2 = \vartheta_1 e^{-i\omega_0 t} + \psi_2 e^{-i\omega_0 t} = \vartheta_1 e^{-i\omega_0 t} + \vartheta_2 e^{-i2\omega_0 t}, \\ \Psi_B &= \Psi_2 + \bar{\Psi}_1 = \vartheta_2 e^{+i\omega_0 t} + \psi_1 e^{-i\omega_0 t} = \vartheta_2 e^{+i\omega_0 t} + \vartheta_1 e^{-i2\omega_0 t}. \end{aligned} \quad (39)$$

That is, we have as total wave equation, of the semitoroids A and B, the wave functions of Equation (37), already analyzed, with additional wave functions $\bar{\Psi}_i \equiv \psi_i e^{-i\omega_0 t}$ that share carrier, in which the wave functions ψ_i (obtained from the original ones) play the role of envelopes that in turn contain the carrier (or, if desired, an already modulated carrier).

We can realize that we arrive at the same result if we do not consider the second terms of Equation (39) as part of the same channel but ascribed to the other channel, being in this case the carrier of a WP that goes from one channel to another to form the compiled carrier $e^{-i\omega_0 t} \times e^{-i\omega_0 t}$, which would have no other effect than to provide the modulating function with a double sampling rate. That is, in the latter case, as we said for WP-I and WP-II in the previous section, each carrier travels from one WP to the other, forming two independent circuits, although equivalent to the superposition of the same developed above, as shown in **Figure 4**.

A change of perspective of the process that can make it easier for us to understand the resulting term, that is, not having to resort to the transmission of the

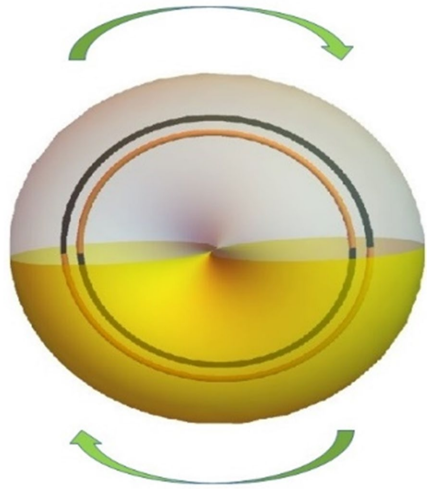


Figure 4. Zenithal view of the toroid for phase-decoupled circulation $\mathrm{e}^{-i\omega t}\times\mathrm{e}^{-i\omega t}$.

ψ_i forms between the channels, since, in this case, Ψ_1 receives carrier $\mathrm{e}^{+i\omega_0 t}$ from Ψ_2 in reverse direction of circulation $\left[\rightarrow\left(\mathrm{e}^{+i\omega_0 t}\right)^*=\mathrm{e}^{-i\omega_0 t}\right]$ to construct $\bar{\Psi}_1$, and Ψ_2, in reverse direction of circulation $\left[\rightarrow\Psi_2^*\propto\left(\mathrm{e}^{+i\omega_0 t}\right)^*=\mathrm{e}^{-i\omega_0 t}\right]$, receives carrier $\mathrm{e}^{-i\omega_0 t}$ from Ψ_1 to construct $\bar{\Psi}_2$. As a consequence, the wave equations Ψ_A and Ψ_B would not be, even being the same, the equations of each part of the toroid (channels A and B) but of two different branches (I and II) along the toroid ($\Psi_A\rightarrow\Psi_I$ and $\Psi_B\rightarrow\Psi_{II}$).

This aside and alternative treatment done, if we carry out on the wave equations of Equation (39) a difference of equations we obtain:

$$\begin{aligned}\Psi_B-\Psi_A&=\left(\vartheta_2\mathrm{e}^{+i\omega_0 t}+\vartheta_1\mathrm{e}^{-i2\omega_0 t}\right)-\left(\vartheta_1\mathrm{e}^{-i\omega_0 t}+\vartheta_2\mathrm{e}^{-i2\omega_0 t}\right)\\&=\left(\vartheta_2\mathrm{e}^{+i\omega_0 t}-\vartheta_1\mathrm{e}^{-i\omega_0 t}\right)+\left(\vartheta_1-\vartheta_2\right)\mathrm{e}^{-i2\omega_0 t},\end{aligned}\tag{40}$$

on which we only have to carry out an extrapolation between the envelopes of group ϑ_i and their velocities $\upsilon_{\omega i}$, to reach:

$$\dot{x}_k\left(\vartheta_i\right)=\left(\upsilon_{\omega2}-\upsilon_{\omega1}\right)+\left[\upsilon_{\omega1}-\upsilon_{\omega2}\right]\mathrm{e}^{-i\omega_0 t}\times\mathrm{e}^{-i\omega_0 t}.\tag{41}$$

