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A Dark Energy Hypothesis IX

—Summary of a DEH: An Alternative Cosmology

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Abstract

Dark energy density decreases with time in a DEH in contrast to its constancy in the standard ΛCDM theory. Hence, the former is an alternative to the latter. This summary traces the implications of this idea as they appear in this journal in eight papers.

Keywords

Dark Energy, Dark Matter, Cosmological Entropy

1. A Dark Energy Hypothesis

A DEH [1] begins with a state in which two comoving coordinates, the conformal time, η, and the cosmic latitude, χ, are coupled:

$$\mathrm{d}s^2 = a^2\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - 2i\,\mathrm{d}\eta\,\mathrm{d}\chi\cos(b) - f(\chi)\left(\mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\varphi^2\right)\right]$$

Two elements of the metric tensor $\boldsymbol{g}$ are $g_{01} = g_{10} = i\cos(b)$, $f(\chi)$ depends on the spatial geometry, and "a" is the scale factor, $a\mathrm{d}\eta = c\mathrm{d}t$. The details in DEH I will not be reproduced here. Decoupling occurs in the limit $b \to \pi/2$ that results in two products. The obvious one is

$$\mathrm{d}s^2 = a^2\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - f(\chi)\left(\mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\varphi^2\right)\right]$$

The line element in Einstein's theory is a geometric representation of gravity. By hypothesis, this line element is that of dark matter.

The second product of decoupling is the dark energy hypothesis per se, that is, a quantity with the units of 1/length^2, which by hypothesis is dark energy, $\Lambda = 1/\eta^2 a^2$. The symbol Λ is that of Einstein's cosmological constant, but now is to be replaced by this variable cosmological parameter. The parameter corresponds to

*Retired.

the dark energy density, $\Lambda = \kappa\varepsilon_\lambda$ where κ is the Einstein gravitational constant:

$$\kappa = \frac{8\pi G}{c^4} = 2.076 \times 10^{-43}\ \mathrm{m \cdot J^{-1}}$$

The total dark energy is

$$U_\lambda = a^3\varepsilon_\lambda = \frac{a}{\kappa\eta^2}$$

In the limit $\eta \to \infty$ $\Lambda \to 0$, while U_λ increases without limit in hyperbolic space.

2. A DEH vis-à-vis the ΛCDM Theory

In one sense, a DEH is allied with the standard ΛCDM theory, which includes the big bang, the fixing of the chemical composition of the cosmos in the first three minutes, the formation of the cosmic microwave radiation, and the need for dark matter to account for rotation curves in galaxies and to stabilize galactic clusters. A DEH is compatible with inflation. In numerical work, it is assumed that the ratio of energies of dark energy, dark matter and baryonic matter in the present epoch is 70/25/5, which is compatible with ΛCDM thought.

But here a divergence of outlook emerges that will be considered in detail below. One difference is replacing the cosmological constant with a variable, which will mean that the ratio of dark energy to dark matter varies with time, although the baryonic fraction will remain at 5% as in the ΛCDM theory. Another is that the dark energy density in a DEH is pressureless whereas it exerts a negative pressure in the standard model. Another is the possible origin of dark matter, whether it is a product of cosmic evolution or a part of spacetime structure.

3. Energy Conservation in a DEH [2]

Let U represent the total energy, which is conserved with the proviso cited below. Its three components are dark energy, dark matter and baryonic matter:

$$U = U_\lambda + c^2\left[M(dm) + M(b)\right] \tag{1}$$

A conservation law is a powerful analytic tool, so a variety of formulations will prove to be of use. Divide Equation (1) by U to express conservation by dimensionless parameters.

$$1 = \lambda + \chi(dm) + \chi(b) \tag{2}$$

These are given for the present epoch.

$$1 = 0.70 + 0.25 + 0.05$$

Multiply Equation (1) by the Einstein gravitational constant to give conservation of cosmological length.

$$\Gamma = \frac{a}{\eta^2} + \kappa c^2\left[M(dm) + M(b)\right] \tag{3}$$

Divide Equation (3) by Γ to get another formula of dimensionless quantities.

$$1 = \frac{a}{\Gamma\eta^2} + \frac{\kappa c^2}{\Gamma}\left[M(dm) + M(b)\right] \tag{4}$$

Compare Equations (2) and (4) to get the following useful formulas.

$$a = \lambda\eta^2\Gamma \,\&\, M(dm) = \frac{\chi(dm)\Gamma}{\kappa c^2} \,\&\, M(b) = \frac{\chi(b)\Gamma}{\kappa c^2} \quad (5)$$

Subsequent analysis will show how to find the numerical value of the conserved total length, Γ, which means that the dark matter and baryonic masses can be calculated.

