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General Maxwell Theory of Fields (3): Lorentz Covariance, Quantization and Gauge Transformation

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Abstract

This paper first reviews our previous research on the general Maxwell theory of fields, then discusses the topological significance of special relativity and deduces the Lorentz invariant of the gravitowagnetic field based on the field theory established in 3-dimensional space. By means of the successful path of electromagnetic field quantization, the quantization of gravitowagnetic field including gravitational field and wagnetic field is derived. Under the condition of ensuring the invariance of the 4-dimensional spacetime interval and the positive direction of time, a parity 4-dimensional spacetime coordinate system is established, and the tensor form of the vacuum field compatible with electromagnetic field and gravitowagnetic field is derived in the system. It is proved that the excited-state vacuum field generated by stationary mass and moving mass is Lorentz covariant. The unified expression of the electromagnetic field and the gravitowagnetic field is obtained, which is a representative formula of general Maxwell theory of fields. Finally, the gauge transformation of the field is discussed.

Keywords

Gravitowagnetic Field, Lorentz Covariance, Quantization, Gauge

1. Introduction

1.1. Brief Review of the Previous Research

Research on the general Maxwell theory of fields consists of three articles. This paper is the third one. The previous two are “General Maxwell Theory” [1], and “General Maxwell theory (2): Explanations and Predictions of Some Phenomena” [2]. The reference [1] reviews Maxwell’s idea of comparing the lines of force of the electromagnetic field to an incompressible fluid, and notes that his idea was successfully applied to fluid mechanics: when the velocity field is analogized to an incompressible fluid field, the velocity field formula is derived by applying the Biot-Savart law of the electromagnetic field. Noticing the above facts, we believe that since all fluids have mass and charged particles also have mass, therefore, the Biot-Savart law can be used to derive an incompressible fluid generated by mass flow density. We use a new term "wagnetic field" to describe it, and another new word "gravitowagnetic field" to describe the gravitational field and wagnetic field. We discover that the algebraic form of wagnetic field has a parity relationship with the algebraic form of magnetic field. At the same time, we also notice that the field excited by the rest mass (Newton's gravitation formula) and the field excited by the static charge (Coulomb's law) also have a parity relationship in the algebraic forms. We think that in the real world, there are no mathematical tools such as coordinate systems. All physical laws are described in a mathematical space. In this abstract space there are no charges and masses, only the concepts of charges and masses are introduced into the space, which cannot change the inherent symmetry of the mathematics space. Therefore, the parity of the electromagnetic field and the gravitowagnetic field must be the large-scale parity displayed by the excited vacuum itself. In reference [2], we explained the reason for allowing local symmetry breaking in the case of large-scale parity symmetry: anti-particles (positrons and anti-electron neutrinos) emerged. According to Feynman's quantum electrodynamics rules, the anti-particles correspond to local time reversal. This rare phenomenon does not affect the fact that the overall direction of time in the large-scale configuration space is positive. We also relate this phenomenon to the overall magnetic order (corresponding to the overall time order) that occurs during the phase transition of a paramagnetic system, where local magnetic disorder is allowed to exist, that is, there is a reverse relationship between local spins and the total spins (corresponding to a reverse relationship between local time direction and overall time direction), because as long as the temperature is above absolute zero, local disorder states always persist under the condition of large-scale order. Lee T. D. pointed out that the breaking of symmetry must involve force and field that can be observed, which are responsible for the asymmetry; there is weak force field in local area so far, which is responsible for the local parity breaking. The weak interaction force cannot be directly observed in the macroscopic world. Its effective range is only about 10^{-18} meters, which is roughly one-thousandth the diameter of an atomic nucleus. It primarily affects fermions, quarks, and neutrinos (including their anti-particles), and governs radioactive phenomena like beta decay. Its effects are mainly evident in the decay processes of microscopic particles. The so-called large-scale space refers to the macroscopic world, in which only electromagnetic force and gravitowagnetic force exist. Up to now, no a force or a field has been found which is responsible for the large-scale parity breaking, so the large-scale parity must exist. Reference [2] states that since humans face the only real world, it cannot be described mathematically using paradoxes: the electromagnetic field theory persists that the space is flat, while the general relativity theory suggests that the space is curved. The reference [2] states that when the points on a curved surface have a one-to-one correspondence with the points on a plane in both directions, the curved surface and the plane are topological homeomorphism, they are considered equivalent. This allows us to describe the field of mass excitation in a flat space. Helmholtz's theorem is an important principle in vector field theory, which has been repeatedly proved and expounded in theory [3]-[6]. This theorem states that as long as any differentiable vector field decays rapidly (with the distance r) to zero (for example, in the form of $1/r^2$) than $1/r$ when $r \to \infty$, the vector field must be composed of the gradient field of a scalar potential and the curl field of a vector potential. The gravitowagnetic field, like the electromagnetic field, meets the requirements of this theorem, and thus this theorem once again proves that the field excited by electric charges and those excited by masses have parity [1] [2].