This is, the same result that we reached in Equation (20), in which analogously the phase functions of the first term are canceled (validating the premise with respect to υ) and those of the second term are maintained, on which it is convenient to emphasize that it is the phase opposition in the first term that makes them cancel through geometry, and not that these phases are single or double. A result that allows us to obtain easily, as we did in Equation (27), the components of the velocities:

$$\begin{aligned}\dot{x}_{k1}&=\upsilon_{\omega1}+\upsilon_{\omega2}\mathrm{e}^{-i2\omega_0 t},\\\dot{x}_{k2}&=\upsilon_{\omega2}+\upsilon_{\omega1}\mathrm{e}^{-i2\omega_0 t}.\end{aligned}\tag{42}$$

It should be noted that the first term in the last member of Equation (40) corresponds through Equation (34) to the energy value of the SWP and, consequently, of the particle, from which we draw that the corpuscular velocity obtained in the first term of Equation (41), which we also recognize as the classical speed υ, has a biunivocal correspondence with that energy balance and that, therefore, the velocity of the second term of Equation (41) does not have that corpuscular nature or energy development. With this, we can conclude that the WPs themselves give us the energy balance of the materialization process and the geometry, but tell us nothing of the secondary velocities, irrelevant in that balance, which however we can only interpret through the interaction of those WPs (by-products of that interaction) in that geometry. Being that geometry also the one that will allow us to speak of angular velocity in the particle in charge of $\upsilon_\omega e^{-2i\omega_0 t}$, as a consequence of the establishment of a circular path along the toroid, and the one that will allow us to define the nature of the fluctuation $e^{-2i\omega_0 t}$ and its effect in that application space, as opposed to the linear oscillating motion that would be by default in the trajectory of any free particle. With all this, Equation (40) can be considered a more general and complete expression of symmetrization, that is, one that contains the main function and secondary currents or byproducts (unsymmetrized WPs), one that represents the final state reached in it, that of the symmetrized object.

That Equation (41) arises from Equation (40), that is, from the SWP itself and its geometry, is as much as to say that it arises from the corresponding intrinsic spinor, which implies establishing, as we have already advanced, a functional or more direct connection between these spinors and the Dirac spinors, linked in turn the latter to any solution derived from the relativistic dispersion equation (such as that relative to the velocity operator that we have discussed). That is, the formal connection between the intrinsic spinors and the Dirac spinors has already been established [5], now we are seeing (and hence the functional connection) that the former are wave functions whose group velocity, explicitly stated in Equation (35a), is that associated with the velocity $\upsilon = (\upsilon_{\omega 2} - \upsilon_{\omega 1})$ derived from H_D, whose wave functions are the Dirac spinors. While shows that the solutions of the Dirac equation cannot be the Dirac spinors, as we anticipated, since they are insufficient (due to lack of one phase capable of relating to another phase) to explain the speed of the second term of the Equation (5), but these intrinsic spinors, which contain, explain and connect with the first [5].

More explicitly, the zitter derived from the Dirac equation refers to carriers that Dirac spinors (the solution to the Dirac equation) do not account for, since they do not appear explicitly to justify the degeneration of states attributable to spin. Contrary to intrinsic spinors, which, as we said, carry a more complete representation, capable of explaining the phenomenology that derive from the existence of carriers, such as zitter or the presence of spin, as well as constructing and differentiating the material phases, for which the carriers represent precisely the most hidden part of physical reality (the least material phase of it) which, as we have seen, cannot be represented by quadri-momentums of a single sign, in which the phase ends up being a mere accompanying factor.

Indeed, on Equation (40) we clearly see differentiated the two material phases, which are two different natures, and consequently the two velocities associated with these material phases, which are carried out in different spaces, one pre-existing and the other external generated through the process of symmetrization, which allows the relation of symmetrized objects, of toroids, of particles in space. From which we also conclude that the zitter is an internal movement or vibration in the toroid, regardless of the fact that individually the two phase functions, as plane waves that they are, are not confined in that space. This external-internal, three-dimensional and one-dimensional distinction is the classical-quantum distinction.

From this image, we can liken the process of symmetrization to that carried out in an electrical capacitor, in which we put two plates with different signs or potentials in a virtual contact that gives rise to a flow and accumulation of charge, on which an electric current and a circuit is set in motion. The first term represents that charge accumulation, which can give rise to an initial continuous flow, until it is charged, and the second represents a circuit superimposed on the capacitor that allows it to fluctuate, as occurs with $\upsilon \mathrm{e}^{-i2\omega_0 t}$, at the rate of a minimum alternating current. The residual character of this current, of this phase function that diffuses from one WP to another, without participating in the energy balance but leaving its presence evident, makes us conceive the second term, in effect, as a by-product, *i.e.* as the consequence of a coupling between the summands of the first term that is not perfect or not instantaneous, *i.e.* one that one that opens a channel of diffusion before being carried out totally.