A theme developed below is that total energy and total length are conserved as long as dark matter is present, that is, if $\chi(dm) \neq 0$, although eventually $\chi(dm) \to 0$.

4. Evolution in a DEH [2]

It is governed by the Friedmann-Lemaître equation:

$$\frac{1}{a^2}\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \quad (6)$$

The effort now is to incorporate the conservation laws into the differential equation and then integrate it subject to the boundary condition that $a = 0$ when $t = 0$. This is done in [2] but won't be repeated here. The results for the three cosmological spaces are

$$\begin{aligned} &k = +1 \qquad a = \frac{\Gamma}{6}\left(1-\cos(\eta)\right) \text{ and } ct = \frac{\Gamma}{6}\left(\eta - \sin(\eta)\right) \\ &k = 0 \qquad a = \frac{\Gamma}{12}\eta^2 \text{ and } ct = \frac{\Gamma}{36}\eta^3 \\ &k = -1 \qquad a = \frac{\Gamma}{6}\left(\cosh(\eta)-1\right) \text{ and } ct = \frac{\Gamma}{6}\left(\sinh(\eta)-\eta\right) \end{aligned} \quad (7)$$

These solutions look like the cycloid, hyperbolic, and Einstein-de Sitter models, but they aren't: in those models Λ = 0 and there's only baryonic matter. The question arises as to the appropriate space for a DEH. It is surprisingly easy to answer.

5. Space Is Hyperbolic

Only hyperbolic space is compatible with dark energy dominance in the present epoch. Compare Equations (5) and (7): the dark energy parameters are

$$k = +1 \qquad \lambda = \frac{1-soc(\eta)}{6\eta^2}$$

$$k = 0 \qquad \lambda = \frac{1}{12}$$

$$k = -1 \qquad \lambda = \frac{\cosh(\eta)-1}{6\eta^2}$$

The dark energy parameter for curved spaces at $\eta = 0$ is also $\lambda = 1/12$: they look flat in the very early universe. For spherical space, λ decreases monotonically to a minimum of 7×10^{-11} at $\eta = 6.283$. For flat space, dark matter is stuck at $\lambda = 1/12$. For hyperbolic space, the parameter evolves to

$$\lambda_0 = \frac{7}{10} \text{ at } \eta_0 = 5.571$$

6. Numerical

The numerical value for the conformal time at the present epoch is a constant of observational cosmology. The Hubble parameter for hyperbolic space is

$$H = \frac{\mathrm{d}\ln(a)}{\mathrm{d}t} = c\frac{\mathrm{d}\ln(a)}{\mathrm{d}\eta} = \frac{6c\sinh(\eta)}{\Gamma(\cosh(\eta)-1)^2}$$

by Equation (7) for $k = -1$. It follows that

$$\Gamma H_0 = 1.390\times 10^7 \text{ m}\cdot\text{s}^{-1}$$

Hence, the conserved length Γ can be found from the current value of the Hubble parameter. At this time in cosmological research, there are two candidates [3], a situation known as the Hubble tension:

$$H_0 = 73.5 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1} = 2.37\times 10^{-18} \text{ s}^{-1}$$

and

$$H_0 = 67.4 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1} = 2.18\times 10^{-18} \text{ s}^{-1}$$

In the DEH I-VIII series, the value of the Hubble parameter used (for no compelling reason) is $H_0 = 2.20 \times 10^{-18}\ \text{s}^{-1}$ giving $\Gamma = 6.32 \times 10^{24}$ m, which leads to the age of the universe by Equation (7) as t_0 = 14.0 Gyr. The occurrence of the Hubble tension represents an unsatisfactory state of cosmological science, but the differences are so small that any computations will give results similar to those cited in the DEH series.