Theoretically, the Lorenz gauge under conservation of mass, Lorentz force, Faraday's law and Maxwell's displacement current law in the gravitowagnetic field are derived [1]. The difference from the electromagnetic field is that the former laws show "compliance" rather than "resistance", while the later laws show "resistance" rather than "compliance". Using the Reynolds transport theorem from fluid mechanics, it is proved that these two laws are not merely effects but also are themselves transport processes. Without them, there would be no wave phenomena [1]. Although the fluctuation of the mass excitation field has been confirmed in the experiments, neither theoretically nor experimentally can we rule out the possibility of the existence of the wagnetic field excited by moving mass.

In the reference [1], we proposed a physics experiment plan to verify the existence of wagnetic field: In the helium-4 ^4He superfluid experiment, the phenomenon of liquid climbing the wall in a specific direction: When superfluid occurs at low temperatures and is affected by the wagnetic field of the Earth, due to the "compliance" effect corresponding to Faraday's law, the wagnetic field generated by the directional movement of superfluid molecules is in the same direction as the Earth's wagnetic field. In the Earth's wagnetic field, the superfluid molecules near the wall are subjected to an upward Lorentz force and thus exhibit the climbing wall phenomenon. Due to the constant direction of the Earth's wagnetic field, the climbing wall phenomenon shows a directional characteristic.

In references [1] [2], using our theory, we explained: 1. The trajectory of Mercury's perihelion; 2. The weak field approximation of general relativity; 3. The gravitational lensing; 4. The dark energy. Especially, we have reveal the fundamental cause of the weak field approximation: Because the mass of objects on the Earth is too small and their falling speed in the gravitational field is too slow, the Lorentz force they experience is extremely weak to the extent that this deflection cannot be detected at all by the observers' sense on the Earth. Therefore, the free fall motion on the Earth is often considered as linear motion. Obviously, this situation meets the condition of weak wagnetic field or weak Lorentz force, and thus the equivalence principle of general relativity holds. Something similar happened in the early days of the theory of universal gravitation force. It was questioned by many people: Why can't we see the phenomenon of objects on the Earth attracting each other? Newton responded as follows: I answer, that since the gravitation towards these bodies is to the gravitation towards the whole earth as these bodies are to the whole earth, the gravitation towards them must be far less than to fall under the observation of our sense [7].

We also predicted four physical phenomena [2]: 1) Chromatic dispersion in the gravitational lensing; 2) Self-focusing effect of light; 3) Electromagnetic field and garvitowagnetic field can generate each other and transform into each other. 4) The vector potential of wagnetic field can participate in the A-B effect.

The above review presents the main important theoretical results, four predictions and a physics experiment plan for testing our theory. For detailed theoretical derivation, analysis, and discussion, please refer to the contents of these two articles [1] [2].