If we consider that Equation (42) is the solution sought for each of the channels or semitoroids, and that in them there is part of a WP and the part of the other that is filtered, we can reconfigure this equation in the analogous form, already expressed in Equation (21),

$$\begin{aligned} \dot{x}_{k1} &= \upsilon_{\omega 1} - \upsilon_{\omega 1}\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t} = \upsilon_{g1}^{c} - \upsilon_{g1}^{c}\mathrm{e}^{-i2\omega_0 t}, \\ \dot{x}_{k2} &= \upsilon_{\omega 2} - \upsilon_{\omega 2}\mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t} = \upsilon_{g2}^{c} - \upsilon_{g2}^{c}\mathrm{e}^{-i2\omega_0 t}, \end{aligned} \tag{43}$$

which we can now interpret as branches along the toroid, and not channels, since the solutions themselves are the crossing of the terms of the Ψ_A and Ψ_B channels. That is, in this case each of the solutions is not the sum of WPs (in their different forms) that make up the channel, but the contribution of a WP to the different channels. Naturally, analogous to the previous case, we can change the perspective and say that the second term is in the same channel as the first with a foreign carrier.

This way of expressing the solutions, either with one perspective or the other, allows us to say that the group velocity of the WP in the material processes has, thanks to its geometry, the corrected form υ_C^1 given already in Equation (23), and similar to υ_C of Equation (32). That is, $\upsilon_C = \upsilon_{C2} - \upsilon_{C1}$, for:

$$\upsilon_{Ci} = \dot{x}_{ki} = \upsilon_{gi}^{c} - \upsilon_{gi}^{c}\mathrm{e}^{-i2\omega_0 t}, \tag{44}$$

which is nothing but the variation that the group velocity of each WP undergoes as a consequence of extending to the whole toroid the space of development, that is, of doubling that space.

Continuing with this, we can think of a WP with a certain value λ in channel A and the same value λ in channel B. On the other hand, since we know that both WPs progress through the other channel, we can assume that $\mathrm{d}\lambda = \lambda$, that is, that the possible variation of λ for each WP (what it can extend) is the value λ of the other channel, and that this variation, in this case, is not fixed but is regulated by the phase through the additional wave equations, which are regulated by the exponential, from which it would result $\mathrm{d}\lambda = \lambda \mathrm{e}^{-i\omega_0 t}$. This allows us (following a reverse process) to obtain Equation (31) from Equation (44), or to give greater foundation to the factors of the former in order to obtain the latter:

$$\upsilon_C = \upsilon_g^c - \upsilon_g^c \left(\frac{\lambda \mathrm{e}^{-i\omega_0 t}}{\lambda} \right)^2 = \upsilon_g^c - \upsilon_g^c \mathrm{e}^{-i2\omega_0 t}, \tag{45}$$

revalidating through this procedure the result we already achieved in Equation (32) with a wave treatment, which strengthens us in the idea that zitter is a wave phenomenon derived from a wave reality and a geometry such as the one we have presented. A wave reality that, thanks to this geometry, is quantum and internal.

This wave treatment of the phenomenon serves to emphasize that the variations of a wave packet are carried out in the other wave packet, which is why there is no dispersion of the WPs, contrary to what happens theoretically and experimentally with all WPs, enabling the possibility of representing a stable material structure by means of this configuration, as we have already developed [2].

5. Etiology of Particle Creation

Since matter is described by SWP, most material properties and corpuscular behaviors must be characterized by it, that is, linked to or determined by its topology and the presence of carriers, as is the case with zitter. We will not delve into many of them here, as they would require a more elaborate scenario, but only those that arise directly from the zitter and can be dealt with more immediately, in order to present this link and incorporate some elements and insights from the SWP perspective, which we believe to be of interest for scientific discussion and further study.

5.1. Etiology of the Corpuscular Speed

In the zitter phenomenon what is striking is obviously what departs from classical operation, that is, that there is a secondary velocity, which also involves an oscillation double to that which, in the best case, could be expected. A phenomenon that can be explained, as we have done, through a process of deconstruction, the contest of two WP and a procedure of material formation. A process of deconstruction and procedure that, on the other hand, have revealed an unexpected fact, more primordial and surprising if possible, that does not involve the second term of Equation (5) but the first one, which is the conversion of the wave velocity of the WP into classical corpuscular velocity or generation of the one through the other, since, as we have seen, not only matter is created but the relative corpuscular velocity $\upsilon \in [0, c[$ of the same: inertial frame of reference. That is, a corpuscular velocity field is created, which always finds light at a speed c, as a consequence of the confrontation or association of two WPs and, more specifically, of the difference $(\upsilon_1 - \upsilon_2)$ of the decrements of the group velocities of the two WPs, which makes the classical corpuscular velocity υ, likewise, a consequence of the wave or non-corpuscular phenomenology, and to this phenomenology its etiological foundation. This, which has been highlighted to us through Equation (20) of the study, is something that is already actually present in the SWP equations for material creation, that is, in how only one corpuscular velocity υ appears in Equation (35a) despite having two WPs in Equation (34).