7. The Hubble Law Approach to Cosmological Entropy

Introduction. This section derives from [4], which is a later version of [5]. In thermodynamics, the search for a parameter of spontaneous change is for a physical quantity that changes monotonically with time. In cosmology that's easy to find: the Hubble Law scale factor, which is easy to convert into entropy. This avoids the problem posed by the Kelvin and Clausius approaches where thermodynamic space divides into system and surroundings, but globally the universe can have no surroundings. The Hubble Law approach has close similarities to that of Carathéodory, which is briefly discussed.

Cosmological second law. *The scale factor increases with time*. This is the case in a DEH with its hyperbolic space and in the standard ΛCDM theory whose space is Euclidean.

Cosmological entropy in a DEH. Dark energy is proportional to the scale factor and increases monotonically with time:

$$U_\lambda = \frac{a}{\kappa\eta^2} = \frac{\lambda\Gamma}{\kappa} \tag{8}$$

Divide by a temperature to get a quantity with the units of entropy, that behaves like entropy, and can rightly be called dark entropy.

$$S_\lambda = \frac{U_\lambda}{T} \tag{9}$$

The entropy difference between two epochs $\lambda_2 > \lambda_1$ is

$$\Delta S = S_{\lambda 2} - S_{\lambda 1} = \frac{U_{\lambda 2}}{T_2} - \frac{U_{\lambda 1}}{T_1} > 0 \tag{10}$$

because

$$U_{\lambda 2} > U_{\lambda 1} \text{ and } T_2 < T_1$$

There cannot be equilibrium states such that

$$S_{\lambda 2} = S_{\lambda 1}$$

meaning that entropy increases irreversibly without limit. If the temperature is that of the cosmic microwave radiation, then it obeys the conservation law

$$aT = a_0 T_0 = 3.73 \times 10^{26}\ \text{m} \cdot \text{K}$$

The numerical value has been computed from values for the present epoch. Given this temperature dependence, it follows that

$$S_\lambda \propto \left(\frac{a}{\eta}\right)^2 \propto (\lambda\eta)^2$$

so that dark entropy depends only on the time—it is rightly named time's arrow.

$$S_\lambda = C[\eta\lambda]^2 \tag{11}$$

The constant is

$$C = \frac{\Gamma^2}{\kappa a_0 T_0} = 5.142 \times 10^{65}\ \text{J} \cdot \text{K}^{-1}$$

Dark entropy could appropriately be called the *Hubble entropy*.

Equation (10) is the entropy of expansion and Equation (11) gives the absolute entropy at any time. For example, given the conformal time for this epoch above, the absolute entropy is

$$S_\lambda(\eta_0) = 7.80 \times 10^{66}\ \text{J} \cdot \text{K}^{-1}$$

This is absolute in the sense that by L'Hopital's Rule $S \to 0$ as $T \to 0$.

Table 1 illustrates numerically what may not be apparent in the above argument: the entropy increase accompanying the conversion of dark matter into dark energy is the same as the entropy of expansion. The comparison must between two epochs, the choice here being $\lambda_1 = 3/10$, $\eta_1 = 4.166$ and $\lambda_2 = 4/10$, $\eta_2 = 4.670$. The following numbers result.

Table 1. Thermodynamic change in a change of epochs.

λ	U_λ (J)	$M(dm)c^2$ (J)	T (K)	S_λ (J·K^{-1})
3/10	9.112×10^{66}	1.974×10^{67}	11.37	8.013×10^{65}
4/10	1.215×10^{67}	1.671×10^{67}	6.786	1.790×10^{66}

Entropies found by both U/T and Equation (11) are in good agreement.

$$\Delta U_{\lambda} = U_{\lambda 2} - U_{\lambda 1} = 3.04 \times 10^{66} \text{ J and } \Delta\left[M(dm)c^2\right] = -3.03 \times 10^{66} \text{ J}$$

as required.

$$\Delta S = S_{\lambda 2} - S_{\lambda 1} = 9.89 \times 10^{65} \text{ J} \cdot \text{K}^{-1}$$

which by the table must be the entropy change accompanying the conversion of dark matter into dark energy, but it is also the entropy of expansion starting with Hubble's Law.