1.2. Preface of This Paper

An important conclusion of Maxwell's theory of electromagnetic fields is that the speed of electromagnetic waves in a vacuum is a constant that is independent of the reference frame. However, according to the Galilean transformation of classical mechanics, the velocity of an object is dependent on the choice of the inertial frame of reference, which means that the speed of light should vary depending on the choice of inertial frame. Experiments have confirmed that the speed of light remains constant in all inertial frames of reference, thereby disproving the existence of a hypothetical medium called "ether". This implies that the relativistic version of classical mechanics needs to be revised. Based on his firm belief in the correctness of the theory of electromagnetic fields, Einstein proposed a new theory, special relativity, which is grounded on the principle of the constancy of the speed of light. Einstein hypothesized that the laws of physics remain unchanged in all inertial reference frames. Experiments have confirmed this hypothesis. For electromagnetic fields, Maxwell's equations remain invariant in all inertial frames, which is called Lorentz invariance, while the magnitudes of the electric field intensity and the magnetic field strength may vary with different reference frames.

For the above content, we can put it in another way. The so-called vacuum we refer to is the mathematical space before the concept of matter was introduced. We merely introduce the concepts of charge and mass into the mathematical space. The vacuum, after the introduction of these concepts, is in an excited state. Clearly, these artificially introduced concepts cannot change the inherent nature of the mathematical space: its topological properties. Therefore, the essence of physical laws expressed in mathematics is the topological properties of space. Abstract topological properties cannot be observed in the real space; however, they are manifested in the form of physical laws and can be perceived by humans through their own senses or measurement instruments. Two characteristics of light: the speed of light remains constant in a vacuum, and the movement speed of all objects does not exceed the speed of light. This makes light a measurement method commonly used by all inertial reference frames, thereby ensuring that the points on the movement trajectory of the observed object have a bidirectional one-to-one correspondence with all reference frames. For instance, there are total i points on the trajectory of the observed object, where $i = 1, 2, \cdots, n$. The coordinate of the i-th point has a coordinate of x_i in the first reference frame and x_i' in the second reference frame. The x_i corresponds to the x_i', and vice versa. Although the numerical values of the coordinates are different, they correspond to the same observed point. The coordinate values can be different because the measurement is not a topological property. This means that all inertial frames are topological homeomorphic spaces, various metrics, including the metric (length interval, time interval), are not topological properties. Two homeomorphic topological spaces may redundantly have different metrics. The pattern revealed by this measurements difference can be described by special relativity. Physical laws with topological properties remain unchanged under the homeomorphism, making their representations Lorentz covariant and having identical 4-dimensional algebraic forms in different inertial systems.

The Lorentz covariance of the electromagnetic field is a well-established theory. By leveraging its parity relationship with the gravitowagnetic field, a sensible path is provided for the quantization of the gravitowagnetic field. However, discussion on this quantization process vary, although the conclusions are the same [8]-[11]. This paper adopts the argumentation method of reference [11], which is concise and highlights the key points. Our main purpose is to explain the reason why the gravitowagnetic field can be quantized.

The content is arranged as following: Section 2 is theory, introduce 4-dimensional mass density, mass current density and 4-dimensional potentials. Base on the existing classical theory, derive the covariant form of the gravitowagnetic field, the gravitational field and the wagnetic field are no longer independent vector fields, together they form an anti-symmetric tensor field. Then, following the path of quantization of the electromagnetic field, the quantization of gravitowagnetic field is derived. Section 3 is discussion, after establishing the 4-dimensional coordinate system of parity, the tensor form of the excited state vacuum that is compatible with both electromagnetic field and gravitowagnetic fields obtained, and it is proved to be covariant. Using the same gauge transformation as that of the electromagnetic field, the gauge invariance of the gravitowagnetic field is proved. Section 4 is conclusion.

