Journal of High Energy Physics, Gravitation and Cosmology, 2026, 12(1), 626-638
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Inflationary Study of the Universe in $f(R)$ Gravity Theory of Brans-Dicke Assisted by a Scalar Field

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How to cite this paper: Florentin, A.K., Yakouba, T., Etienne, B.H., Inès, S.G., Stéphane, H.M.J. and Antonin, K.D. (2026) Inflationary Study of the Universe in $f(R)$ Gravity Theory of Brans-Dicke Assisted by a Scalar Field. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 626-638.
https://doi.org/10.4236/jhepgc.2026.121031

Received: October 28, 2025
Accepted: January 27, 2026
Published: January 30, 2026

Abstract

In this paper, we have studied cosmic inflation in the context of modified $f(R)$ gravity, by integrating a scalar field. The general objective is to understand the primordial phases of the universe and to test whether alternative models to general relativity can better correspond to recent observations (especially those of the Planck satellite). In this work, we used an approach based on the Brans-Dicke hypothesis, which introduces a scalar field that is minimally coupled to gravity. Two types of inflation are explored: The first approach is inflation related to the power law. Here, we have chosen a scale factor of the form $a(t) = At^b$, we then derived the evolution equations for the scalar field and the inflation potential. We analytically expressed observational parameters such as the spectral index η_s and the tensor-to-scalar ratio r in terms of the parameter b and the folds N. Specific values of b (between 60 and 70) correspond to observational data. Chaotic inflation for the second approach, we chose from the outset a specific form of potential defined as $V(\phi) = \frac{1}{2}m^2\phi^2$. The objective set is to study the cosmological implications of this form and verify the consistency of the predictions with the current constraints (for example, Planck 2018). The results obtened prove that models with high values of the parameter r lead to predictions consistent with observations (spectral index close to 0.965 and low ratio r). Pacs numbers: 04.50.Kd, 95.36.+x, 98.80.-k.

Keywords

Modified Gravity $f(R)$, Brans-Dicke Theory, Scalar Field, Cosmic Inflation, Inflation in Power Law

1. Introduction

The current evolution of the universe seems to us day by day like mysteries. We are still very impressed by these mysteries of the universe when we dive into them deeply. Cosmology, which is a part of astrophysics, seeks to explain how the universe began, its evolution over time up to today. Questions such as the nature of the Big Bang where the universe was considered to be a point, cosmic inflation, dark matter, and dark energy highlight the complexities of the universe's history. The Big Bang theory, which describes the formation of the universe, dates back 13 millions years. The best evidence we have of this very dense and very hot period of our universe is called the cosmic microwave background (CMB), discovered by Penzias and Wilson [1]. Temperature measurements of the CMB indicate that the scalar perturbations in the early universe were nearly scale-invariant, and this scale invariance is a fundamental requirement that must be met by any cosmological model to be accepted as viable. We generally obtain the scale invariant when using the scalar field and this, through their perturbations [2] [3]. In the Big Bang model, the scalar field is primordial. This model, often still called inflaton or quantum vacuum, is the name given to the form of a hypothetical matter responsible for cosmic inflation, a period during which the universe significantly grew after the Big Bang [4] [5]. From the perspective of particle physics, it is a hypothetical scalar field, like the Higgs field, but with very different dynamics. We observe a negative and almost unchanging pressure throughout the inflation phase as well as its energy density, which takes a constant but opposite value. Thus, the scalar field behaves similarly to a cosmological constant Λ [6] [7]. Larger studies on the inflation of the universe allow us to have several observational data [8]-[13]. These observational data in the Big Bang model have shifted cosmology towards a new, more scientific concept called the standard model of cosmology. An overview of inflationary cosmology and its state was provided by the author in [14] after the observational data from Planck in 2013. The author particularly emphasized the new broad class of theories, cosmic attractors, which have predictions that do not heavily depend on the model converging to the ideal point of Planck in the (η_s, r) plane where η_s and r represent the spectral index and the tensor-to-scalar ratio, respectively. Furthermore, they discussed the problem of initial conditions for theories driven by scalar fields and favored by the Planck data. The implication of the inflationary scenario of the scalar field is at the heart of several investigations. Cosmic inflation over cosmic time describes the phase of the evolution of the universe during which primordial perturbations can arise naturally due to quantum fluctuations of the scalar field, referred to as inflation and assumed to be minimally coupled to gravity. Under other skies, the scalar field also slowly moves down a functional effective scalar potential [13] [15]-[17]. Thus, it can be stated that the dominance of the scalar field in the primitive universe leads to the contribution of curvature that can be seen through the additional degrees of freedom contained in the modified theory of gravity [18]. In [19], a dynamic cosmological system in $f(R)$ gravity in the presence of a canonical scalar field ϕ with an exponential potential is studied where R denotes the scalar curvature. In this work, the author has extensively explained the role of the scalar field in the exit from inflation within the framework of modified $f(R)$ theory.