The disappearance of dark matter correlates to the expansion. Does this mean that dark matter drives the expansion? This is discussed in [4] in some detail, but suffice to say here that it will depend on whether temperamentally the reader is a positivist or a realist. A widespread view associated with the standard ΛCDM theory is that dark energy, represented by Einstein's cosmological constant, drives the expansion. An alternative view is that the "reason" for the expansion is anthropic, that biological evolution could not occur in a collapsing universe. Parentheses have been used because it is not uncommon for some cosmologists to hold that an anthropic argument is never a reason because it is contrary to physical thought, which must be grounded in physical necessity.

Dark matter disappears completely when $\lambda = 19/20$, $\eta = 6.033$ at which point λ and η decouple, and entropy undergoes a second-order phase change, meaning that the slope of the entropy vs. time curve changes discontinuously:

$$S_{\lambda} = (0.95)^2 C\eta^2$$

From this point entropy increases without limit quadratically with the conformal time.

Notice the connection between § 5 and § 7. In the former, space must be hyperbolic because dark energy dominates in the present epoch. This is because $U_{\lambda} \propto a$, but that's just the Hubble version of the second law. Hence there is a necessary connection between hyperbolic space and the increase of entropy with expansion.

Reference [4] compares the dark/Hubble entropy to the entropy of the cosmic microwave radiation entropy and to the Barrow-Tipler entropy invariant, ***S***. A common view in the literature is that the CMR is the principal source of entropy in the cosmos, but Equation (11) challenges that view. The early universe, defined as $\eta < 1$, is causally disconnected, which increases the entropy relative to what it would be in a causally connected space.

8. Dark Matter as Helmholtz Free Energy [6]

Total energy is conserved as long as dark matter is present:

$$U = U_{\lambda} + c^2\left[M(dm) + M(b)\right]$$

By the preceding section, dark/Hubble entropy is given by

$$U_{\lambda} = TS_{\lambda}$$

But $U = TS + F$, defines the Helmholtz free energy, F (A in the chemist's notation). Hence

$$F = c^2\left[M\left(dm\right) + M\left(b\right)\right] = F\left(dm\right) + F\left(b\right)$$

that is, it is the sum of dark free energy and baryonic free energy. Of course, the two free energies differ in quality: baryons are the building material of cosmological structures such as galaxies and galactic clusters, whereas the gravity of dark matter stabilizes the structures formed from baryons. (Dark) Helmholtz free energy is a good synonym for dark matter.

9. Evolution of Cosmological Parameters

The Friedmann-Lemaître equation, Equation (6), can be written as an algebraic equation.

$$1 = \Omega_1 + \Omega_m + \Omega_\Lambda - \Omega_k$$

where r, m, Λ, and k refer to radiation, matter (both dark and baryonic), dark energy, and curvature. In hyperbolic space, $k = -1$ and $K < 0$. The Ω refer to the fractional contribution of each at the designated epoch, which are easily found as relative energy densities. The energy densities for dark energy and curvature are Λ/κ and $|K|/\kappa$. For radiation and for matter

$$\varepsilon_r = \frac{\varepsilon_{r0} a_0^4}{a^4} \;\&\; \varepsilon_m = \frac{Mc^2}{a^3}$$

Parameter dynamics. The variation of the dimensionless parameters with conformal time depends on their relationship to the scale factor:

$$\Omega_r \propto a^{-4},\ \Omega_m \propto a^{-3},\ -\Omega_k \propto a^{-2} \;\&\; -\Omega_k = \eta^2\Omega_\Lambda$$

Hence, radiation and matter dominate the early universe as **Figure 1** shows but curvature dominates the late universe as illustrated in **Figure 2**.