2. Theory

2.1. Lorentz Covariance

The following discussion is entirely conducted within the framework of classical theory, and the conclusion obtained can be compared with our existing theory [1] [2]. Our main objective is to verify that the classical gravitowagnetic field in the configuration space can also conform to Lorentz covariance in the 4-dimensional spacetime like the electromagnetic field.

There are four coordinate systems that maintain the 4-dimensional spacetime interval unchanged in mathematics: (x, y, z, ict), $(-x, -y, -z, -ict)$, $(x, y, z, -ict)$, $(-x, -y, -z, ict)$. This means that there are four local coordinate systems on the complex manifold. The first one has been used to describe the electromagnetic field in tensor form [6]. In the 3-dimensional space, when describing the gravitowagnetic field, the same coordinate system as the the electromagnetic field is adopted. This approach significantly highlights its parity with the electromagnetic field in algebraic form in the 3-dimensional configuration space [1]. Both types of vacuum excitation fields are uniformly described using the right-handed coordinate system. The Equations (25), (26), (27), and (28) in the reference [1] are characterized by the fact that the algebraic expressions on the right side of the equal signs in the two equations with the same number of formula are exactly the same forms and positive or negative signs, while the algebraic forms on the left side of the equal signs are the same but differ a positive or negative sign: the electromagnetic field is positive, and the gravitowagnetic field is negative. This clearly demonstrates the parity of the two excitation fields. On the other hand, in a three-dimensional configuration space, time is a parameter rather than a coordinate quantity, and there is no parity. In a four-dimensional complex space, time appears in a one-dimensional imaginary space with parity properties. Time is implicitly contained within the imaginary coordinate quantity. Therefore, if we want to use those formulas in the reference [1] to describe the Lorentz covariance of gravitowagnetic field mentioned above in the 4-dimensional spacetime and show its parity with the electromagnetic field, only the third coordinate system can be chosen, and rewritten as $(x_1 = x, x_2 = y, x_3 = z, x_4 = -ict)$. Next, we first write out the main formulas of the gravitowagnetic field in the configuration space to omit the subscript for all physical quantities [1]:

$$\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\frac{4\pi}{c}\boldsymbol{j} \tag{1}$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -4\pi\rho \tag{2}$$

$$-\nabla \cdot \boldsymbol{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0 \tag{3}$$

$$\boldsymbol{G} = \nabla \varphi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} = 0 \tag{4}$$

$$\boldsymbol{W} = -\nabla \times \boldsymbol{A} \tag{5}$$

Introduce the 4-dimensional potential $A\left(A_1 = A_x, A_2 = A_y, A_3 = A_z, A_4 = i\varphi\right)$, and 4-dimensional current $j\left(j_1 = j_x, j_2 = j_y, j_3 = j_z, j_4 = ic\rho\right)$. Let the coordinate axes of two inertial reference frames be parallel to each other and move in the x direction, and take the time origin when the origins coincide. Under the special relativistic conditions, the length and mass vary with the relative velocity v, so

$$j=\gamma^2 j_0,\quad \rho=\gamma^2\rho_0 \tag{6}$$

Among them, $\gamma^2=\left(1-v^2/c^2\right)^{-1}$, j_0 represents the mass density, which physical meaning is the mass within a unit volume. ρ_0 represents the mass flow density, it is a vector, its definition is the total mass passing through a unit cross-sectional area perpendicular to the direction of flow per unit time, and they are under the non-relativistic condition $(\gamma=1)$. For two definite reference frames in relative motion, the velocity keeps constant, so γ is a constant and does not change the algebraic form of the original formulas. The covariant form of the combination of formulas (1) and (2) is

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A_\mu=-(4\pi/c)\,j_\mu \tag{7}$$