Discussion: Speed and Conservation Principles

Having accepted the SPW mechanism for the creation of matter and taken into consideration the way in which velocity is created within the creation process, there are additional reasons to understand and accept that the SWP mechanism is the relevant, or even inescapable, mechanism for such creation. Indeed, starting from a WP with $\upsilon_{g1}^{c} = \upsilon_{\omega 1} \approx c$, it is evident that we cannot lead to a result $\upsilon_{g}^{0} = 0$ for the particle from that single WP, and that we need a second WP with $\upsilon_{g2}^{c} = \upsilon_{\omega 2} \approx c$ that achieves this change of reference in υ. That is, it is emphasized in Equation (36) that the radical variation of that reference, by means of two opposing waves that cancel their phase, giving rise to a null reference resultant ($c \to 0$), is the physical foundation of the change of material state or densification of the matter, since it is the only way in which it can preserve the principles of conservation, and the only way to achieve in a continuous way what done in any other way would be an abrupt jump between two disparate states.

Consequently, since there must be some way to create particles and the aforementioned principles must be preserved, as in any pair creation-annihilation phenomenon, the plausibility of such a preserving dynamic in this particular and primordial case is not so strange. More precisely, it must be preserved in the SWP, which involves not only the confrontation of the two velocities referred to but the whole energy balance involved in the symmetrization process in which the final resultant of two individual WPs satisfies, utilizes and benefits (making virtue of the obligation) from the most basic principle of the principles, that of the conservation of momentum.

That is to say, by this particular, and well thought (without taking into account our development), the symmetrized imbrication of two opposite waves is the only way in which it is possible to achieve, according to the principles, what is finally achieved, that is, to bring to rest two constituents with a high momentum p, and to create another different thing from them, without the need (for these purposes) of a mediating mass. The difference between the creation of matter through a pair of photons and the one we are dealing with here is that by the SWP we do not need to create a pair of particles because the conservation of momentum is managed by the WP in the form seen, which to these effects behave or are two pseudoparticles.

5.2. Wave-Matter Duality

In the same way that through the solution of H_D a zitter wave kinetic development associated with the classical behavior is revealed, ETE, as an energetic solution of the SWP, also reveals a whole wave energy development associated with this classical behavior, precisely because it is not developed from classical corpuscular but wave assumptions. The classical phenomenology indicates that the speed υ is associated with the mass of the particle and we find it in its kinetic energy, while by means of ETE, although we find the same in the corpuscular part of Equation (35a), we also find that there is a wave term, which is the phase factor $\mathrm{sen}[\Phi]$, for the phase [2].

$$\begin{aligned}[\Phi] &= \left[\Delta k\left(\upsilon t - x\right)\right] = \left[\Delta k\left(a\gamma^{-1}\right)\right] \\ &= \left[\beta_i\left(\left(\frac{1}{m}\right)^{1/3}\gamma^{-1}\right)\right] = \left[\beta_i\left(\left(\frac{1}{m}\right)^{1/3}\left(1-\upsilon^2/c^2\right)^{1/2}\right)\right],\end{aligned} \tag{46}$$

which also accounts for the corpuscular speed υ. In the same manner, by the way, that it accounts for the charge, since in fact $[\Phi] = [\Phi]^{-q} \equiv \phi$ for $q = -1$, which is but a consequence of the introduction of the same exponent q in the starting function, that is $\Psi_i = \vartheta_i^{-q}\mathrm{e}^{-i\omega_0 t}$ [5].

The question is that when we treat $\upsilon = \upsilon_g^0$ in Equation (46), we are not treating υ of the kinetic term of corpuscular nature, but υ_g^0 of wave nature, which is associated with mass and certain dynamical variables, as expressed in Equation (8), and therefore the phase factor $\mathrm{sen}[\Phi]$ is the matter wave, and is the precursor of the wave-corpuscle duality and of all phenomena involving such duality [3].

That is to say, the matter wave is the first manifestation that in the energy balance there is something that is not corpuscular in the corpuscle itself, just as occurs in zitter for dynamic purposes. In one case by the envelopes and the other by the carriers. Both cases are expressing the same thing and they are expressing it for the same reasons, which are no other than the internal construction of the electrons or their separability into two differentiated entities, apart from the obvious difficulty of distinguishing or achieving such separability. This is perhaps clearer if we take into account that:

$$\mathrm{sen}[\Phi] = \mathrm{sen}[M] = \frac{\mathrm{e}^{iM} - \mathrm{e}^{-iM}}{2i}, \tag{47}$$

that is, if we notice that even in the matter wave is reflected the combination of the two wave envelopes, of two entities, as it occurs in the zitter by means of $\exp\left(-2iHt/\hbar\right)$. In the zitter, the exponential establishes a relationship between defined speed ($\dot{x}_k$) and energy (H), while in the exponential of the matter wave, it establishes a relationship between defined energy (E_k) and its group velocity

($M = \left[\Delta k\left(\upsilon t - x\right)\right]$) [2].