The conformal time scale of $\eta = 0$ - 6 spans about 22 Gyr. The entry for $\eta = 0$ is actually for $t = 3$ minutes, $\eta = 6.75 \times 10^{-5}$. The radiation energy density formula given above is for the cosmic microwave radiation, which is not valid here: instead

$$\varepsilon_r = \frac{3c^2}{32\pi G t^2}$$

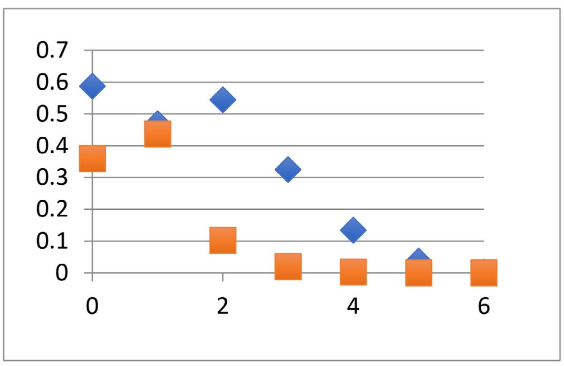

Figure 1. Ω_r (squares) and Ω_m (diamonds) vs. conformal time, η.

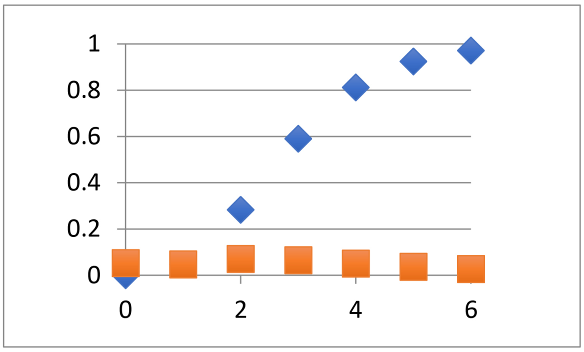

Figure 2. Ω_Λ (squares) & $-\Omega_k$ (diamonds) vs. η (The two are equal at $\eta = 1$).

Dark matter/dark energy dynamic. Dark matter exists in the range $0 \le \lambda \le 19/20$ during which it continually flows into dark energy. Since it is essential for the structural integrity of galaxies and galactic clusters, its loss destabilizes them suggesting that their dissociation in time is likely.

Total energy as a function of time. It is conserved as long as dark matter is present.

$$U = U_D + F(b)$$

$$U_D = \text{constant}, U = \text{constant} \,\&\, \frac{\mathrm{d}U}{\mathrm{d}\eta} = 0$$

But when dark matter vanishes U is no longer constant because dark energy continues to grow. Hence, the total energy, like the entropy, undergoes a second-order phase transition, meaning that the slope of the energy curve changes discontinuously.

$$\frac{\mathrm{d}U}{\mathrm{d}\eta} = \frac{1}{\kappa\eta^2}\left[\frac{\mathrm{d}a}{\mathrm{d}\eta} - \frac{2a}{\eta}\right]$$

A calculation for $\eta = 6.1$ shows that $\mathrm{d}U/\mathrm{d}\eta > 0$ as expected.

Curvature. It dominates the current epoch $\eta_0 = 5.571$ with $-\Omega_k = 0.956$ and dominates the dynamics of the late universe. As $t \to \infty$, the Friedmann-Lemaître equation, Equation (6), becomes

$$\left(\frac{\mathrm{d}\ln a}{\mathrm{d}t}\right)^2 - \left(\frac{c}{a}\right)^2 = 0$$

Hence $\mathrm{d}a/\mathrm{d}t = c$ and $a = ct$, whereas $\mathrm{d}a/\mathrm{d}t > c$ in earlier epochs. Then

$$U = \frac{a}{\kappa\eta^2} + F(b)$$

Eventually the baryonic free energy becomes negligible relative to dark energy.

10. A DEH and Type Ia Supernovae

The theory and observation of Type Ia supernovae as standard candles won a Nobel Prize in physics for the principal investigators. The analysis of their observations led them to conclude that space is flat and that the dark energy/matter (dark and baryonic) ratio is 70/30:

$$\Omega_\Lambda = 0.70, \Omega_m = 0.30 \text{ and } \Omega_k = 0$$

A subset of their data fits a DEH as Figure 3 indicates; the blue circles are DEH calculated values and the orange are observed values. The details and literature reference won't be repeated here. From the viewpoint of a DEH, the cosmos evolved into this flat state, but the graph indicates that the results are also consistent with negative curvature. This raises the issue of how a flat universe that does not evolve got itself into this dark energy/matter distribution.