The covariant of formula (3) is

$$-\partial A_\mu/\partial x_\mu=0 \tag{8}$$

All the formulas in this paper are subject to the Einstein summation convention: when a certain indication in the same term appears repeatedly, the sum of all possible values of that indicator is calculated by default. From formula (4), in the 4-dimensional coordinate system $G_1=G_x=\frac{1}{i}\partial(i\varphi)/\partial x+(i/c)\partial A_1/\partial(-it)$, so

$$G_1=G_x=-i\left(\frac{\partial A_4}{\partial x_1}-\frac{\partial A_1}{\partial x_4}\right) \tag{9}$$

Similarly,

$$G_2=G_y=-i\left(\frac{\partial A_4}{\partial x_2}-\frac{\partial A_2}{\partial x_4}\right) \tag{10}$$

$$G_3=G_z=-i\left(\frac{\partial A_4}{\partial x_3}-\frac{\partial A_3}{\partial x_4}\right) \tag{11}$$

From formula (5), in the 4-dimensional coordinate system $W_1=W_x=\frac{\partial}{\partial x_2}A_3(-\boldsymbol{j}\times\boldsymbol{k})-\frac{\partial}{\partial x_3}A_2(\boldsymbol{k}\times\boldsymbol{j})$, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the basis vectors of x, y and z, so

$$W_1=W_x=-\left(\frac{\partial A_3}{\partial x_2}-\frac{\partial A_2}{\partial x_3}\right) \tag{12}$$

$$W_2=W_y=-\left(\frac{\partial A_1}{\partial x_3}-\frac{\partial A_3}{\partial x_1}\right) \tag{13}$$

$$W_3=W_z=-\left(\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2}\right) \tag{14}$$

A_μ constitutes a 4-dimensional vector, and $\partial A_\nu/\partial x_\mu$ and $\partial A_\mu/\partial x_\nu$ constitute a 4-dimensional tensor,

$$F_{\mu\nu}=\frac{\partial A_\nu}{\partial x_\mu}-\frac{\partial A_\mu}{\partial x_\nu},\quad \mu,\nu=1,2,3,4. \tag{15}$$

Thus, there exists a tensor matrix

$$F=\begin{pmatrix} 0 & -W_3 & W_2 & iG_1 \\ W_3 & 0 & -W_1 & iG_2 \\ -W_2 & W_1 & 0 & iG_3 \\ -iG_1 & -iG_2 & -iG_3 & 0 \end{pmatrix} \tag{16}$$

The significance of F is that in the 4-dimensional spacetime the gravitational field and the wagnetic field are no longer independent vector fields, together, they form an anti-symmetric tensor field.

Under non-relativistic condition [1]

$$-\nabla\cdot\boldsymbol{G}=4\pi\rho \tag{17}$$

$$-\nabla\times\boldsymbol{W}=\frac{1}{c}\frac{\partial\boldsymbol{G}}{\partial t}+\frac{4\pi}{c}\boldsymbol{j} \tag{18}$$

Under the relativistic condition, Equations (17) and (18) are combined

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=-\frac{4\pi}{c}j_\nu \tag{19}$$

Under the non-relativistic condition [1]

$$-\nabla\times\boldsymbol{G}=-\frac{1}{c}\partial\boldsymbol{W}/\partial t \tag{20}$$

$$\nabla\cdot\boldsymbol{W}=0 \tag{21}$$

Under the relativistic condition, equations (20) and (21) are combined

$$\frac{\partial F_{\mu\nu}}{\partial x_\lambda}+\frac{\partial F_{\nu\lambda}}{x_\mu}+\frac{\partial F_{\lambda\mu}}{\partial x_\nu}=0 \tag{22}$$

Equation (22) is completely anti-symmetric with respect to μ, ν, and λ, and actually corresponds to four independent equations, corresponding to (μ,ν,λ) taking $(2,3,4)$, $(3,4,1)$, $(4,1,2)$ and $(1,2,3)$.

2.2. Quantization

The equations (16) and (17) in the reference [1] are the wave equations of the radiation fields for the gravitowagnetic field. The quantization of radiation field can be carried out by imitating the mature theory of the quantization of electromagnetic field. The following mainly explains the reason why the gravitowagnetic field can be quantized [11].