5.3. Spin

All the models that have tried to overcome what Schrödinger proposed [8], concerning spin, have done so on the assumption of wanting to solve an inconsistency or answer to a conception of the corpuscular motion associated with the zitter that contravened the most elementary aspects of physics. In this regard, we take it for certain that there is no zitter associated with the particle along its classical trajectory, there is a zitter along the closed circuit of the toroid, which is quantum (not classical) in the sense of being internal and underlying the material states. A zitter that does not initially represent a circular motion but an oscillation with respect to the main motion of the carriers, which will entail a transverse displacement with respect to them, on an equilibrium position (this is, if we remember, what a phase does on the stationary pattern). A transverse displacement which, according to each of the phase factors $\mathrm{e}^{-i\omega_0 t}$, we can understand to be double frequency (or double action), paradoxically giving rise to a circular movement of the envelopes and their virtual axis of rotation at that rate, to an ultra-fast vibration or beat.

Once we have seen that the velocity associated with zitter does not imply a translation of the particle but a fluctuation that ends up being a rotation in a second order (which we will address after this preliminary explanation), and that this rotation is not in itself of the particle but a movement of the phase through it, which does not violate relativistic principles, we can try to understand spin from the same initial perspective or initial working hypothesis of the foundation of that internal property, and its connection with the zitter.

From this initial perspective, it was postulated that zitter may be the basis of the intrinsic angular momentum $s = (1/2)\hbar$ of fundamental particles, where, in particular, the doubling of the angular frequency of Equation (5) with respect to the frequency associated with the particle in question would give rise to the ratio ½ or spin value that provides a measure of said angular momentum. According to this hypothesis, said angular momentum would be motivated by a circular motion of radius $\lambda\!\!\!^{-}_D \equiv c/\omega_D = \hbar/2m_e c$ and a value $p = m_e c = \hbar\omega_0/c$, that is:

$$S = p\lambda\!\!\!^{-}_D = \hbar\omega_0/\omega_D = \hbar\left(\frac{1}{2}\right). \tag{48}$$

on which it should be noted that the invariance of the quotient, that is, of the spin, is guaranteed in the face of energy fluctuations affecting the frequency, since they would affect both the numerator and the denominator of the equation, of which, beyond this, we will have to verify its physical coherence.

Concerning this coherence, we see that the momentum p, as it is defined, of the whole mass m on a radius $\lambda\!\!\!^{-}_D$, faces two problems. On the one hand, the mass is not punctual (toroid) which prevents defining a unique distance $\lambda\!\!\!^{-}_D$. On the other hand, this distance $\lambda\!\!\!^{-}_D$ is much smaller than the radius of the toroid [5], which disqualifies it as an axis of rotation. The solution to this set of impossible physical behaviors is exposed in some way already, and we derive it equally (or complement it) from the correct interpretation of Equation (48). This equation says that m gives rise to a spin momentum over a distance $\lambda\!\!\!^-_D$, but it does not say that all the mass m establishes that distance over the same point, *i.e.*, the momentum we handle may be $p=\sum m_i c=\hbar\omega_0/c$, where each m_i gives rise to an p_i over an identical distance $\lambda\!\!\!^-_D$ established over a different point $O_i\left(m_i\right)$. What we are saying, according to the zitter motion itself, is that each m_i of the toroid follows the fluctuation of the carriers, that is, an oscillating motion according to those carriers, equivalent to a rotation, which generates infinite internal angular moments, and intrinsic to the material nature itself, whose sum is the spin, which is presented as a pulse or vibration of resonance throughout the toroid.

Discussion: Quantization

Having said this, we look again at the final quotient of Equation (48) to realize that apart from the initial terms, the spin $s=1/2$ is reduced to the ratio of the starting frequency to the double frequency, depending on the number of phase functions $e^{-i\omega_0 t}$ or equivalently, on the number of WP put into play in the corpuscular conformation, which in this case is 2, consubstantial to the SWP. That is, we are saying that the quantization of spin, as an intrinsic property, is a consequence of an internal relation as prosaic as the number of pieces that compose the material puzzle. Put another way, and in a slightly more sophisticated manner, and taking into account that the energy of the corpuscle represented by the SWP is $E=\hbar\omega_0=mc^2$ and that of the photon represented by each WP is E_p, $s=1/2$ gives us the ratio E_p/E, from which we obtain the number of photons that make up the corpuscle.

From there we can ask ourselves what motivates other physical entities, whether corpuscular or not, to have a different spin value. In particular, we can ask ourselves why photons have the spin value $s=1$, which, according to the above, would be related to a single phase function of a single WP, with the ability to flow in a circular manner. We can also ask ourselves (in reverse) what might happen, with respect to what we saw in the SWP, if it were a single WP that, joining its ends, achieved that type of typically corpuscular geometric configuration.