Among others, the authors in [20]-[24] perform works on $f(R)$ gravity theory where the inflation at the beginning of the universe, the late cosmic acceleration, and other gravitational phenomena without introducing exotic fields such as quintessence or dark energy are addressed. One of their motivations for this study was that General Relativity alone does not allow for an explanation of cosmic acceleration, and the other was that generalized $f(R)$ theories provide a geometric framework to reproduce this cosmic history without dark matter or dark energy. Starting from well-recognized models in $f(R)$, they managed to prove that in modified gravity, more precisely the $f(R)$ theory constitutes a solid and elegant alternative to dark energy and standard inflation models, allowing for a complete geometric description of the evolution of the universe, from early times to the present day.

Recently, other authors have studied a perfect fluid induced by a scalar field and the comparison with observational data is analyzed in [25] where the observables of inflationary models, in particular, the spectral index of curvature perturbations, the tensor-scalar ratio, and the evolution of the spectral index are examined within the framework of $F(R)$ gravity theory through reconstruction methods. The description of the inflationary scenario and the comparison with observational data remain a problem in cosmology.

It is in this context that we studied cosmic inflation in this article, within the framework of modified gravity $f(R)$, integrating a scalar field. Our general objective is to understand the primordial phases of the universe and to check whether alternative models to general relativity can better match recent observations (especially those from the Planck satellite).

The approach used in this work is based on the Brans-Dicke hypothesis, which introduces a scalar field that is non-minimally coupled to gravity.

This article is organized according to the following outline. In Section 2, we start from the generalities in $f(R)$ gravity that we have detailed the useful equations for our work. Section 3 is dedicated to our first model, which is inflation according to the power law, followed by analyses and interpretations of the results obtained in this model. Section 4 is reserved for our second model in this work, which is chaotic inflation, where we have also interpreted the obtained results. And finally the conclusion in Section 5.

2. Action in Gravity Theory $f(R)$

The total action S of the gravitational field in $f(R)$ gravity theory is defined by

$$S=\frac{1}{2\kappa^2}\int_\Omega f(R)\sqrt{-g}\,\mathrm{d}x^4+\int_\Omega £_m\sqrt{-g}\,\mathrm{d}x^4, \tag{1}$$

where $f(R)$ is a function of the Ricci scalar R.

The variation of S with respect to the contravariant range $g^{\mu\nu}$ gives us the general equation of motion in $f(R)$:

$$f_R R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}f(R)+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)f_R=\kappa^2T_{\mu\nu}. \tag{2}$$

With f_R is the partial derivative of $f(R)$ with respect to R.

In this regard and assuming that the Lagrangian density can be chosen as $\mathcal{L}_m=-\overline{p}$, we obtain the FRW equations defined as

$$3H^2=\frac{\kappa^2}{f_R}\rho+\frac{1}{f_R}\left[\frac{1}{2}(f-Rf_R)-3\dot{R}Hf_{RR}\right], \tag{3}$$

$$-2\dot{H}-3H^2=\frac{\kappa^2}{f_R}P+\frac{1}{f_R}\left[2H\dot{R}f_{RR}+\ddot{R}f_{RR}+\dot{R}^2f_{RRR}-\frac{1}{2}(f-Rf_R)\right], \tag{4}$$

The Equation (3) can be rewritten as

$$f(R)=6H^2f_R-\kappa^2\rho+6H\dot{R}f_{RR}+Rf_R \tag{5}$$

By replacing (5) in (4) we have

$$-\left(2\dot{H}-R\right)f_R=\kappa^2(\rho+P)+\left(\ddot{R}-H\dot{R}\right)f_{RR}+\dot{R}^2f_{RRR} \tag{6}$$

or

$$f_R=\phi(R)+1, f_{RR}=\frac{\dot{\phi}}{\dot{R}}, f_{RRR}=\frac{\ddot{\phi}\dot{R}-\ddot{R}\dot{\phi}}{\ddot{R}\dot{R}^2} \tag{7}$$