The Hamiltonian for a set of harmonic oscillators, each having unit mass is

$$H_{OSC}=\sum_m\frac{1}{2}\left(P_m^2+\omega_m^2Q_m^2\right) \tag{23}$$

With Q_m being the coordinate, $P_m=\mathrm{d}Q_m/\mathrm{d}t$ being the momentum or velocity, and m being natural number. Dirac first transformed the system into a quantum field. On the other hand, the Hamiltonian of the radiation field is

$$H_{GW}=(8\pi)^{-1}\int\left(G^2+W^2\right)\mathrm{d}^3x=(8\pi)^{-1}\sum_m\left(f_m^2+k_m^2h_m^2\right) \tag{24}$$

where f_m, k_m, and h_m are obtained through the following approach: The solution of the radiation field can be facilitated by representing the gravitational field as a sum of (wave) mode functions $\boldsymbol{u}_m(x)$, which are defined by the eigenvalue equation:

$$\nabla^2 \boldsymbol{u}_m(x) = -k_m^2 \boldsymbol{u}_m(x) \tag{25a}$$

Here, x is a vector coordinate with three components x_1, x_2, and x_3.

$$\nabla \cdot \boldsymbol{u}_m(x) = 0 \tag{25b}$$

$$\boldsymbol{n} \times \boldsymbol{u}_m(x) = 0 \quad \text{on any conducting surface.} \tag{25c}$$

where $\boldsymbol{n}$ is the unit normal to the surface. The latter condition is imposed because the tangential component of $\boldsymbol{G}$ must vanish on a conducting surface. It can be readily be shown that Eqs (25) describe a Hermitian eigenvalue problem, and so the mode functions that correspond to unequal eigenvalues must be orthogonal. The gravitational field is represented as a sum of the functions

$$\boldsymbol{G}(x,t) = \sum_m f_m(t) \boldsymbol{u}_m(x) \tag{26}$$

Similarly, the radiation wagnetic field can be represented as

$$\boldsymbol{W}(x,t) = \sum_m h_m(t) \nabla \times \boldsymbol{u}_m(x) \tag{27}$$

Note that the direction of the mode function here is opposite to its direction in the reference [11], because the direction of gravitowagnetic field is opposite to that of electromagnetic field. According to the formula (23) of reference [1], $h_m(t)$ and $f_m(t)$ should satisfy the following equation

$$\frac{\mathrm{d}h_m(t)}{\mathrm{d}t} = -cf_m(t) \tag{28}$$

According to Dirac's theory, the Hamiltonian operator for a system of independent oscillators is of the form

$$H = \sum_m \hbar\omega_m \left(a_m^+ a_m + \frac{1}{2} \right) \tag{29}$$

The Q_m and P_m in the formula (23) are

$$Q_m = \left(\frac{\hbar}{2\omega_m} \right)^{1/2} \left(a_m^+ + a_m \right) \tag{30}$$

$$P_m = i\left(\frac{\hbar\omega_m}{2} \right)^{1/2} \left(a_m^+ - a_m \right) \tag{31}$$

Making corresponding permutation,

$$Q_m \leftrightarrow \frac{f_m}{2\omega_m\sqrt{\pi}} \tag{32}$$

Thus

$$f_m = \left(2\omega_m\sqrt{\pi} \right) Q_m = \left(2\pi\hbar\omega_m \right)^{1/2} \left(a_m^+ + a_m \right) \tag{33}$$

Substituting equation (33) into equation (26), the gravitational field operator is

$$\boldsymbol{G}(x,t)=\sum_{m}(2\pi\hbar\omega_m)^{1/2}\left\{a_m^{+}(t)+a_m(t)\right\}\boldsymbol{u}_m(x) \tag{34}$$

Similarly,

$$Q_m \leftrightarrow \frac{h_m}{2c\sqrt{\pi}} \tag{35}$$

Combining equation (27), the wagnetic field operator is

$$\boldsymbol{W}(x,t)=\sum_{m} ic\left(\frac{2\pi\hbar}{\omega_m}\right)^{1/2}\left\{a_m^{+}(t)-a_m(t)\right\}\nabla\times\boldsymbol{u}_m(x) \tag{36}$$

At this point, the gravitowagnetic fields have been quantized.