A priori, it seems clear that in this case we would be talking about only one of the elements of Equation (25), without the possibility, therefore, of relating elements and doubling the frequency in the second term, as appears in Equation (27) and subsequent ones. A second term that could only be given in this case by the progression and contribution of the first term to itself through the only existing union, which in relation to υ_g^c would be indistinguishable, as well as ineffective and fictitious (since what comes out is equal to what goes in).

Whether or not this propagation exists (which we will now see), it seems clear that in this case, according to Equation (48), we would have a single WP curved upon itself and a single carrier or phase, which would give rise to the unit value as the inverse of the number of phases or, in other words, to a frequency $\omega_i=\omega_0$, which would also lead us, as we anticipated, to an integer spin value:

$$S = \hbar\omega_0/\omega_i = \hbar(1). \tag{49}$$

This leads us to believe that, in particular, photons are simply WP twisted into that quasi-corpuscular form of $s = 1$, which they can circumstantially take on or abandon (hence the duality), while for entirely material particles it would imply the coupling of toroids with spin ½ (with which the value $s = 1$ could obviously also be achieved). We see with this that the analysis not only leads us to understand the particles but also to understand what photons are and to differentiate them, through geometry, from their elementary or wave-like expression.

We need to put ourselves in context to better understand this. First, an electromagnetic wave that can circumstantially become a photon is ideally an infinite wave. It is because it is infinite that it is a flat wave to which we can only associate a phase velocity $\upsilon_p = c = \lambda\nu$. We can assume (assure) that it is not actually infinite, and that it has a beginning and an end, which would make it a pulse that, being equivalent to a WP [10], would have a group velocity $\upsilon_g^c \simeq c$ because it is practically indistinguishable from the ideal case, although there are already experiments that prove, precisely because of this, that there is a reduction in group velocity even for a single photon as soon as we can introduce a spatial structure [13]. More explicitly, the experiment associates υ_g^c with a photon under certain conditions, and the more restrictive theory associates υ_g^c in any case (the flat wave is an ideal) regardless of whether we are able to verify and measure it. In any case, the speed of the photon in that toroidal arrangement is υ_g^c, that is, the one expressed by Equation (7), with no evidence of an indistinguishable and ineffective additional term which, moreover, would have no reason to exist based on the equations for $m = 0$.

That said, we conclude that flat waves are an ideal expression of a WP which, as such WP, can adopt a deployed (flat) arrangement or a closed circular geometry (non-deployed arrangement) in which the beginning and end of the wave are joined, as is the case with the SWP. For a single WP to adopt a closed circular geometry (not necessarily toroidal) implies, to begin with, that it adopts a spatial and corpuscular configuration, since it is this characteristic, and no other, that establishes a local change in material phase and a differentiation of material phases, and which determines that the WP remains stable and does not disperse systematically and endlessly (with limit asymptotic zero).

We must bear in mind, with regard to this systematic dispersion, that if individual WP's were dispersed (which is what they tend to do due to entropy and what they would do without a specific topology), they would reach an ideal or infinite flat wave situation, and thus the entire universe would be bathed solely by the same background radiation. This leads us to believe that this topology (or individual quasi-corpuscular form) is the effective way that photons have found to achieve this, that is, to preserve their integrity and not generate entropy without doing effective work, neutralizing (through the configuration of another system) the systemic or inherent dispersion of the WP.

That is to say, we know that energy is more usable (and its application more reversible) when it is concentrated than when it is dispersed. The incorporation of energetic photons corrects, minimizes, or neutralizes the natural increase in entropy (2nd law of thermodynamics), that is, the decrease in usable energy. This is what happens in relation to the energy value of the photons that arrive from the sun compared to the much more numerous low-energy photons that we emit to the outside, which leads to an increase in entropy in the universe, which would be greater (and this is the question) if these last photons underwent the gradual and natural dispersion of the WPs. We therefore say that this topology (or alternative configuration of the WP) improves the purpose and effectiveness of this exchange mechanism in that it makes any photon (such as the outgoing photon of the mentioned interaction) a closed system, which preserves the useful energy charge when it does not interact (passive or inactive phase) and prevents the unnecessary increase in entropy of the universe. Which, in other words, allows for the viability of the universe in time (the existence of temporal transit), as indeed does the SWP and its systemic non-dispersion for the corpuscular entity for the same reasons.

6. Discussion

This paper consists of two distinct parts on two different frameworks, plus an additional phenomenological section which contains its own discussion or projection of results. In the first part (divided into two sections), we use what we know about the zitter problem and physics in general to address this problem, which, despite being elementary (or precisely because of this), we have not omitted anything so as not to test the reader's memory of basic concepts and the details that make up the study scenario. In the second part, we use our own resources, that is, the knowledge that we have achieved in relation to the SWP.