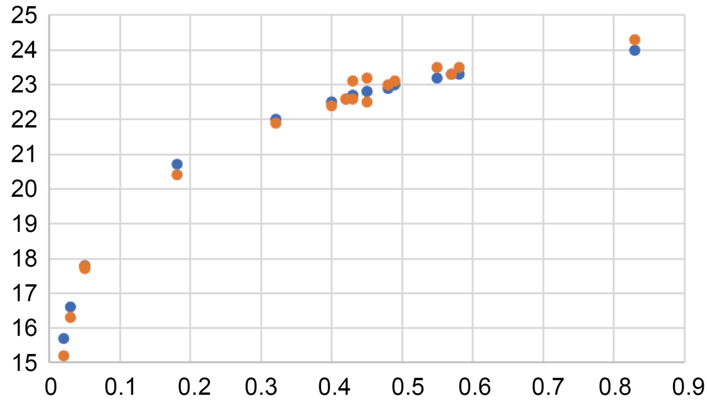

Figure 3. Type Ia Supernovae Apparent magnitude vs. redshift in a DEH.

11. Structure Formation in the Early Universe [7]

Structure means galaxy and galactic cluster, but here the focus is on their masses. In the ΛCDM theory, structure formation begins with density fluctuations that increase density relative to the background average after the radiation/matter equality. Radiation/matter equality never occurs in a DEH because the early universe, defined as $\eta < 1$, is so massive; Figure 1 illustrates this.

In a DEH, because the early universe is so massive, the Jeans criterion for fragmentation into cosmic structures is met immediately. As noted previously, the early universe is causally disconnected. In the model employed, the cosmos divides into disconnected cells of equal size. The earlier the epoch, the smaller the size and the greater their number. At a given epoch, a cell has a characteristic mass. As evolution occurs, these characteristic masses increase through the mass of an irregular (dwarf) galaxy, a regular (Hubble's "tuning fork") galaxy and a galactic cluster. The model of equal cells sizes in a given epoch is a simplistic one that nonetheless suggests that the characteristic mass of a cosmic structure is a relic of the early universe.

12. Models of Dark Matter

Hypothesis: dark matter is structural rather than evolutionary. This model, worked out in [8] and summarized here, is perhaps the most radical. Three other models follow that seem independent of this one.

The most common view is that dark matter is evolutionary. Just as the chemical composition of the universe is a relic of the first three minutes of cosmological history, frozen out at mainly hydrogen with a lesser amount of helium, and still smaller amounts of a few light isotopes, so dark matter consists of frozen relics, the principal candidates being WIMPs, axions, primordial black holes and massive neutrinos.

This structural model for dark matter is that it is coupled to spacetime, that it is part of the spacetime structure. Beginning with the Klein-Gordon equation, the procedure is to quantize a sphere whose radius is the particle horizon distance, which is the locus of all causal activity. To couple dark matter to spacetime means to assume that both obey the same quantum structure, the former at the scale of length given by the Compton wavelength,

$$\lambda_C = \frac{h}{2\pi mc}$$

and the latter at the global scale, which is the scale factor. The root-mean-square momentum of a dark particle is

$$P_{rms} = \frac{3\pi nmc}{\eta} = \frac{3nh}{2\eta\lambda_c} = \frac{h}{\lambda}$$

where η is the conformal time of the epoch, $\eta = \chi_{PH}$ is the comoving coordinate of the particle horizon, $n = 1, 2, \cdots$ is a quantum number and λ is the de Broglie wavelength. Hence,

$$\lambda = \frac{2\eta\lambda}{3n}$$

The numerator illustrates the coupling by the multiplication of a cosmological parameter and a particle parameter. The two wavelength scales, those of the scale factor and Compton wavelength, are analogs of what is seen on any large body of water where short wavelength, slow-moving capillary waves are superposed on long wavelength, fast-moving gravity waves. In the present epoch, $2\eta/3 = 3.714$. Dark matter and baryonic matter can couple either by a resonance condition, $\lambda(dm) = \lambda(b)$ or because dark matter particles vastly outnumber baryonic.