When they are substituted in (6), we find

$$4\left(3H^2+\dot{H}\right)(\phi+1)\dot{R}\ddot{R}=\kappa^2(P+\rho)\dot{R}\ddot{R}+\ddot{R}\left(\ddot{R}-H\dot{R}\right)\dot{\phi}+\left(\ddot{\phi}\dot{R}-\ddot{R}\dot{\phi}\right)\dot{R} \tag{8}$$

where $R=6\left(2H^2+\dot{H}\right)$, $\dot{R}=6\left(4H\dot{H}+\ddot{H}\right)$, $\ddot{R}=6\left(4\dot{H}^2+4\ddot{H}H+\dddot{H}\right)$ and $P+\rho=\dot{\phi}^2\epsilon$ avec

$$P=\frac{1}{2}\dot{\phi}^2\epsilon-V(\phi), \rho=\frac{1}{2}\dot{\phi}^2\epsilon+V(\phi) \tag{9}$$

Limiting ourselves to the raw derivatives of H and ϕ and then assuming that $\phi\gg1$ and $H\gg1$, (6) becomes

$$6912H^2\dot{H}+2304\dot{H}^2\phi=-576H\epsilon\dot{H}\dot{\phi}-576H\dot{H}^2\dot{\phi}-576H\dot{H}\dot{\phi} \tag{10}$$

This equation can be solved (either by imposing ϕ or by giving a significant cosmological expression to H). The main objective of this work is to describe the inflationary era in the modified gravity theory $f(R)$ under the Brans-Dike hypothesis. In fact, in literature, it is not easy to point the finger at him a typical functional form of $\phi(t)$ that describes the inflationary era. In contrast, several of the cosmological works have focused on certain functional forms of the scalar factor (the Hubble parameter, respectively) that can replicate the era of power-law inflation.

3. Power-Law Inflation

The general solution described by the power law function of the scale factor in cosmology is to describe a power law inflation [15]. This expansion is provided by the following scalar factor and its corresponding Hubble parameter.

$$a(t) = At^b \Rightarrow H = \frac{b}{t} \tag{11}$$

where A and b are constants with this expression of the Hubble parameter H, Equation (10) becomes

$$9216\phi t - 576\left(\epsilon t^2 - bt^3 + b\right)\dot{\phi} = 0 \tag{12}$$

The simple solutions of this equation are

$$\ln\phi = \int \frac{16t}{\epsilon t^2 - bt^3 + b}\mathrm{d}t + C \tag{13}$$

When we assume $t \gg 1$, (13) becomes

$$\ln\phi = -\int \frac{16}{bt^2}\mathrm{d}t + C \tag{14}$$

either

$$\phi(t) = A\exp\left(\frac{-16}{bt}\right) \tag{15}$$

A and b are constants that can be determined explicitly by identification.

Using the 1-order expansion of the exponential function, ϕ becomes:

$$\phi(t) = A\left(\epsilon t^2 - bt^3 + b\right) + 1 + C \tag{16}$$

According to the Klein-Gordon equation, we have:

$$\ddot{\phi} + 3H\dot{\phi} + V'\left(\phi(t)\right) = 0 \tag{17}$$

Inserting Equation (16) into Equation (17), we find after integration

$$V\left(\phi(t)\right) = \frac{3b}{2}(2+3b)t^2 - 2\epsilon(1+3b)t + C \tag{18}$$

By expressing cosmic time t as a function of the potential q according to Equation (16), Equation (18) becomes

$$V\left(\phi(t)\right) = \frac{384(2+3b)}{b\ln^2\left(\frac{A}{\phi(t)}\right)} - \frac{32(1+3b)}{b\ln\left(\frac{A}{\phi(t)}\right)} + C \tag{19}$$

For the purposes of simplification, let us set $\ln\left(\frac{A}{\phi(t)}\right) = x$

$$V\left(\phi(t)\right) = \frac{384(2+3b)}{bx^2} - \frac{32(1+3b)}{bx} + C \tag{20}$$