Leaving aside the initial presentation of the problem, each part contributes something to the solution or understanding of the zitter. In the scenario described in the first part, we postulate (Section 2) the possibility that the term zitter corresponds to the coupling of the carriers of two WPs, which can be decoupled in a certain way. A decoupled form that is consistent, according to optical physics (Section 3), with the propagation of any WP, or the form that a WP would take in a circular topology. We cannot ensure that this circular topology exists in the material environment or that these two postulated WPs are present. However, by the second part (Section 4), we have WPs with initially separated carriers, in accordance with the first point above, and a topology that makes the second point possible. That is, overall, more precisely and from a canonical perspective, we are postulating (for the first point) that if matter were made up of WP's in a circular arrangement, these would achieve the precise form to give rise to the zitter if we impose the weak conditions $\lambda_1 = \lambda_2$ (innate to symmetry) and $d\lambda_i = \lambda_i \mathrm{e}^{-i\omega_0 t}$ (the natural variation in time of the variable responsible for the phase). Meanwhile, on the SWP side (for the second), we are providing all the requirements, which may (or may not) be understood as sufficiently accredited through the references.

The second, in this case, cannot invalidate the first; it can only validate it or not instrumentalize it. However, since the first is true, regardless of the second, and since the second is the promoter of the material reality associated with the SWP, it seems difficult for both processes to be dissociated, that is, for there to be no causal relationship between them. In other words, given that zitter can be reached under certain conditions, and zitter occurs, it seems reasonable or natural that those conditions should occur, insofar as it can also be justified by the SWP, and since SWP deals precisely, apart from the zitter, with the corpuscular environment or the inmost form of matter. This leads us to place the burden of proof of the results in its proper place and to open our minds to this line of argument, which is also relevant to many other issues, insofar as we cannot ignore or even invalidate the SWP's instrumentalization of the zitter based on the SWP's ignorance, or a priori (and unfounded) disregard for it, since the lines of argument (zitter and SWP) reinforce each other.

From another perspective, in the first part we connect the problem with a possible solution, and in the second part we connect the SWP scenario of an energetic nature (and, consequently, unrelated to the problem) with that possible solution, and with the underlying phenomenon of the problem itself. In other words, the physical environment of the first part provides us with the details of the problem and the solution (although not conclusive) through recognizable and unimpeachable physics. Meanwhile, from the SWP, and the progression of the carriers as a condition, we achieve a proper and conclusive development of the solution, and a better understanding of the nature of the problem. A condition that does not seem to be a great demand, that is, it does not seem that transmission between two equal media (without a potential barrier) could be anything other than the transparent progression we have represented, without prejudice to the possibility of later arriving at a more formal or rigorous representation of this solution with the help of a wave equation linked to the SWP. It is to endorse this total transmission of the phase through the circuit that we have introduced the analogous behavior, in this regard, of standing waves.

We can say that there are three lines of argument (a result of the Dirac equation, optical equations, and SWP equations) that point in the same direction and make up a very simple and impeccable outline of the zitter question, that is, an irrefutable logic, achieved from a simple additional assumption or premise. **We are establishing that:**

(1) The corpuscular speed could be a composition of speeds with physical meaning (premise): $\upsilon = \upsilon_1 - \upsilon_2$.

[Note: This composition of corpuscular speeds is equivalent to a difference in wave speeds $\upsilon_1 - \upsilon_2 \equiv \left(c - \upsilon_{g1}^c\right) - \left(c - \upsilon_{g2}^c\right) = \upsilon_{g2}^c - \upsilon_{g1}^c$ on a speed c.]

(2) The zitter speed, which is a wave speed, accompanied by factor $\mathrm{e}^{-2i\omega_0 t}$, can also be (premise) a composition of wave speeds

$\left(c - \upsilon_g^0\right) = \upsilon_g^c = \upsilon_{g1}^c - \upsilon_{g2}^c = \left(c - \upsilon_{g1}^0\right) - \left(c - \upsilon_{g2}^0\right)$ accompanied by that factor $\mathrm{e}^{-2i\omega_0 t} = \mathrm{e}^{-i\omega_0 t} \times \mathrm{e}^{-i\omega_0 t}$, on a speed c common to all of them (which naturally operates differently on υ_{gi}^c than on υ_{gi}^0).

[Note: The total speed (sum of the two) will in this case take the final form $\dot{x}_k = \left(\upsilon_{g2}^c - \upsilon_{g1}^c\right) - \left[\upsilon_{g2}^c - \upsilon_{g1}^c\right] e^{-i2\omega_0 t} = \upsilon_g^c - \upsilon_g^c e^{-i2\omega_0 t} \equiv \upsilon_C$ given in Equation (20).]

(3a) In this way, the total corpuscular speed $\dot{x}_k = \upsilon_g^c - \upsilon_g^c e^{-i2\omega_0 t} = \upsilon_C$ is analogous to that of a wave packet (WP) in a circular and closed topology expressed in Equation (31).