3. Discussion

3.1. Unified Expression

In the special relativity, time appears in the fourth dimension. Noticing that $(\pm i)^2=-1$, and the parity of the fourth dimension has a different meaning from that of time: Under the condition that time is positive $(t>0)$ and there is no absolute zero point in time, the 1-dimensional imaginary space containing time and the speed of light and the 3-dimensional configuration space form a complex 4-dimensional spacetime. The 1-dimensional imaginary space has two characteristics: 1. No matter how the coordinate direction of the configuration space is chosen, it always exists as an independent space; 2. Ensure that the 4-dimensional spatiotemporal spacing is always in the form of $x^2+y^2+z^2-c^2t^2$. This 4-dimensional space should still exhibit compactness, which leads to the spacetime having large-scale parity. In the fourth dimension, ict is in parity with $-ict$. The essence of electromagnetic field and gravitowagnetic field is tensor fields in the 4-dimensional spacetime, and the parity of the two fields reflects the parity of 4-dimensional spacetime. The physical properties of the vacuum excitation fields are a true reflection of the topological properties of spacetime. The magnitude of the field strength, the density, the space interval. etc., are all measurements and must be real numbers. When using the variable of time, the modulus of the fourth dimensional coordinate quantity is actually adopted: $|\pm ict|=ct$. The existence of the speed of light makes the dimension consistent with the space.

Just like the electromagnetic field, the 4-dimensional tensor field equations truly describe the gravitowagnetic field. To really reflect the parity of spacetime together with the electromagnetic field in form, the $(-x,-y,-z,-ict)$ coordinate system, which is different from the one adapted in section 2, must be adopted. The actual operation is quite easy: As long as the tensor defined by equation (15) is applicable to both the (x,y,z,ict) coordinate system and the $(-x,-y,-z,-ict)$ coordinate system, the resulting tensors with the same foot label imply parity. Such an expression truly demonstrates that the properties of the fields are manifestation of the topological properties of the complex spacetime. However, in such coordinate system, the gravitowagnetic field expressed by the redefined $F_{\mu\nu}$ no longer corresponds to the algebraic formulas of the fields in the references [1] [2].

In quantum field theory, the natural system of units, $c = \hbar = 1$, is often used. This is to highlight the symmetry between the algebraic expressions of the fields and ignore their differences in measurements. Based on this approach, in order to highlight the parity between the electromagnetic field and the gravitowagnetic field, when defining the field strength using Helmholtz theorem, we adopted the same form of scalar potential and vector potential, which already implicitly contained the metric differences between the two fields [2]. Thus, the vacuum field in the excited state can be expressed in a unified form,

$$F = \begin{pmatrix} 0 & \Lambda_3 & -\Lambda_2 & -i\Omega_1 \\ -\Lambda_3 & 0 & \Lambda_1 & -i\Omega_2 \\ \Lambda_2 & -\Lambda_1 & 0 & -i\Omega_3 \\ i\Omega_1 & i\Omega_2 & i\Omega_3 & 0 \end{pmatrix} \tag{37}$$

For the electromagnetic field: $(x_1 = x, x_2 = y, x_3 = z, x_4 = ict)$, $\Omega_i = E_i$, $\Lambda_i = B_i$, $i = 1,2,3$. For the gravitowagnetic field: $(x_1 = -x, x_2 = -y, x_3 - z, x_4 = -ict)$, $\Omega_i = -G_i$, $\Lambda_i = -W_i$, $i = 1,2,3$. The Lorentz invariant is obtained from formula (37),

$$\frac{1}{2} FF = \Lambda^2 - \Omega^2 \tag{38}$$

The topological significance of formula (38) is that the excited state vacuum maintains the parity of the complex 4-dimensional spacetime, and its law exists in the same way in all inertial reference frames.