Thus

$$V'\left(\phi(t)\right) = \frac{1}{\phi(t)}\left(\frac{768(2+3b)}{bx^3} - \frac{32(1+3b)}{bx^2}\right) \tag{21}$$

and

$$V''(\phi(t))=\frac{1}{\phi^2(t)}\left[\left(\frac{768(2+3b)}{bx^3}-\frac{32(1+3b)}{bx^2}\right)-\left(\frac{2304(2+3b)}{bx^4}-\frac{64(1+3b)}{bx^3}\right)\right] \tag{22}$$

with the integration constant C. To better provide detailed and proper explanations of the origin and evolution of the universe in cosmology, in the Big Bang Model, was added to the inflation model to form the Standard Model of Cosmology. The study of inflation in this set gives the possibility of fitting the theoretical results to the observational data. In [20]-[23], it is established that in the ordinary; The scalar field model of inflation, the inflationary observables namely the spectral index, the scalar ratio tensor, and the unfolding of the spectral index are represented using the $V(\phi)$ potential of the scalar field. Depending on the parameters of slow rolling, these observables can also be expressed in terms of term of the Hubble parameter. The expression of the slow rolling parameters is that with respect to the canonical scalar field potential $V(\phi)$ (40).

$$\varepsilon=\frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2=\frac{1}{2\kappa^2}\left(\frac{\frac{1}{\phi}\left(\frac{768(2+3b)}{bx^3}-\frac{32(1+3b)}{bx^2}\right)}{\frac{384(2+3b)}{bx^2}-\frac{32(1+3b)}{bx}}\right)^2 \tag{23}$$

and

$$\eta=\frac{1}{\kappa^2}\left(\frac{V''(\phi)}{V(\phi)}\right)=\frac{1}{\kappa^2}\left(\frac{\frac{1}{\phi^2(t)}\left[\left(\frac{768(2+3b)}{bx^3}-\frac{32(1+3b)}{bx^2}\right)-\left(\frac{2304(2+3b)}{bx^4}-\frac{64(1+3b)}{bx^3}\right)\right]}{\frac{384(2+3b)}{bx^2}-\frac{32(1+3b)}{bx}}\right) \tag{24}$$

We address the most signifiant observational quantities, the spectral index of primordial scalar perturbations η_s, and the tensor-to-scalar ratio r.

$$\eta_s=1-6\epsilon+2\eta, r=16\epsilon \tag{25}$$

or

$$\eta_s=1-6\frac{1}{2\kappa^2}\left(\frac{\frac{1}{\phi}\left(\frac{768(2+3b)}{bx^3}-\frac{32(1+3b)}{bx^2}\right)}{\frac{384(2+3b)}{bx^2}-\frac{32(1+3b)}{bx}}\right)^2+2\frac{1}{\kappa^2}\left(\frac{\frac{1}{\phi^2(t)}\left[\left(\frac{768(2+3b)}{bx^3}-\frac{32(1+3b)}{bx^2}\right)-\left(\frac{2304(2+3b)}{bx^4}-\frac{64(1+3b)}{bx^3}\right)\right]}{\frac{384(2+3b)}{bx^2}-\frac{32(1+3b)}{bx}}\right) \tag{26}$$

and

$$\eta_s = 1 - 6\epsilon + 2\eta, r = 16\epsilon \tag{27}$$

In the same approximation as [25], we can express these observables as a function of the number e.fold in order to track their evolutions and extract their values at the end of inflation. Thus, using (23), the e.fold number can be expressed separately as a function of cosmic time and the scalar field as follows.

$$N = \int_{t_i}^{t_f} H(t)\,\mathrm{d}t = \int_{t_i}^{t_f} \frac{b}{t}\mathrm{d}t = 16Ab\int_{\phi_i}^{\phi_f} \ln\left(\frac{A}{\phi(t)}\right)\mathrm{d}\phi(t) \Rightarrow N = b\frac{\ln\left(\frac{A}{\phi_i}\right)}{\ln\left(\frac{A}{\phi_f}\right)} \tag{28}$$

According to the typical results of power-law inflation, one can approximate

$$\epsilon \simeq \frac{1}{2bN}, \eta \simeq \frac{1}{bN}, n_s \simeq 1 - \frac{1}{bN}, r \simeq \frac{8}{bN} \tag{29}$$

What needs to be remembered here is that the obtained expression depends on the parameter b of the power law model, which is not the case in the Sitter description. We can study the evolution of the parameters η_s and r around the end of inflation, namely when $N = 60$. The curves in figure reveal certain values of the parameter b for which the observation n_s and r can recover their values determined by observation. What should be noted regarding this **Figure 1** is that when $b \in [60;80]$, we have the possibility to track the evolution of the observables as a function of the number of e.fold by fixing it. The corresponding curves are illustrated in **Figure 2** for three different choices of b within the range $[60;80]$. These curves provide in this work the opportunity to achieve the best fit with the observational data. We are also undertaking the study of the correlation between the spectral index and the scalar-tensor ratio r.