[Note: Here we already have a result, if you want to see it ($\dot{x}_k = \upsilon_C$).]

(3b) Strictly speaking, $\dot{x}_k$ is not analogous to a WP, but rather to two WPs ($\Rightarrow \upsilon_C = \upsilon_{C2} - \upsilon_{C1}$), according to the second member of the equation, which suggests that the premise, and in particular $\upsilon = \upsilon_1 - \upsilon_2$, makes sense as the corpuscular representation of the coupling of two WPs (susceptible to being decoupled) in the aforementioned topology.

[Note: This is already a complete wave characterization of the corpuscular result (without using the SWP)]

(4) Once this has been established, and having speculated on how to decouple this combination of WP, we verify that the result achieved and everything stated is compatible with the SWP and its intrinsic topology, or, to put it better, that the SWP is capable of accounting for all of this on its own, since it is based on the existence of two WPs in a toroidal topology [(4) $\Rightarrow$ (3)].

(5) From which we can conclude that if (1+2) occurs, then (3) would be fulfilled, that is, we would be faced with two WPs, which in turn would justify (1+2), or otherwise [WPs:(3) $\Leftrightarrow$ WPs:(1+2)]. Being able to escape the loop (twice) through (4), which, on the one hand, directly fulfills the starting condition, (4) $\Rightarrow$ (3) [$\Rightarrow$ (1+2)], and on the other hand validates the premise in (1+2) [$\Rightarrow$ (3)].

It should be noted that although the SWP fulfills the conditions required by the zitter from that canonical perspective, the SWP alone has been able to arrive at the zitter result through the aforementioned propagation of carriers and simple extrapolation between envelopes and group velocities. This also implies that, despite the zitter not being an energetic phenomenon, at some point in a future analysis we could have arrived at the idea of the zitter through the SWP without prior knowledge of the zitter, if through that hypothetical analysis we had become convinced that the SWP carriers propagate from one half of a toroid to the other, giving rise to Equation (39) and following in the development, that is, if we realize that these carriers do not disappear in the material conformation.

We would also like to highlight that the treatment carried out by dissecting the solution and the existing equations is the most analytical and rigorous that can be performed. In other words, the analytical treatment of the zitter is what is given until Equation (5) is reached. From there, there is no choice but to carry out the construction process used to link with the SWP (which has its own prior analytical development), since, as we said, H_D does not deal with WPs or half particles. A more rigorous or formal representation would involve the use of a wave equation attached to the SWP that provides Equation (20) in some of its variants. All the work done so far confronts us with the need to achieve this.

In the third part of this work, we address issues that are directly related to the zitter (spin) or the treatment carried out (corpuscular speed), as well as other immediately related particularities, which we have consequently been unable to avoid or leave out of the study. We have not left them out because of this direct relationship and because they serve to show that the result achieved confronts us with something that goes far beyond the zitter; that is to say, the zitter has served to reveal these issues, which have a much greater significance than the zitter itself (the movement of the carriers). What is revealed, if one cares to see it, is the true importance of this work.

The real importance lies not in the zitter (which is just one more effect) but in the consequences of the existence of two elements in the construction of the material entity and, therefore, in the very existence of those two elements (SWP). We have highlighted this importance in the two cases presented, but we could have presented similar results regarding kinetic energy or momentum, which, not being so immediate and requiring specific frameworks, we have left aside. We could have presented them because the SWP, as the foundation of the structure and dynamics of the fermion, which accounts for everything that happens to it, offers them. Let us consider, for example, that with the intrinsic spinor (SWP) we have not only justified spin and corpuscular speed, but we have also surreptitiously justified the positive energy differential between matter and antimatter, which has already been addressed from another perspective [2], in that each energetically positive WP supports a negative one, *i.e.*, according to this, there would not strictly (or only) be a differential between matter and antimatter, but rather a material or immaterial composition between both types of energy. A composition that would camouflage the negative energy it uses, which, once quantified, could be all that is missing in this material universe. This alone is a rich vein or niche for study.

We see that SWP is not a result; it is an element on which we can build an entire physics because, as in the previous case, each finding leads us to a discussion and each discussion to a finding or interpretation of physical reality. This is why we maintain a continuous discussion, interpretation, or expansion in line with the results, not as final considerations glossed over in a standard epigraph, but as an adjustment or modeling of the theoretical framework that emerges from them. Ultimately, we are sketching out a paradigm.

To accept the SWP as possible and well-articulated (I am not even saying to take it as true) is to accept another possible way of understanding matter and physics itself. There are essays that, regardless of objections and particularities, must be kept in mind because what they contribute, beyond what they seem to offer at first glance through their results, is a different working framework that needs to be deployed and explored, a tool. A tool with a very different scope, as has been shown here with regard to the zitter.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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