3.2. Gauge Transformation

The action of electromagnetic force exists between charged particles, and the gravitowagnetic force exists between particles with masses. The action of these two fields is in the $(-\infty, \infty)$, and within this range, there are also exist scalar field and spinor field. Therefore, the electromagnetic field and the gravitowagnetic field must be involved in the interaction between these fields. At this point, gauge transformation must be taken into account. Due to the parity relationship between the gravitowagnetic field and the electromagnetic field, the algebraic expressions are completely similar, so the gauge transformation of the electromagnetic field is also applicable to the gauge transformation of the gravitowagnetic field. Therefore, under the same gauge transformation, we only need to consider the gauge transformation of the gravitowagnetic field. The gauge properties of the field are mainly determined by the gauge properties of the tensor $F_{\mu\nu}$, as this tensor determines the properties of the field. Introduce a gauge transformation of the same form as the electromagnetic field,

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{g} \partial_\mu \theta(x) \tag{39}$$

Here, g is the coupling constant (including conserved mass and charge), $\theta(x)$ is any differentiable scalar function, and the corresponding gauge transformation of $F_{\mu\nu}$ is

$$\begin{aligned} &F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ &\to F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu = \left[\partial_\mu A_\nu - \partial_\nu A_\mu\right] + \frac{1}{g}\left[\partial_\mu \partial_\nu - \partial_\nu \partial_\mu\right]\theta(x) \end{aligned} \tag{40}$$

Due to $\partial_\mu \partial_\nu = \partial_\nu \partial_\mu$, so

$$F'_{\mu\nu} = F_{\mu\nu} \tag{41}$$

It can be seen that the excited-state vacuum with electromagnetic field and gravitowagnetic field has gauge invariance.

4. Conclusion

Like the electromagnetic field, the gravitowagnetic field can be quantized. The Lorentz covariant of the excited-state vacuum composed electromagnetic field and gravitowagnetic field not only demonstrates the parity of 4-dimensional spacetime, but also remains unchanged under the gauge transformation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Feng, Y.G. (2025) General Maxwell Theory of Fields. *Journal of High Energy Physics, Gravitation and Cosmology*, **11**, 8-18. https://doi.org/10.4236/jhepgc.2025.111002

[2] Feng, Y.G. (2025) General Maxwell Theory of Fields (2): Explanations and Predictions of Some Phenomena. *Applied Physics Research*, **17**, 32-39. https://doi.org/10.5539/apr.v17n2p32

[3] Morse, P.M. and Feshbach, H. (1953) Methods of Theoretical Physics, Part 1. The McGraw-Hill Companies, Inc.

[4] Arfken, G.B., Weber, H.J. and Harris, F.E. (2013) Mathematical Methods for Physicists. 7th Edition, Elsevier.

[5] Jackson, J.D. (1999) Classical Electrodynamics. 3rd Edition, John Wiley & Sons, Inc.

[6] Griffiths, D.J. (2013) Introduction to Electrodynamics. 4th Edition, Pearson.

[7] Newton, I. (2021) Mathematical Principles of Natural Philosophy and Its System of the World. Shanghai Translation Publishing House.

[8] Kaku, M. (1993) Quantum Field Theory: A Modern Introduction. World Publishing Corporation.

[9] Srednicki, M. (2007) Quantum Field Theory. Cambridge University Press. https://doi.org/10.1017/cbo9780511813917

[10] Schiff, L.I. (1968) Quantum Mechanics. 3rd Edition, McGraw-Hill.

[11] Ballentine, L.E. (1998) Quantum Mechanics: A Modern Development. World Scientific. https://doi.org/10.1142/3142