Estimate of the mass of a dark matter particle. The procedure is to adapt Einstein's method of fluctuations as given by Max Born to dark matter, but here the term dark Helmholtz free energy is more appropriate. The notion is that two fluctuations contribute to fluctuations in the mean Helmholtz free energy, $\langle F(dm)\rangle$, namely temperature and number fluctuations. The method gives a mass in the range of 10^{-40} kg.

Equation of state for dark matter. Assume that dark matter is a spinless boson obeying Bose-Einstein statistics. The result is

$$\frac{p}{nk_B T} = \frac{2}{\sqrt{\pi}} n\Lambda_B^3 \tag{12}$$

where n is the number density and Λ_B is the de Broglie thermal wavelength:

$$\Lambda_B = \frac{h}{\sqrt{2\pi m k_B T}}$$

The right-hand side of Equation (12) may be rewritten with

$$n\Lambda_B^3 = \varphi = \exp(\beta\mu)$$

where ϕ is the fugacity and μ the chemical potential. If the right-hand side were unity, it would be the equation of state of an ideal, classical gas, where ideal means that there are no interparticle forces. Equation (12) is the equation of state of an ideal, non-classical gas, because there are no interparticle forces between bosons, but their symmetry properties produce quantum effects. An application is the study of the equilibrium between the ideal BE gas and the BE condensate.

Interaction between dark matter and baryonic matter. This is the subject of [9]. The specific baryonic system is the recombination of a proton and electron into a hydrogen atom and a photon: p + e = H + γ. At what temperature will recombination occur in the absence of dark matter and in its presence? A modified Saha equation provides the answer.

$$1 = \frac{D}{T^{2.5}} \exp\left(\gamma\left(\frac{T}{m}\right)^{2.5} - \frac{\theta}{T}\right)$$

D and γ are constants whose numerical values are in the reference. The recombination temperature is the root of the equation.

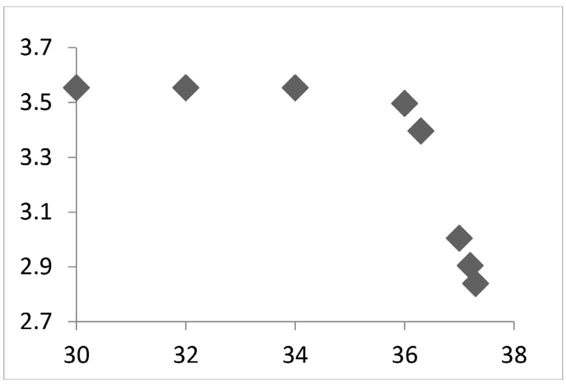

Figure 4. $\log_{10}(T)$ vs $-\log_{10}(m)$.

The ionization temperature of an isolated hydrogen atom is Θ. If dark matter is absent, the other term in the exponential is absent. If present, there will be a strong dependence on the dark matter particle's mass, m, as shown in **Figure 4**. In the absence of dark matter or in its presence for $m > 10^{-36}$ kg, the recombination temperature $T = 3579$ K corresponding to a redshift of $z = 2653$ in hyperbolic space. At $m = 10^{-36}$ kg, an interaction begins and the recombination temperature begins to fall. Suppose that the mass of a dark particle $m = 10^{-37}$ kg: then $T = 1012$ K with a redshift of $z = 491$.

The universe on the backward light cone is opaque if hydrogen is ionized. This model shows that for a sufficiently small dark particle mass the opacity begins at a smaller redshift than would be the case in its absence.

13. Dark Energy and Dark Matter: A Summary

Dark energy is a geometric quantity, a length composed of elements of space and time. Division by the Einstein constant converts it into an energy:

$$U_{\lambda} = \frac{a}{\kappa \eta_2}$$

Dark matter is somewhat enigmatic since astronomers have yet to measure its particle mass. However, it too is a length that is converted into a Helmholtz free energy on division by the Einstein constant:

$$F = \frac{\chi(dm)\Gamma}{\kappa}$$

An important distinction between dark energy and dark matter is that the former is a global quantity whereas the latter is local. Local means that dark matter concentrates in galaxies and galactic clusters. The transformation of dark matter into dark energy is a delocalization accompanied by an increase in cosmological entropy.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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