The expression of ns as a function of r is given by

$$n_s \simeq 1 - \frac{r}{8} \tag{30}$$

The relationship allows us to plot the spectral index as a function of the scalar-to-tensor ratio r for an appropriate choice of the parameter *b*. The result obtained is presented in **Figure 3** which includes three different observation data limits already presented in the previous section.

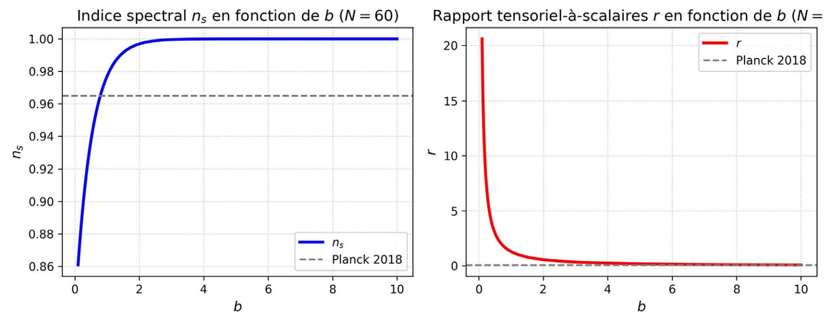

Figure 1. Evolution of spectral index η_s (blue color) and tensor-to-scalar r (red color) versus the parameer of the power-law model b. The curves are obtained for $\kappa = 1$ and $N = 60$.

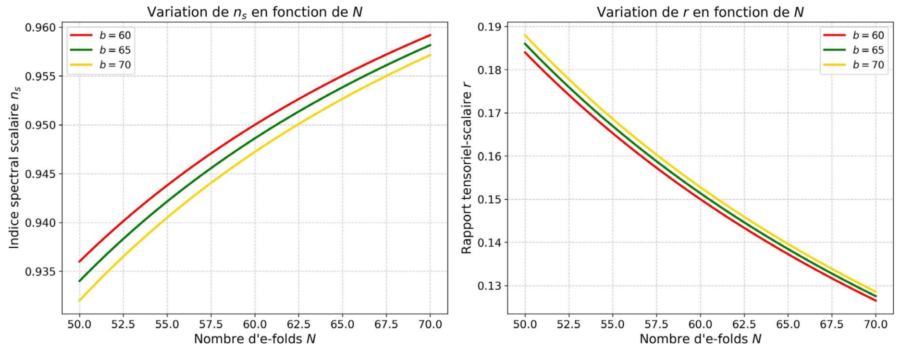

Figure 2. Evolution of spectral index η_s and tensor-to-scalar r versus the e.fold number for three values of the power-law model b: $b = 60$ (red color); $b = 65$ (green color) and $b = 70$ (yellow color).

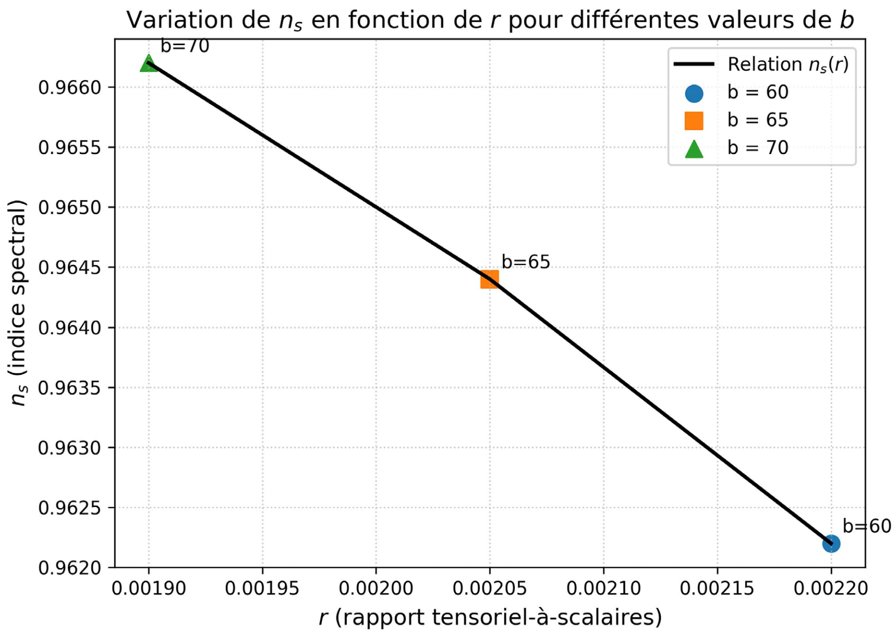

Figure 3. Correlation between the spectral index n_s and the tensor-to-scalar ratio r in the power-law expansion under the Brans Dicke construction. The green, the blue and the red tasks mean the observational values of these observables suggested in the following works [25]-[28] respectively. Since the correlation Equation (30) depends of the parameter b the obtained curve results from the superpostion of the three lines curves corresponding to the three promising values of the parameter $b : b = 60$; $b = 65$ and $b = 70$.

We notice here that, when b increases:

- r decreases significantly, which means that the amplitude of primordial gravitational waves becomes negligible. This result is in agreement with the Planck 2018 data which impose $r < 0.07$.
- η_s increases and tends towards 1, which corresponds to a nearly invariant scale spectrum, also in agreement with cosmological observations that give approximately $\simeq 0.965$.

When $b \in [60; 70]$, this range of values for b allows for predictions that are consistent with the observations.

We can partially conclude that **Figure 1** validates the power law inflation model within the framework of modified gravity $f(R)$ coupled with a scalar field, for high values of b (60 to 70). It shows that this model can reproduce the values of η_s and r consistent with recent observational constraints, particularly those from Planck.

In relation to our behavior $\eta_s(N)$, we can note that:

- For each curve (each value of b), η_s increases slightly with N.
- This translates to a spectrum of scalar perturbations increasingly close to an invariant spectrum (*i.e.*, $\eta_s \to 1$).
- $b = 70$ gives $\eta_s \simeq 0.976$ for $N = 60$, very close to the Planck observations ($\eta_s \simeq 0.965$).

For the behavior of r as a function of N, it should be noted that:

- The report r decreases with N for all values of *b*, which means that the influence of primordial gravitational waves becomes increasingly negligible.
- The higher *b* is, the smaller *r* is.
- This reinforces the compatibility with currrent observations that impose $r < 0.07$.

We note here that when b increases, the tensor ratio r decreases. This means that primordial gravitational waves are less significant in models with larger values of b. Also, a greater value of b gives a spectral index closer to 1, thus a "flatter" scalar spectrum, which is consistent with observational data(such as those from Planck indicating $\eta_s = 0.965$).

In short, models with larger b (like $b = 70$) provide values of η_s and r that are more compatible with current cosmological data. Notably:

- $\eta_s \to 0.976$ (what is realistic)
- $\to 0.0019$ (very weak, which is also consistent with the strict observational constraints on r).

4. Chaotic Inflation

Chaotic cosmological inflation is often studied in the standard theory of gravity. It is fueled by a massive scalar field whose potential is given by

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \tag{31}$$

The approach in this section is the opposite of those developed in the previous section. There, we choose the type of expansion through the scale factor, namely the Hubble parameter and we extract the corresponding potential $V(\phi)$ from which the observables are provided. Here, the starting point is $V(\phi)$ and we aim to investigate some cosmological implications of this choice. Under this condition, we have

$$ns = 1 - \frac{1}{4} r \tag{32}$$

Numerically, we have (**Figure 4**)

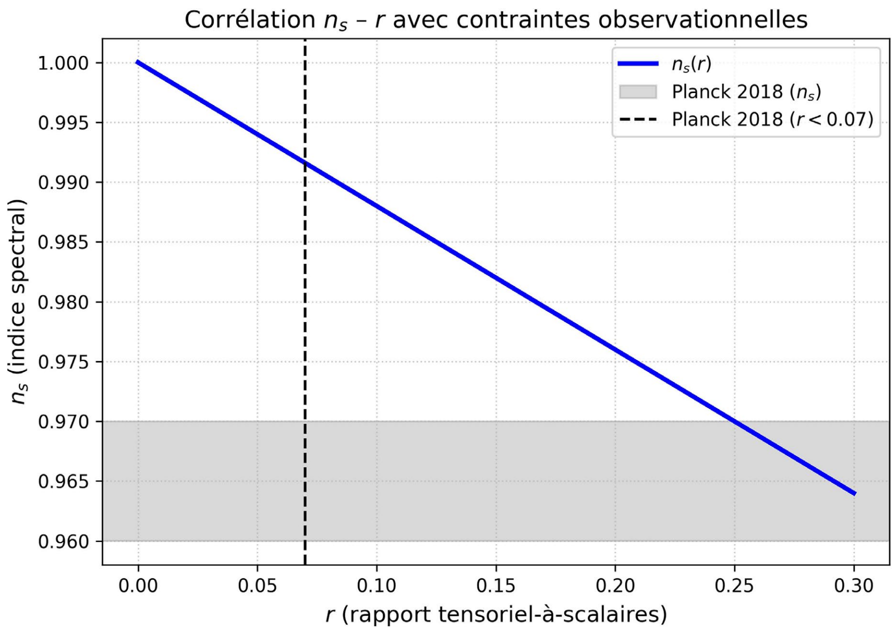

Figure 4. Correlation between the spectral index and the scalar tensor ratio r for the different values of $b = 60$, 65, and 70 overlaid on the observation areas of Planck 2018.

It should be noted that:

- The horizontal gray band represents the confidence interval for $n_s = 0.9650004$.
- The vertical gray line is the upper bound of $r < 0.07$ The interpretation of this figure is as follows.
- The curves for $b = 60$, $b = 65$, and $b = 70$ cut through the gray area, which means they can reproduce the observed values of η_s.
- The portions to the left of the vertical line (where $r < 0.07$) are the only ones compatible with current observations from Planck.
- This suggests that, under certain conditions, high values of b allow this inflationary model to be consistent with observational data.

5. Note

In this section, we compared the inflationary predictions from our model, based on a modified $f(R)$ theory with a scalar field under the Brans–Dicke hypothesis, with those of other works, including those by Odintsov *et al.* (2014) [29].

We can summarize the main results as follows:

- In this work, we predicted very low tensor/scalar ratio values r ($r \approx 0.0019$ for $b = 70$) and a spectral index $\eta_s \simeq 0.976$, in excellent agreement with the observations of the Planck 2018 satellite ($\eta_s = 0.9649 \pm 0.0042$, $r < 0.07$).
- The chaotic inflation model explored in the same article provides a relation $\eta_s = 1 - \frac{1}{4} r$, also consistent with observational data for certain low values of r.
- The work of Odintsov *et al.* (2014) offers a more general framework based on

universal attractors, allowing robust predictions: $\eta_s \sim 0.96 - 0.967$ and $r \sim 0.003 - 0.05$. These models are also in very good agreement with Planck data.

- The work of Odinsov *et al.* provides a numerical reconstruction of inflationary models that also respect current constraints ($n_s \sim 0.963$, $r < 0.07$).

We can partially conclude that all the models examined are consistent with current observations, but our model is particularly interesting for its analytical simplicity and its ability to reproduce accurate observational results with a reduced number of parameters.

On the other hand, by referring to one of the works of Dicong Liang and al. in [30], we can also inform you that gravitational wave astronomy, which began about 10 years ago, could in principle be fundamental in distinguishing general relativity from $f(R)$ gravity. In fact, $f(R)$ theories generally allow for more polarizations of gravitational waves and, consequently, different interferometric response functions compared to standard general relativity.

6. Conclusions

In this article, we studied scenarios of cosmological inflation within the modified gravity theory $f(R)$ with a scalar field under the Brans-Dicke hypothesis. Two types of approaches were explored: power-law inflation and chaotic inflation.

The analytical approach based on a scale factor of type $a(t) = At^b$ allowed to derive expressions of the observable parameters of inflation (spectral index ns, tensor-scalar ratio r, etc.), as a function of the parameter b and the number of e-folds N. It has been shown that certain ranges of b (notably between 60 and 70) allow the proposed models to align with current cosmological observations (in particular the Planck 2018 data), by reproducing realistic values for ns and r.

The study of the chaotic scenario, with a quadratic potential of the scalar field, also showed good consistency with observational constraints under certain conditions.

In short, this work shows that inflationary models in modified gravity.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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